ANL-Osaka Model for extracting nucleon resonances:past,present and future

Toru Sato

RCNP, University of Osaka

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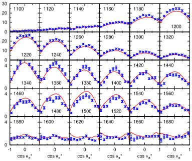
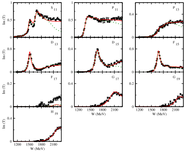


Data of meson production reaction

$$\pi N, \gamma^* N \rightarrow \pi N, \eta N, KY, \pi\pi N, ..$$

$$\frac{d\sigma_T}{d\Omega} + \epsilon \frac{d\sigma_L}{d\Omega}, \ P, \ \ T, \ \ ,,$$

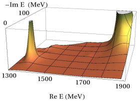


Partial wave amplitude

$$\mathcal{F}^{J^\pi, I}(W, Q^2)$$

Effective theory of QCD

LQCD



Resonance parameter

$$M_R - i\Gamma_R/2, \ \ g_R(Q^2)$$

Analysis of meson production reaction with
Dynamical Coupled Channel reaction model(DCC model)

- reaction : meson-baryon $\pi N, \eta N, KY, \pi\pi N, \omega N, \eta' N, ...$ production
- hadron dynamics : meson,baryon exchange picture
- 3-dim 2body(MB)-3body($\pi\pi N$) coupled channel equation

“Dynamical coupled-channel models for hadron dynamics”, M. Döring, J. Haidenbauer, M. Mai, TS, Prog. Part. Nucl. Phys. 146 (2026) 104213

Juelich-Bonn-Washington and ANL-Osaka approaches   M. Döring, (Talk Tue)

In this talk: ANL-Osaka approach

- SL: 1996-2002 (πN)
- JLMS(EBAC): 2007-2011($\pi N, \eta N, \pi\pi N(\rho N, \pi\Delta, \sigma N)$)
- ANL-Osaka: 2012 $\sim$ $(+K\Lambda, K\Sigma)$

- Study electromagnetic $\gamma^* N \Delta_{33}(1232)$ transition
- Model: $\pi N, \gamma N, \Delta$

$$H_{eff} = H_0 + v_{\pi N, \pi N} + v_{\pi N, \gamma^* N} + \Gamma_{\pi N, \Delta} + \Gamma_{\gamma^* N, \Delta}$$

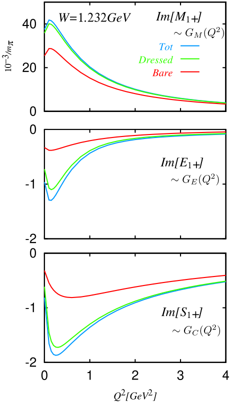
H_{eff} using modified Ohkubo-Nishijima method starting from 'chiral Lagrangian'.
3-dim, Lippmann-Schwinger equation for $N(\pi, \pi)N$,$N(\gamma^*, \pi)N$.

- step 1: construct model of πN scattering

  step 2: analysis of $N(\gamma, \pi)N$ data : determine $G_M^B(0), G_E^B(0)$('bare' form factor)

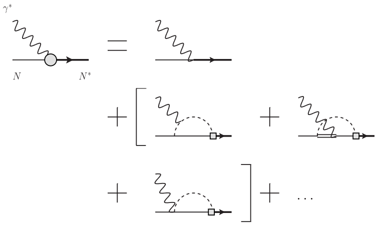
  step 3: analysis of $N(e, e'\pi N)$ data : determine $G_M^B(Q^2), G_E^B(Q^2), G_C^B(Q^2)$

  step 4: prediction of $G_M(Q^2), G_E(Q^2), G_C(Q^2)$ with pion cloud.



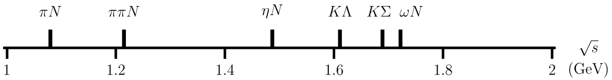
Few-body Problems in Physics(Taipei, 2000)

- $G^B_{M,E}(Q^2)$ from data at $Q^2 = 2.8$ and 4GeV^2
  $G^B_{M,E}(Q^2) = G^B_{M,E}(0)G_D(Q^2)(1 + aQ^2)e^{-bQ^2}$
  G^B_C from G^B_E using Siegert theorem
- SL-model predicted characteristic Q^2 dependence due to pion cloud.



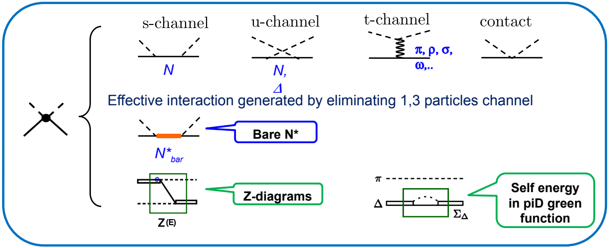
(vertex correction for (γ, π). Tanabe-Ohta)

- Opening of $\eta N, \pi\pi N$... in N^* region beyond $\Delta_{33}(1232)$



- Effective Hamiltonian for $MB - \pi\pi N$ system $(\pi N, \eta N, \rho N, \sigma N, \pi\Delta, \pi\pi N)$

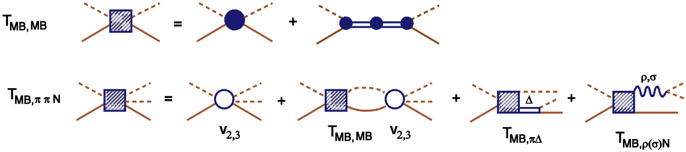
  2(MB)-2(MB) Hamiltonian eliminating bare N^* and $\pi\pi N$ Fock space



$(+\sigma/\rho \leftrightarrow \pi\pi)$

JLMS(EBAC) model 2

- MB-$\pi\pi N$ amplitude



- Key

1. spline interpolation method to solve 3-body scattering with moon-shape singularity of particle exchange mechanism.

2. characterize resonance with pole position and residue

Analytic continuation of amplitude: choose appropriate integration contour of integral equation

two-body: choose appropriate branch ($G_0(E,\boldsymbol{r}) \sim e^{ikr}/r$ out going boundary condition)
three-body: access nearest un-physical sheet.(nnn Glöckle, $\bar{K}pp$ Ikeda, TS)

- JLMS(EBAC) model

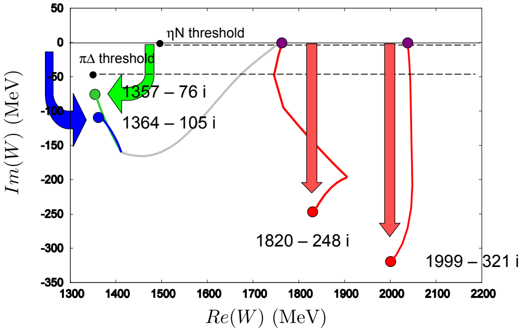
step 1 fit πNscattering($W < 2GeV$): $\pi N, \eta N, \rho N, \sigma N, \pi\Delta, \pi\pi N$

step 2 $p(\gamma,\pi)N$ ($W < 1.6GeV$)):bare helicity amplitude of resonance

step 3 $p(e,e'\pi)N$ and predict $p(\gamma,\pi\pi)N, p(\pi,\pi\pi)N$

Trajectory of pole (P11 amplitude)

$$det[(W - M_i)\delta_{ij} - x\Sigma_{ij}(W)] = 0, \quad x : 0 \to 1$$

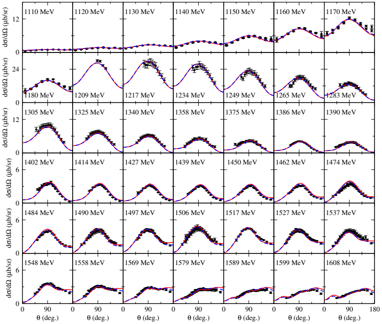


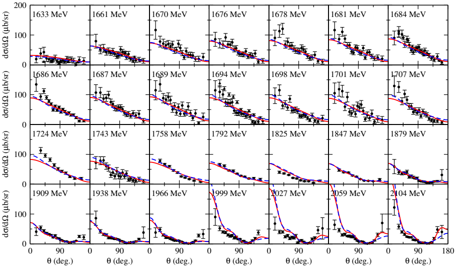
multi-poles, mixing of excited nucleon states through MB rescattering

Model		Meson-Baryon Fock space
SL	1996-2002	πN
JLMS(EBAC)	2007-2011	$\pi N, \eta N, \pi\pi N, (\pi\Delta, \rho N, \sigma N)$
ANL-Osaka	2012 $\sim$	$\pi N, \eta N, \pi\pi N, (\pi\Delta, \rho N, \sigma N) + K\Lambda, K\Sigma$

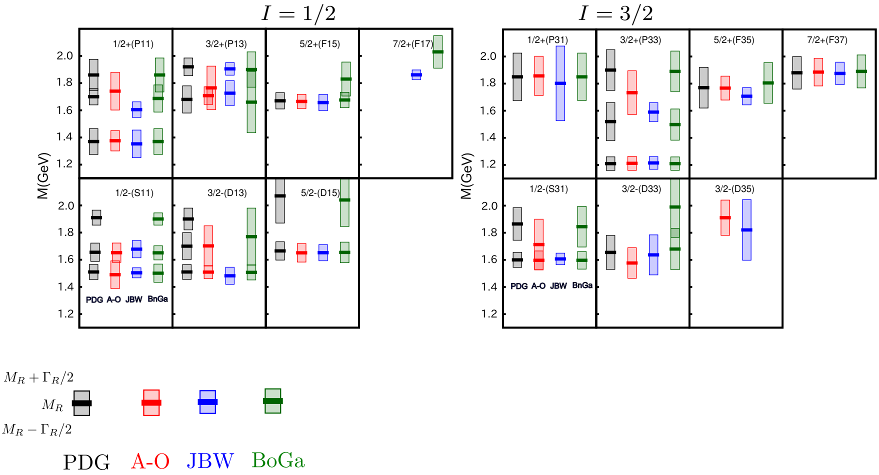
	SL	JLMS	ANL-Osaka
W	$<$ 1.3GeV	$<$ 2GeV($<$ 1.6 for γp)	$<$ 2.1GeV
$\pi p \to \pi N$	O	O	O
$\gamma p \to \pi N$	O	O	O
$\pi^- p \to \eta n$	-	O	O
$\gamma p \to \eta N$	-	-	O
$\pi p \to K\Lambda, K\Sigma$	-	-	O
$\gamma p \to K\Lambda, K\Sigma$	-	-	O
$\gamma n \to \pi N$	-	-	O
$ep \to e'\pi N$	O	O	O ($Q^2 < 3$)
$\pi N \to \pi\pi N$	-	O	O
$\gamma p \to \pi\pi N$	-	O	O

O : full fit O : fit only 'bare' $\gamma^* NN^*$ O : prediction of model

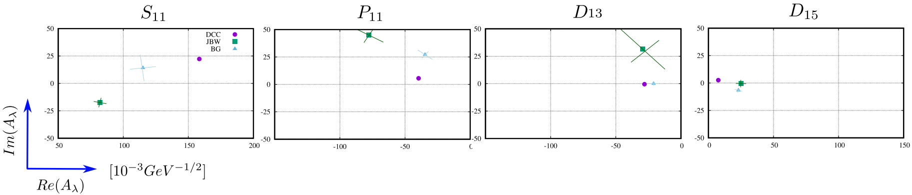

$d\sigma/d\Omega$: $\gamma p \to \pi^0 p$


$\pi^- p \to K^0 \Lambda$

ANL-Osaka Model 3



Helicity amplitude $A_{1/2}(Q^2 = 0)$ from residue $\sim < N^* | \boldsymbol{J}_{em} \cdot \boldsymbol{\epsilon} | N >$

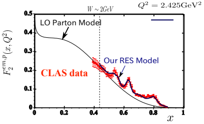


Further development of DCC approach

- Combined MB and $\pi\pi N$ analysis

  analyze sensitivity of N^* on $\pi\pi N$ ($\sigma, M_{\pi N}, M_{\pi\pi}$), (arXiv2601.17389, H. Kamano, T. -S. H. Lee)

- Q^2 structure of resonance transition
  revisit transition form factor, inclusive response



$\rightarrow$ arXiv 2602.29690, (Talk K. Joo, Mon)

- Heavy Flavour system: $u, d \rightarrow s, c, ..$

  DCC model of $K^- p \rightarrow \bar{K}N, \pi\Sigma, \pi\Lambda, \eta\Lambda, K\Xi$ (2014,2015)

  $\pi N \rightarrow K^* MB$ to probe low-lying Λ^*, Σ^* PRC114(2026) H. Kamano, T.-S. H. Lee

- Meson resonances and DCC approach (Talk S. X. Nakamura, Tue)

- Neutrino induced meson production reaction

- Toward model independent study of resonance

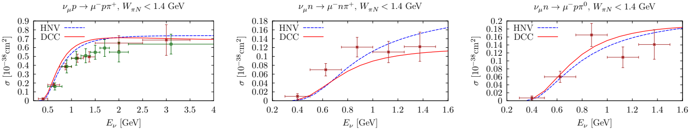
Motivation of GeV neutrino reaction mechanism:

Leptonic CP, Mass hierarchy: Long Base Line experiment (T2K, Dune,..), Atmospheric neutrino

Extension of DCC model $J^\mu_{EM} \rightarrow V^\mu - A^\mu$
axial vector: $\partial^\mu \pi \rightarrow A^\mu$ PCAC

vector : iso-spin decomposition of $I = 1/2$ resonance parameters

E_ν dependence of $\nu - N$ CC single pion production cross section



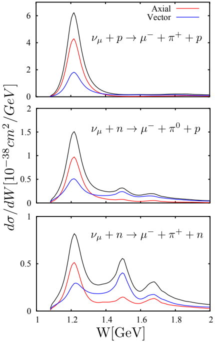
(PRD98, J.E.Sobczyk et al. ANL-Osaka DCC and HNV for $W < 1.4$GeV)

Apply ν-nucleus reaction

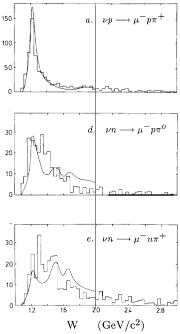
νd (J.J. Wu et al, Nakamura et al.)

Neutrino event generator NEUT and AO model(Talk S. Abe, Wed.)

$\Delta_{33}(1232)$ plays central role



$E_\nu = 40GeV$



BEBC NP343 (1990)

$N\Delta$ Axial transition form factors

C_A^5 'Gamow-Teller', C_A^6 Pion pole
C_A^3, C_A^4 ('deformation')

Theoretical inputs:
LQCD, hadron model(C. Chen et al. PRL133)

Exp. information: PV Asymmetry of electron scattering

$$A = -Q^2(8.99 \times 10^{-5})(1.075 + \Delta_V + \Delta_A)$$
$$\Delta_A \sim 0.1\ (E_e = 1\text{GeV}, \theta_e = 110^o)$$

Towards model independent extraction of resonance parameters

- Deep learning (D. Sombillo, Y. Ikeda, TS, A. Hosaka)

  Classifying the pole of an amplitude using a deep neural network, PRD102,016024(2020)

  Model independent analysis of coupled-channel scattering, PRD104,036001(2021)

- Unfirmized Mittag-Leffler expansion (W. A. Yamada, O. Morimatsu, TS, K. Yazaki)

  New method to extract information of near threshold reonances, PRC102,055201 (2020)

  Application of the uniformized Mittag-Leffler expansion to $\Lambda(1405)$, PRC103,045201 (2021)

  Near threshold spectrum from a unifomized Mittag-Leffler expansion, PRD105,014034 (2022)

  Analytic Map of three-channel S-matrix, PRL 129, 192001(2022)

  Survival probability of unstable states in coupled-channels, PRD108, L071502 (2023)

  Pole-expansion of two-hadron imaginary-time correlation function, PRD112, 034040 (2025)

Uniformized Mittag-Leffler expansion of coupled channel S-matrix

express S-matrix as a sum of pole terms(Mittag-Leffler expansion) under an appropriate variable where the S-matrix is made single-valued(uniformization)

Mittag-Leffler expansion J. Humbelet, L. Rosenfeld(1961), W. Romo(1978)
Uniformization J. R. Cox(1962), M. Kato(1965) ,H. Weidenmuller(1964)

Single channel case: Mittag-Leffler expansion of Green function with uniformization variable $z = p$

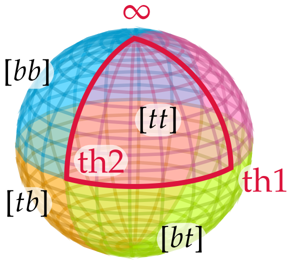
$$\mathcal{G}(p) = \sum_R [\frac{r_R}{p - p_R} - \frac{r_R^*}{p + p_R^*}]$$

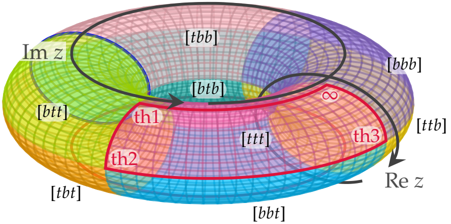
Uniformization variable

$$\text{2ch: } z_{12} = \frac{p_1 + p_2}{\Delta_{21}} \qquad \text{3ch: } z = \frac{1}{4K(1/\gamma^2)} sn^{-1}(\gamma/z_{12}, 1/\gamma^2)$$

$\Delta_{ij} = \sqrt{\epsilon_i^2 - \epsilon_j^2}, \gamma = (\Delta_{13} + \Delta_{23})/\Delta_{12}, sn$(Jacobi's elliptic function), ..

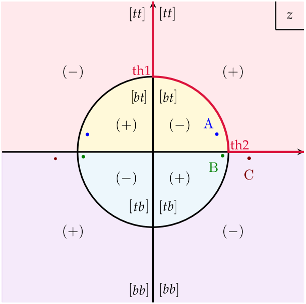
Riemann surface

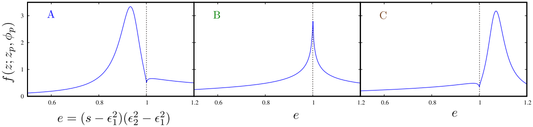




pole near threshold(two-channel)

- 'conventional pole': $[bt]$ between th2 and th1, $[bb]$ above th2
- pole on $[tb]$ (E_R positive imaginary part)can be close to real s.



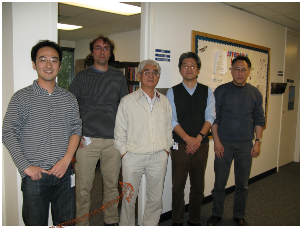


$$f(z;z_p,\phi_p) = -\frac{1}{\pi} Im[\frac{\exp(i\phi_p)}{z-z_p} - \frac{\exp(-i\phi_p)}{z+z_p^*}]$$

- ANL-Osaka model is developed to extract N^*, Δ properties from π, γ, e induced meson production reactions.
- Systematic set of data (cross section, polarization) is important to construct, constrain model and extract amplitudes.
- Reaction model for $Q^2 < 3GeV^2$, $W < 2GeV$ is constructed and resonance parameters are obtained.
- Combined analysis of MB and $\pi\pi N$, Q^2 structure of resonances, analysis of the resonance properties within the model, pole structure of the amplitudes.. are needed to be completed.
- DCC model is extended for KY system, meson resonances, neutrino reactions.

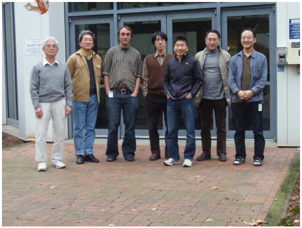
Collaborators

2013@NSTAR



T.-S.H. Lee

2008~2009@Jlab



J. Durand (Saclay)
B. Julia-Diaz (Barcelona)
H. Kamano (RCNP,JLab)
T.-S. H. Lee (ANL,JLab)
A. Matsuyama(Shizuoka)
S. Nakamura (YITP,Kyoto,JLab)
B. Saghai (Saclay)
T. Sato (Osaka)
C. Smith (Virginia, Jlab)
N. Suzuki (Osaka)
K. Tsushima (Adelaide,JLab)