On-shell Matrix Elements of EMT-trace and Heavy Quark Masses

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Take-home Messages of the Talk

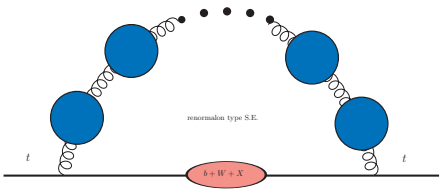
- In perturbative Quantum Gauge Theories, the relation

$$\boxed{\langle \vec{\mathbf{p}} | -\frac{\epsilon}{2}\left[F^a_{\rho\sigma}\, F^{a\,\rho\sigma}\right]_B + \sum \left[m_f \bar{\psi}_f \psi_f\right]_B | \vec{\mathbf{p}} \rangle = \bar{u}(\vec{\mathbf{p}})\, m_{\rm os}\, u(\vec{\mathbf{p}})}$$

  is **proved** to hold as an *identity* for all elementary particles to any loops and to all orders in ϵ

  (without any ref. to prelaid *operator* renormalization conditions).

- The contribution from EMT trace anomaly is not only indispensable, but interestingly captures the entire leading IR-renormalon discovered in *pole* masses of **heavy quarks**.



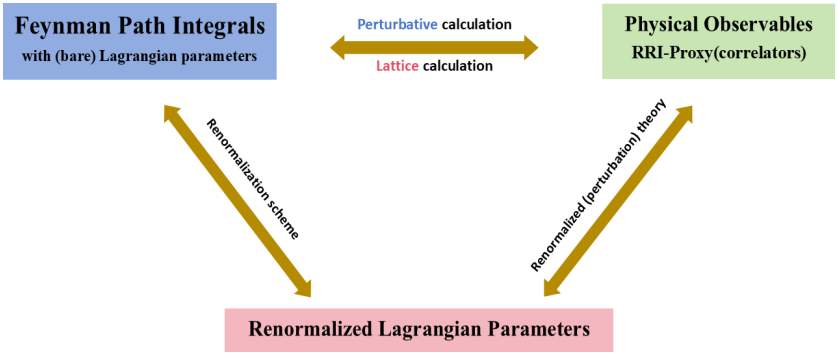
- A novel trace-anomaly-subtracted σ-mass definition for heavy quarks is introduced

  (which is scheme/scale and gauge invariant and free from the leading IR-renorm. ambiguity)

$$\boxed{\frac{m_\sigma}{m_{\rm os}} = \frac{1 + 2\,\beta\,\frac{\partial \ln\left(C_{\overline{m}}\left(\alpha_s, \mu/\overline{m}\right)\right)}{\partial \ln\left(\alpha_s\right)}}{1 - 2\gamma_m}}$$

Quark Masses: Indispensable yet Intermediate Parameters

Quark masses are *fundamental* theoretical parameters of the Standard Model



- Find physical observable(s) sensitive to the quark-mass parameters in the *renormalized* Lagrangian (but insensitive to the unknown parameters or uncontrollable aspects)
- Determine the relations using perturbation techniques and/or Lattice-based approaches
- Different definitions for the **intermediate renormalized quark masses** exist (Pole, $\overline{\text{MS}}$, Kinetic, PS, 1S, (m)SMOM, ...)

Regularization-Independent Renormalization

Workflow for determining $\overline{\text{MS}}$ quark mass using Lattice with RI-renormalization:

- Determine a bare quark mass $m(a)$ in **Lattice** QCD (with spacing a) tuned non-perturbatively to reproduce the mass of a given hadron from experiment

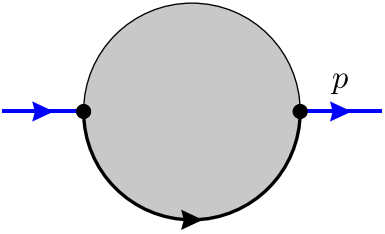
- Compute non-perturbatively in **Lattice** QCD the mass renormalization factor Z_m^{RI} that converts $m(a)$ at each a to a mass defined in the RI-scheme at subtraction scale μ_s

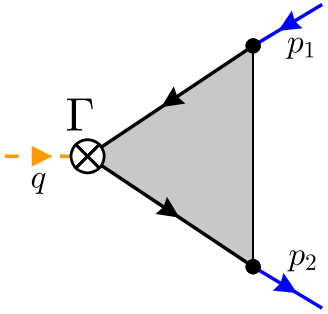
- Compute perturbatively solely in *continuum* QCD in DR the mass conversion factor $Z_m^{\overline{\text{MS}}/\text{RI}}$ that transforms the RI-mass at μ_s to a $\overline{\text{MS}}$ mass $\overline{m}(\mu)$; Schematically,

$$\boxed{\overline{m}(\mu) = Z_m^{\overline{\text{MS}}/\text{RI}}(\alpha_s(\mu), \mu, \mu_s)\, Z_m^{\text{RI}}(\mu_s, a)\, m(a)}$$

RI-ren. is very effective in suppressing the discretization (latt. cutoff) errors in extrapolation to the continuum limit $a \to 0$

RI Momentum Subtraction Schemes





$$Z_{\mathcal{O}}\,\Gamma^{\mu}_{\alpha\,\beta}\,\hat{\mathcal{P}}^{\alpha\,\beta}_{\mu}\Big|_{p^2=\mu_s^2} = \left[\Gamma_{\text{tree}}\right]^{\mu}_{\alpha\,\beta}\,\hat{\mathcal{P}}^{\alpha\,\beta}_{\mu}\Big|_{p^2=\mu_s^2}; \qquad Z_A\,G^{-1}_{\mu\nu}(q)\Big(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\Big)/(D-1)\Big|_{p^2=\mu_s^2} = \mu_s^2\,.$$

- **RIMOM** [Martinelli et al. 95] : $p^2 = p_1^2 = p_2^2 = -\mu_s^2 < 0$ and $q = 0$
- **RISMOM** [Aoki 08; Sturm et al. 10] : $p^2 = p_1^2 = p_2^2 = q^2 = -\mu_s^2 < 0$ and $q = p_2 - p_1$
- **RImSMOM** [Boyle, Debbio et al. 16] : same external kinematics as in RISMOM but at $m_Q^R \neq 0$

Definitions of Pole and $\overline{\text{MS}}$ Mass in (p)QFT

The (perturbative) pole-mass m_{os} of a fermion is **defined** by:

$$Z_\psi \left(\not{p} - m_B - \Sigma_B(\not{p}, m_B) \right) = \left(\not{p} - m_R - \Sigma_R(\not{p}, m_R) \right)$$

with the on-shell R. C. :

$$\Sigma_R(\not{p} = m_{\text{os}}, m_R = m_{\text{os}}) = 0\,; \quad \frac{\partial \Sigma_R(\not{p}, m_{\text{os}})}{\partial \not{p}}\Big|_{\not{p}=m_{\text{os}}} = 0\,.$$

The $\overline{\text{MS}}$-mass is **defined** by:

$$m_B = Z_{\overline{m}}(\alpha_s, \epsilon)\, \overline{m}(\mu) = Z_m^{os}(\alpha_s, \epsilon, \mu)\, m_{\text{os}}\,.$$

- m_{os}:
  - gauge-invariant and IR-finite, reg./ren. scheme- and scale-independent [Veltman 92, Breckenridge at al 94, Kronfeld 98, Gambino at al 99]
  - plagued by the IR-renormalon, and defined to **any but finite** perturbative orders
- $\overline{m}$:
  - gauge-invariant and IR-finite, pertaining to DR and scale-independent
  - free from the IR-renormalon, but physical interpretation unclear
- The pole-to-$\overline{\text{MS}}$ matching factor [Marquard et al 15] ($n_f = n_l + 1 = 6$, and $\mu = \overline{m}$):

$$\begin{aligned} \text{C}_{\overline{m}} \equiv \frac{m_{\text{os}}}{\overline{m}} &= \frac{Z_{\overline{m}}}{Z_m(\alpha_s,\, \mu/m_{\text{os}})}\Big|_{\epsilon\to 0} \\ &= 1. + 0.424413\,\alpha_s + 0.834538\,\alpha_s^2 + 2.37489\,\alpha_s^3 + 8.615(18)\,\alpha_s^4 + \mathcal{O}(\alpha_s^5) \end{aligned}$$

Alternative Quark Mass definitions in QFT

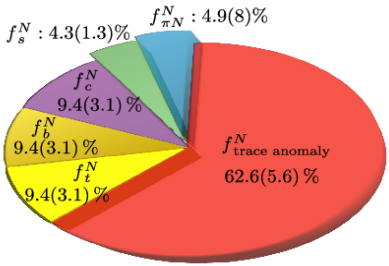
Several (*renormalon-free*) "quasi on-shell" threshold mass definitions for heavy quarks

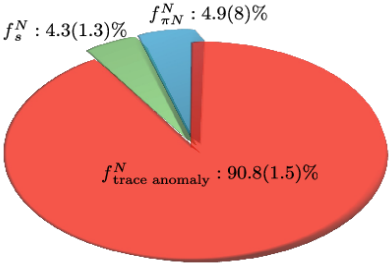
An incomplete list includes:

- For heavy-light Meson (decay)
  Kinetic mass [Bigi:1994ga,Bigi:1996si,Czarnecki:1997sz,Fael:2020iea]
- For Quarkonium
  Potential-subtracted mass [Beneke:1998rk] and **1S** mass [Hoang:2008yj]
- For $t\bar{t}$-productions at high-energy colliders
  MSR mass [Hoang:1998ng,Hoang:2017suc]
- For quark-mass extraction from heavy-light Meson
  (minimal) **Renormalon-Subtracted** mass [Pineda:2001zq,Komijani:2017vep] ,
- For Lattice QCD:
  RI/MOM mass [Martinelli:1994ty] and **RI/(m)SMOM** mass [Aoki:2007xm,Sturm:2009kb,Boyle:2016wis]

Their perturbative relations to $m_{\rm os}$ or $\overline{m}$ are currently known, at least, to three loops [→RunDec]

Rest-mass(energy) Viewed from EMT-trace Formula





(plots taken from 2103.15768)

- Proton mass decomposition in terms of the nucleon σ-terms of different quark flavors and trace anomaly
- *Trace anomaly* does contribute a significant portion, about 2/3 to M_{proton} (not vanishing in the chiral limit)
- Continued interests in perturbation theories and Lattice-QCD [Adler et al. 77; Collins et al. 77; Nielsen 77; Shifman et al. 78; Kashiwa 79; Cheng 91; Ji 94; Polyakov 02; Hatta 18 ... Yang et al. 15,18 .....] , further renewed by the recent **EIC** and **EIcC** proposals

Masses from EMT Trace: *the role of anomaly*?

- Pole mass of a fermion (i.e. quark) may be claimed equal to its rest-mass/energy when isolated, but this is **NOT** its *definition* existing in QFT:

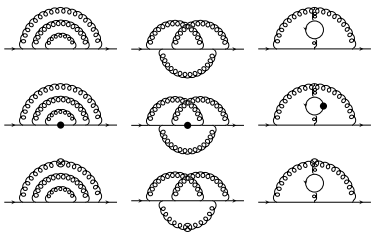
$$\not{p} - m_B - \Sigma_B(\not{p}, m_B)\Big|_{\not{p}=m_{os}} = 0\,.$$

- Why the forward form-factor c_1 of EMT [Pagels 66] is always 2 (to all-orders in QFT)?

$$\langle \mathbf{p}, s \big| \, \Theta^{\mu\nu}(0) \, \big| \mathbf{p}, s \rangle \Big|_{r.n.} = c_1 \, p^\mu p^\nu + c_2 \, g^{\mu\nu}$$

  Is it **true** by (renormalization) definition (condition) or rather a theorem (identity)?

- Feynman diagrams for $\Sigma_B(\not{p}, m_B)$ and on-shell matrix elements of $g_{\mu\nu}\Theta^{\mu\nu}$ are quite different from each other $\Longrightarrow$



- What is the role of the trace anomaly in all these considerations?

To promote a claim to a theorem requires more than hand-waving analogs, only a rigorous proof will suffice.

Masses from EMT Trace: *the role of anomaly*?

Could EMT trace anomaly lead to a **new** mass different from ***pole mass*** for some *elementary* particles?

- The physical (symmetric and conserved) EMT in QCD reads

$$\Theta^{\mu\nu} \equiv -F^{a\,\mu}{}_{\rho} F^{a\,\nu\rho} + \frac{1}{4} g^{\mu\nu} F^{a}_{\rho\sigma} F^{a\,\rho\sigma} + \frac{i}{4} \sum_{q} \bar{q} \left(\gamma^{\mu} \overleftrightarrow{D}^{\nu} + \gamma^{\nu} \overleftrightarrow{D}^{\mu} \right) q \,,$$

- The all-order **EMT-trace** formula: [Adler, Collins, Duncan, Joglekar 77; Nielsen 77]

$$\left[\Theta^{\mu}_{\mu}\right]_{\mathrm{B}} = -\frac{\epsilon}{2} \left[F^{a}_{\rho\sigma} F^{a\,\rho\sigma}\right]_{B} + \sum_{f} \left[m_f \bar{\psi}_f \psi_f\right]_{B} = \frac{\beta}{2} \left[F^{a}_{\rho\sigma} F^{a\,\rho\sigma}\right]_{\mathrm{R}} + \left(1 - 2\gamma_m\right) \left[\sum_{q} m_q \bar{\psi}_q \psi_q\right]_{\mathrm{R}} .$$

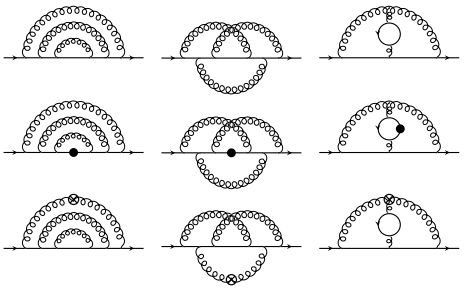
with $\frac{\mathrm{d} \ln a_s}{\mathrm{d} \ln \mu^2} \equiv -\epsilon + \beta$, $\frac{\mathrm{d} \ln m}{\mathrm{d} \ln \mu^2} \equiv \gamma_m$.

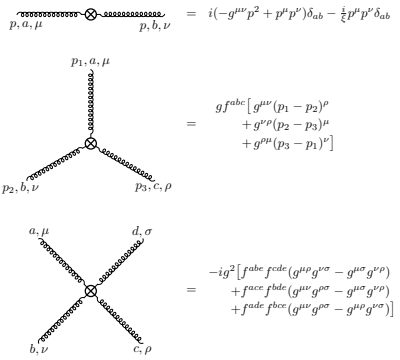
An operator subtraction scheme was specifically formulated [Adler, Collins, Duncan 77] to *define* $\left[\sum_q m_q \bar{\psi}_q \psi_q\right]_{\mathrm{R}}$ and $\left[F^{a}_{\rho\sigma} F^{a\,\rho\sigma}\right]_{\mathrm{R}}$ to **manually ensure** the condition $\langle e(\mathbf{p}) | \Theta^{\mu}_{\mu} | e(\mathbf{p}) \rangle |_{n.n.} = \mathbf{m_e}$ (and $\langle \gamma(\mathbf{p}) | \Theta^{\mu}_{\mu} | \gamma(\mathbf{p}) \rangle |_{n.n.} = 0$).

- In general, **UV-finite operator $\neq$ No quantum corrections to matrix elements** !

Could there be *regularization-dependent* remnant **artifact**, to be removed by additional **finite** renormalizations?

Explicit Verification up-to Three Loops in QCD





- We completed an explicit perturbative verification [arXiv: 2509.03580] of

$$\langle \vec{\mathbf{p}}| -\frac{\epsilon}{2}\big[F^a_{\rho\sigma}\, F^{a\,\rho\sigma}\big]_B + \sum_f \big[m_f \bar{\psi}_f \psi_f\big]_B |\vec{\mathbf{p}}\rangle = \bar{u}(\vec{\mathbf{p}})\, m_{\rm os}\, u(\vec{\mathbf{p}})$$

  in both QCD and QED for quarks, electron, photon and gluons up-to three loops [LC, Li, Niggetiedt 25] .

- Trace-anomaly contribution is finite, non-zero, for massive fermions, but vanishing for gauge bosons.

A Direct All-order Diagrammatic Proof

Take-home messages: [LC, Li, Niggetiedt 25]

- A novel **diagrammatic proof of the *identity***

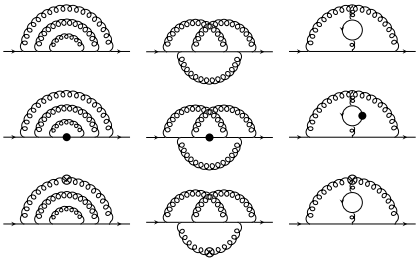
$$\langle\vec{\mathbf{p}}| - \frac{\epsilon}{2}\left[F^a_{\rho\sigma}\,F^{a\,\rho\sigma}\right]_B + \sum_f \left[m_f\bar{\psi}_f\psi_f\right]_B|\vec{\mathbf{p}}\rangle = \bar{u}(\vec{\mathbf{p}})\,m_{\rm os}\,u(\vec{\mathbf{p}})$$

  to any loops in perturbative gauge theories (and all orders in ϵ) without any ref. to prelaid *operator* renormalization conditions.

- The proof is based on:
  - equation of mass-dimensional analysis in DR
  - topological properties of contributing diagrams
  - the pole-mass definition in OS-ren.

$$m_{\rm os} = Z_\psi\left(m_B + \Sigma_B(\not{p}, m_B, \hat{\mu}) - \not{p}\,\frac{\partial\,\Sigma_B(\not{p}, m_B, \hat{\mu})}{\partial\,\not{p}}\right)\Big|_{\not{p}=m_{\rm os}}$$

[arXiv: 2509.03580]



$$\Sigma_B(\not{p}, m_B, \hat{\mu}) = \not{p}\,\frac{\partial\,\Sigma_B(\not{p}, m_B, \hat{\mu})}{\partial\,\not{p}} + m_B\,\frac{\partial\,\Sigma_B(\not{p}, m_B, \hat{\mu})}{\partial\,m_B} + \hat{\mu}\,\frac{\partial\,\Sigma_B(\not{p}, m_B, \hat{\mu})}{\partial\,\hat{\mu}}$$

$$\bar{u}(\mathbf{p}, s)\left(\hat{\mu}\,\frac{\partial\,\Sigma_B(\not{p}, m_B, \hat{\mu})}{\partial\,\hat{\mu}}\right)u(\mathbf{p}, s) = \sum_{L=1}^{\infty} 2\epsilon\,L\,\hat{\alpha}^L\,\hat{\mu}^{2\epsilon\,L}\,\Sigma_B^{(L)}$$

$$= \langle\mathbf{p}, s|\,2\epsilon\Big[-\frac{1}{4}F_{\mu\nu}\,F^{\mu\nu}\Big]_B\,|\mathbf{p}, s\rangle\big|_{\rm 1PI}$$

A Direct All-order Diagrammatic Proof

Take-home messages: [LC, Li, Niggetiedt 25]

- A novel **diagrammatic proof of the *identity***

$$\langle \vec{\mathbf{p}} | - \frac{\epsilon}{2} \left[F^a_{\rho\sigma} F^{a\,\rho\sigma} \right]_B + \sum_f \left[m_f \bar{\psi}_f \psi_f \right]_B | \vec{\mathbf{p}} \rangle = \bar{u}(\vec{\mathbf{p}})\, m_{\rm os}\, u(\vec{\mathbf{p}})$$

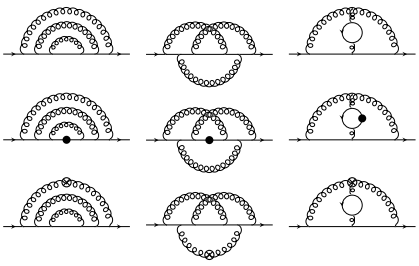
  to any loops in perturbative gauge theories (and all orders in ϵ) without any ref. to prelaid *operator* renormalization conditions.

- The proof is based on:
  - equation of mass-dimensional analysis in DR
  - topological properties of contributing diagrams
  - the pole-mass definition in OS-ren.

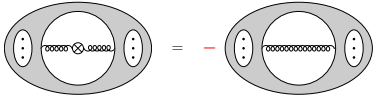
- The role of the trace anomaly is clarified

  (crucial for massive fermions, albeit vanishing for gauge bosons)

[arXiv: 2509.03580]



Reduction of diagrams with degree-2 vertex



For arbitrary vacuum diagrams in gauge theories:

$$N_g^{\rm eff} = N_g - V_3 - V_4 = N_L - 1$$

A Topological Relation for QCD Diagrams

Consider a connected QCD Feynman diagram with E_F fermion legs and E_B gauge-boson legs, and let N_g and N_f denote the number of the internal gauge-boson and fermion propagators, and V_f, V_3 and V_4 for the number of fermion-gauge, triple-gauge and quartic-gauge coupling vertices.

- Following from the topological property of a linear graph,

$$\left(E_F + E_B\right) + 2\left(N_g + N_f\right) = 3\,V_f + 3\,V_3 + 4\,V_4\,,$$

- the number of loops

$$N_L = \left(N_g + N_f\right) - \left(V_f + V_3 + V_4\right) + 1\,,$$

- the number of internal fermion propagators

$$2\,N_f + E_F = 2\,V_f\,,$$

we then have

$$\boxed{N_g^{\text{eff}} = N_g - V_3 - V_4 = N_L - 1 + E_F/2}$$

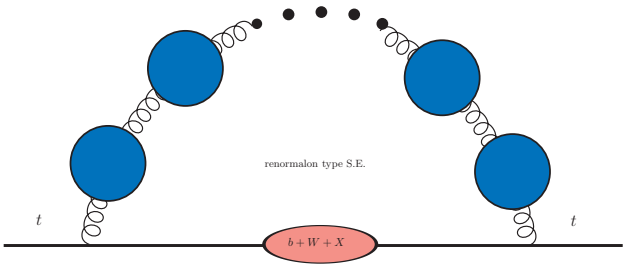
which is **independent** of E_B.

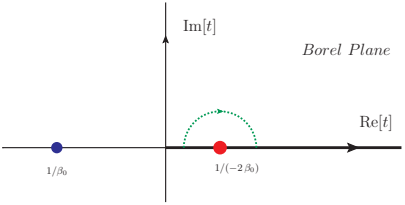
IR Renormalon in Pole Mass

The pole mass of a quark in p. QCD admits the following series result: [Bigi et al. 94; Beneke et al. 94]

$$m_{\rm os} = \overline{m}(\mu)\, C_{\overline{m}}(\alpha_s,\, \mu/\overline{m}) = \overline{m}(\mu) + \sum_{n=0}^{\infty} r_n\, \alpha_s^{n+1}(\mu)$$

with the large-order behavior $r_n\big|_{n\to\infty} \longrightarrow C_F\, \frac{e^{5/6}}{\pi}\, \mu\, \left(-2\beta_0\right)^n\, n!$ (at the large n_f limit).





$$\sum_{n=0}^{\infty} n!\, x^{n+1} \;\sim\; \int_0^{\infty} \frac{x}{1-x\,t}\, e^{-t}\mathrm{d}t$$

The so-called leading IR-renormalon ambiguity: [Bigi et al. 94; Beneke et al. 94]

$$\Lambda_{\rm LIR} \equiv e^{5/6}\, \frac{C_F}{-\beta_0}\, \mu\, e^{\ln(\Lambda_{\rm QCD}/\mu)} = e^{5/6}\, \frac{C_F}{-\beta_0}\, \Lambda_{\rm QCD}\,.$$

A Scheme/scale-invariant Trace-anomaly-subtracted σ-mass

Take-home messages: [**LC**, Li, Niggetiedt 25; **LC**, Zhao 25]

- **A discovery**:

  The leading IR-renormalon divergence in *pole-mass* of massive quarks **resides entirely** in the EMT-trace anomaly contribution!

  - $$\langle p,s|\, 2\epsilon\big[-\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu}\big]_B\,|p,s\rangle\big|_{\text{ampu.}} = \bar{u}(p,s)\Big(\hat{\mu}\frac{\partial\,\Sigma_B(\not{p},m_B,\hat{\mu})}{\partial\,\hat{\mu}}\Big)\,u(p,s)\,,$$

  - $$\hat{\mu}\frac{\partial\,\Sigma_B(\not{p},m_B,\hat{\mu})}{\partial\,\hat{\mu}}\Big|_{\not{p}\to m_{\text{os}}} = \hat{\mu}\frac{\partial\,m_{\text{os}}(m_B,\hat{\mu})}{\partial\,\hat{\mu}} \text{ and } \mu\frac{\partial\,m_{\text{os}}(m_B,\,\mu)}{\partial\,\mu}\Big|_{\text{LIR}} = \overline{m}\,\mathrm{C}_{\overline{m}}(\alpha_s,\,\mu/\overline{m})\Big|_{\text{linear-}\mu} = m_{\text{os}}(m_B,\,\mu)\Big|_{\text{LIR}}.$$

- **A proposal**:

  A novel trace-anomaly-subtracted σ-mass definition for heavy quarks, scheme/scale and gauge invariant, and free from the leading IR-renorm. ambiguity. [arXiv:2509.03580, 2509.10786]

  (In other words, it naturally combines the merits of both pole and $\overline{\text{MS}}$ mass definitions!)

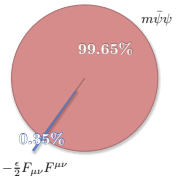
A Scheme/scale-invariant Trace-anomaly-subtracted σ-mass

The fraction Z_{TA} of the trace-anomaly contribution to m_{os} and m_σ of elementary fermions: [LC, Li, Niggetiedt 25]

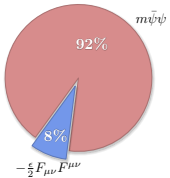
	electron	t-quark	b-quark	c-quark
Z_{TA}	0.347 %	7.9 %	20.4 %	34.3 %
m_σ	0.509 MeV	159.0 GeV	3.96 GeV	1.17 GeV

- Z_{TA} increase for lighter heavy quarks (evaluated via rewriting m_{os} i.t.o. $\overline{m}$ for b-, c-quark)
- Trace-anomaly contribution will vanish in chiral limit $m_q = 0$

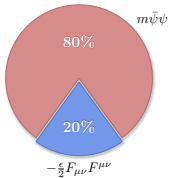
m_e^{os} -composition



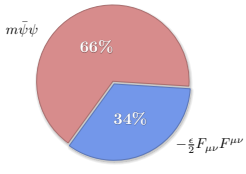
m_t^{os}-composition



m_b^{os}-composition



m_c^{os}-composition



Compact Formula and Results for m_σ in QCD

- A compact formula [LC, Zhao 25] [arXiv: 2509.10786]

$$Z_\sigma = \frac{m_\sigma}{m_{\rm os}} = \frac{1 + 2\,\beta\,\dfrac{\partial \ln\left(C_{\overline{m}}\left(\alpha_s\,,\mu/\overline{m}\right)\right)}{\partial \ln\left(\alpha_s\right)}}{1 - 2\gamma_m}$$

i.t.o. anomalous dimensions of α_s and $\overline{m}$ and the pole-to-$\overline{\rm MS}$ mass ratio $C_{\overline{m}}\left(\alpha_s\,,\,\mu/\overline{m}\right)$.

  - **A nice feature**: Z_σ at $\mathcal{O}(\alpha_s^N)$ involves perturbatively $C_{\overline{m}}$ only up to $\mathcal{O}(\alpha_s^{N-1})$, i.e. one loop-order less!

- Result for the 5-loop relation to $m_{\rm os}$ at $\mu = m_{\rm os}$:

$$\begin{aligned} m_\sigma/m_{\rm os} = 1 &+ \alpha_s\left(-0.636620\right) + \alpha_s^2\left(-1.11735 + 0.0731764\,n_l\right) \\ &+ \alpha_s^3\left(-4.98197 + 0.800055\,n_l - 0.0206485\,n_l^2\right) \\ &+ \alpha_s^4\left(-31.2996 + 6.70684\,n_l - 0.405322\,n_l^2 + 0.00658157\,n_l^3\right) \\ &+ \alpha_s^5\left(-243.76(11) + 68.515(5)\,n_l + 6.4963(2)\,n_l^2 + 0.240658\,n_l^3 - 0.00295411\,n_l^4\right) + \mathcal{O}(\alpha_s^6) \end{aligned}$$

- Result for the perturbative relation to $\overline{m}(\mu = \overline{m})$ at 4-loop:

$$\begin{aligned} m_\sigma/\overline{m} = 1 &+ \alpha_s\left(-0.212207\right) + \alpha_s^2\left(-0.0254365 - 0.0323361 n_l\right) \\ &+ \alpha_s^3\left(0.268010 + 0.00994659\,n_l + 0.000401805\,n_l^2\right) \\ &+ \alpha_s^4\left(1.162(17) - 0.29899(37)\,n_l + 0.0240154\,n_l^2 - 0.000380218\,n_l^3\right) + \mathcal{O}(\alpha_s^5) \end{aligned}$$

How to Implement In Practice

- Scenario-1: **Only propagator-masses redefined** (external momenta kept on-shell)

  For $\Sigma_R(\not{p}, m_\sigma)$ defined by $Z_\psi^\sigma\left(\not{p} - m_B - \Sigma_B(\not{p}, m_B, \hat{\mu})\right) = \not{p} - m_\sigma - \Sigma_R(\not{p}, m_\sigma)$ with $m_B = Z_m^\sigma\, m_\sigma$ subtracted at the O.S.-momentum for defining S-matrix elements, there is

$$\Sigma_R(\not{p}, m_\sigma)\Big|_{\not{p}=m_{\rm os}} = m_{\rm os} - m_\sigma\,; \quad \frac{\partial\,\Sigma_R(\not{p}, m_\sigma)}{\partial\,\not{p}}\Big|_{\not{p}=m_{\rm os}} = 0$$

  with $m_\sigma = \langle p,s|\left[m\bar{\psi}\psi\right]_B|p,s\rangle / \bar{u}(p,s)\, m_{\rm os}\, u(p,s)\Big|_{\not{p}=m_{\rm os}}$.

  - Only the propagator-mass counter-term $(Z_m^\sigma - 1)\, m_\sigma$ changed compared to O.S.

- Scenario-2: **ALL $m_{\rm os}$-dependence** in S-matrix elements $\mathcal{S}(m_{\rm os})$ **re-expanded** around $m_\sigma = Z_\sigma\, m_{\rm os}$
  - Compute $\mathcal{S}(m_{\rm os})$ in the O.S.-scheme up to $\mathcal{O}(\alpha_s^N)$ of interest;
  - Perturbative re-expand and truncate $S(m_{\rm os} = m_\sigma/Z_\sigma)$ in α_s up to $\mathcal{O}(\alpha_s^N)$

Summary and Outlook

The masses of elementary fields, e. g. leptons and quarks, are fundamental parameters of SM, and a better understanding of them is of utmost importance.

☑ Presented a novel **diagrammatic all-order proof of the *identity***

$$\langle \vec{\mathbf{p}} | -\frac{\epsilon}{2} \left[F^a_{\rho\sigma} F^{a\,\rho\sigma} \right]_B + \sum_f \left[m_f \bar{\psi}_f \psi_f \right]_B | \vec{\mathbf{p}} \rangle = \bar{u}(\vec{\mathbf{p}})\, m_{\text{os}}\, u(\vec{\mathbf{p}})$$

in perturbative gauge theories (to all orders in ϵ without any ref. to prelaid *operator* ren. conditions).

☑ Discovered that the leading IR-renormalon observed in *pole-mass* of heavy quarks **resides entirely** in the EMT-trace anomaly contribution!

☑ Proposed a novel trace-anomaly-subtracted σ-mass definition for heavy quarks (which is scheme/scale and gauge invariant and free from the leading IR-renorm. ambiguity).

☑ Derived a compact formula for the relation between σ-mass and pole-mass, exhibiting nice features!

$$\frac{m_\sigma}{m_{\text{os}}} = \frac{1 + 2\,\beta\, \frac{\partial \ln\left(\mathrm{C}_{\overline{m}}\left(\alpha_s, \mu/\overline{m}\right)\right)}{\partial \ln\left(\alpha_s\right)}}{1 - 2\gamma_m}$$

☒ Investigating the effects of additional (virtual) massive quarks, QCD⊕QED mixed corrections ... applications to Higgs and heavy-quark decays.

Thank you for listening!