Applying strong dynamics to dark matter phenomenology

Xiaoyong Chu (ICTP-AP, UCAS)
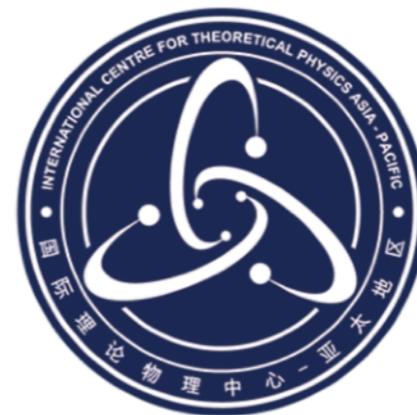
With Josef Pradler, Daris Samart 2512.08517

University of Chinese Academy of Sciences
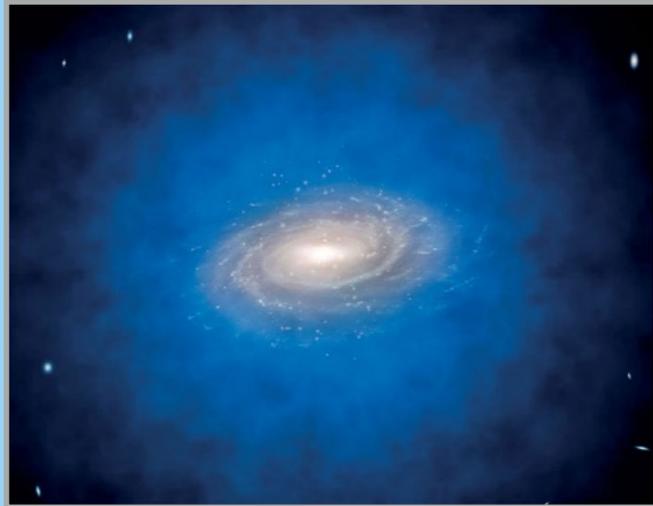
unesco
Centre
Under the auspices
of UNESCO

ICTP-AP
International Centre
for Theoretical Physics Asia-Pacific
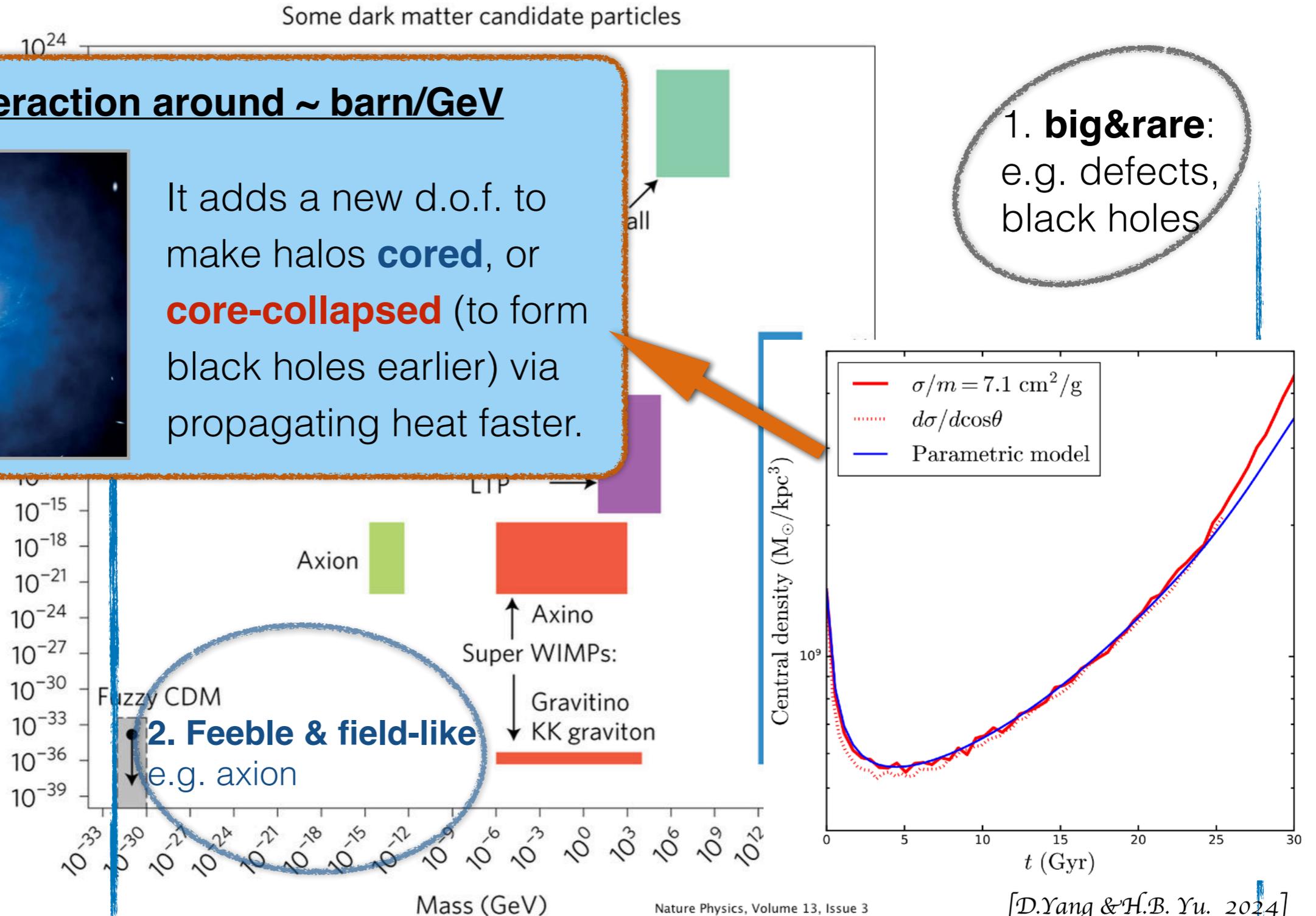国际理论物理中心-亚太地区

1. If **strong dynamics** in dark sector

Some dark matter candidate particles

10^{24}

3. Self-interaction around ~ barn/GeV

It adds a new d.o.f. to make halos **cored**, or **core-collapsed** (to form black holes earlier) via propagating heat faster.

1. **big&rare**: e.g. defects, black holes

$\sigma/m = 7.1\ \mathrm{cm^2/g}$
$d\sigma/d\cos\theta$
Parametric model

LTP

10^{-15}

Axion

10^{-18}

10^{-21}

Axino

10^{-24}

Super WIMPs:

10^{-27}

10^{-30}

Fuzzy CDM

Gravitino

10^{-33} **2. Feeble & field-like**

KK graviton

10^{-36} e.g. axion

10^{-39}

Central density $(\mathrm{M_\odot/kpc^3})$

10^9

10^{-33} 10^{-30} 10^{-27} 10^{-24} 10^{-21} 10^{-18} 10^{-15} 10^{-12} 10^{-9} 10^{-6} 10^{-3} 10^{0} 10^{3} 10^{6} 10^{9} 10^{12}

Mass (GeV)

0   5   10   15   20   25   30

t (Gyr)

Nature Physics, Volume 13, Issue 3
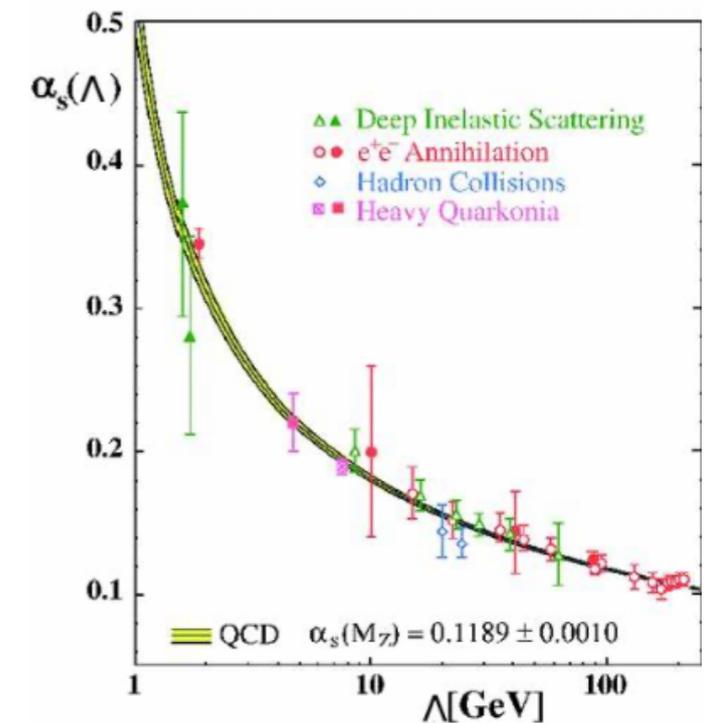
[D. Yang & H.B. Yu. 2024]

Dark strong dynamics has rich phenomenology

A dark-QCD DM can be **the stable composite state**

- dark baryon (if it is like the SM);

- dark pion (light-quark limit);

J. M. Cline, Z. Liu, et al., 2013,
G. D. Kribs &E.T. Neil 2016

- dark glueball (heavy-quark limit);

- large-N nuclei, dark atom,….

Production mechanism:

- DM could be asymmetric, to prevent significant self-annihilation.

  5 GeV to explain coincidence, or other co-genesis?

- Or one needs N-body annihilation (N > 2)

Self-interaction: exchanging one or more **dark pion**s.

Interesting SIDM energy-scale: sub-GeV QCD phase-transition

Dark strong dynamics has **rich phenomenology**

A dark-QCD DM can be **the stable composite state**

- dark baryon (if it is like the SM);
- dark pion (light-quark limit);
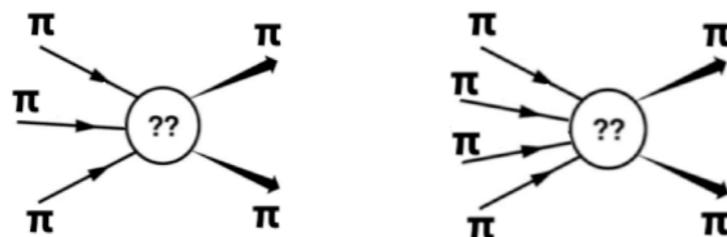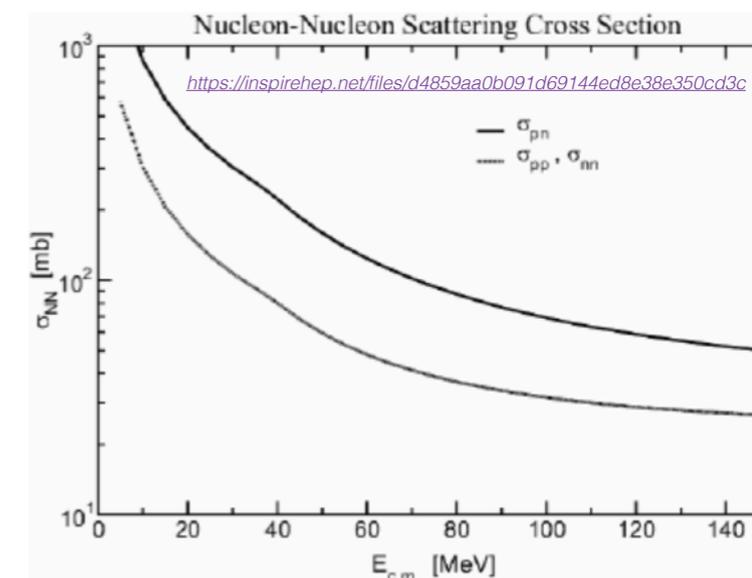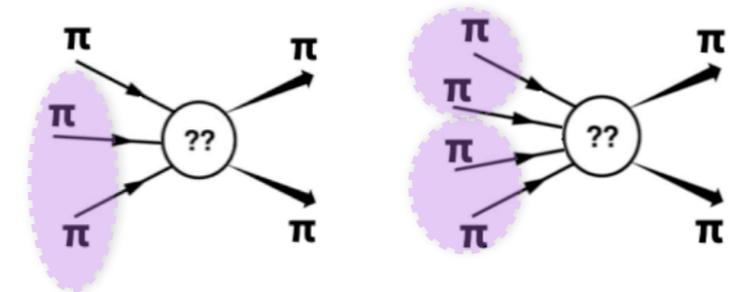- dark glueball (heavy-quark limit);
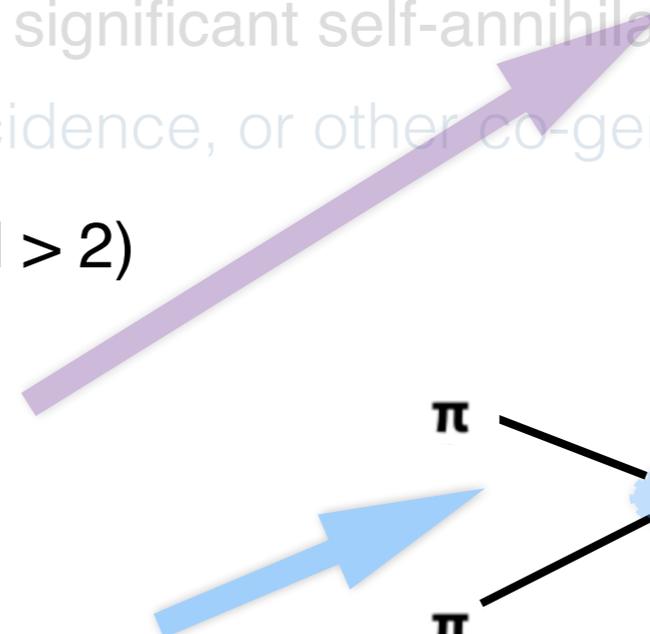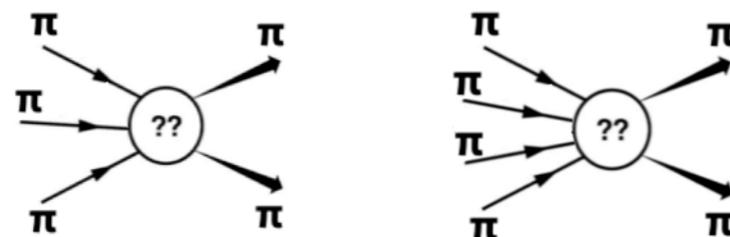- large-N nuclei, dark atom,….

J. M. Cline, Z. Liu, et al., 2013,
G. D. Kribs &E.T. Neil 2016

[XC, M. Nikolic, J.Pradler 2024]

Production mechanism:

- DM could be asymmetric, to prevent significant self-annihilation.

  5 GeV to explain coincidence, or other co-genesis?

- Or one needs N-body annihilation (N > 2)

Self-interaction: exchanging one or more **dark pion**s.

[XC, C. Garcia-Cely, H. Murayama 2018]

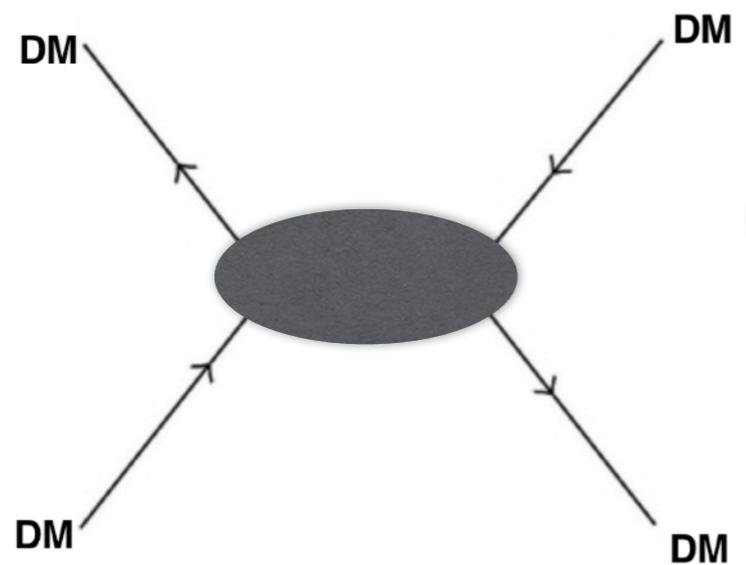Interesting SIDM energy-scale: sub-GeV QCD phase-transition

3. **Dark self-interactions**:

universally

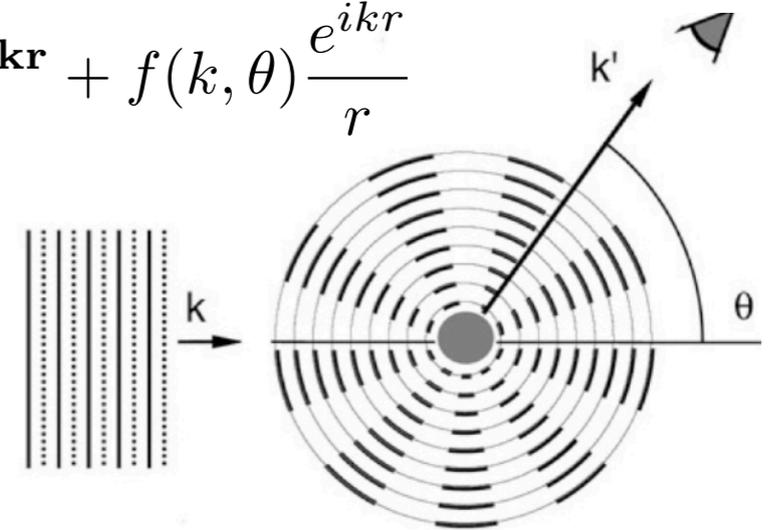# Dark strong dynamics has **rich phenomenology**

Quantitative statements need lattice, and depend on concrete models.

How far can we go with universal/analytical methods?

[XC, C. García-Cely, H. Murayama 2019]

$$\psi(r) \simeq e^{i\mathbf{k}\mathbf{r}} + f(k,\theta)\frac{e^{ikr}}{r}$$

DM, DM, DM, DM

$$f_\ell(k) \equiv \frac{e^{2i\delta_\ell(k)} - 1}{2ik} = \frac{1}{k\left(\cot\delta_\ell(k) - i\right)}$$

boundary conditions of **finite-range potential** allows an expansion of the phase shift at very low velocities *[Schwinger, Blatt&Jackson, Bethe, 1940s]* :

$$k\cot\delta_0 = -\frac{1}{a_0} + \frac{k^2}{2r_{e,0}} + \mathcal{O}(k^4)$$

(strong non-relativistic scattering, suggesting $Im[\delta_0] << Re[\delta_0]$, and neglected)
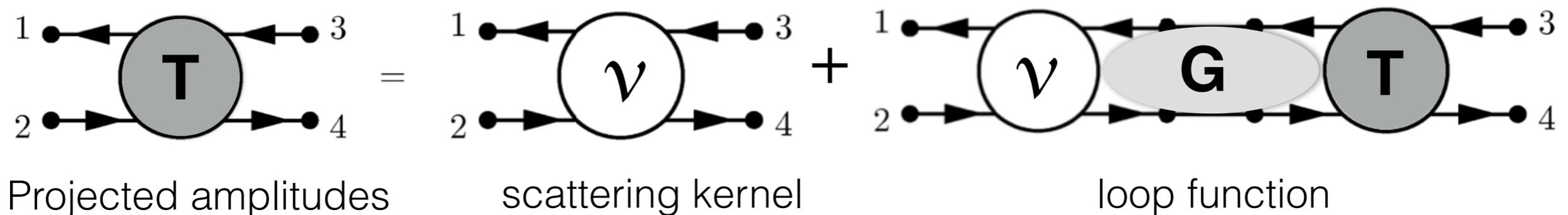
A quantitative way?

Adopt various **unitarization methods** (N/D, IAM etc.) and impose s/p-wave projections, but essentially:

$$T(s) = \frac{v(s)}{1 + v(s)\,G(s)}$$

(Sign depends on convention)

Similar to Lippmann-Schwinger equation, it is simply

Projected amplitudes   scattering kernel   loop function

At non-relativistic limit $\tilde{G}(E) \equiv \int d^3p \, \dfrac{\Theta(\Lambda - p)}{E - 2m_\pi - \frac{\vec{p}^2}{m_\pi}}$

A quantitative way?

Adopt various **unitarization methods** (N/D, IAM etc.) and impose s/p-wave projections, but essentially:

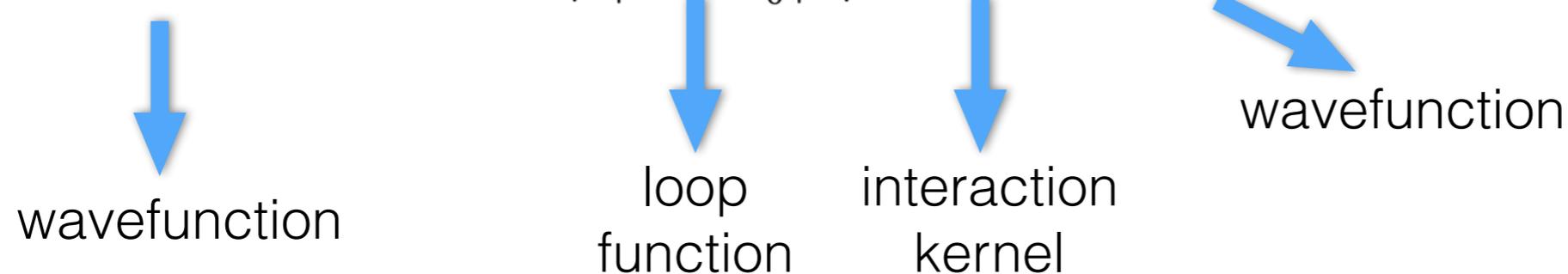$$T(s) = \frac{v(s)}{1 + v(s)\, G(s)}$$

(Sign depends on convention)

For **bound state-dominated** scattering amplitudes:

[following *D. Gamermann, J. Nieves, E. Oset, & E. Ruiz Arriola, 2009*]

1. first work out the bound wavefunction

$$\langle \vec{p} \mid \Psi \rangle = \int d^3k \int d^3k' \left\langle \vec{p} \left| \frac{1}{E - \hat{H}_0} \right| \vec{k} \right\rangle \left\langle \vec{k} | \hat{V} | \vec{k}' \right\rangle \left\langle \vec{k}' \mid \Psi \right\rangle$$

wavefunction

loop function

interaction kernel

wavefunction

π   resonance   π

π   π

A quantitative way?

Adopt various **unitarization methods** (N/D, IAM etc.) and impose s/p-wave projections, but essentially:

$$T(s) = \frac{v(s)}{1 + v(s)\,G(s)}$$

(Sign depends on convention)

Bound state solution
$$1 + \tilde{v}\,\tilde{G}(E) = 0$$

For **bound state-dominated** scattering amplitudes:

[following *D. Gamermann, J. Nieves, E. Oset, & E. Ruiz Arriola, 2009*]

1. first work out the bound wavefunction

$$\langle \vec{p} \mid \Psi \rangle = \int d^3k \int d^3k' \left\langle \vec{p} \left| \frac{1}{E - \hat{H}_0} \right| \vec{k} \right\rangle \left\langle \vec{k} | \hat{V} | \vec{k}' \right\rangle \left\langle \vec{k}' \mid \Psi \right\rangle$$

wavefunction

loop function

interaction kernel

wavefunction

π resonance π

π π

$$|\Psi(0)| = \frac{|\tilde{G}(E)|}{(2\pi)^{3/2}} \left| \frac{d\tilde{G}(E)}{dE} \right|^{-1/2} \quad \text{at} \quad E = 2m_\pi - E_B$$

A quantitative way?

Adopt various **unitarization methods** (N/D, IAM etc.) and impose s/p-wave projections, but essentially:

$$T(s) = \frac{v(s)}{1 + v(s)\, G(s)}$$

(Sign depends on convention)

Bound state solution

$$1 + \tilde{v}\, \tilde{G}(E) = 0$$

For **bound state-dominated** scattering amplitudes:

[following *D. Gamermann, J. Nieves, E. Oset, & E. Ruiz Arriola, 2009*]

1. first work out the bound wavefunction;

2. then take another form of the amplitude;

$$T(E^2) = \frac{g_{X\pi\pi}^2}{E - (2m_\pi - E_B)}$$

Both 1/T and $E - (2m_\pi - E_B)$ go to zero at the BS pole **(L'Hôpital's rule)**

$$g_{X\pi\pi}^2 = 32\pi^3 m_\pi^2 \left(\frac{d\tilde{G}(E)}{dE}\right)^{-1}\Bigg|_{E=2m_\pi - E_B}$$

Adopt various **unitarization methods** (N/D, IAM etc.) and impose s/p-wave projections, but essentially:

$$T(s) = \frac{v(s)}{1 + v(s)\,G(s)}$$

(Sign depends on convention)

Bound state solution

$$1 + \tilde{v}\,\tilde{G}(E) = 0$$

For **bound state-dominated** scattering amplitudes:

[following *D. Gamermann, J. Nieves, E. Oset, & E. Ruiz Arriola, 2009*]

1. first work out the bound wavefunction;

2. then take another form of the amplitude;

π ⟶ resonance ⟶ π

π ⟶ resonance ⟶ π

Nature choice of parameters gives astrophysically interesting values (sub-GeV SIDM): (whatever dark group structures)

$$\frac{\langle \sigma_{\text{self}} \rangle}{m_\pi} \simeq 1\,\text{cm}^2/\text{g} \left(\frac{0.1}{\kappa}\right)^2 \left(\frac{0.5\,\text{GeV}}{m_\pi}\right)^3 \left(\frac{|\Psi(0)|}{0.14 m_\pi^{3/2}}\right)^4 \left(\frac{0.02}{|G(s)|_{s=(2m_\pi - \kappa m_\pi)^2}}\right)^4$$
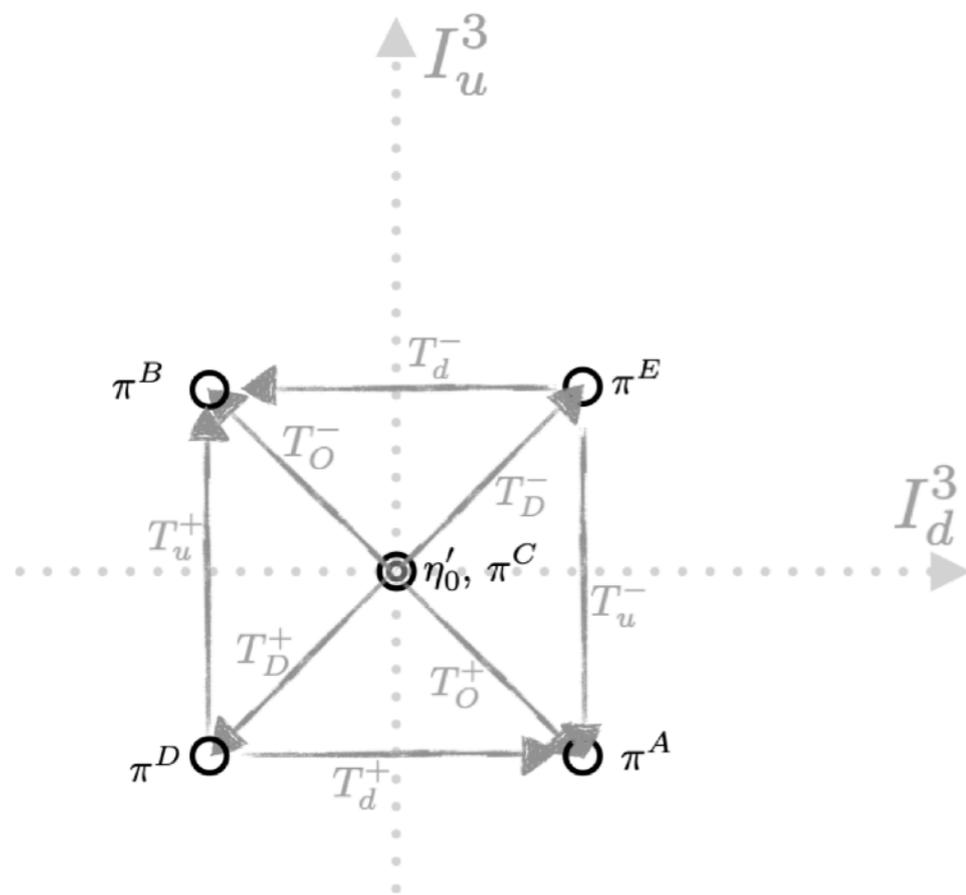
3. **Dark self-interactions:**

a toy model

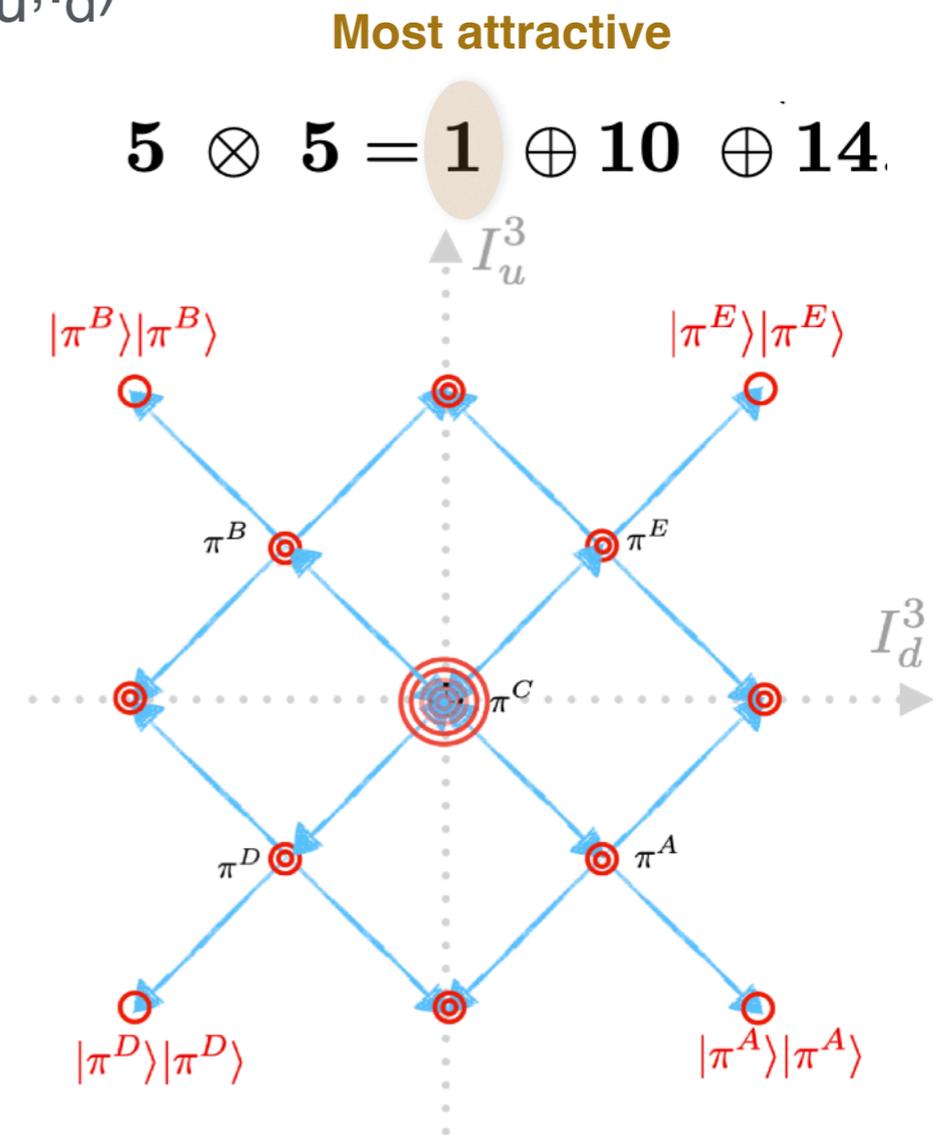Take SIMP motivated: 2-quark case, global symmetry SU(4) → Sp(4)

(five dark pions as dark matter)

For rank-2 group Sp(4), weight vectors labeled by (I_u, I_d)

Traceless part of anti-symmetric rank-2 tensor **5**

Most attractive

$$\mathbf{5} \otimes \mathbf{5} = \mathbf{1} \oplus \mathbf{10} \oplus \mathbf{14}$$
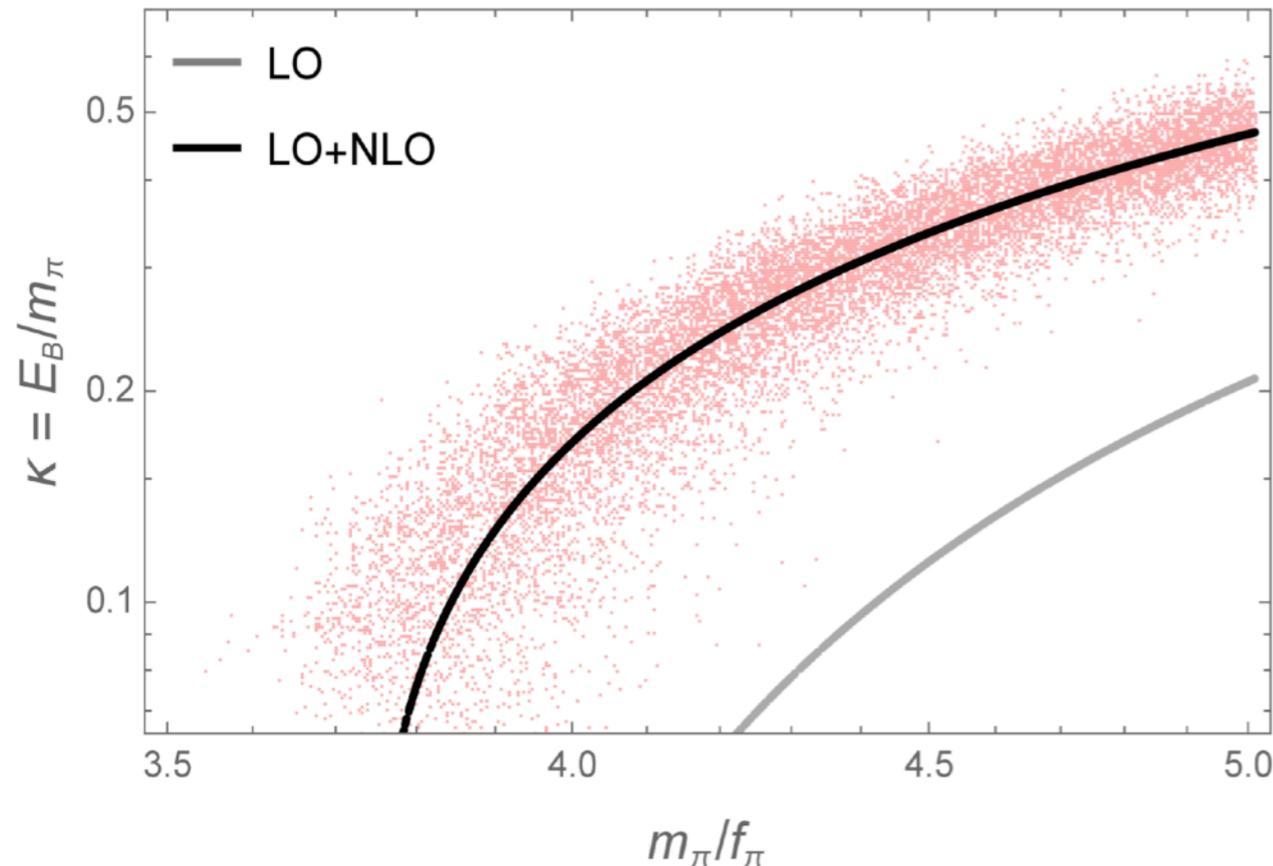
$$\pi^{a_1}(p_1) + \pi^{a_2}(p_2)$$

A quantitative way?

Take SIMP motivated: 2-quark case, global symmetry SU(4) → Sp(4)

(five dark pions as dark matter)

→ Singlet (isoscalar) channel of $\pi^{a_1}(p_1) + \pi^{a_2}(p_2) \to \pi^{a_3}(p_3) + \pi^{a_4}(p_4)$

use amplitudes from *M. Albaladejo & J. A. Oller 2012*, to work out the s-wave projected interaction kernel to NLO.

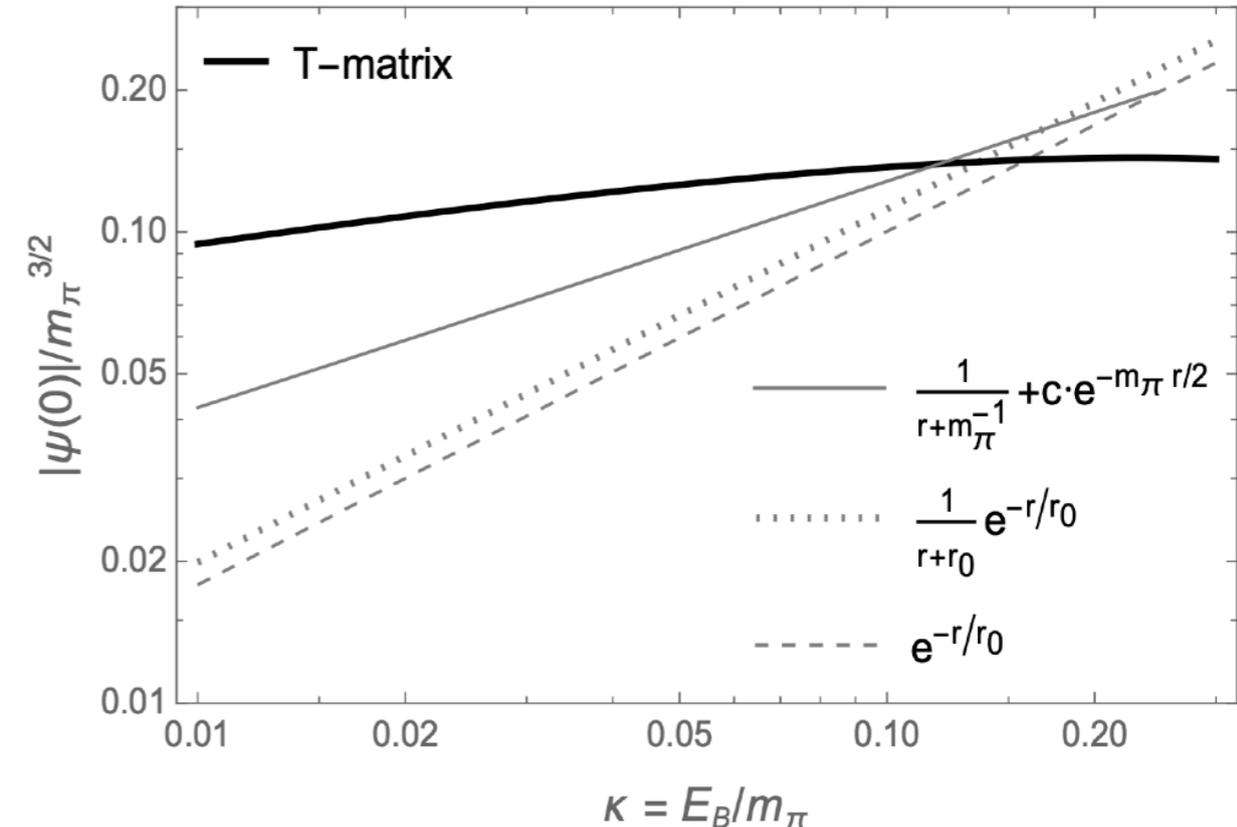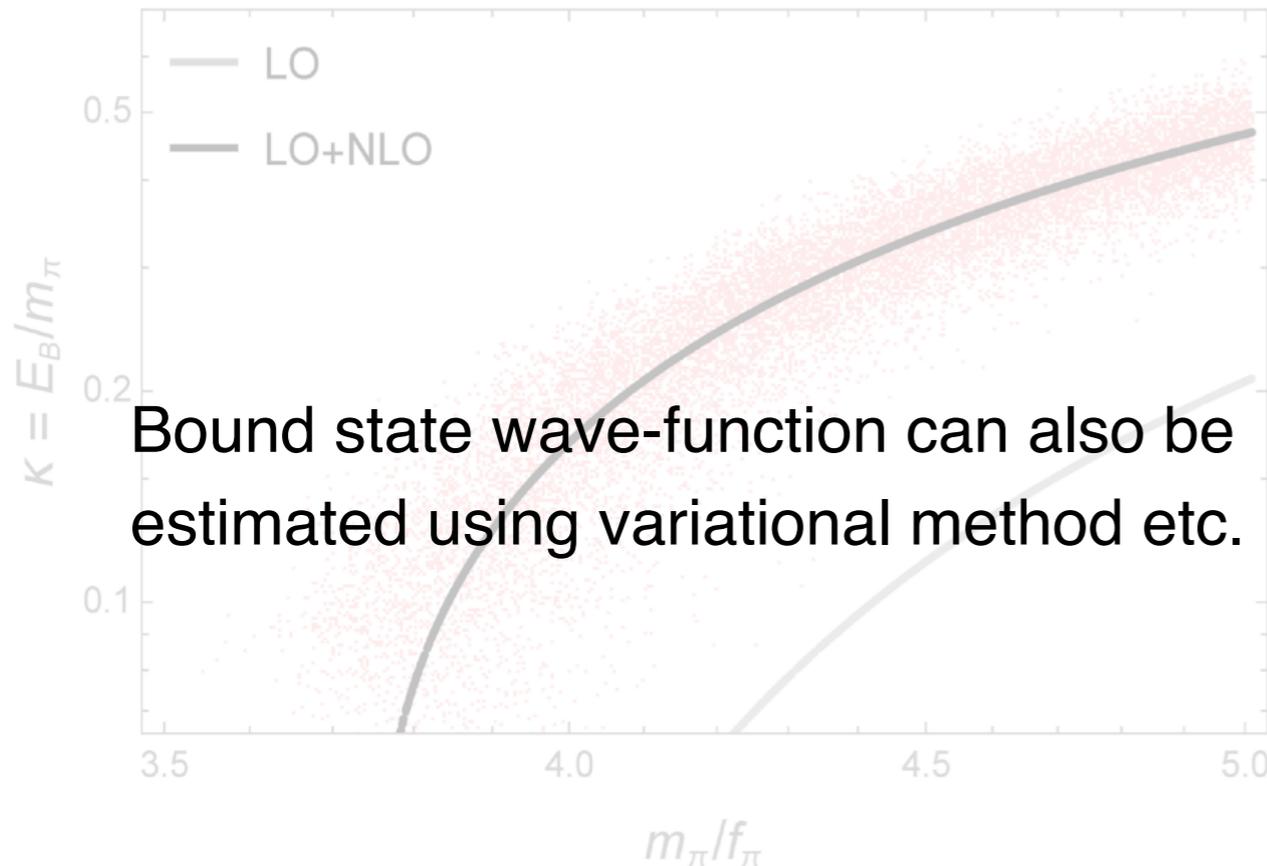**Only s-wave pole exists (sigma meson becomes a bound state, rho meson cannot)**

Take SIMP motivated: 2-quark case, global symmetry SU(4) → Sp(4)

(five dark pions as dark matter)

Singlet (isoscalar) channel of $\pi^{a_1}(p_1) + \pi^{a_2}(p_2) \to \pi^{a_3}(p_3) + \pi^{a_4}(p_4)$
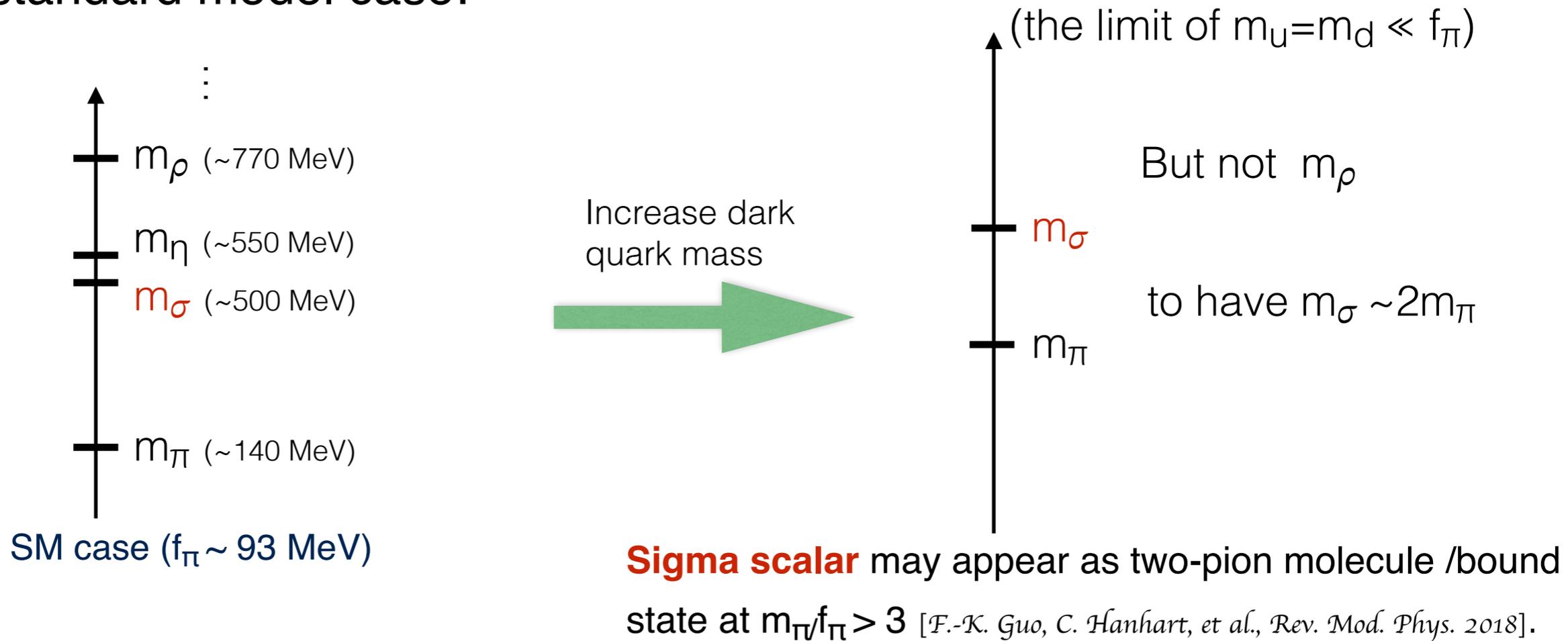
use amplitudes from *M. Albaladejo & J. A. Oller 2012*, to work out the s-wave projected interaction kernel to NLO.

Only s-wave pole exists (sigma meson becomes a bound state, rho meson cannot)

Bound state wave-function can also be estimated using variational method etc.

Dark strong dynamics has rich phenomenology

In the standard model case:

$\vdots$

m_ρ (~770 MeV)

m_η (~550 MeV)

m_σ (~500 MeV)

m_π (~140 MeV)

SM case ($f_\pi \sim$ 93 MeV)

Increase dark quark mass →

(the limit of $m_u = m_d \ll f_\pi$)

But not m_ρ

m_σ

m_π

to have $m_\sigma \sim 2m_\pi$

Sigma scalar may appear as two-pion molecule /bound state at $m_\pi/f_\pi > 3$ [F.-K. Guo, C. Hanhart, et al., Rev. Mod. Phys. 2018].
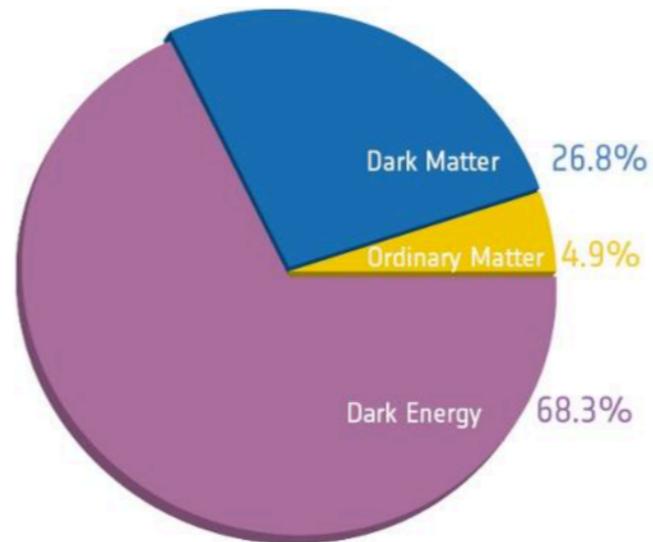
These results of SU(4)/Sp(4) are not a surprise.

Actually **the attraction is weaker** in this group structure.

5. Perspectives

In the end

A dark-QCD DM can be **the stable composite state**

Precisely-measured relic abundance
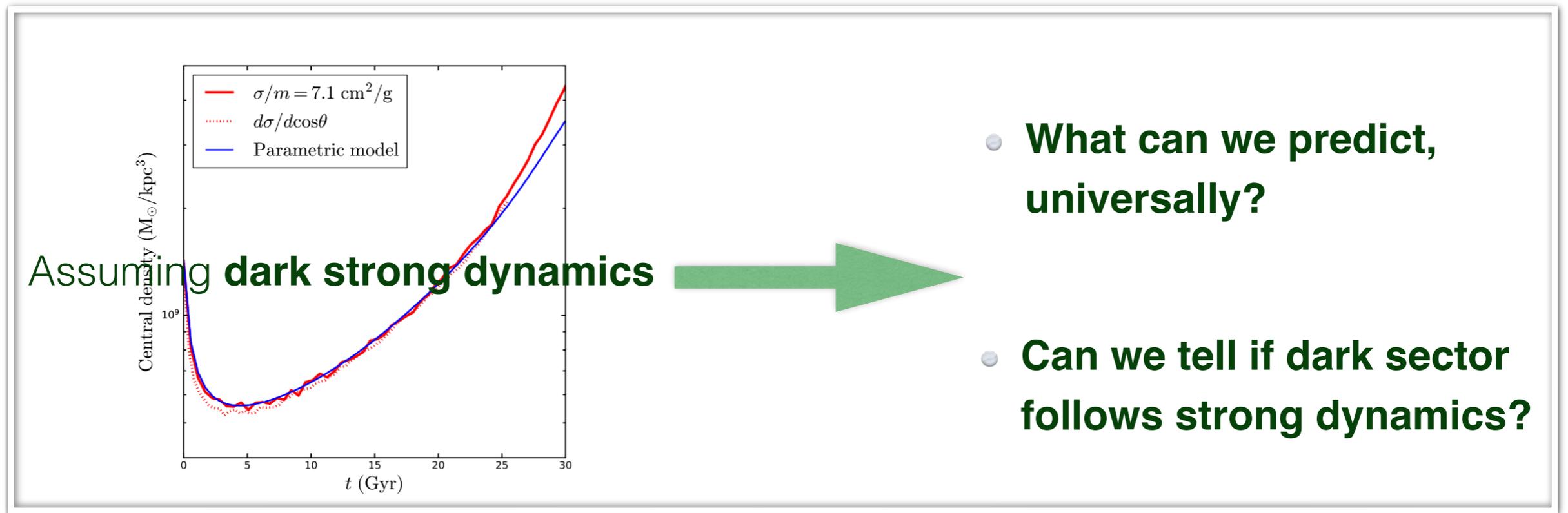
Without known particle properties

Assuming **dark strong dynamics**

- What can we predict, universally?

- Can we tell if dark sector follows strong dynamics?

Thanks!