

Two-pion distribution amplitudes and the $H_{/4}$ decays

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Overview

I: Light-cone distribution amplitudes

II: LCDAs of two-pion system and their phenomena

i: LCDAs of two-pion system (2π DAs)

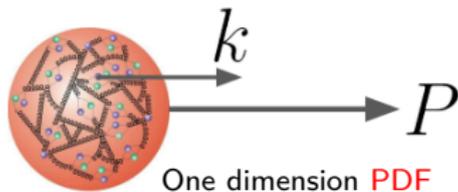
ii: Energy dependent partonic structure of $f_0(980)$

III: Summary and Prospect

Emergence of QCD, PDF/TMD/GPD

QCD is believed to confine, that is, its physical states are color singlets with internal quark and gluon degrees of freedom

Definitions of pion distribution



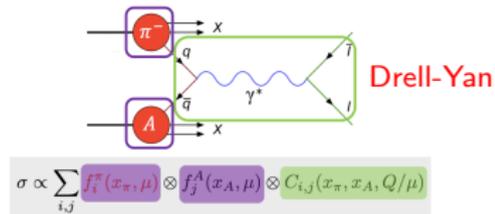
$$\Delta f_i(\zeta) = \int \frac{dz^-}{4\pi} e^{-i\zeta P^+ z^-} \langle \pi | \bar{\psi}_i(0, z^-, \mathbf{0}_T) \gamma^+ \psi_i(0) | \pi \rangle$$

$\zeta = \frac{k^+}{P^+}$, the parton momentum fraction

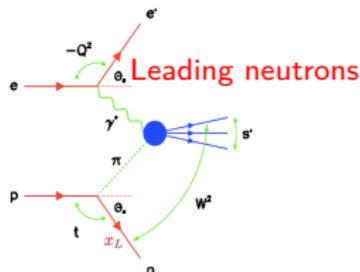
$$f_i(\zeta) \sim \sum_{\alpha} \int dk_T^2 \langle \pi | b_{k, \alpha}^{\dagger} b_{k, \alpha} (\zeta P^+, k_T, \alpha) | \pi \rangle$$

number operator

- △ transversal momentum distributions (TMD) $f(\zeta, k_T)$
- △ Generalized parton distributions (GPD) $f(\zeta, b_T)$



Extracted from fixed target πA data



Deeply virtuality meson production

- △ TDIS at 12GeV JLab, leading proton observable, fixed target instead of collider (HERA);
- △ EIC, EICc, great integrated luminosity to reduce the systematics uncertainties;
- △ COMPASS++/AMBER give π -induced DY data.

Light-cone distribution amplitudes (LCDAs)

Exclusive QCD processes with larger momentum transfers

- The Lorentz and gauge invariant ME (infinity momentum frame)

$$\langle 0 | \bar{u}(x) \gamma_\mu \gamma_5 d(-x) | \pi^-(p) \rangle = f_\pi \int_0^1 du e^{i(2u-1)p \cdot x} \left[i p_\mu \left(\phi(u, \mu) + \frac{x^2}{4} \phi_1^4(u, \mu) \right) + \dots \right]$$

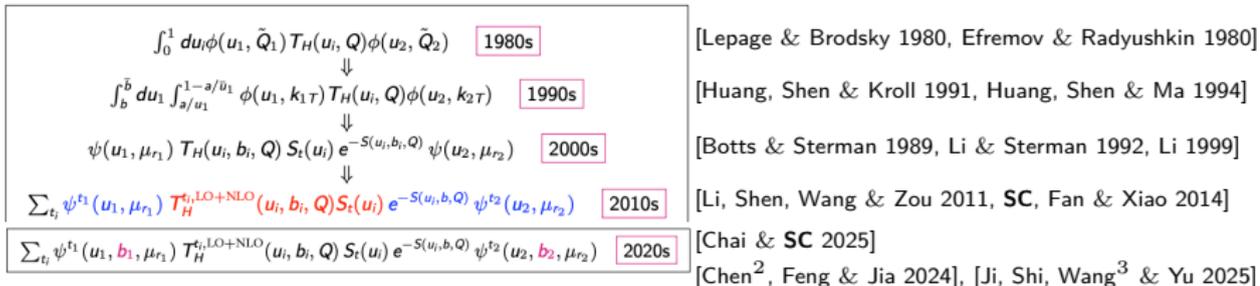
$$\langle 0 | \bar{u}(x) \gamma_\mu \gamma_5 d(-x) | \rho^-(p) \rangle = f_\rho m_\rho \int_0^1 du e^{i(2u-1)p \cdot x} \left[p_\mu \frac{\epsilon^{(\lambda)} \cdot x}{p \cdot x} \left(\phi_{\parallel}(u, \mu) - \phi_{\perp}^3(u, \mu) \right) + \dots \right]$$

$$\langle 0 | \bar{q}(x) \gamma_\mu \gamma_5 q(-x) | f_0(p) \rangle = p_\mu \int_0^1 du e^{i(2u-1)p \cdot x} [\phi(u, \mu) + \dots]$$

- LCDAs are dimensionless functions of u and renormalization scale μ
- the probability amplitudes to **find the meson** in a state with minimal number of constituents and have small transversal separation of order $1/\mu$
- The LCDAs description achieved **great success** in describing large momentum transferred processes involving stable hadrons

- LCDAs of pion achieved great success in describing large Q^2 processes.

△ the establishment and development of the pQCD factorization $F_\pi(Q^2)$



- LCDAs of proton serves as the fundamental input to explain ep scattering
[Chen², Feng, Hu, Jia 2025], [Huang, Shi, Wang, Zhao 2025], [Yu, SC, Han, Li, Yu 2025]

- The non-perturbative input for HFP theoretical studies that determines the precision and accuracy predicted the CPVs in the $B \rightarrow \pi\pi, K\pi$ decays and et.al.,

- ...

The limitations of the single particle picture

- V, S meson LCDAs reveal certain limitations in the precision testing era
 - △ the probability amplitudes to **find the light meson** in a state with ...
 - △ the $\pi\pi$ invariant mass spectra in $[0.554, 0.996]$ GeV is selected to identify candidates for the $\rho(770)$ resonance, more complicated for $f_0(980)$

- A second-best approach frequently employed in phenomenology is the **cascade decay framework**

$$\mathcal{M}(B^0 \rightarrow \pi^0 \pi^- l^+ \nu_l) = \mathcal{M}(B^0 \rightarrow \rho^- l^+ \nu_l) \text{BW}(s) \mathcal{M}(\rho \rightarrow \pi\pi)$$

- *model dependence, large uncertainties* (out of control in the f_0 case)
- How to accurately describe the **width effects** of unstable intermediate particles, the contributions and **interference effects** of different partial waves, and the **QCD backgrounds** from non-resonant states
- Dipion LCDAs (**2π DAs**) provide a most general description of $\pi\pi$ spectral

- **LCDAs of two-pion system (2π DAs)**
- Energy dependent partonic structure of $f_0(980)$

[arXiv: 2509.15659]

- $B \rightarrow \pi^- \pi^0 l^+ \nu_l$ decay and $|V_{ub}|$ extraction

[see PRD 112. L111301(2025)]

2πDAs

- Chiral-even LC expansion with gauge factor $[x, 0]$ [Polyakov 1999, Diehl 1998]

$$\langle \pi^a(k_1) \pi^b(k_2) | \bar{q}_f(zn) \gamma_\mu \tau q_f(0) | 0 \rangle = \kappa_{ab} k_\mu \int dx e^{iuz(k \cdot n)} \Phi_{||}^{ab, ff'}(\mu, u, \zeta, k^2)$$

$$\langle 0 | \bar{q}(x) \gamma_\mu \gamma_5 q(-x) | f_0(p) \rangle = p_\mu \int_0^1 du e^{i(2u-1)p \cdot x} \phi(\mu, u)$$

- 2πDAs is decomposed in terms of $C_n^{3/2}(2u-1)$ and $C_\ell^{1/2}(2\zeta-1)$

$$\Phi^{l=1}(u, \zeta, k^2, \mu) = 6u(1-u) \sum_{n=0, \text{even}}^{\infty} \sum_{l=1, \text{odd}}^{n+1} B_{nl}^{l=1}(k^2, \mu) C_n^{3/2}(2u-1) C_\ell^{1/2}(2\zeta-1)$$

$$\Phi^{l=0}(u, \zeta, k^2, \mu) = 6u(1-u) \sum_{n=1, \text{odd}}^{\infty} \sum_{l=0, \text{even}}^{n+1} B_{nl}^{l=0}(k^2, \mu) C_n^{3/2}(2u-1) C_\ell^{1/2}(2\zeta-1)$$

- Evolution from $4m_\pi^2$ to large k^2** via the Watson theorem of $\pi\pi$ scattering amplitudes

$$B_{n\ell}^l(k^2) = B_{n\ell}^l(0) \text{Exp} \left[\sum_{m=1}^{N-1} \frac{k^{2m}}{m!} \frac{d^m}{dk^{2m}} \ln B_{n\ell}^l(0) + \frac{k^{2N}}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{\delta_\ell^l(s)}{s^N(s-k^2-i0)} \right]$$

△ 2πDAs in a wide range of energies is given by δ_ℓ^l and a few subtraction constants

2πDAs

- The subtraction constants of $B_{n\ell}(k^2)$ at low k^2 (around the threshold)

(nl)	$B_{n\ell}^{\parallel}(0)$	$c_1^{\parallel,(nl)}$	$\frac{d}{dk^2} \ln B_{n\ell}^{\parallel}(0)$	$B_{n\ell}^{\perp}(0)$	$c_1^{\perp,(nl)}$	$\frac{d}{dk^2} \ln B_{n\ell}^{\perp}(0)$
(01)	1	0	1.46 \rightarrow 1.80	1	0	0.68 \rightarrow 0.60
(21)	-0.113 \rightarrow 0.218	-0.340	0.481	0.113 \rightarrow 0.185	-0.538	-0.153
(23)	0.147 \rightarrow -0.038	0	0.368	0.113 \rightarrow 0.185	0	0.153
(10)	-0.556	-	0.413	-	-	-
(12)	0.556	-	0.413	-	-	-

△ firstly studied in the effective low-energy theory based on instanton vacuum [Polyakov 1999]

△ updated with the kinematical constraints and the new a_2^{π}, a_2^{ρ} [SC 2019, 2023]

- 2πDAs were introduced at leading twist [Polyakov 1999, Diehl 1998]

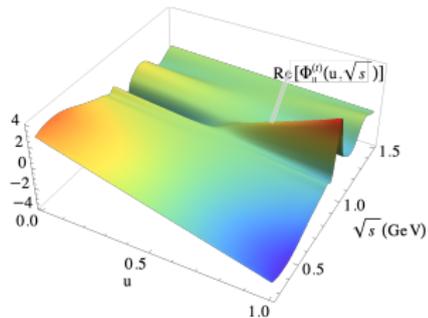
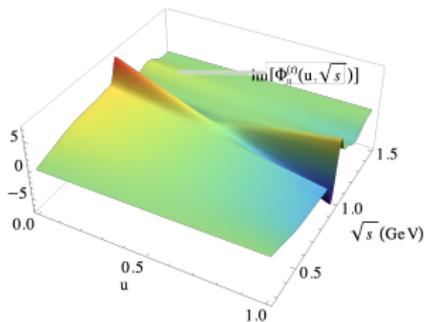
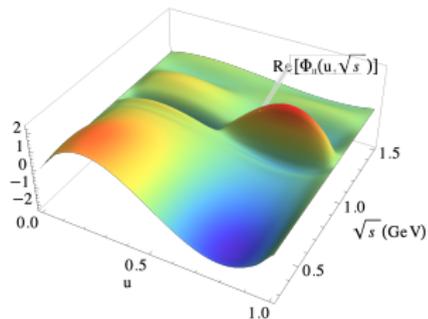
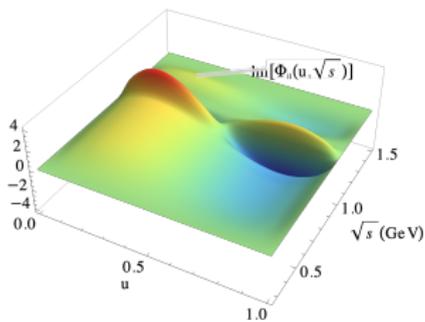
$$\langle \pi^a(k_1) \pi^b(k_2) | \bar{q}_f(zn) \gamma_{\mu} \tau q_f(0) | 0 \rangle = \kappa_{ab} k_{\mu} \int dx e^{iuz(k \cdot n)} \Phi_{\parallel}^{ab,ff'}(u, \zeta, k^2)$$

- improved to twist-three level recently [SC 2502.07333]

$$\langle \pi(k_1) \pi(k_2) | \bar{q}(0) q(x) | 0 \rangle = \int du e^{i\bar{u}k \cdot x} \frac{ik^2(k \cdot x)}{2f_{2\pi}^{\perp}} \Phi_{\parallel}^{(s)},$$

$$\langle \pi(k_1) \pi(k_2) | \bar{q}(0) \sigma^{\mu\nu} q(x) | 0 \rangle = -\frac{i}{f_{2\pi}^{\perp}} \int du e^{i\bar{u}k \cdot x} \left[\frac{k_{\mu} \bar{k}_{\nu} - k_{\nu} \bar{k}_{\mu}}{2\zeta - 1} \Phi_{\perp} - k^2 \frac{k_{\mu} x_{\nu} - k_{\nu} x_{\mu}}{k \cdot x} \Phi_{\parallel}^{(t)} \right].$$

2π DAs $[\pi\pi]_S$



- asymmetry of the twist-3 2π DAs to the parton momentum fraction u
- symmetric for f_0 obtained under the single particle approximation
- where QCD sum rules dictate that the asymmetric component vanishes

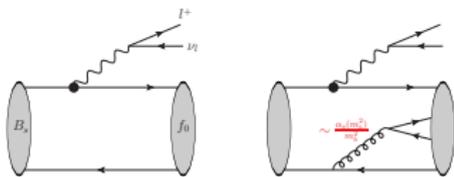
The controversy of $f_0(980)$ structure

- **A longstanding topic in hadron physics**
- Experimental identification is particularly challenging (large width)
- From the quantum theory, it is a superposition of all possible Fock states

$$|f_0(980)\rangle, \quad |[\pi\pi]_S\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}\rangle + \dots$$

- **Hadron spectroscopy** provides clear evidence for the complex config
conventional resonant with possible gluonball component
exotic multiparticle state (compact tetraquark state, molecular state)
- **The underlying partonic dynamics** can not be extracted directly from spectral analysis, even though it reveals these exotic configurations
- **Semileptonic B, D decays** are powerful probe of the underlying structure
- In terms of LCDAs, **scale-dependent functions**

- Semileptonic B, D decays are powerful probe of the underlying structure
- **color transparency mechanism** in $B_{(s)} \rightarrow f_0 l^+ \nu_l$ decays



† high Fock states' contribution is doubly suppressed by α_s and $\mathcal{O}(1/Q^2)$, FSI is weak

- **the mechanism fails in $D_s \rightarrow f_0 l^+ \nu_l$ decays**

† the produced $q\bar{q}$ is nonrelativistic, likely proceeds through the creation of additional g or dynamical $q\bar{q}$

- while the energy-dependence of partonic configurations is a QCD result
- **the cascade decay analyses of $D_s \rightarrow (f_0 \rightarrow \pi\pi) e\nu$ under $q\bar{q}$ ansatz consists well with data**, with $D_s \rightarrow f_0$ FFs and Flatté model of f_0 resonant
- QCD calculation have revealed the scale dependence in $D_s \rightarrow f_0$ form factor
- the seemingly agreement in D_{14} decays is attributed to the cascade framework
- the Flatté parameterization of the intermediate resonance disrupts the assessment of color transparency
- a model-independent study directly from the $\pi\pi$ signal state

$H \rightarrow \pi\pi$ from factors in H_{I4} decays

- **QCD dynamics of D_{I4} decays is incorporated in $H \rightarrow \pi\pi$ form factors** instead of the $H \rightarrow f_0$ ffs followed by $f_0 \rightarrow \pi\pi$ in the cascade decay

$$\begin{aligned} i\langle \pi^+(k_1)\pi^-(k_2) | \bar{s}\gamma_\nu(1 - \gamma_5)c | D_s(p) \rangle = & F_\perp(q^2, k^2, \zeta) \frac{2}{\sqrt{k^2}\sqrt{\lambda_B}} i\epsilon_{\nu\alpha\beta\gamma} q^\alpha k^\beta \bar{k}^\gamma \\ & + F_t(q^2, k^2, \zeta) \frac{q_\nu}{\sqrt{q^2}} + F_0(q^2, k^2, \zeta) \frac{2\sqrt{q^2}}{\sqrt{\lambda_B}} \left(k_\nu - \frac{k \cdot q}{q^2} q_\nu \right) \\ & + F_\parallel(q^2, k^2, \zeta) \frac{1}{\sqrt{k^2}} \left(\bar{k}_\nu - \frac{4(q \cdot k)(q \cdot \bar{k})}{\lambda_B} k_\nu + \frac{4k^2(q \cdot \bar{k})}{\lambda_B} q_\nu \right) \end{aligned}$$

† $\lambda = \lambda(m_B^2, k^2, q^2)$ is the Källén function, $q \cdot k = (m_B^2 - q^2 - k^2)/2$,
 $q \cdot \bar{k} = \sqrt{\lambda}\beta_\pi(k^2) \cos \theta_\pi/2 = \sqrt{\lambda}(2\zeta - 1)$, $\beta_\pi(k^2) = \sqrt{1 - 4m_\pi^2/k^2}$

- **QCDF** (QCD factorization) in the large two-pion mass
 - [P. Böer, T. Feldmann and D. van Dyk, JHEP02, 133(2017)] $T_I \propto F_{Bto\pi} \otimes \phi_\pi$.
- **SU(3) flavor symmetry/breaking** with the intermediate resonance
 - [R.M. Wang, Y.G. Xu, J.H. Sheng, X.D. Cheng and et.al., 2301.00090, PRD 112, 033002 (2025)]

$H \rightarrow \pi\pi$ form factors

- **LQCD** (Lattice QCD) in the ρ resonance region with a simple BW model
 - [L. Leskovec and et.al, PRL 134.161901 (2025), **Editors' Suggestion**]
- **HChPT** (Heavy-meson Chiral Perturbative Theory) in the large q^2 by taking dispersive methods in terms of Omnés functions
 - [X.-W. Kang, B. Kubis, C. Hanhart, and U.-G. Meißner, PRD 89. 053015 (2014)]in the full phase-space by a novel parameterization with unitarity
 - [F. Herren, B. Kubis and R. van Tonder, PRD 112, 014037 (2025), **Editors' Suggestion**]
- **LCSRs** (Light-cone sum rules) in the small and intermediate q^2
 - [**SC**, A. Khodjamirian and J. Virto, JHEP 05(2017)157] **B-meson LCSRs**, [S. Descotes-Genon, A. Khodjamirian, J. Virto and K.K. Vos, JHEP 12(2019)083, 06(2023)034] **$B \rightarrow K\pi$**
 - [C. Hambrock and A. Khodjamirian, NPB 905. 379-390(2016)] **2π DAs LCSRS of $F_{\parallel, \perp}$**
 - [**SC**, A. Khodjamirian and J. Virto, PRD(R) 96. 051901(2017)] **timelike-helicity FF F_{\parallel} and F_0**
 - [**SC**, PRD 99. 053005(2019)] **2π DAs updates and $B \rightarrow [\pi\pi]_{S,P}$ FFs**
 - [**SC** and J.M Shen, EPJC 6:554(2020), **SC** and S.L Zhang, EPJC 84:379(2024)] **Pheno**
 - [**SC**, PRD 112. L111301(2025)] **first study of twist-three 2π DAs and $|V_{ub}|$ extraction**
 - [**SC**, L.Y. Dai, J.M. Shen and S.L. Zhang, arXiv:2509.15659] **$D_s \rightarrow [\pi\pi]_S e\nu$, minor $q\bar{q}$ contribution**

$D_s \rightarrow [\pi\pi] e^+ \nu_e$ decay

- $D_s \rightarrow f_0 e^+ \nu$ [CLEO '09], $D_{(s)} \rightarrow a_0 e^+ \nu$ [BESIII '18, '21], $D^+ \rightarrow f_0/\sigma e^+ \nu$ [BESIII '19]
- $D_s \rightarrow f_0 (\rightarrow \pi^0 \pi^0, K_S K_S) e^+ \nu$ [BESIII 22], $D_s \rightarrow f_0 (\rightarrow \pi^+ \pi^-) e^+ \nu$ [BESIII 23]

$$\mathcal{B}(D_s \rightarrow f_0 (\rightarrow \pi^0 \pi^0) e^+ \nu) = (7.9 \pm 1.4 \pm 0.3) \times 10^{-4}$$

$$\mathcal{B}(D_s \rightarrow f_0 (\rightarrow \pi^+ \pi^-) e^+ \nu) = (17.2 \pm 1.3 \pm 1.0) \times 10^{-4}$$

$$f_+^0(0) |V_{cs}| = 0.504 \pm 0.017 \pm 0.035$$

- single particle (narrow width limit) $D_s \rightarrow f_0 e^+ \nu$

$$\frac{d\Gamma(D_s^+ \rightarrow f_0 l^+ \nu)}{dq^2} = \frac{G_F^2 |V_{cs}|^2 \lambda^{3/2}(m_{D_s}^2, m_{f_0}^2, q^2)}{192\pi^3 m_{D_s}^3} |f_+(q^2)|^2, \quad D_s \rightarrow f_0 \text{ FF}$$

- improvement with the width effect by resonant models $D_s \rightarrow [f_0 \rightarrow] \pi\pi e^+ \nu$

$$\frac{d\Gamma(D_s^+ \rightarrow [\pi\pi]_S l^+ \nu)}{dsdq^2} = \frac{1}{\pi} \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s}^3} |f_+(q^2)|^2 \frac{\lambda^{3/2}(m_{D_s}^2, s, q^2) g_1 \beta_\pi(s)}{|m_S^2 - s + i(g_1 \beta_\pi(s) + g_2 \beta_K(s))|^2}, \quad \text{BESIII}$$

- calculate directly the signal channel $D_s \rightarrow [\pi\pi]_S e^+ \nu$

$$\frac{d^2\Gamma(D_s^+ \rightarrow [\pi\pi]_S l^+ \nu)}{dsdq^2} = \frac{G_F^2 |V_{cs}|^2 \beta_\pi(s) \sqrt{\lambda_{D_s} q^2}}{3(4\pi)^5 m_{D_s}^3} \sum_{\ell=0}^{\infty} |F_0^{(\ell)}(q^2, s)|^2, \quad D_s \rightarrow \pi\pi \text{ FF}$$

$D_s \rightarrow f_0$ FFs and cascade decay $D_s \rightarrow (f_0 \rightarrow) \pi\pi e^+ \nu$

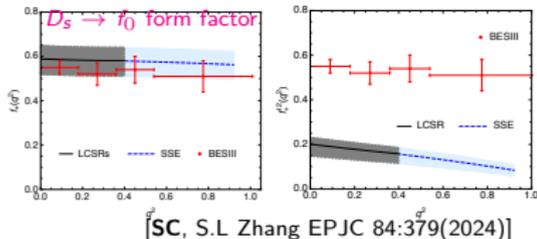
- $\{M^2, s_0\} = \{5.0 \pm 0.5, 6.0 \pm 0.5\} \text{GeV}^2$

this work	3pSRs(07)	LFQM(09)	CLFD/DR(08)	LCSRs(10)
0.63 ± 0.04	0.96	0.87	0.86/0.90	0.30 ± 0.03

- the BESIII result in the $\pi^+ \pi^-$ system $f_+(0) = 0.518 \pm 0.018 \pm 0.036$ [BESIII 23]

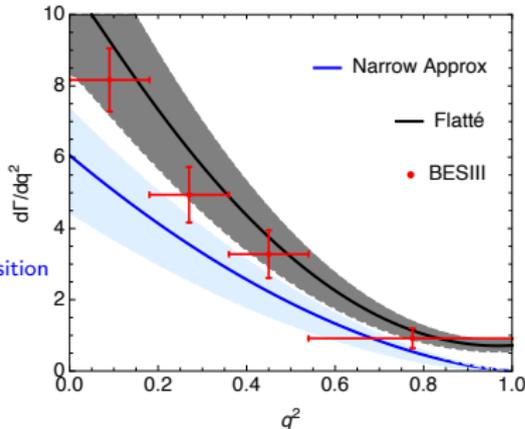
different input of the decay constant $\tilde{f}_{f_0} = 335 \text{ MeV}$, much larger than 180 MeV in LCSRs(10)
we add the first gegenbauer expansion terms in the LCDAs, up-to-date parameters

$\bar{s}s - \bar{n}n$ mixing scenario of f_0 with $\theta = 20^\circ \pm 10^\circ$



- Twist-3 LCDAs give dominate contribution in $D_s \rightarrow f_0$ transition

- the uncertainty estimation is conservative
- without NLO correction
- we need a model independent calculation
- for the QCD understanding
- and the future partial-wave measurement

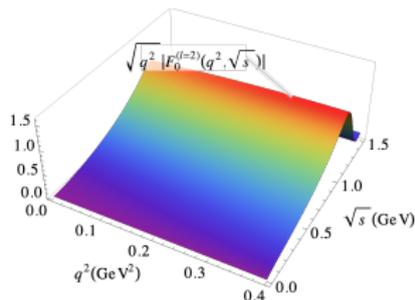
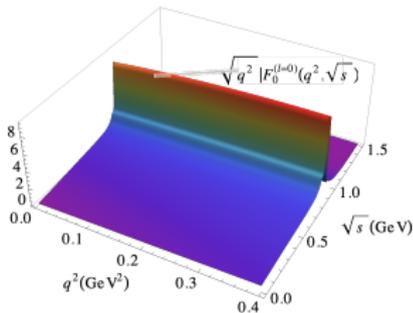


Differential decay width of $D_s^+ \rightarrow (f_0 \rightarrow) [\pi\pi]_S e^+ \nu_e$

$D_s \rightarrow [\pi\pi]_S$ FFs and $D_s \rightarrow [\pi\pi]_S e^+ \nu$ decay

- The LCSRs ℓ' -wave $D_s \rightarrow [\pi\pi]_S$ form factors ($\ell' = \text{even} \ \& \ \ell' \leq n + 1$)

$$\sqrt{q^2} F_0^{(\ell')} (q^2, k^2) = \frac{m_c(m_c + m_s) \sqrt{q^2} \sqrt{\lambda_{D_s}}}{m_{D_s}^2 f_{D_s}} \sum_{n=1, \text{odd}}^{\infty} \frac{\beta_{\pi}(k^2)}{\sqrt{2\ell' + 1}} J_n^0(q^2, k^2, M^2, s_0) B_{n\ell', \parallel}^{l=0}(k^2) I_{\ell\ell'}$$

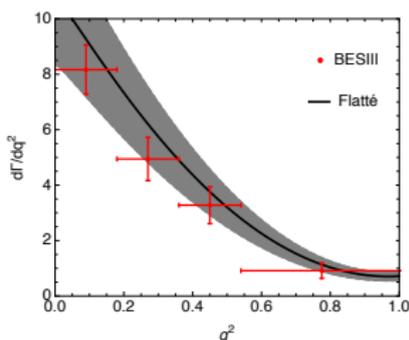
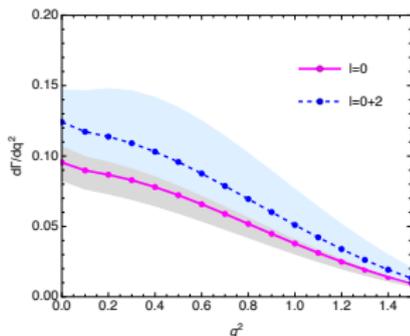


- Twist-2 and twist-3 contributions to $D_s \rightarrow \pi\pi, f_0$ form factors at $q^2 = 0$
under the $q\bar{q}$ approximation

Form Factors	Twist-2	Twist-3	Total
$\sqrt{q^2} F_0^{(l=0)}(0) = \sqrt{q^2} F_t^{(l=0)}(0)$	$0.20_{-0.02}^{+0.02} - i0.24_{-0.02}^{+0.02}$	$-0.41_{-0.05}^{+0.04} + i0.51_{-0.04}^{+0.02}$	$-0.21_{-0.01}^{+0.02} + i0.27_{-0.02}^{+0.03}$
$\sqrt{q^2} F_0^{(l=2)}(0) = \sqrt{q^2} F_t^{(l=2)}(0)$	$0.27_{-0.02}^{+0.03} + i0.21_{-0.01}^{+0.02}$	$-0.55_{-0.03}^{+0.02} - i0.41_{-0.04}^{+0.05}$	$-0.28_{-0.02}^{+0.02} - i0.20_{-0.01}^{+0.02}$
$f_+(0) = f_0(0)$	$0.20_{-0.05}^{+0.03}$	$0.41_{-0.06}^{+0.04}$	$0.61_{-0.07}^{+0.05}$

$D_s \rightarrow [\pi\pi]_S$ FFs and $D_s \rightarrow [\pi\pi]_S e^+ \nu$ decay

- The differential decay width on momentum transfers



$D_s \rightarrow [\pi\pi]_S e^+ \nu_e$	$D_s \rightarrow [f_0 \rightarrow \pi\pi] e^+ \nu_e$ [23]	Data [25]
$0.81^{+0.34}_{-0.14}$	$18.8^{+4.5}_{-3.8}$	17.2 ± 1.6

[SC, L.Y Dai, J.M Shen and S.L Zhang, 2509.15659]

- Differential widths $d\Gamma/dq^2$ is two-order in magnitude smaller than the data
- non- $q\bar{q}$ Fock states are the dominate component of $[\pi\pi]_S$ in charm decays
- in consistent with the assessment of color transparency is severe disrupted by the Flatté model
- much different in B decays leading twist dominated [SC, arXiv: 2502.07333]
- go further to multi-particle DiPion LCDAs in CHARM ($q\bar{q}g$, $q\bar{q}q\bar{q}$)

Summary and Prospect

- 2π DAs provide a general description of the $\pi\pi$ mass spectrum
 - † accurately clarifying the contributions of different partial waves and of different resonant states within the same partial wave,
 - † effectively separating and clearly describing the contributions of the non-resonant background
- 2π DAs are a crucial input for the study of H_{I4} decays
- 2π DAs have been studied at the three-twist level, explaining the multi-particle picture of scalar mesons
- wishlists related to the (future) colliders
 - * possible anomalies in the FCNC processes of the $D \rightarrow \pi\pi l^+ l^-$ decay
 - * the processes e^+e^- annihilation $\gamma^* \rightarrow \pi\pi\gamma$ and $\gamma^*\gamma \rightarrow \pi\pi$

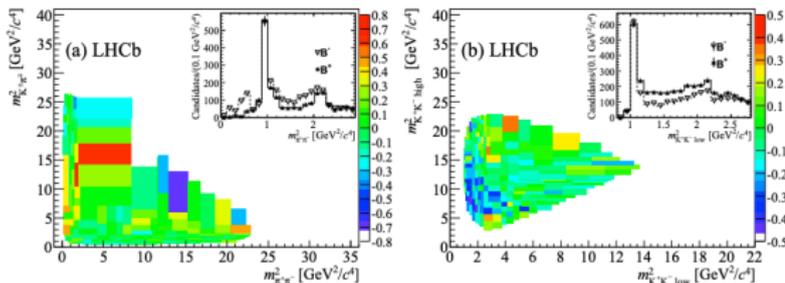
Thank you for your patience.

More opportunities/phenomena of 2π DAs

pion EMFF widely used in the three-body B decays studied from pQCD and QCDF are the asymptotic formula of 2π DAs

[J. Chai, SC and A.J Ma PRD 105. 033003 (2022)]

normalized to unit as $\Gamma_{M_1 M_2}^{J=1}(0) = 1$. When the invariant mass of dimeson system is small, the higher $\mathcal{O}(s)$ terms in the expansion of coefficient $B_{n1}(s, \mu)$ around the resonance pole can be safely neglected due to the large suppression $\mathcal{O}(s/m_b^2)$ in contrast to the energetic dimeson system in B decay, so the relation $B_{n1}(s, \mu) \rightarrow a_n(\mu)\Gamma_{M_1 M_2}^{J=1}(s)$ can be obtained in the lowest partial wave approximation. This argument induces the basic assumption in PQCD that the energetic dimeson DAs can be deduced from the DAs of resonant meson by replacing the decay constant by the timelike form factor.



$B \rightarrow K\pi\pi, KKK$

[PRL 111.101801(2013) LHCb]

- * Non-resonant contributions are small ($< 10\%$) in three-body D decays
- * large/dominate in the penguin dominated three-body B decays, the theoretically unresolved problem
- * $|V_{cb}|$ in the $B \rightarrow D^* l\nu$ processes, B anomalies in $B \rightarrow K^* l^+ l^-$ processes