# Symmetry energy dependence of the bulk viscosity of nuclear matter

PRL 135 (2025), arXiv:2504.07805

Maurício Hippert

In collaboration with Yumu Yang, Enrico Speranza and Jorge Noronha

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V Simpósio do INCT-FNA

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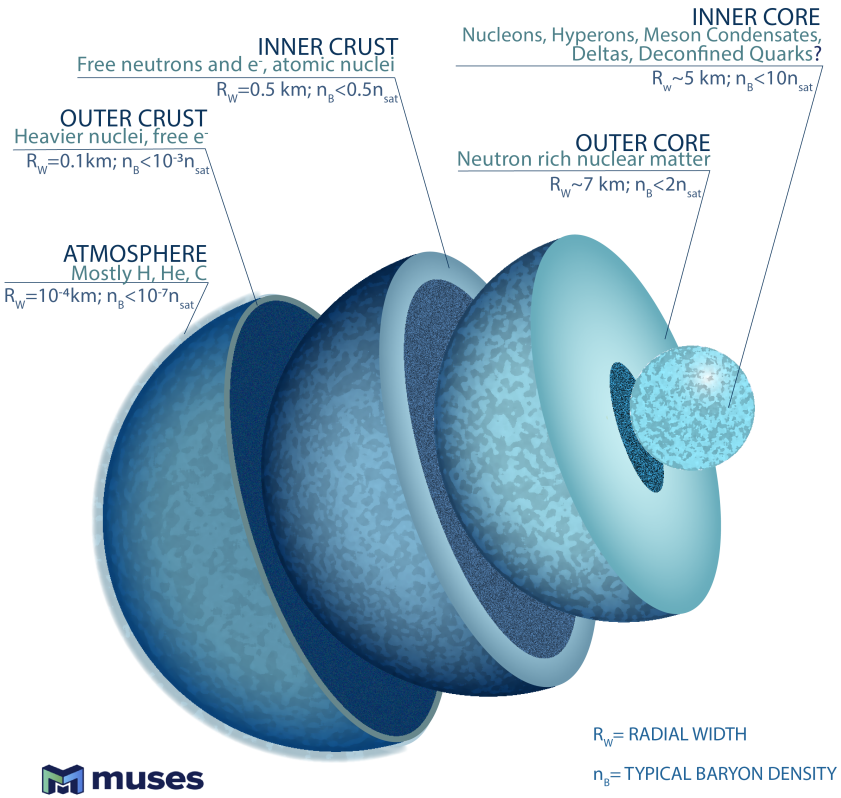
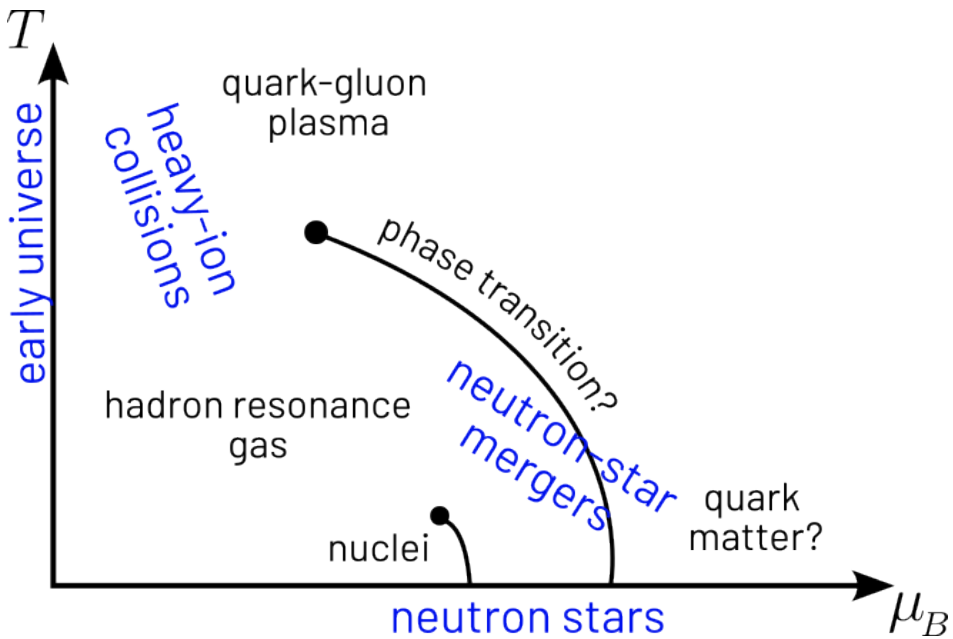
# Motivation

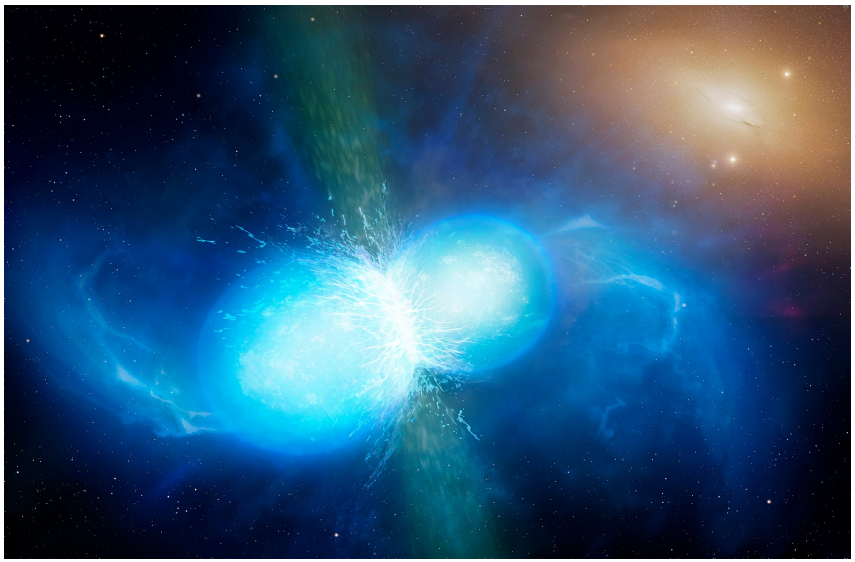


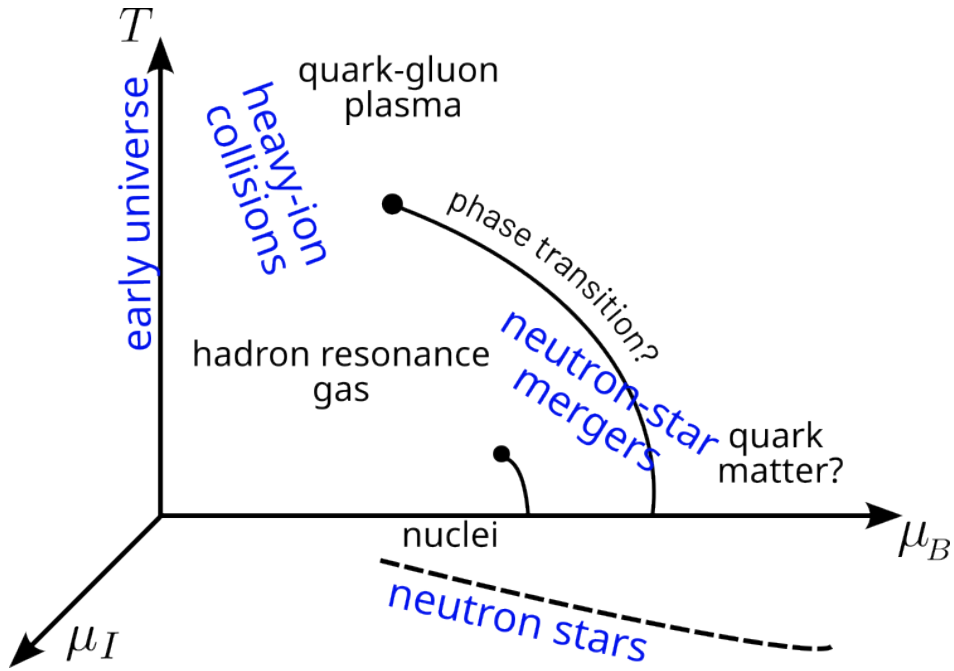




Left Fig.: R. Kumar *et al.* [MUSES], Living Rev. Rel. **27** (2024)

- Neutron stars: QCD equation of state at extreme densities.
- Isolated stars: equation of state in $\beta$-equilibrium.

# Motivation






Left Fig.: R. Kumar *et al.* [MUSES], Living Rev. Rel. **27** (2024)

- Neutron stars: QCD equation of state at extreme densities.
- Isolated stars: equation of state in $\beta$-equilibrium.
- Mergers: evolution out of $\beta$-equilibrium $\rightarrow$ dissipation.

E. R. Most, A. Haber, S. P. Harris, Z. Zhang, M. G. Alford and J. Noronha, AJL (2024)

# Weak interactions

- In vacuum, neutron unstable under $\beta$ decay:

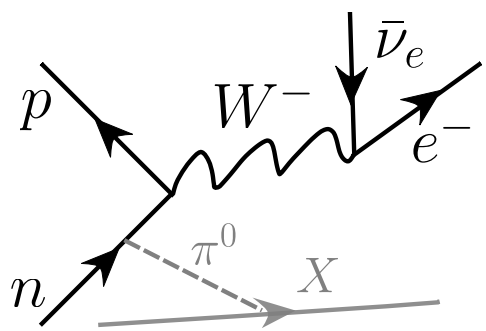
$$n \longrightarrow p + e^- + \bar{\nu}_e$$

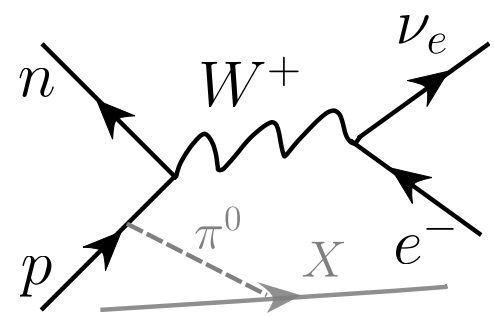
- In neutron stars, Urca and modified Urca processes.
- In $\beta$ equilibrium, detailed balance:

$$p + e^- \leftrightarrow n + \nu_e$$

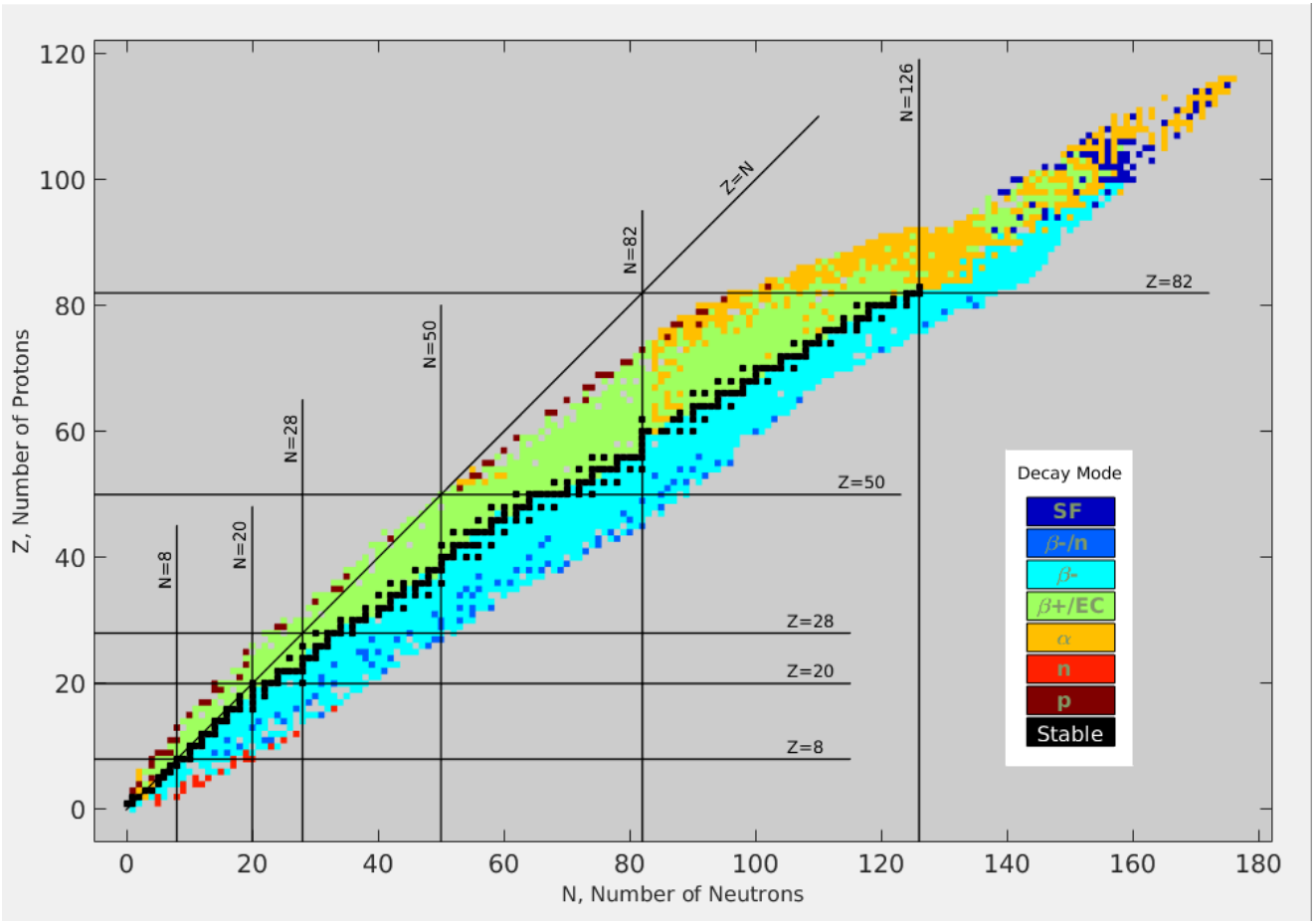
- In binary mergers: dynamics out of $\beta$ equilibrium.

M. G. Alford, L. Bovard, M. Hanauske, L. Rezzolla and K. Schwenzer, PRL (2018)
E. R. Most, A. Haber, S. P. Harris, Z. Zhang, M. G. Alford and J. Noronha, AJL (2024)



# In nuclei: Valley of stability




- Decays $\longrightarrow$ proportion that minimizes energy
- Charge fraction $Y_Q \approx 0.3 - 0.5$

Fig.: Bdushaw, DecayModeNuDat2, CC BY-SA 4.0

# Neutron stars: neutrino transparent $npe$ matter

- Nuclear matter + electrons.
- Charge neutrality: $n_e = n_p$
- Cannot decrease free energy via $n \leftrightarrow p + e^-$:

$$\sum_i \mu_i \, \Delta N_i = \underbrace{\mu_p + \mu_e - \mu_n}_{\delta\mu} = 0$$

- Small fraction of protons $Y_p = n_p/n_B \sim 0.1$

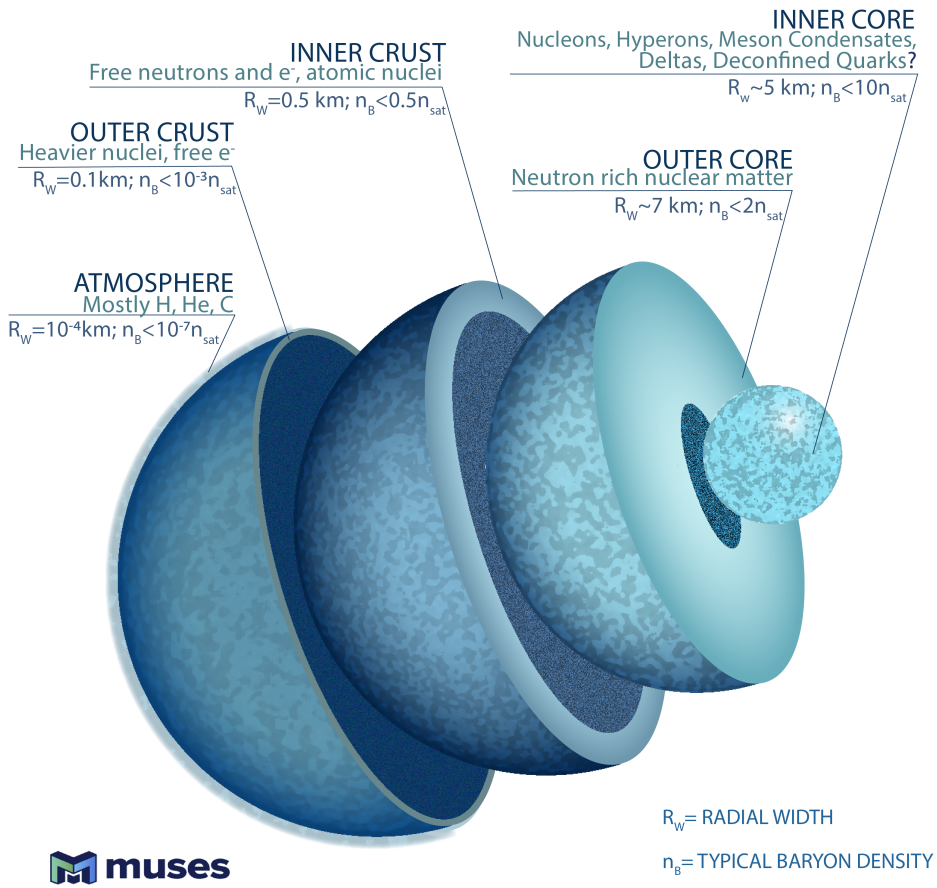
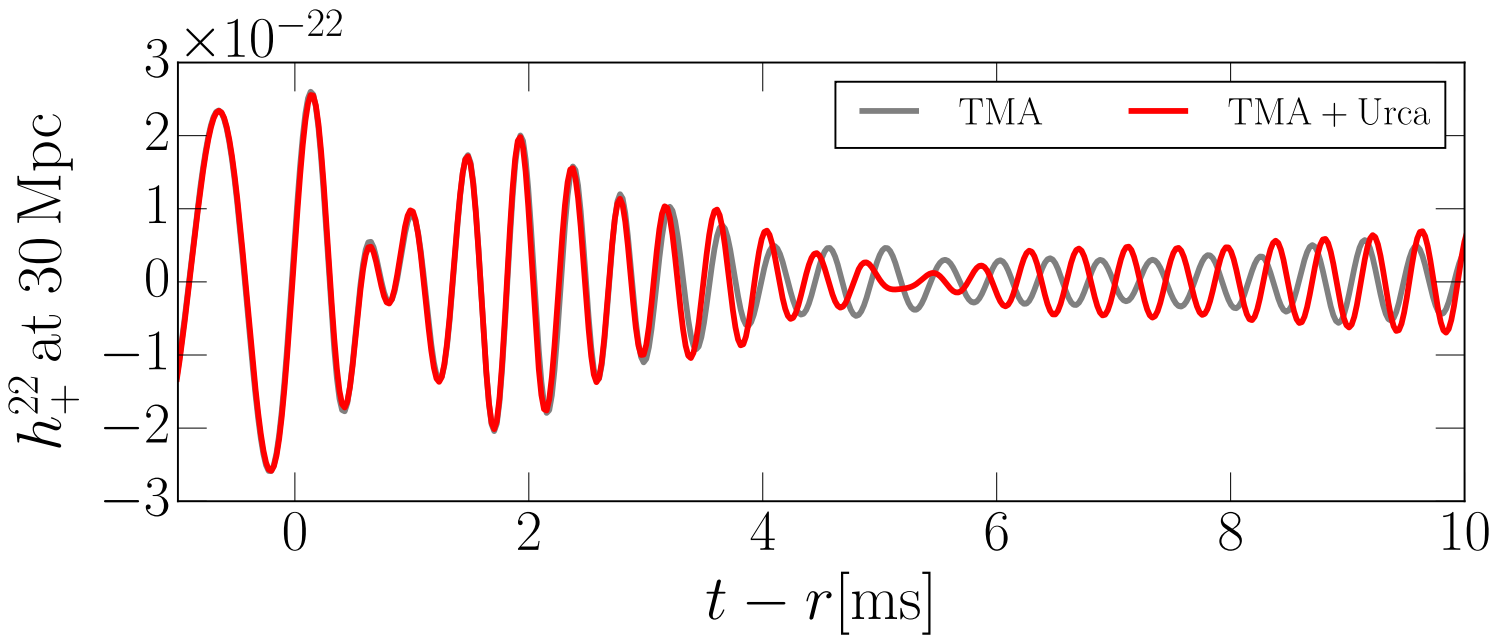



Fig.: R. Kumar *et al.* [MUSES], Living Rev. Rel. **27** (2024)

# In mergers

- Out-of-equilibrium evolution of the proton fraction:

$$n_B \dot{Y}_p = \Gamma_p \equiv \Gamma_{\rm dU} + \Gamma_{{\rm mU},n} + \Gamma_{mU,p},$$




E. R. Most, A. Haber, S. P. Harris, Z. Zhang, M. G. Alford and J. Noronha, AJL (2024)

- Relevant for postmerger graviational-wave signal.

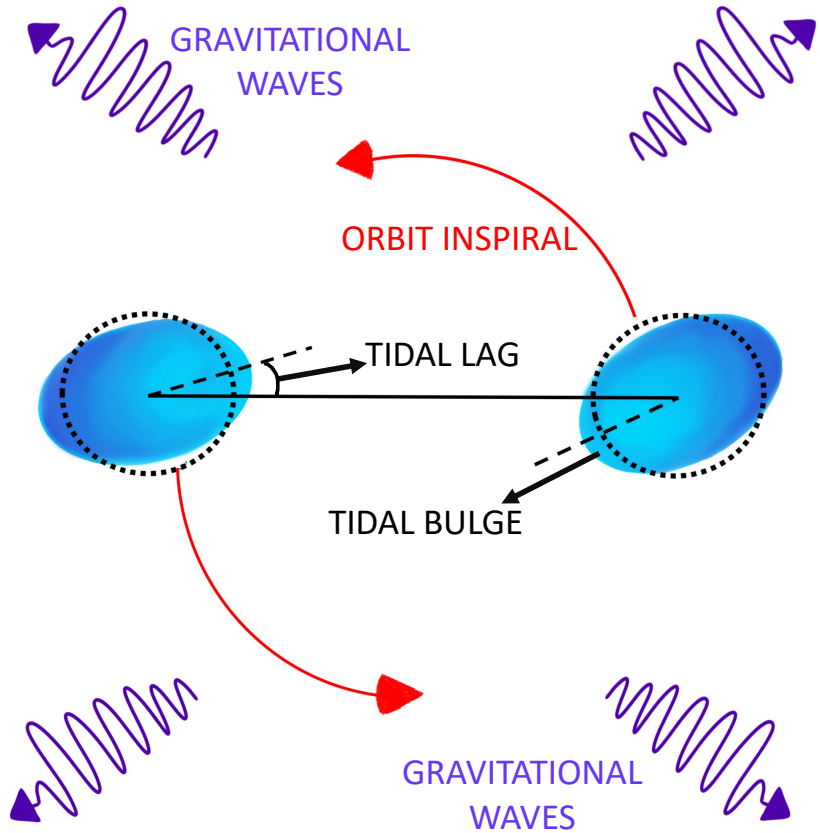
# In the inspiral phase

- Out-of-equilibrium dynamics
  $\rightarrow$ dissipative effects.

  R. F. Sawyer, PRD (1989)
  P. Haensel and R. Schaeffer, PRD (1992)
  M. G. Alford, L. Bovard, M. Hanauske, L. Rezzolla and K. Schwenzer, PRL (2018)

- Dissipation constrained by **dissipative tidal deformability**.

  Ripley, Hegade K. R., Yunes, PRD **108** (2023)
  Hegade K. R., Ripley, Yunes, PRD **110** (2024)




Ripley, Hegade K. R., Chandramouli, Yunes, Nature Astron. **8** (2024)

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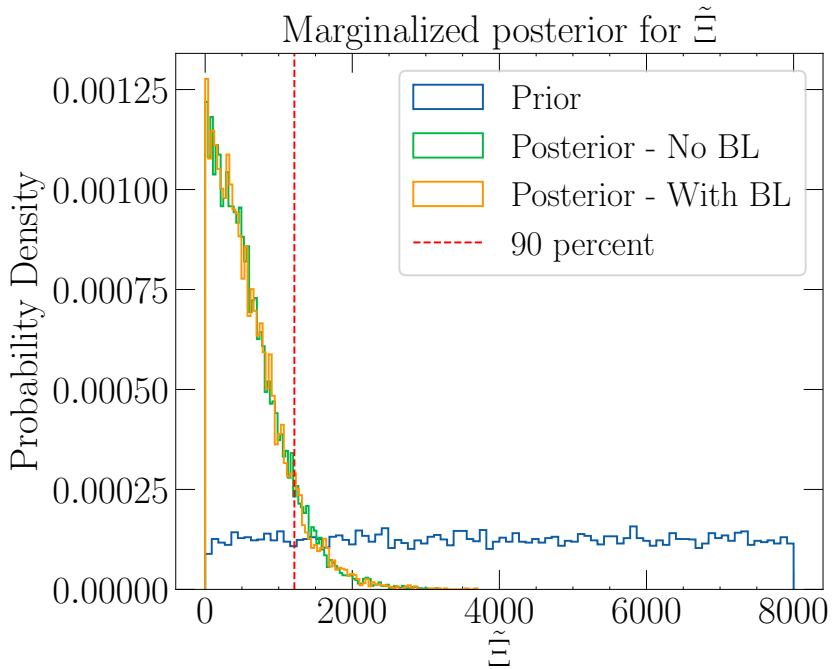



Ripley, Hegade K. R., Chandramouli, Yunes, Nature Astron. **8** (2024)

# Symmetry energy: connecting regimes

- Characterize nuclear binding-energy dependence on asymmetry

$$\delta = \frac{n_n - n_p}{n_B} = 1 - 2\frac{n_p}{n_B} = 1 - 2\,Y_p$$

- Symmetric matter for $\delta = 0$, pure neutron matter for $\delta = 1$.
- Connection between nuclear physics and neutron stars.

## Symmetry energy expansion

- Expansion of the energy per baryon around symmetric nuclear matter:

$$\frac{E}{A}(n_B, \delta) \approx \left.\frac{E}{A}\right|_{Y_p=1/2} + E_{\rm sym}(1 - 2\,Y_p)^2 + \mathcal{O}\left((1 - 2\,Y_p)^4\right)$$

# Density dependence of $E_{\rm sym}$

## Expansion around nuclear saturation

- Usual to expand around nuclear saturation, $n_B = n_{\rm sat}$:

$$E_{\rm sym}(n_B) \approx \boldsymbol{S} + \boldsymbol{L}\left(\frac{n_B - n_{\rm sat}}{3n_{\rm sat}}\right) + \frac{K_{\rm sym}}{2}\left(\frac{n_B - n_{\rm sat}}{3n_{\rm sat}}\right)^2 + \ldots,$$

- Described by symmetry energy $\boldsymbol{S}$ and slope $\boldsymbol{L}$ at saturation.

C. Drischler, V. Soma and A. Schwenk, PRC **89** (2014)

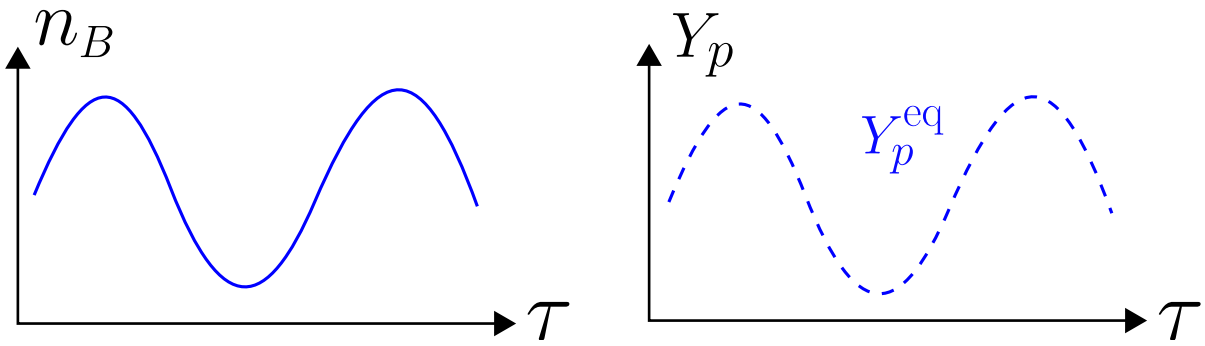
- Important properties of nuclear matter, used to constrain models.
- Crucial for the structure of neutron stars.

J. M. Lattimer and M. Prakash, ApJ **550** (2001)

- Value for slope $\boldsymbol{L}$ still uncertain/debated.

B. T. Reed, F. J. Fattoyev, C. J. Horowitz and J. Piekarewicz, PRL **126** (2021)
J. M. Lattimer, Particles **6** (2023)

# Dynamics of the proton fraction




- Equilibrium fraction of protons $Y_p \equiv n_p/n_B = Y_p^{\rm eq}(n_B)$.

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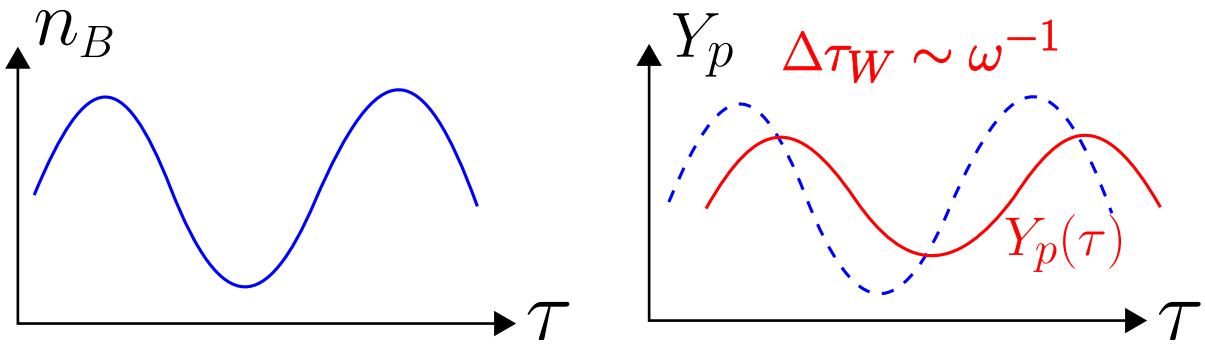


- Equilibrium fraction of protons $Y_p \equiv n_p/n_B = Y_p^{\rm eq}(n_B)$.
- Equilibration depends on weak interactions on timescales $\Delta\tau_W$.
- Mergers timescale $\omega^{-1} \sim \Delta\tau_W \longrightarrow$ phase shift

M. G. Alford, L. Bovard, M. Hanauske, L. Rezzolla and K. Schwenzer, PRL (2018)

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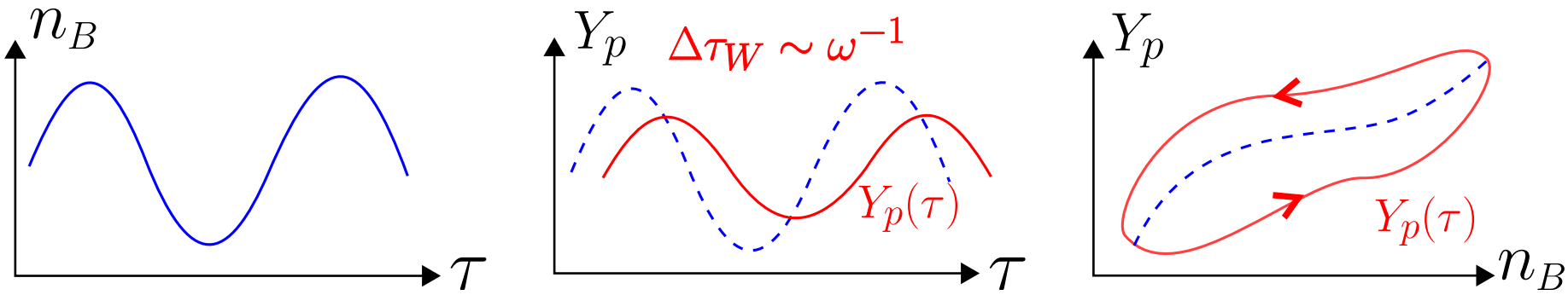


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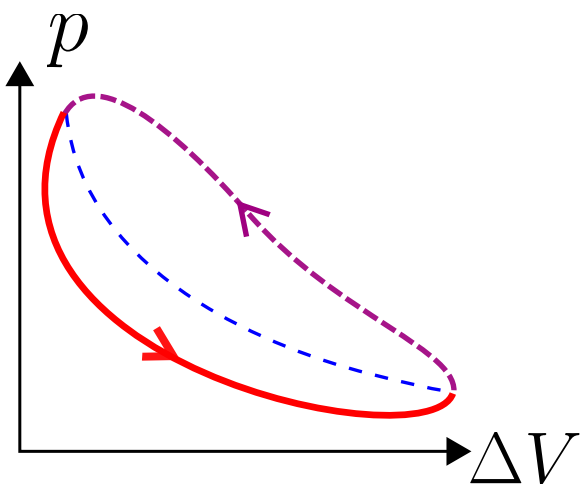
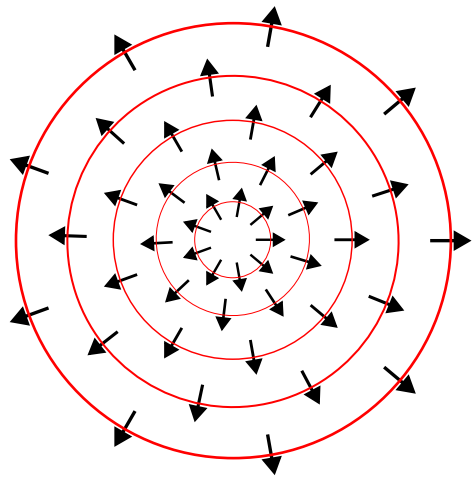
# Dissipative effects



$\Delta V \equiv$ *volume per baryon*

- Expansion: $p < p_{\rm eq}$.

R. F. Sawyer, PRD (1989)
P. Haensel and R. Schaeffer, PRD (1992)

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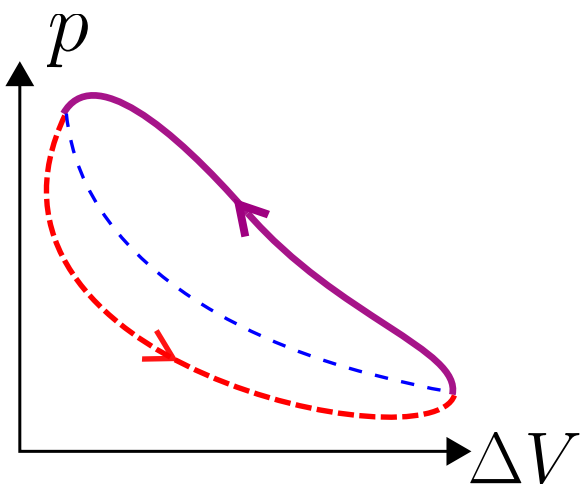


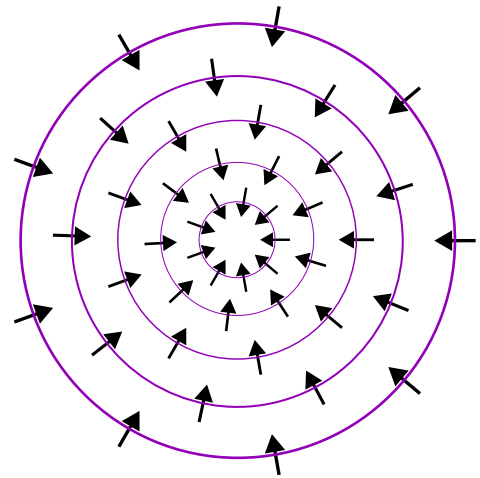

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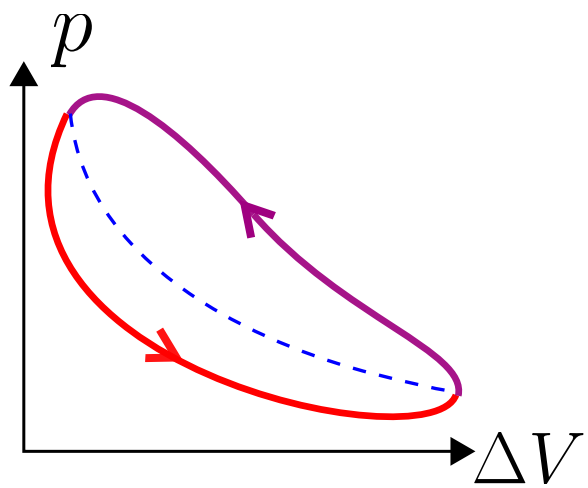


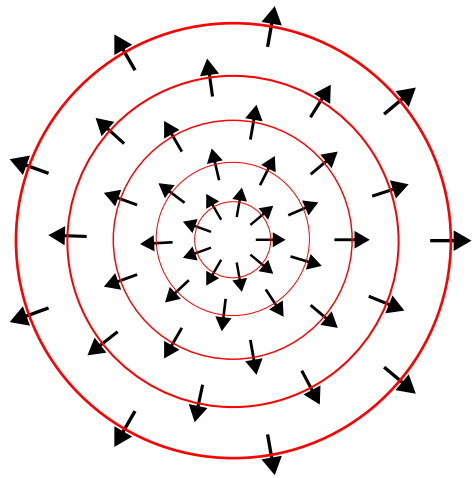

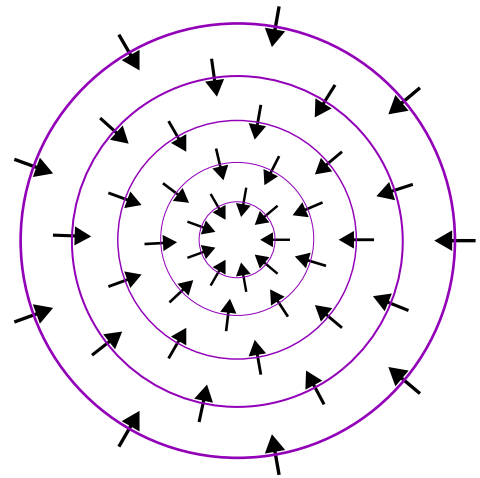

$\Delta V \equiv$ *volume per baryon*

- Expansion: $p < p_{\rm eq}$.
- Compression: $p > p_{\rm eq}$.

$\Rightarrow$

- Resistance to volume changes.

R. F. Sawyer, PRD (1989)
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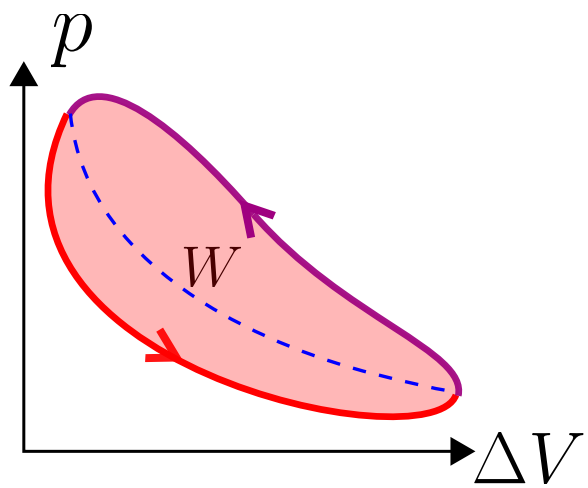
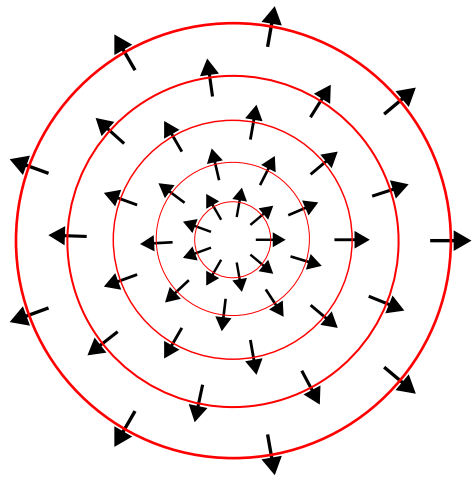
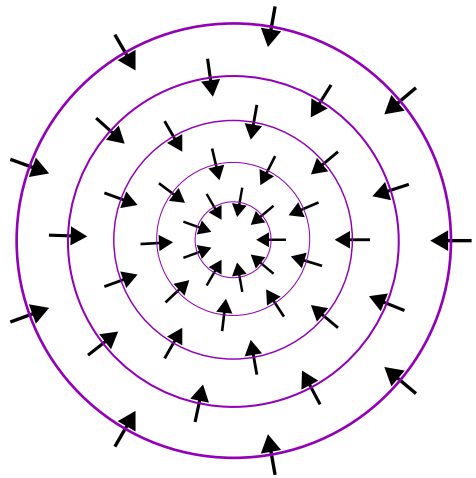
# Dissipative effects





$\Delta V \equiv$ *volume per baryon*

- Expansion: $p < p_{\rm eq}$.
- Compression: $p > p_{\rm eq}$.

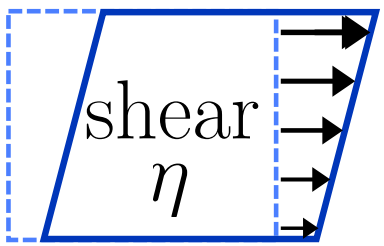
$\Rightarrow$

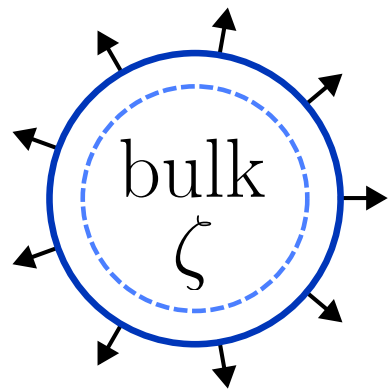
- Resistance to volume changes.
- **Dissipative work.**

R. F. Sawyer, PRD (1989)
P. Haensel and R. Schaeffer, PRD (1992)

# Bulk viscosity

- Resistance to velocity gradients: viscosity.
- Response to expansion rate $\theta = \nabla_\mu u^\mu$: **bulk viscosity $\zeta$**.




- Bulk-viscous effects in neutron star mergers.

B. A. Sa'd and J. Schaffner-Bielich, arXiv:0908.4190 [astro-ph.SR]
M. G. Alford and S. P. Harris, PRC (2018)
J. Cruz Rojas, T. Gorda, C. Hoyos *et al.* PRL (2024)
...

# From reactive to viscous hydrodynamics

- Shift in the pressure from $\delta Y_p = Y_p - Y_p^{\beta-\mathrm{eq}}(\varepsilon, n_B)$.

$$P = \underbrace{P(\varepsilon, n_B, Y_p^{\beta-\mathrm{eq}}(\varepsilon, n_B))}_{\text{equilibrium } P_{\mathrm{eq}}} + \Pi(\varepsilon, n_B, \delta Y_p)$$

- Equation of motion from chain rule $\sim$ expansion rate $\nabla_\mu u^\mu$

T. Celora, I. Hawke, P. C. Hammond, N. Andersson and G. L. Comer, PRD (2022)
G. Camelio, L. Gavassino, M. Antonelli, S. Bernuzzi and B. Haskell, PRD (2023)

$$\tau_\Pi \, u^\mu \nabla_\mu \Pi + \underbrace{\Pi = -\zeta_0 \, \nabla_\mu u^\mu}_{\text{Navier-Stokes}}$$

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$$\tau_\Pi\, u^\mu \nabla_\mu \Pi + \underbrace{\Pi = -\zeta_0\, \nabla_\mu u^\mu}_{\text{Navier-Stokes}}$$

- Israel-Stewart equation. Relaxes to Navier-stokes at long times.

I. Muller, Z. Phys. **198** (1967), W. Israel, Annals Phys. **100** (1976)
W. Israel and J. M. Stewart, Annals Phys. **118** (1979)

# Transport coefficients for neutrino-transparent *npe* matter

- Relaxation time $\tau_\Pi$ and **bulk viscosity** $\zeta$.

- Linear approximation in $\delta\mu \equiv \mu_p + \mu_e - \mu_n$:

$$\tau_\Pi = -\frac{n_B}{\lambda}\left(\frac{\partial \delta\mu}{\partial Y}\right)^{-1}_{n_B}, \quad \zeta_0 = -\frac{n_B^2}{\lambda}\left.\frac{\partial P}{\partial \delta\mu}\right|_{n_B}\left(\frac{\partial \delta\mu}{\partial Y}\right)^{-1}_{n_B}\left.\frac{\partial \delta\mu}{\partial n_B}\right|_{Y_p}$$

  with reaction rate $\Gamma \approx -\lambda\,\delta\mu$.

- Transport coefficients scale with symmetry energy and first derivative:

$$\tau_\Pi \sim \frac{1}{E_{\rm sym}(n_B)}, \qquad \zeta_0 \sim \left(\frac{\partial E_{\rm sym}(n_B)}{\partial n_B}\Big/ E_{\rm sym}(n_B)\right)^2.$$

- At nuclear saturation density, determined $S$ and $L$:

$$\tau_\Pi(n_{\rm sat}) \sim \boldsymbol{S}^{-1}, \qquad \zeta(n_{\rm sat}) \sim (\boldsymbol{L}/\boldsymbol{S})^2$$

# Transport coefficients for neutrino-transparent *npe* matter

- Relaxation time $\tau_\Pi$ and **bulk viscosity** $\zeta$.
- Linear approximation in $\delta\mu \equiv \mu_p + \mu_e - \mu_n$:

wi

Direct connection between dissipation in mergers and properties of nuclear matter.

Y. Yang, MH, E. Speranza and J. Noronha, PRL **135** (2025)

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$$\tau_\Pi \sim \frac{1}{E_{\rm sym}(n_B)}, \qquad \zeta_0 \sim \left(\frac{\partial E_{\rm sym}(n_B)}{\partial n_B}\middle/ E_{\rm sym}(n_B)\right)^2.$$

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$$\tau_\Pi(n_{\rm sat}) \sim \boldsymbol{S}^{-1}, \qquad \zeta(n_{\rm sat}) \sim (\boldsymbol{L}/\boldsymbol{S})^2$$

# Symmetry energy slope constraints

1. $L \approx 50 \pm 15.5$ MeV

Fan, Dong and Zuo, PRC **89** (2014)

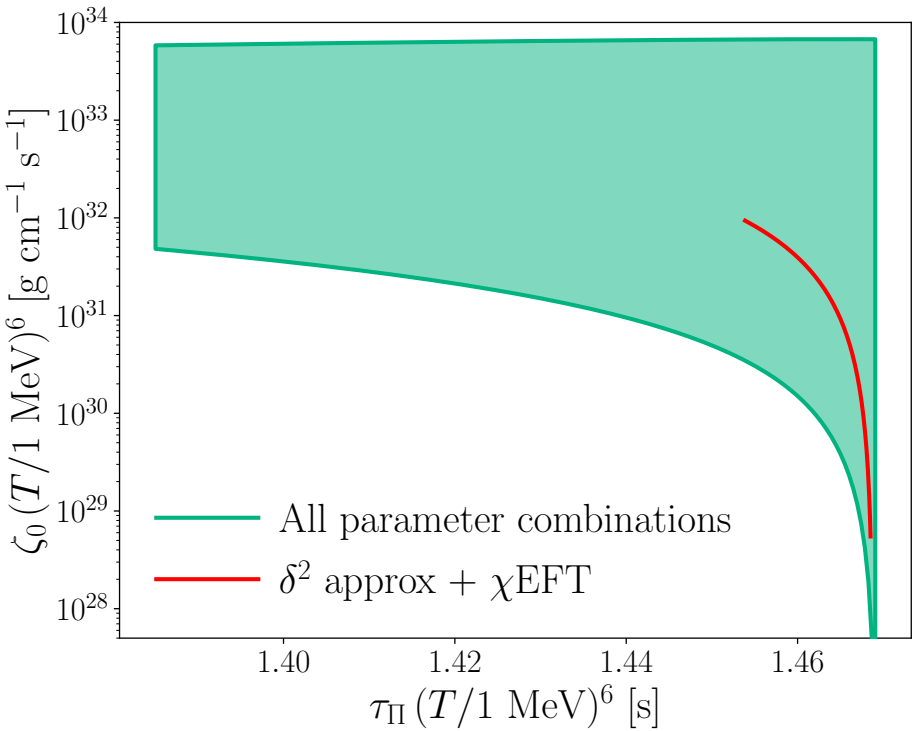
(Li, Krastev, Wen, Zhang, EPJ A **55** (2019), Reinhard, Roca-Maza, Nazarewicz, PRL **127** (2021))

2. $L = 106 \pm 37$ MeV

Reed, Fattoyev, Horowitz and Piekarewicz, PRL **126** (2021)
Adhikari *et al.* [PREX], PRL **126** (2021)

- $\tau_\Pi$ well constrained by $S$.
- $\zeta$ very sensitive to $L$ determination.
- Large uncertainties even from chiral effective theory ($\chi$EFT).

Y. Yang, MH, E. Speranza and J. Noronha, PRL **135** (2025)



# Nuclear matter as visco-elastic fluid

## Physical regimes

1. Navier-Stokes regime $\omega\,\tau_\Pi \ll 1$, $\zeta(\omega) \approx \zeta_0$
2. Resonant regime: $\omega\,\tau_\Pi \approx 1$, maximum dissipation.
3. Frozen regime: $\omega\,\tau_\Pi \gg 1$, $\zeta(\omega) \approx 0$, $\Upsilon_p \approx$ const.

L. Gavassino, M. Antonelli and B. Haskell, Class. Quant. Grav. **38** (2021)

- In the frozen regime, time-reversal symmetry

$$\tau_\Pi\,\partial_\tau \Pi = -\zeta_0\,\nabla_\mu u^\mu$$

- Expansion rate $\nabla_\mu u^\mu = \partial_\tau \Delta V/V \Rightarrow$ Hooke's law:

$$\Pi = -\frac{\zeta_0}{\tau_\Pi}\Delta V$$

- Bulk modulus $K = \zeta_0/\tau_\Pi$ independent of rates.

G. Montefusco, M. Antonelli and F. Gulminelli, Astron. Astrophys. **694** (2025)

# Low-temperature regime

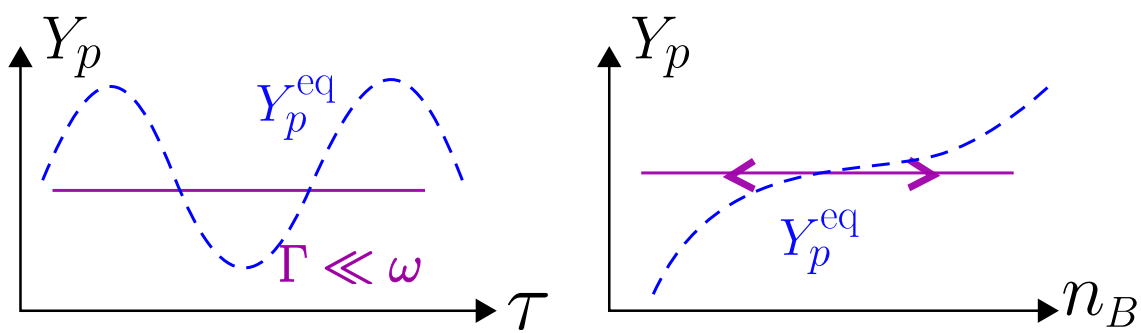
- Relaxation time and bulk viscosity diverge at low $T$

$$\tau_\Pi,\ \zeta_0 \sim \left(\frac{\partial \Gamma}{\partial \delta\mu}\right)^{-1} \sim T^{-6}$$

- Frozen regime: $Y_p \neq Y_p^{\rm eq}(n_B)$ is fixed, but out of equilibrium as matter compresses or expands.

- Relevant for early inspiral phase, with $T \sim 10^5$ K and $\omega \sim$ kHz.

Y. Yang, MH, E. Speranza and J. Noronha, PRL **135** (2025)



# Low-temperature regime

- Relaxation time and bulk viscosity diverge at low $T$

$$\tau_\Pi,\ \zeta_0 \sim \left(\frac{\partial \Gamma}{\partial \delta\mu}\right)^{-1} \sim T^{-6}$$

- Frozen regime: $Y_p \neq Y_p^{\rm eq}(n_B)$ is fixed, but out of equilibrium as matter compresses or expands.

- Relevant for early inspiral phase, with $T \sim 10^5$ K and $\omega \sim$ kHz.

Y. Yang, MH, E. Speranza and J. Noronha, PRL **135** (2025)

- Nonlinear corrections: weaker suppression → important effects?

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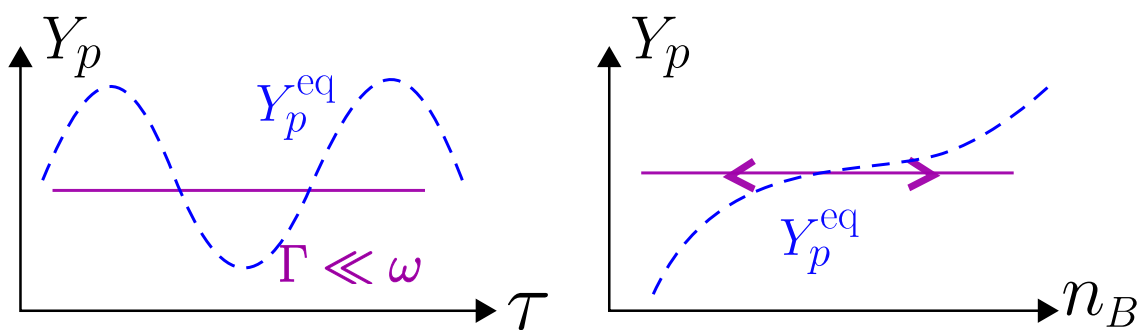

# Conclusions

- Dissipative effects in neutron star mergers from weak interactions.
- Transport coefficients depend on nuclear matter properties related to neutron/proton asymmetry.
- Constraints from dynamical tidal response in inspiral phase?

## Outlook

- Estimate constraints from GW170817 and future detections.
- Nonlinear effects at low $T$ in inspiral.

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# Backup slides...

## Deviation from weak equilibrium

- Neutrino transparent regime: mean free path $\gtrsim$ system size.
- First law of thermodynamics

$$
\begin{aligned}
T\,ds &= d\varepsilon - \mu_n \overbrace{dn_n}^{dn_B - dn_p} - \mu_p\, dn_p - \mu_e \overbrace{dn_e}^{dn_p} \\
&= d\varepsilon - \mu_n\, dn_B - \underbrace{(\mu_p + \mu_e - \mu_n)}_{\delta\mu}\, dn_p
\end{aligned}
$$

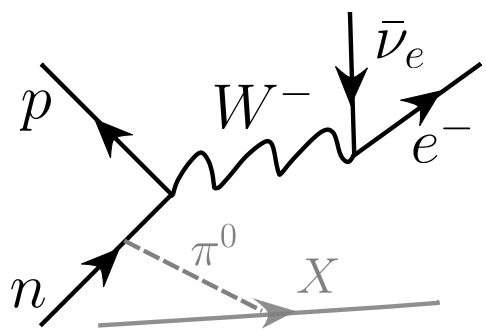
- In equilibrium $\delta\mu \equiv \mu_p + \mu_e - \mu_n = 0$.

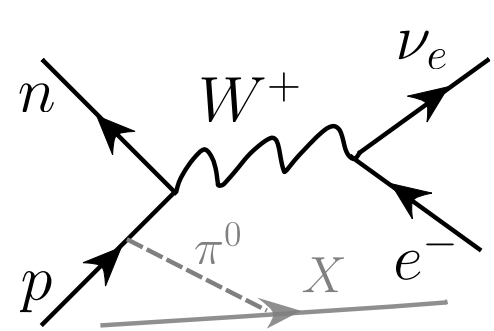
# Reaction rate

Evolution of the proton fraction:

$$n_B \dot{Y}_p = \Gamma_p \equiv \Gamma_{\rm dU} + \Gamma_{{\rm mU},n} + \Gamma_{mU,p},$$

- $\Gamma_{\rm dU/mU} = \Gamma_{n\to p+e} - \Gamma_{p+e\to n}$.
- $\Gamma_{\rm dU/mU}$ odd in $\delta\mu \equiv \mu_p + \mu_e - \mu_n = 0 \Rightarrow$ entropy increases.
- Fermi's four-fermion interaction. Perturbation theory justified.
- Calculation in the Fermi surface approximation, $T \lesssim 5$ MeV.



# Conventional approach to the bulk-viscosity

- Linear analysis:

$$\Gamma_p \approx -\lambda\,\delta\mu \qquad \delta Y_e = f(\omega)\,\delta n_B \qquad \delta P, \delta\mu \sim g(\omega)\,\delta n_B$$

- Dissipative work per oscillation

$$W = -\oint p\,dV$$

- Match to Navier-Stokes hydrodynamics:

$$W_{\rm NS} = -\zeta \int_0^T dt\,(\nabla\cdot\vec{v})^2$$

- Bulk viscosity found as function of frequency $\zeta_{\rm AC}(\omega)$.
- Computed for a variety of equations of state,

R. F. Sawyer, PRD (1989)
P. Haensel and R. Schaeffer, PRD (1992)
B. A. Sa'd and J. Schaffner-Bielich, arXiv:0908.4190 [astro-ph.SR]
M. G. Alford and S. P. Harris, PRC (2018)
J. Cruz Rojas, T. Gorda, C. Hoyos *et al.* PRL (2024)
...

# Urca rates

$$\Gamma_{dU} = \frac{G^2(1+3g_A^2)}{240\pi^5} E^*_{Fn} E^*_{Fp} p_{Fe} \vartheta_{dU} \delta\mu (17\pi^4 T^4 + 10\pi^2 \delta\mu^2 T^2 + \delta\mu^4)$$

$$\Gamma_{mU,n} = \frac{1}{5760\pi^9} G^2 g_A^2 f^4 \frac{(E^*_{Fn})^3 E^*_{Fp}}{m_\pi^4} \frac{p_{Fn}^4 p_{Fp}}{(p_{Fn}^2 + m_\pi^2)^2} \vartheta_n \delta\mu (1835\pi^6 T^6 + 945\pi^4 \delta\mu^2 T^4 + 105\pi^2 \delta\mu^4 T^2 + 3\delta\mu^6)$$

$$\Gamma_{mU,p} = \frac{1}{40320\pi^9} G^2 g_A^2 f^4 \frac{E^*_{Fn} (E^*_{Fp})^3}{m_\pi^4} \frac{p_{Fn}(p_{Fn} - p_{Fp})^4}{((p_{Fn} - p_{Fp})^2 + m_\pi^2)^2} \vartheta_p \delta\mu \times \times (1835\pi^6 T^6 + 945\pi^4 \delta\mu^2 T^4 + 105\pi^2 \delta\mu^4 T^2 + 3\delta\mu^6)$$

# Viscous relativistic hydrodynamics

Can include viscous stresses, energy and (baryon) charge diffusion as corrections to currents. For instance,

$$T^{\mu\nu} = T^{\mu\nu}_{\rm ideal} + \Pi^{\mu\nu}$$

## Bulk viscosity

- Viscous correction to the pressure: $\Pi^{\mu\nu} = \Pi\,\Delta^{\mu\nu}$.
- Relativistic Navier-Stokes theory $\Pi = -\zeta\,\theta$.

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## Bulk viscosity

- Viscous correction to the pressure: $\Pi^{\mu\nu} = \Pi\,\Delta^{\mu\nu}$.
- Israel-Stewart-like theories $\Rightarrow$ promote $\Pi$ to degree of freedom:

$$\overbrace{\tau_\Pi}^{\text{relaxation time}}\, u^\mu \nabla_\mu \Pi + \Pi = -\zeta \overbrace{\theta}^{\text{expansion rate } \nabla_\mu u^\mu}$$

- Toy-version of theories applied to relativistic heavy ion collisions.