A Non-linear Representation of General Scalar Extensions of the Standard Model for HEFT Matching

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Based on work with
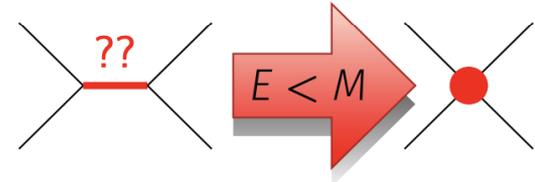**X. Wan [arXiv:2412.00355, 2503.00707]**
Z. Ge, and X. Wan [arXiv:2601.xxxxx]

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A CHOICE OF TWO EFTS FOR THE SM

At low energy scale, heavy new physics looks like some new contact interactions.

One can encode the contact interactions as EFT operators:

- SMEFT: built out of the Higgs doublet Φ, …
$$\Phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}} \left(v_{\mathrm{H}} + h + iG^0 \right) \end{pmatrix}$$

$$\kappa_f \sim |\Phi|^2 \overline{Q}_L \Phi d_R ; \quad \kappa_V \sim |\Phi|^2 |D\Phi|^2 ; \quad \kappa_\lambda \sim |\Phi|^6 .$$

- HEFT: built separately out of the Higgs h and $U \equiv \exp\left(\frac{i\pi_i \sigma_i}{v}\right)$ Goldstones π_i, …

$$\kappa_f \sim h \bar{f}_L f_R ; \quad \kappa_V \sim h \partial\pi^+ \partial\pi^- ; \quad \kappa_\lambda \sim h^3 .$$
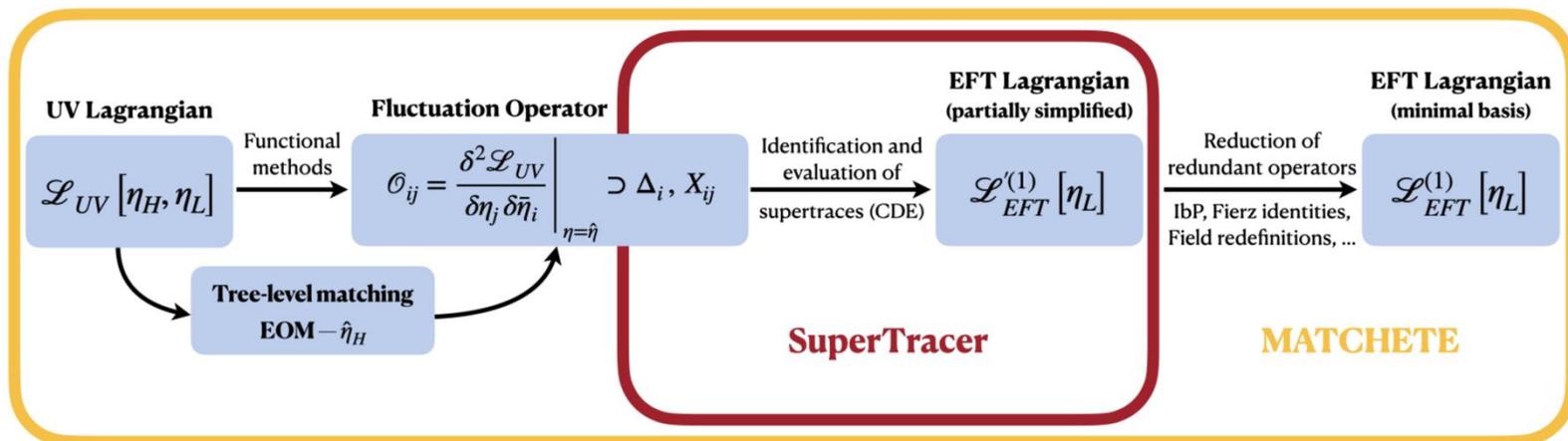
Motivation: ● modification of EWSB ● improve EFT description if not decoupling

SMEFT Matching

The UV Lagrangian is a combination of the SM Lagrangian and the one describing the interactions among the BSM states and between them and the SM

- Scalar singlet

$$\mathcal{L}_s = \frac{1}{2}(D_\mu S)(D^\mu S) - \frac{1}{2}M_S^2 SS - (\kappa_s)SH^\dagger H$$
$$- (\lambda_s)SSH^\dagger H - \kappa_{S^3}SSS - (\kappa_{S^4})SSSS$$

- Scalar triplet

$$\mathcal{L}_\Xi = \frac{1}{2}(D_\mu \Xi^a)(D^\mu \Xi^a) - \frac{1}{2}M_\Xi^2(\Xi^a \Xi^a) - \kappa_\Xi H^\dagger \Xi^a \sigma^a H +$$
$$- \lambda_\Xi(\Xi^a \Xi^a)(H^\dagger H) - \frac{1}{4}\eta_\Xi(\Xi^a \Xi^a)^2 \,,$$

SMEFT Matching

- ## Scalar doublet (2HDM)

$$\mathcal{L}_{\mathrm{kin}} = (D_\mu H_1)^\dagger (D^\mu H_1) + (D_\mu H_2)^\dagger (D^\mu H_2)$$

$$
\begin{aligned}
V = {} & Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + \left(Y_3 H_1^\dagger H_2 + \mathrm{h.c.} \right) \\
& + \frac{Z_1}{2} \left(H_1^\dagger H_1 \right)^2 + \frac{Z_2}{2} \left(H_2^\dagger H_2 \right)^2 + Z_3 \left(H_1^\dagger H_1 \right) \left(H_2^\dagger H_2 \right) + Z_4 \left(H_1^\dagger H_2 \right) \left(H_2^\dagger H_1 \right) \\
& + \left\{ \frac{Z_5}{2} \left(H_1^\dagger H_2 \right)^2 + Z_6 \left(H_1^\dagger H_1 \right) \left(H_1^\dagger H_2 \right) + Z_7 \left(H_2^\dagger H_2 \right) \left(H_1^\dagger H_2 \right) + \mathrm{h.c.} \right\}
\end{aligned}
$$

$$
H_j = e^{i\xi_j} \begin{pmatrix} \varphi_j^+ \\ (v_j + \eta_j + i\chi_j)/\sqrt{2} \end{pmatrix}, \quad j = 1, 2.
$$

Neither H_1 nor H_2 is the Higgs doublet H in the SM.

SMEFT Matching

- ## Scalar doublet (2HDM)

The canonically normalized (gauge-covariant) kinetic energy terms of the scalar fields are invariant under arbitrary global U(2) transformations. Therefore one can perform an appropriate global U(2) transformations to the so-called *Higgs-basis.*

$$H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}} \left(v + h_1^{\mathrm{H}} + i G_0 \right) \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}} \left(h_2^{\mathrm{H}} + i A \right) \end{pmatrix}$$

Pure Goldstones

Pure Physical States upto Mixing among Physical States

Identified as the SM Higgs doublet

BSM state that can be integrated out as a single object

HEFT Matching

What happens in the matching between the HEFT and the UV model?

Not easy, but doable!

Trivial? In the UV model, find π_i and h (matrices diagonalization), then integrate out all the BSM particles.

Problems: ● vevs ● mixing ● complex UV parameter space

However, this is another difficulty, that usually the operators in the HEFT are not written in the form of π_i but U.

$$\langle D_\mu U^\dagger D^\mu U \rangle \quad \langle U^\dagger D_\mu U \sigma_3 \rangle \langle U^\dagger D^\mu U \sigma_3 \rangle$$

Further though the HEFT is an EFT with only $SU(3)_c \times U(1)_{em}$ symmetry, its operators are written in an $SU(3)_c \times SU(2)_L \times U(1)_Y$ invariant way.

This talk: some algebraic tricks for the scalars.

HEFT Matching

Non-linear Representation

In the SM $\quad H = \exp\left(i\dfrac{\pi^a \tau^a}{v} \right)\left(\begin{array}{c} 0 \\ \frac{v}{\sqrt{2}} + \frac{h}{\sqrt{2}} \end{array} \right)$ **Goldstones**

- Scalar singlet, trivial, just use the above expression for the doublet, diagonalize the two neutral scalars, identify the light one as the SM h and integrate out the heavy one H

- Scalar doublet $\quad \mathcal{H}_1 = \dfrac{v + h_1^H}{\sqrt{2}}\, U(\omega) \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \mathcal{H}_2 = U(\omega) \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(h_2^{\mathrm{H}} + iA) \end{pmatrix}$

$$
\begin{aligned}
V = {}& Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + \left(Y_3 H_1^\dagger H_2 + \text{h.c.} \right) \longrightarrow H^+ \pi^- + \text{h.c.} \\
& + \frac{Z_1}{2}\left(H_1^\dagger H_1 \right)^2 + \frac{Z_2}{2}\left(H_2^\dagger H_2 \right)^2 + Z_3 \left(H_1^\dagger H_1 \right)\left(H_2^\dagger H_2 \right) + Z_4 \left(H_1^\dagger H_2 \right)\left(H_2^\dagger H_1 \right) \\
& + \left\{ \frac{Z_5}{2}\left(H_1^\dagger H_2 \right)^2 + Z_6 \left(H_1^\dagger H_1 \right)\left(H_1^\dagger H_2 \right) + Z_7 \left(H_2^\dagger H_2 \right)\left(H_1^\dagger H_2 \right) + \text{h.c.} \right\}
\end{aligned}
$$

HEFT Matching

Non-linear Representation

In the SM $\quad H = \exp\left(i\dfrac{\pi^a \tau^a}{v} \right)\left(\begin{array}{c} 0 \\ \frac{v}{\sqrt{2}} + \frac{h}{\sqrt{2}} \end{array} \right)$ **Goldstones**

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- Scalar doublet $\quad \mathcal{H}_1 = \dfrac{v + h_1^H}{\sqrt{2}} U(\omega) \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \mathcal{H}_2 = U(\omega) \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(h_2^{\mathrm{H}} + iA) \end{pmatrix}$

An $SU(2)_L \times U(1)_Y$ rotation parametrized by the Goldstones

$$
\begin{aligned}
V = \; & Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + \left(Y_3 H_1^\dagger H_2 + \text{h.c} \right) \qquad \longrightarrow \; H^+\pi^- + \text{h.\,c.} \\
& + \frac{Z_1}{2} \left(H_1^\dagger H_1 \right)^2 + \frac{Z_2}{2} \left(H_2^\dagger H_2 \right)^2 + Z_3 \left(H_1^\dagger H_1 \right)\left(H_2^\dagger H_2 \right) + Z_4 \left(H_1^\dagger H_2 \right)\left(H_2^\dagger H_1 \right) \\
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\end{aligned}
$$

HEFT Matching

What happens in other cases?

Can we do this trick for other scalar multiplets?

- "Unitary basis" excises Goldstone modes from real triplet. (Cohen, Craig, Lu, and Sutherland 2021)
- "broken phase EFT" (unitary gauge) removes the would-be Goldstone fields from complex triplet. (Liao, Ma, and Uchida 2025)

HEFT Matching

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Do we have a "Higgs basis" representation for other scalar multiplets, or at least a representation similar to the "Higgs basis" in which the Goldstones can be factored out?

HEFT Matching

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Lesson learned from the 2HDM case

$$\mathcal{H}_1 = \frac{v + h_1^H}{\sqrt{2}} \, U(\omega) \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \mathcal{H}_2 = U(\omega) \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(h_2^{\mathrm{H}} + iA) \end{pmatrix}$$

After splitting the Goldstones, there is no mass mixing in the potential.

- DFSZ models for axion
- 2HDM parametrized in another way (P. Ciafaloni, and D. Espriu hep-ph/9612383)

HEFT Matching

Scalar triplet as an example

Rotated by Goldstone matrix

$$\Phi = \phi_i \sigma_i / 2 = \frac{1}{2}\begin{pmatrix} v' + \phi^0 & \phi_1 - i\phi_2 \\ \phi_1 + i\phi_2 & -v' - \phi^0 \end{pmatrix} \qquad \Sigma = U(\pi)\Phi U(\pi)^\dagger$$

No mixing terms (except between neutral states) in the potential

$$V = Y_1^2 \mathrm{H}^\dagger \mathrm{H} + Z_1 (\mathrm{H}^\dagger \mathrm{H})^2 + Y_2^2 \mathrm{Tr}(\Sigma^\dagger \Sigma) + Z_2 \left(\mathrm{Tr}(\Sigma^\dagger \Sigma)\right)^2 + Z_3 \mathrm{H}^\dagger \mathrm{H}\,\mathrm{Tr}(\Sigma^\dagger \Sigma) + 2Y_3 \mathrm{H}^\dagger \Sigma \mathrm{H}$$

$$= Y_1^2 \frac{(v+h^0)^2}{2} + Z_1 \frac{(v+h^0)^4}{4} + Y_2^2 \mathrm{Tr}(\Phi^2) + Z_2 \mathrm{Tr}(\Phi^2)^2 + Z_3 \frac{(v+h^0)^2}{2}\mathrm{Tr}(\Phi^2) - Y_3 \frac{(v+h^0)^2}{2}\phi_3$$

However, kinetic mixing exists

$$\mathcal{L}_{\mathrm{kin}} = D_\mu \mathrm{H}^\dagger D^\mu \mathrm{H} + \mathrm{Tr}(D_\mu \Sigma D^\mu \Sigma)$$

$$= \frac{\partial_\mu h^0 \partial^\mu h^0}{2} + \frac{(v+h^0)^2}{4}\mathrm{Tr}(D_\mu U^\dagger D^\mu U) + \mathrm{Tr}(D^\mu \Phi D_\mu \Phi) + 2\mathrm{Tr}(\Phi\Phi D^\mu U^\dagger D_\mu U)$$

$$+ 2\mathrm{Tr}(\Phi U^\dagger D^\mu U \Phi U^\dagger D_\mu U) + 2\mathrm{Tr}(U^\dagger D^\mu U(\Phi D_\mu \Phi - D_\mu \Phi\Phi))\,,$$

$$-v'\epsilon_{3jk}\partial_\mu \phi_j \partial_\mu \pi_k / v + \dots$$

HEFT Matching

Scalar triplet as an example

Rotated by Goldstone matrix

$$\Phi = \phi_i \sigma_i / 2 = \frac{1}{2} \begin{pmatrix} v' + \phi^0 & \phi_1 - i\phi_2 \\ \phi_1 + i\phi_2 & -v' - \phi^0 \end{pmatrix}$$

$$\Sigma = U(\pi) \Phi U(\pi)^\dagger$$

Not in "*Higgs basis*"

No mixing terms (except between neutral states) in the potential

U is not the real would-be Goldstone matrix

$$V = Y_1^2 H^\dagger H + Z_1 (H^\dagger H)^2 + Y_2^2 \mathrm{Tr}(\Sigma^\dagger \Sigma) + Z_2 \left(\mathrm{Tr}(\Sigma^\dagger \Sigma) \right)^2 + Z_3 H^\dagger H \mathrm{Tr}(\Sigma^\dagger \Sigma) + 2Y_3 H^\dagger \Sigma H$$

$$= Y_1^2 \frac{(v + h^0)^2}{2} + Z_1 \frac{(v + h^0)^4}{4} + Y_2^2 \mathrm{Tr}(\Phi^2) + Z_2 \mathrm{Tr}(\Phi^2)^2 + Z_3 \frac{(v + h^0)^2}{2} \mathrm{Tr}(\Phi^2) - Y_3 \frac{(v + h^0)^2}{2} \phi_3$$

However, kinetic mixing exists

$$\mathcal{L}_{\mathrm{kin}} = D_\mu H^\dagger D^\mu H + \mathrm{Tr}(D_\mu \Sigma D^\mu \Sigma)$$

$$= \frac{\partial_\mu h^0 \partial^\mu h^0}{2} + \frac{(v + h^0)^2}{4} \mathrm{Tr}(D_\mu U^\dagger D^\mu U) + \mathrm{Tr}(D^\mu \Phi D_\mu \Phi) + 2\mathrm{Tr}(\Phi \Phi D^\mu U^\dagger D_\mu U)$$

$$+ 2\mathrm{Tr}(\Phi U^\dagger D^\mu U \Phi U^\dagger D_\mu U) + 2\mathrm{Tr}(U^\dagger D^\mu U (\Phi D_\mu \Phi - D_\mu \Phi \Phi)) \,,$$

$$-v' \epsilon_{3jk} \partial_\mu \phi_j \partial_\mu \pi_k / v + \ldots$$

HEFT Matching

Scalar triplet as an example

$$
\begin{pmatrix} \phi^- \\ i\sqrt{1+2\xi^2}\pi^- \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{1+2\xi^2}}{\sqrt{1-2\xi^2}} & 0 \\ \frac{-2\xi}{\sqrt{1-2\xi^2}} & 1 \end{pmatrix} \begin{pmatrix} H^- \\ i\sqrt{1+2\xi^2}\pi'^- \end{pmatrix} \qquad \xi \equiv v'/v
$$

$$
U = \exp\left(\frac{i\sigma_i \pi_i'}{v} + \frac{2\xi}{\sqrt{1-4\xi^4}} i(-\sigma_1 H_2 + \sigma_2 H_1) \right)
$$

$$
= \boxed{U'} + \xi M_1(\pi', H^\pm) + \xi^2 M_2(\pi', H^\pm) + \dots
$$

U' is the real would-be Goldstone matrix

M_i can be solved order by order using Baker–Campbell–Hausdorff formula

HEFT Matching

Lessons learned

- After splitting the Goldstones, there is no mass mixing in the potential.

- But kinetic mixing generally exists.

Convert the mass mixing to kinetic mixing

$$\langle U^\dagger D^\mu U (D_\mu \hat{\rho} + D_\mu \hat{\rho}^\dagger) \rangle \quad \longrightarrow \quad D^\mu \pi$$

We can do more!

HEFT Matching

Scalar triplet

DoFs: 3

DoFs: 1

DoFs: 3

$$S_R \equiv U R, \qquad R = \frac{1}{\sqrt{2}} \left((v_H + h)\mathbb{I} + i\hat{\rho} \right), \quad \hat{\rho} = \rho_i \sigma_i,$$

$$\Sigma_\Phi \equiv U\Phi U^\dagger, \quad \Phi = \frac{1}{2}\phi_i\sigma_i = \frac{1}{2}\begin{pmatrix} v_\Sigma + \phi^0 & \sqrt{2}\phi^+ \\ \sqrt{2}\phi^- & -v_\Sigma - \phi^0 \end{pmatrix},$$

DoFs: 3

$$\mathcal{L}_{S_R}^{\text{kin}} = \frac{1}{2}\langle D^\mu S^\dagger D_\mu S \rangle$$

$$= \frac{1}{2}\left(\langle D_\mu R^\dagger D^\mu R \rangle + \langle (RR^\dagger) D^\mu U^\dagger D_\mu U \rangle + \langle U^\dagger D_\mu U (R D^\mu R^\dagger - D^\mu R R^\dagger) \rangle \right)$$

$$= -\frac{i}{4}v_{\text{H}}\langle U^\dagger D^\mu U (D_\mu\hat{\rho} + D_\mu\hat{\rho}^\dagger) \rangle + ...,$$

$$\mathcal{L}_{\Sigma_\Phi}^{\text{mixing}} = \frac{1}{2}v_\Sigma \langle U^\dagger D^\mu U ([\sigma_3, D_\mu\Phi] + [\sigma_3, D_\mu\Phi^\dagger]) \rangle$$

Cancel each other

$$\hat{\rho} = -2i\frac{v_\Sigma}{v_{\text{H}}}[\sigma_3, \Phi] = -2\frac{v_\Sigma}{v_{\text{H}}}(\phi_2\sigma_1 - \phi_1\sigma_2)$$

$$\rho^\pm = 2\frac{v_\Sigma}{v_{\text{H}}}\phi^\pm, \quad \rho_3 = 0$$

HEFT Matching

Lessons learned

- After splitting the Goldstones, there is no mass mixing in the potential.

- But kinetic mixing generally exists.

$$\langle \underbrace{U^\dagger D^\mu U}(D_\mu \hat{\rho} + D_\mu \hat{\rho}^\dagger)\rangle \longrightarrow D^\mu \pi$$

Convert the mass mixing to kinetic mixing

- Total degrees of freedom should be correct, which indicates that the singly charged (CP-odd neutral) particle is just the physical singly charged (CP-odd neutral) scalar up to a normalization factor.

HEFT Matching

General multiplets

Unlike using the vector representation of SU(2), we introduce the tensor representation

$$\Phi_{ijklm\cdots} \equiv \Phi_{(ijklm\cdots)}$$

Under a SU(2) transformation, $\phi_{ijklm\cdots}$ transforms as

$$\Phi_{ijklm\cdots} \longrightarrow U_i^{i_1} U_j^{j_1} U_k^{k_1} U_l^{l_1} U_m^{m_1} \cdots \Phi_{i_1 j_1 k_1 l_1 m_1 \cdots}$$

We can promote U to the Goldstone matrix, and split the Goldstones and physical states

$$\Phi_{ijklm\cdots} = U_i^{i_1} U_j^{j_1} U_k^{k_1} U_l^{l_1} U_m^{m_1} \cdots \phi_{i_1 j_1 k_1 l_1 m_1 \cdots}$$

where $\phi_{i_1 j_1 k_1 l_1 m_1 \cdots}$ is composed of physical states.
The Kinetic term becomes

$$(D\Phi^{*i_1 i_2 i_3 i_4 i_5 \cdots})(D\Phi_{i_1 i_2 i_3 i_4 i_5 \cdots})$$

$$=(D\phi^{*i_1 i_2 i_3 i_4 i_5 \cdots})(D\phi_{i_1 i_2 i_3 i_4 i_5 \cdots}) + (DU^{*i_n}_{k_n} DU^{j_n}_{i_n})\phi^{*\cdots i_{n-1} k_n i_{n+1} \cdots}\phi_{\cdots i_{n-1} j_n i_{n+1} \cdots}$$

$$+ (U^{*i_n}_{k_n} DU^{j_n}_{i_n} D\phi^{*\cdots i_{n-1} k_n i_{n+1} \cdots}\phi_{\cdots i_{n-1} j_n i_{n+1} \cdots} + DU^{*i_n}_{k_n} U^{j_n}_{i_n}\phi^{*\cdots i_{n-1} k_n i_{n+1} \cdots}D\phi_{\cdots i_{n-1} j_n i_{n+1} \cdots})$$

$$+ (U^{*i_m}_{k_m} DU^{j_m}_{i_m} DU^{*i_n}_{k_n} U^{j_n}_{i_n} + DU^{*i_m}_{k_m} U^{j_m}_{i_m} U^{*i_n}_{k_n} DU^{j_n}_{i_n})$$

$$\phi^{*\cdots i_{m-1} k_m i_{m+1} \cdots i_{n-1} k_n i_{n+1} \cdots}\phi_{\cdots i_{m-1} j_m i_{m+1} \cdots i_{n-1} j_n i_{n+1} \cdots}$$

HEFT Matching

General multiplets

Without loss of generality, we assume that $y \geq 0$. The index of the neutral component is given as

$$\underbrace{1 \cdots 1}_{j-y} \underbrace{2 \cdots 2}_{j+y}$$

and the indices of the charged components with unit charge therefore can be

$$\underbrace{1 \cdots 1}_{j-y+1} \underbrace{2 \cdots 2}_{j+y-1} \qquad \text{for positive charge}$$

$$\underbrace{1 \cdots 1}_{j-y-1} \underbrace{2 \cdots 2}_{j+y+1} \qquad \text{for negative charge}$$

HEFT Matching

$$\mathrm{H}_i = U_i^j \mathfrak{h}_j, \quad \mathfrak{h} = \begin{pmatrix} \chi^+ \\ \frac{1}{\sqrt{2}}\left(v_\mathrm{H} + h + i\chi^0\right) \end{pmatrix}$$

General multiplets

The mixing in the neutral sector

$$\chi^0 = -\frac{2yv_\phi}{v_H}\eta^0$$

$$U^{*\,i_n}_{k_n} DU^{j_n}_{i_n} D\phi^{*\cdots i_{n-1}k_n i_{n+1}\cdots}\phi_{\cdots i_{n-1}j_n i_{n+1}\cdots} + DU^{*\,i_n}_{k_n} U^{j_n}_{i_n}\phi^{*\cdots i_{n-1}k_n i_{n+1}\cdots} D\phi_{\cdots i_{n-1}j_n i_{n+1}\cdots}$$

$$\supset U^{*\,i_{j-y+1}}_2 DU^2_{i_{j-y+1}} D\phi^{*\,\overbrace{1\cdots 1}^{j-y}{}_2\overbrace{2\cdots 2}^{j+y-1}}\phi_{\underbrace{1\cdots 1}_{j-y}{}_2\underbrace{2\cdots 2}_{j+y-1}} + DU^{*\,i_{j-y+1}}_2 U^2_{i_{j-y+1}}\phi^{*\,\overbrace{1\cdots 1}^{j-y}{}_2\overbrace{2\cdots 2}^{j+y-1}} D\phi_{\underbrace{1\cdots 1}_{j-y}{}_2\underbrace{2\cdots 2}_{j+y-1}}$$

$$+ U^{*\,i_{j-y}}_1 DU^1_{i_{j-y}} D\phi^{*\,\overbrace{1\cdots 1}^{j-y-1}{}_1\overbrace{2\cdots 2}^{j+y}}\phi_{\underbrace{1\cdots 1}_{j-y-1}{}_1\underbrace{2\cdots 2}_{j+y}} + DU^{*\,i_{j-y}}_1 U^1_{i_{j-y}}\phi^{*\,\overbrace{1\cdots 1}^{j-y-1}{}_1\overbrace{2\cdots 2}^{j+y}} D\phi_{\underbrace{1\cdots 1}_{j-y-1}{}_1\underbrace{2\cdots 2}_{j+y}}$$

$$=2jC^{2j-1}_{j-y}\left[(U^\dagger DU)^2_2(D\phi^{0*}/\sqrt{C^{2j}_{j-y}})(\phi^0/\sqrt{C^{2j}_{j-y}}) + (DU^\dagger U)^2_2(D\phi^0/\sqrt{C^{2j}_{j-y}})(\phi^{0*}/\sqrt{C^{2j}_{j-y}})\right]$$

$$+ 2jC^{2j-1}_{j-y-1}\left[(U^\dagger DU)^1_1(D\phi^{0*}/\sqrt{C^{2j}_{j+y}})(\phi^0/\sqrt{C^{2j}_{j+y}}) + (DU^\dagger U)^1_1(D\phi^0/\sqrt{C^{2j}_{j+y}})(\phi^{0*}/\sqrt{C^{2j}_{j+y}})\right]$$

$$=\left[(j+y)(U^\dagger DU)^2_2 + (j-y)(U^\dagger DU)^1_1\right](D\phi^{0*}\phi^0 - D\phi^0\phi^{0*})$$

$$\supset \frac{v_\phi}{\sqrt{2}}\left[(j+y)(U^\dagger DU)^2_2 + (j-y)(U^\dagger DU)^1_1\right]D\left(\phi^{0*} - \phi^0\right)$$

$$=-iv_\phi\left[(j+y)(U^\dagger DU)^2_2 + (j-y)(U^\dagger DU)^1_1\right]D\eta = -i2yv_\phi(U^\dagger DU)^2_2 D\eta^0$$

HEFT Matching

$$\mathrm{H}_i = U_i^j \mathfrak{h}_j, \quad \mathfrak{h} = \begin{pmatrix} \chi^+ \\ \frac{1}{\sqrt{2}} \left(v_{\mathrm{H}} + h + i\chi^0 \right) \end{pmatrix}$$

General multiplets

$$\chi^+ = \frac{v_\phi}{v_H} (\sqrt{(j-y)(j+y+1)}\phi^{-*} - \sqrt{(j+y)(j-y+1)}\phi^+)$$

The mixing in the singly-charged sector

$$U^{*\,i_n}_{k_n} DU^{j_n}_{i_n} D\phi^{*\cdots i_{n-1}k_n i_{n+1}\cdots} \phi_{\cdots i_{n-1}j_n i_{n+1}\cdots} + DU^{*\,i_n}_{k_n} U^{j_n}_{i_n} \phi^{*\cdots i_{n-1}k_n i_{n+1}\cdots} D\phi_{\cdots i_{n-1}j_n i_{n+1}\cdots}$$

$$\supset U^{*\,i_{j-y+1}}_1 DU^2_{i_{j-y+1}} D\phi^{*\overbrace{1\cdots 1}^{j-y}\overbrace{12\cdots 2}^{j+y-1}} \phi_{\underbrace{1\cdots 1}_{j-y}2\underbrace{2\cdots 2}_{j+y-1}} + DU^{*\,i_{j-y+1}}_2 U^1_{i_{j-y+1}} \phi^{*\overbrace{1\cdots 1}^{j-y}\overbrace{2\cdots 2}^{j+y-1}} D\phi_{\underbrace{1\cdots 1}_{j-y}1\underbrace{2\cdots 2}_{j+y-1}}$$

$$+ U^{*\,i_{j-y}}_2 DU^1_{i_{j-y}} D\phi^{*\overbrace{1\cdots 1}^{j-y-1}2\overbrace{2\cdots 2}^{j+y}} \phi_{\underbrace{1\cdots 1}_{j-y-1}1\underbrace{2\cdots 2}_{j+y}} + DU^{*\,i_{j-y}}_1 U^2_{i_{j-y}} \phi^{*\overbrace{1\cdots 1}^{j-y-1}\overbrace{2\cdots 2}^{j+y}} D\phi_{\underbrace{1\cdots 1}_{j-y-1}2\underbrace{2\cdots 2}_{j+y}}$$

$$= 2j C^{2j-1}_{j-y} \left[(U^\dagger DU)^2_1 (D\phi^{+*}/\sqrt{C^{2j}_{j-y+1}})(\phi^0/\sqrt{C^{2j}_{j-y}}) + (DU^\dagger U)^1_2 (D\phi^+/\sqrt{C^{2j}_{j-y+1}})(\phi^{0*}/\sqrt{C^{2j}_{j-y}}) \right]$$

$$+ 2j C^{2j-1}_{j-y-1} \left[(U^\dagger DU)^1_2 (D\phi^{-*}/\sqrt{C^{2j}_{j-y-1}})(\phi^0/\sqrt{C^{2j}_{j+y}}) + (DU^\dagger U)^2_1 (D\phi^-/\sqrt{C^{2j}_{j-y-1}})(\phi^{0*}/\sqrt{C^{2j}_{j+y}}) \right]$$

$$= (U^\dagger DU)^2_1 \left[\sqrt{(j+y)(j-y+1)} D\phi^{+*}\phi^0 - \sqrt{(j-y)(j+y+1)} D\phi^-\phi^{0*} \right]$$

$$+ (U^\dagger DU)^1_2 \left[\sqrt{(j-y)(j+y+1)} D\phi^{-*}\phi^0 - \sqrt{(j+y)(j-y+1)} D\phi^+\phi^{0*} \right]$$

$$\supset v_\phi/\sqrt{2}(U^\dagger DU)^2_1 \left[\sqrt{(j+y)(j-y+1)} D\phi^{+*} - \sqrt{(j-y)(j+y+1)} D\phi^- \right]$$

$$+ v_\phi/\sqrt{2}(U^\dagger DU)^1_2 \left[\sqrt{(j-y)(j+y+1)} D\phi^{-*} - \sqrt{(j+y)(j-y+1)} D\phi^+ \right]$$

HEFT Matching

Complex scalar triplet

$$\mathcal{L}_{\mathrm{CT}}(\mathrm{H}_i, \Delta_{ij}) \supset D^\mu \mathrm{H}^\dagger D_\mu \mathrm{H} + (D_\mu \Delta^{*ij})(D^\mu \Delta_{ij}) - V(\mathrm{H}, \Delta),$$

$$\mathcal{L}_\Delta^{\mathrm{mix}} = 2\langle \phi_{22} \rangle \Big((U^\dagger D_\mu U)_1^2 D^\mu \phi^{*12} - (U^\dagger D_\mu U)_2^1 D^\mu \phi_{12}$$

$$+ (U^\dagger D_\mu U)_2^2 (D^\mu \phi^{*22} - D^\mu \phi_{22}) \Big),$$

$$\chi^+ = -\frac{2v_\Delta}{v_{\mathrm{H}}} \phi_{12} = -\frac{\sqrt{2}v_\Delta}{v_{\mathrm{H}}} \phi^+, \quad \chi^0 = -\frac{2v_\Delta}{v_{\mathrm{H}}} \eta_\Delta$$

HEFT Matching

Quadruplet with Y = 3/2

$$\mathcal{L}_{\text{Quatruplet}}(\text{H}_i, \Theta_{ijk}) \supset D^\mu \text{H}^\dagger D_\mu \text{H} + (D_\mu \Theta^{*ijk})(D^\mu \Theta_{ijk}) - V(\text{H}, \Theta),$$

$$\mathcal{L}_\Theta^{\text{mix}} = 3\langle \phi_{222} \rangle \Big((U^\dagger D_\mu U)_1^2 D^\mu \phi^{*122} - (U^\dagger D_\mu U)_2^1 D^\mu \phi_{122}$$

$$+ (U^\dagger D_\mu U)_2^2 (D^\mu \phi^{*222} - D^\mu \phi_{222}) \Big),$$

$$\chi^+ = -\frac{3v_\Theta}{v_\text{H}} \phi_{122} = -\frac{\sqrt{3}v_\Theta}{v_\text{H}} \phi^+, \quad \chi^0 = -\frac{3v_\Theta}{v_H} \eta_4$$

HEFT Matching

Quadruplet with Y = 1/2

$$\mathcal{L}_{\text{Quatruplet}}(\mathrm{H}_i, \Theta_{ijk}) \supset D^\mu \mathrm{H}^\dagger D_\mu \mathrm{H} + (D_\mu \Theta^{*ijk})(D^\mu \Theta_{ijk}) - V(\mathrm{H}, \Theta),$$

$$\mathcal{L}_\Theta^{\text{mix}} = 3\langle \phi_{122} \rangle \Big((U^\dagger D_\mu U)_1^2 (2D^\mu \phi^{*112} - D^\mu \phi_{222}) + (U^\dagger D_\mu U)_2^1 (D^\mu \phi^{*222} - 2D^\mu \phi_{112})$$

$$+ (U^\dagger D_\mu U)_2^2 (D^\mu \phi^{*122} - D^\mu \phi_{122}) \Big),$$

$$\chi^+ = \frac{\sqrt{3}v_\Theta}{v_\mathrm{H}}(\phi_2^+ - 2\phi_1^+/\sqrt{3}), \quad \chi^0 = -\frac{v_\Theta}{v_H}\eta_4$$

HEFT Matching for real scalar triplet

$$V_\mu \equiv U^\dagger D_\mu U \xrightarrow[\text{gauge}]{\text{unitary}} W_\mu^\pm, Z_\mu$$

$$
\begin{aligned}
\mathcal{L} =& \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{2}m_h^2 h^2 - d_1 h^3 - z_1 h^4 + \frac{1}{4}\left(2v_\Sigma^2 - 4v_\Sigma s_\gamma h + 2s_\gamma^2 h^2\right) V_\mu^3 V_3^\mu \\
& -\frac{1}{4}\left[v_H^2 + 4v_\Sigma^2 + 2\left(v_H c_\gamma - 4v_\Sigma s_\gamma\right)h + \left(c_\gamma^2 + 4s_\gamma^2\right)h^2\right]\langle V_\mu V^\mu\rangle \\
& +\left[-d_2 h^2 - z_2 h^3 - \frac{1}{2}\left(v_H s_\gamma + 4v_\Sigma c_\gamma - 3s_\gamma c_\gamma h\right)\langle V_\mu V^\mu\rangle + \left(v_\Sigma c_\gamma - s_\gamma c_\gamma h\right)V_\mu^3 V_3^\mu\right]K \\
& +\left[\left(v_\Sigma - s_\gamma h\right)V_\mu^1 V_3^\mu - i\left(s_\gamma + v_\Sigma c_\gamma / v_H\right)V_\mu^2\left(h D^\mu - D^\mu h\right)\right]\phi_1 \\
& +\left[\left(v_\Sigma - s_\gamma h\right)V_\mu^2 V_3^\mu + i\left(s_\gamma + v_\Sigma c_\gamma / v_H\right)V_\mu^1\left(h D^\mu - D^\mu h\right)\right]\phi_2 \\
& +\frac{1}{2}\begin{pmatrix} K & \phi_1 & \phi_2 \end{pmatrix}\mathcal{X}\begin{pmatrix} K \\ \phi_1 \\ \phi_2 \end{pmatrix} \\
& -d_4 K^3 - d_6 K\phi^+\phi^- - z_4 h K^3 - z_5 K^4 - z_7 h K\phi^+\phi^- - z_8 K^2 \phi^+\phi^- - z_9\left(\phi^+\phi^-\right)^2
\end{aligned}
$$

$$
\mathcal{X} = \begin{pmatrix}
\begin{array}{c}-\partial^2 - m_K^2 - 2d_3 h - 2z_3 h^2 \\ -\left(1+3c_\gamma^2\right)\langle V_\mu V^\mu\rangle/2 + c_\gamma^2 V_\mu^3 V_3^\mu\end{array} & c_\gamma V_\mu^1 V_3^\mu + 2i(c_\gamma - v_\Sigma s_\gamma / v_H)V_\mu^2 D^\mu & c_\gamma V_\mu^2 V_3^\mu - 2i(c_\gamma - v_\Sigma s_\gamma / v_H)V_\mu^1 D^\mu \\
c_\gamma V_\mu^3 V_1^\mu - 2i(c_\gamma - v_\Sigma s_\gamma / v_H)V_\mu^2 D^\mu & \begin{array}{c}-\left(1+4v_\Sigma^2/v_H^2\right)D^2 - m_{\phi^\pm}^2 - d_5 h - z_6 h^2 \\ -2\left(1+v_\Sigma^2/v_H^2\right)\langle V_\mu V^\mu\rangle + V_\mu^1 V_1^\mu\end{array} & V_\mu^1 V_2^\mu + 2i(1+2v_\Sigma^2/v_H^2)V_\mu^3 D^\mu \\
c_\gamma V_\mu^3 V_2^\mu + 2i(c_\gamma - v_\Sigma s_\gamma / v_H)V_\mu^1 D^\mu & V_\mu^2 V_1^\mu - 2i(1+2v_\Sigma^2/v_H^2)V_\mu^3 D^\mu & \begin{array}{c}-\left(1+4v_\Sigma^2/v_H^2\right)D^2 - m_{\phi^\pm}^2 - d_5 h - z_6 h^2 \\ -2\left(1+v_\Sigma^2/v_H^2\right)\langle V_\mu V^\mu\rangle + V_\mu^2 V_2^\mu\end{array}
\end{pmatrix}
$$

HEFT Matching for real scalar triplet

Primary (Pratical) HEFT $\quad m_{\phi^\pm}^2 \sim m_K^2 \sim \mathcal{O}(t^{-1}), \quad m_h \sim v_H \sim \sin\gamma \sim \xi \sim \mathcal{O}(t^0)$

$$
\begin{aligned}
\mathcal{L}_{\mathrm{HEFT}}^p(t^{-1}) = {} & \frac{v_H^2\left[(4\xi^2+1)m_K^2(s_\gamma+\xi c_\gamma)^2 - \xi^2 m_{\phi^\pm}^2\right]}{8(4\xi^2+1)} - \frac{h^3 m_{\phi^\pm}^2 s_\gamma^2(2\xi c_\gamma+s_\gamma)}{2\xi(4\xi^2+1)v_H} \\
& - \frac{h^4}{8\xi^2(4\xi^2+1)^2 v_H^2 m_K^2}\Big\{ m_{\phi^\pm}^2 s_\gamma^2\Big[(4\xi^2+1)m_K^2\Big(6(2\xi^2-1)c_\gamma^4 + 7c_\gamma^2 + 18\xi c_\gamma^3 s_\gamma - 4\xi c_\gamma s_\gamma - 1\Big) \\
& \qquad + m_{\phi^\pm}^2\Big((9-36\xi^2)c_\gamma^4 + 3(8\xi^2-3)c_\gamma^2 - 36\xi c_\gamma^3 s_\gamma + 12\xi c_\gamma s_\gamma - 4\xi^2\Big)\Big]\Big\} + \mathcal{O}(h^5)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{\mathrm{HEFT}}^p(t^0) = {} & \frac{1}{2}\langle V_\mu\sigma_3\rangle\langle V^\mu\sigma_3\rangle\left\{ \xi^2 v_H^2 - 2h\xi v_H s_\gamma + h^2 s_\gamma\left[s_\gamma^3 - \xi c_\gamma^3 + \frac{c_\gamma m_{\phi^\pm}^2(3s_\gamma(c_\gamma-2\xi s_\gamma)+4\xi)}{(4\xi^2+1)m_K^2}\right] + \mathcal{O}(h^3)\right\} \\
& + \frac{1}{4}\langle V_\mu V^\mu\rangle\bigg\{ -(4\xi^2+1)v_H^2 - 2hv_H(c_\gamma - 4\xi s_\gamma) + \frac{h^2}{\xi}\Big[c_\gamma s_\gamma\Big(4\xi^2 + (1-4\xi^2)s_\gamma^2\Big) \\
& \qquad + \xi(-5s_\gamma^4 + 2s_\gamma^2 - 1) + \frac{m_{\phi^\pm}^2 s_\gamma\big(3(8\xi^2-1)c_\gamma s_\gamma^2 - 16\xi^2 c_\gamma + 2\xi(9s_\gamma^2-8)s_\gamma\big)}{(4\xi^2+1)m_K^2}\Big] + \mathcal{O}(h^3)\bigg\} \\
& + \frac{1}{2}D_\mu h D^\mu h + \frac{1}{8}m_h^2 v_H^2(c_\gamma - \xi s_\gamma)^2 - \frac{1}{2}h^2 m_h^2 + \frac{h^3 m_h^2(s_\gamma^3 - \xi c_\gamma^3)}{2\xi v_H} \\
& + \frac{h^4 m_h^2}{24\xi^2(4\xi^2+1)^2 v_H^2 m_K^4}\Big\{ 4m_\phi^4 s_\gamma^2\big[3s_\gamma(c_\gamma - 2\xi s_\gamma) + 4\xi\big]^2 \\
& \qquad + (4\xi^2+1)^2 m_K^4\Big[s_\gamma^2\Big(38\xi c_\gamma^3 s_\gamma + 25\xi^2 + 19(\xi^2-1)s_\gamma^4 + (16-41\xi^2)s_\gamma^2\Big) - 3\xi^2\Big] \\
& \qquad - 20(4\xi^2+1)m_K^2 m_{\phi^\pm}^2 s_\gamma^2\Big[\xi c_\gamma s_\gamma(7-9s_\gamma^2) - c_\gamma^2\Big((6\xi^2-3)s_\gamma^2 - 4\xi^2\Big)\Big]\Big\} + \mathcal{O}(h^5) \\
& + \bar{Q}_L U\begin{pmatrix} y_u & 0 \\ 0 & y_d \end{pmatrix}Q_R \times \frac{1}{2\sqrt{2}}\bigg\{ -2v_H - 2hc_\gamma \\
& \qquad + \frac{h^2 s_\gamma^2\big[\xi c_\gamma^2\big((4\xi^2+1)m_K^2 - 6m_{\phi^\pm}^2\big) + c_\gamma s_\gamma\big((4\xi^2+1)m_K^2 - 3m_{\phi^\pm}^2\big) + 2\xi m_{\phi^\pm}^2\big]}{\xi(4\xi^2+1)v_H m_K^2} + \mathcal{O}(h^3)\bigg\} + \mathrm{h.c.}
\end{aligned}
$$

HEFT Matching for real scalar triplet

Primary (Pratical) HEFT $\quad m_{\phi^\pm}^2 \sim m_K^2 \sim \mathcal{O}(t^{-1}), \quad m_h \sim v_H \sim \sin\gamma \sim \xi \sim \mathcal{O}(t^0)$

Operators	WCs ($\mathcal{L}^p_{\mathrm{HEFT}}(t^1)$)	WCs ($\mathcal{L}^d_{\mathrm{HEFT}}(t^3)$) $\sin\gamma \sim \xi \sim \mathcal{O}(t^1)$
$\langle V_\mu V^\mu \rangle$	$\frac{1}{2\xi(4\xi^2+1)m_K^4} h^2 m_h^2 s_\gamma (4\xi c_\gamma + s_\gamma)$ $\times \big\{ 2\xi \big[c_\gamma^2 ((4\xi^2+1)m_K^2 - 3m_{\phi^\pm}^2) + m_{\phi^\pm}^2 \big]$ $+ c_\gamma s_\gamma \big[2(4\xi^2+1)m_K^2 - 3m_{\phi^\pm}^2 \big] \big\}$	$\frac{1}{2\xi m_K^4} h^2 m_h^2 s_\gamma (4\xi + s_\gamma)$ $\times \big[2m_K^2(\xi + s_\gamma) - m_{\phi^\pm}^2(4\xi + 3s_\gamma) \big]$
$\langle V_\mu \sigma_3 \rangle \langle V^\mu \sigma_3 \rangle$	$-\frac{1}{(4\xi^2+1)m_K^4} h^2 c_\gamma m_h^2 s_\gamma$ $\times \big\{ 2\xi \big[c_\gamma^2 ((4\xi^2+1)m_K^2 - 3m_{\phi^\pm}^2) + m_{\phi^\pm}^2 \big]$ $+ c_\gamma s_\gamma \big[2(4\xi^2+1)m_K^2 - 3m_{\phi^\pm}^2 \big] \big\}$	$-\frac{1}{m_K^4} h^2 m_h^2 s_\gamma$ $\times \big[2m_K^2(\xi + s_\gamma) - m_{\phi^\pm}^2(4\xi + 3s_\gamma) \big]$

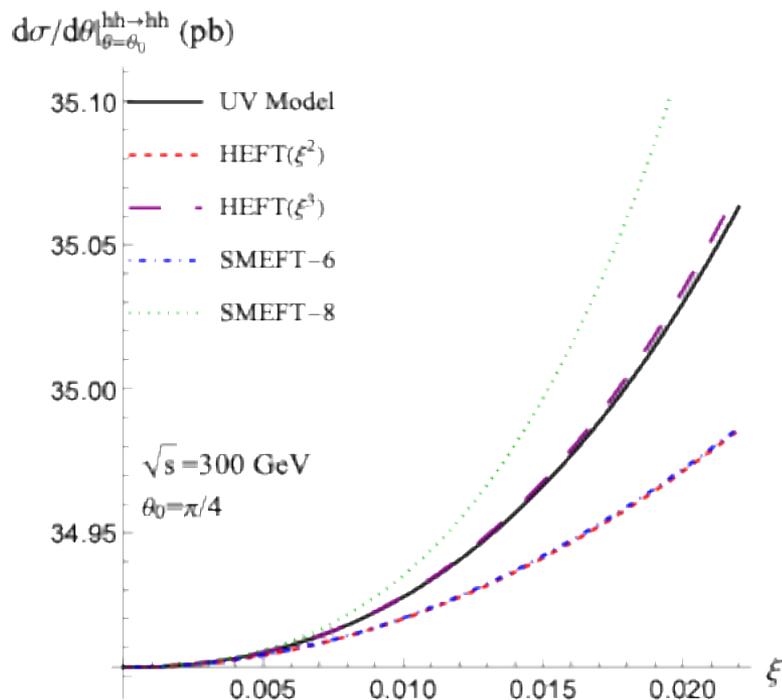| Operators	WCs ($\mathcal{L}^p_{\mathrm{HEFT}}(t^1)$)	WCs ($\mathcal{L}^d_{\mathrm{HEFT}}(t^3)$)
$\langle V_\mu V^\mu \rangle \langle V_\nu V^\nu \rangle$	$v_H^2 (4\xi c_\gamma + s_\gamma)^2 / 8m_K^2$	$v_H^2 (4\xi + s_\gamma)^2 / 8m_K^2$
$\langle V_\mu \sigma_3 \rangle \langle V^\mu \sigma_3 \rangle \langle V_\nu V^\nu \rangle$	$-\xi c_\gamma v_H^2 (4\xi c_\gamma + s_\gamma)/2m_K^2$	$-\xi v_H^2 (4\xi + s_\gamma)/2m_K^2$
$\langle V_\mu \sigma_3 \rangle \langle V_\nu \sigma_3 \rangle \langle V^\mu V^\nu \rangle$	$\xi^2 v_H^2 / (4\xi^2+1) m_{\phi^\pm}^2$	$\xi^2 v_H^2 / m_{\phi^\pm}^2$
$\langle V_\mu \sigma_3 \rangle \langle V^\mu \sigma_3 \rangle \langle V_\nu \sigma_3 \rangle \langle V^\nu \sigma_3 \rangle$	$\frac{1}{2}\xi^2 v_H^2 \left[\frac{c_\gamma^2}{m_K^2} - \frac{1}{(4\xi^2+1)m_{\phi^\pm}^2} \right]$	$\xi^2 v_H^2 (m_{\phi^\pm}^2 - m_K^2)/2m_K^2 m_{\phi^\pm}^2$
$\langle V_\mu V_\nu \sigma_3 \rangle \langle V^\mu \sigma_3 \rangle D^\nu h$	$4\xi v_H (\xi c_\gamma + s_\gamma)/(4\xi^2+1) m_{\phi^\pm}^2$	$4\xi v_H (\xi + s_\gamma)/m_{\phi^\pm}^2$
$\langle V_\mu V_\nu \rangle D^\mu h D^\nu h$	$-4(\xi c_\gamma + s_\gamma)^2/(4\xi^2+1) m_{\phi^\pm}^2$	$-4(\xi + s_\gamma)^2/m_{\phi^\pm}^2$
$\langle V_\mu \sigma_3 \rangle \langle V_\nu \sigma_3 \rangle D^\mu h D^\nu h$	$2(\xi c_\gamma + s_\gamma)^2/(4\xi^2+1) m_{\phi^\pm}^2$	$2(\xi + s_\gamma)^2/m_{\phi^\pm}^2$
$\langle V_\mu \sigma_3 \rangle \langle V^\mu \sigma_3 \rangle D_\nu h D^\nu h$	$\frac{c_\gamma}{(4\xi^2+1)m_K^4} \big\{ c_\gamma s_\gamma^2 \big[(4\xi^2+1)m_K^2 - 3m_{\phi^\pm}^2 \big]$ $+ \xi s_\gamma \big[c_\gamma^2((4\xi^2+1)m_K^2 - 6m_{\phi^\pm}^2) + 2m_{\phi^\pm}^2 \big] \big\}$	$\dfrac{s_\gamma \big[m_K^2(\xi + s_\gamma) - m_{\phi^\pm}^2(4\xi + 3s_\gamma) \big]}{m_K^4}$
$D_\mu h D^\mu h D_\nu h D^\nu h$	$\frac{s_\gamma^2}{2\xi^2(4\xi^2+1)^2 v_H^2 m_K^6} \big\{ \xi c_\gamma^2 \big[(4\xi^2+1)m_K^2 - 6m_{\phi^\pm}^2 \big]$ $+ c_\gamma s_\gamma \big[(4\xi^2+1)m_K^2 - 3m_{\phi^\pm}^2 \big] + 2\xi m_{\phi^\pm}^2 \big\}^2$	$\dfrac{s_\gamma^2 \big[m_K^2(\xi + s_\gamma) - m_{\phi^\pm}^2(4\xi + 3s_\gamma) \big]^2}{2\xi^2 v_H^2 m_K^6}$

HEFT Matching for real scalar triplet

Primary (Pratical) HEFT $\quad m_{\phi\pm}^2 \sim m_K^2 \sim \mathcal{O}(t^{-1}), \quad m_h \sim v_H \sim \sin\gamma \sim \xi \sim \mathcal{O}(t^0)$
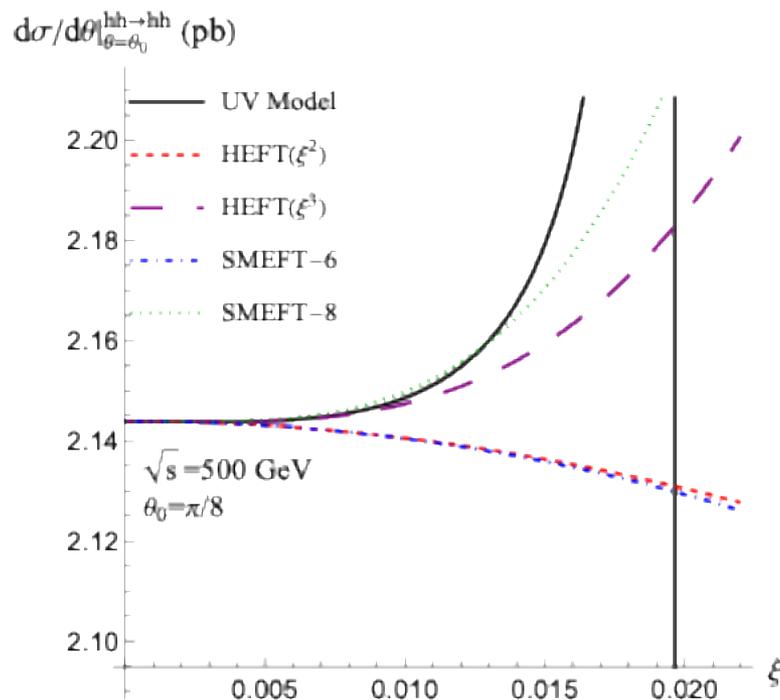
Operator	Wislon coefficients
$(\bar{Q}_L U Q_R)\langle V_\mu V^\mu\rangle + \text{h.c.}$	$\frac{1}{2}(y_u + y_d)\frac{v_H s_\gamma(4\xi c_\gamma + s_\gamma)}{2\sqrt{2}m_K^2}$
$(\bar{Q}_L U \sigma_3 Q_R)\langle V_\mu V^\mu\rangle + \text{h.c.}$	$\frac{1}{2}(y_u - y_d)\frac{v_H s_\gamma(4\xi c_\gamma + s_\gamma)}{2\sqrt{2}m_K^2}$
$(\bar{Q}_L U Q_R)\langle V_\mu\sigma_3\rangle\langle V^\mu\sigma_3\rangle + \text{h.c.}$	$\frac{1}{2}(y_u + y_d)\left(-\frac{\xi c_\gamma v_H s_\gamma}{\sqrt{2}m_K^2}\right)$
$(\bar{Q}_L U \sigma_3 Q_R)\langle V_\mu\sigma_3\rangle\langle V^\mu\sigma_3\rangle + \text{h.c.}$	$\frac{1}{2}(y_u - y_d)\left(-\frac{\xi c_\gamma v_H s_\gamma}{\sqrt{2}m_K^2}\right) - \frac{1}{2}(y_u - y_d)\frac{\sqrt{2}\xi^2 v_H}{(4\xi^2+1)m_{\phi\pm}^2}$
$(\bar{Q}_L U [V_\mu,\sigma_3] Q_R)\langle V^\mu\sigma_3\rangle + \text{h.c.}$	$\frac{1}{2}(y_u + y_d)\frac{\sqrt{2}\xi^2 v_H}{(4\xi^2+1)m_{\phi\pm}^2}$
$(\bar{Q}_L U V_\mu Q_R)\langle V^\mu\sigma_3\rangle + \text{h.c.}$	$(y_u - y_d)\frac{\sqrt{2}\xi^2 v_H}{(4\xi^2+1)m_{\phi\pm}^2}$
$(\bar{Q}_L U Q_R) D_\mu h D^\mu h + \text{h.c.}$	$\frac{1}{2}(y_u + y_d)\frac{s_\gamma^2\left\{m_{\phi\pm}^2\left[3s_\gamma c_\gamma + 2\xi(2-3s_\gamma^2)\right] - (4\xi^2+1)c_\gamma m_K^2(\xi c_\gamma + s_\gamma)\right\}}{\sqrt{2}\xi(4\xi^2+1)v_H m_K^4}$
$(\bar{Q}_L U \sigma_3 Q_R) D_\mu h D^\mu h + \text{h.c.}$	$\frac{1}{2}(y_u - y_d)\frac{s_\gamma^2\left\{m_{\phi\pm}^2\left[3s_\gamma c_\gamma + 2\xi(2-3s_\gamma^2)\right] - (4\xi^2+1)c_\gamma m_K^2(\xi c_\gamma + s_\gamma)\right\}}{\sqrt{2}\xi(4\xi^2+1)v_H m_K^4}$
$(\bar{Q}_L U [V_\mu,\sigma_3] Q_R) D^\mu h + \text{h.c.}$	$\frac{1}{2}(y_u - y_d)\frac{2\sqrt{2}\xi(\xi c_\gamma + s_\gamma)}{(4\xi^2+1)m_{\phi\pm}^2}$
$(\bar{Q}_L U V_\mu Q_R) D^\mu h + \text{h.c.}$	$(y_u + y_d)\frac{2\sqrt{2}\xi(\xi c_\gamma + s_\gamma)}{(4\xi^2+1)m_{\phi\pm}^2}$
$(\bar{Q}_L U \sigma_3 Q_R)\langle V_\mu\sigma_3\rangle D^\mu h + \text{h.c.}$	$-\frac{1}{2}(y_u + y_d)\frac{2\sqrt{2}\xi(\xi c_\gamma + s_\gamma)}{(4\xi^2+1)m_{\phi\pm}^2}$
$(\bar{Q}_L U Q_R) + \text{h.c.}$	$\frac{1}{2}(y_u + y_d)\frac{h^2 m_h^2 s_\gamma^2\left\{2(4\xi^2+1)c_\gamma m_K^2(\xi c_\gamma + s_\gamma) - m_{\phi\pm}^2\left[3s_\gamma c_\gamma + 2\xi(2-3s_\gamma^2)\right]\right\}}{\sqrt{2}\xi(4\xi^2+1)v_H m_K^4}$
$(\bar{Q}_L U \sigma_3 Q_R) + \text{h.c.}$	$\frac{1}{2}(y_u - y_d)\frac{h^2 m_h^2 s_\gamma^2\left\{2(4\xi^2+1)c_\gamma m_K^2(\xi c_\gamma + s_\gamma) - m_{\phi\pm}^2\left[3s_\gamma c_\gamma + 2\xi(2-3s_\gamma^2)\right]\right\}}{\sqrt{2}\xi(4\xi^2+1)v_H m_K^4}$
$(\bar{Q}_L U Q_R)(\bar{Q}_L U Q_R) + \text{h.c.}$	$\frac{1}{4}(y_u + y_d)^2\frac{s_\gamma^2}{4m_K^2}$
$(\bar{Q}_L U Q_R)(\bar{Q}_L U \sigma_3 Q_R) + \text{h.c.}$	$\frac{1}{2}(y_u^2 - y_d^2)\frac{s_\gamma^2}{4m_K^2}$
$(\bar{Q}_L U \sigma_3 Q_R)(\bar{Q}_L U \sigma_3 Q_R) + \text{h.c.}$	$\frac{1}{4}(y_u - y_d)^2\frac{s_\gamma^2}{4m_K^2} + \frac{1}{4}y_u y_d\frac{4\xi^2}{(4\xi^2+1)m_{\phi\pm}^2}$
$(\bar{Q}_L \sigma^I U Q_R)(\bar{Q}_L \sigma^I U Q_R) + \text{h.c.}$	$-\frac{1}{4}y_u y_d\frac{4\xi^2}{(4\xi^2+1)m_{\phi\pm}^2}$
$(\bar{Q}_L \gamma_\mu Q_L)(\bar{Q}_R \gamma^\mu Q_R)$	$\frac{1}{4}(y_u^2 + y_d^2)\frac{s_\gamma^2}{4m_K^2} + \frac{1}{8}(y_u^2 + y_d^2)\frac{4\xi^2}{(4\xi^2+1)m_{\phi\pm}^2}$
$(\bar{Q}_L \gamma_\mu \sigma^I Q_L)(\bar{Q}_R \gamma^\mu U^\dagger \sigma^I U Q_R)$	$\frac{1}{2}y_u y_d\frac{s_\gamma^2}{4m_K^2}$
$(\bar{Q}_L \gamma_\mu U \sigma_3 U^\dagger Q_L)(\bar{Q}_R \gamma^\mu Q_R)$	$\frac{1}{4}(y_u^2 - y_d^2)\frac{s_\gamma^2}{4m_K^2} - \frac{1}{8}(y_u^2 - y_d^2)\frac{4\xi^2}{(4\xi^2+1)m_{\phi\pm}^2}$
$(\bar{Q}_L \gamma_\mu Q_L)(\bar{Q}_R \gamma^\mu \sigma_3 Q_R)$	$\frac{1}{4}(y_u^2 - y_d^2)\frac{s_\gamma^2}{4m_K^2} + \frac{1}{8}(y_u^2 - y_d^2)\frac{4\xi^2}{(4\xi^2+1)m_{\phi\pm}^2}$
$(\bar{Q}_L \gamma_\mu U \sigma_3 U^\dagger Q_L)(\bar{Q}_R \gamma^\mu \sigma_3 Q_R)$	$\frac{1}{4}(y_u - y_d)^2\frac{s_\gamma^2}{4m_K^2} - \frac{1}{8}(y_u^2 + y_d^2)\frac{4\xi^2}{(4\xi^2+1)m_{\phi\pm}^2}$

HEFT Matching for real scalar triplet

Y3 = 39.75 GeV, Z2 = 1 and Z3 = 0.759

HEFT converges faster
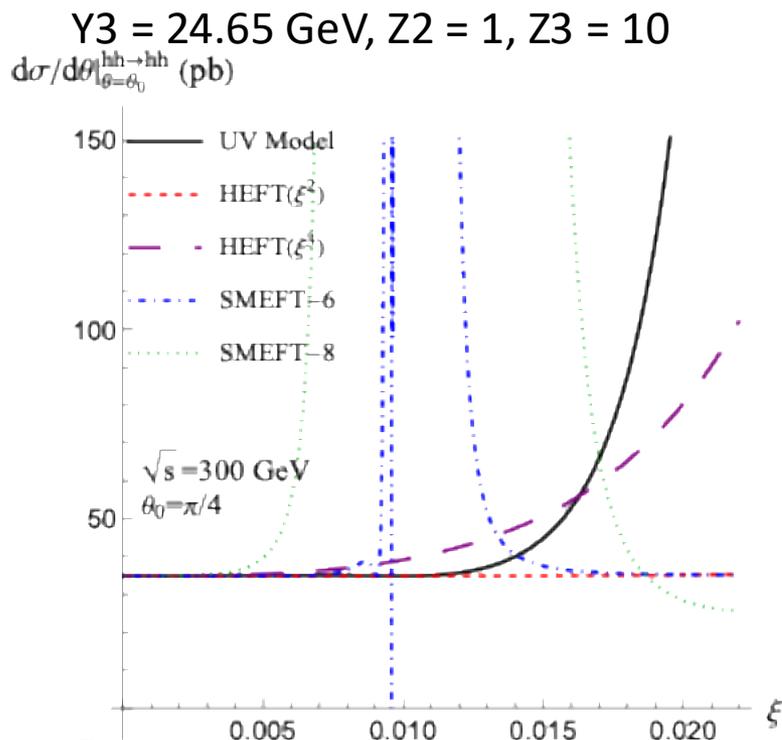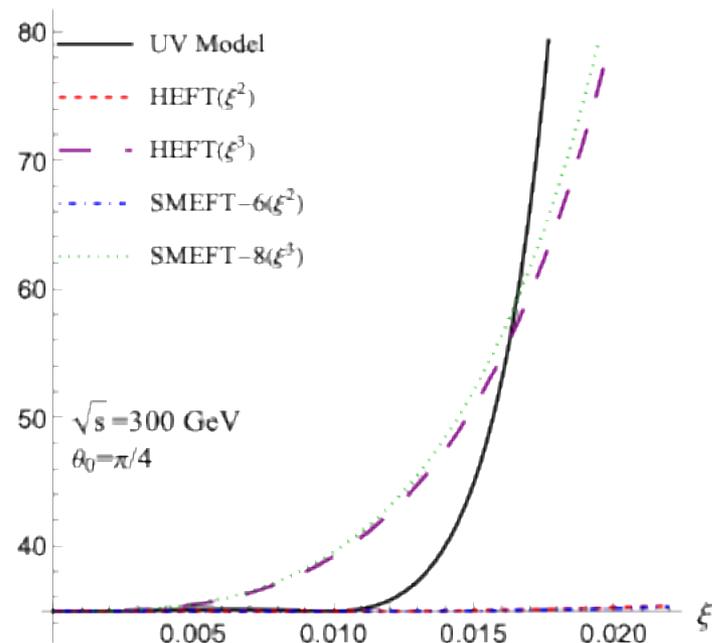

Both EFTs break down near the resonance

HEFT Matching for real scalar triplet

Y3 = 24.65 GeV, Z2 = 1, Z3 = 10

After a second expansion with $1/m_{\phi^\pm}^2$, $1/m_K^2$

Power counting,

SMEFT: $1/Y_2^2 \sim 1/0$

HEFT: ξ or $1/m_{\phi^\pm}^2$

The breakdown of SMEFT is due to the presence of another source of EW symmetry breaking, which is noted by (T. Cohen, Nathaniel Craig, Xiaochuan Lu, Dave Sutherland 2008.08597).
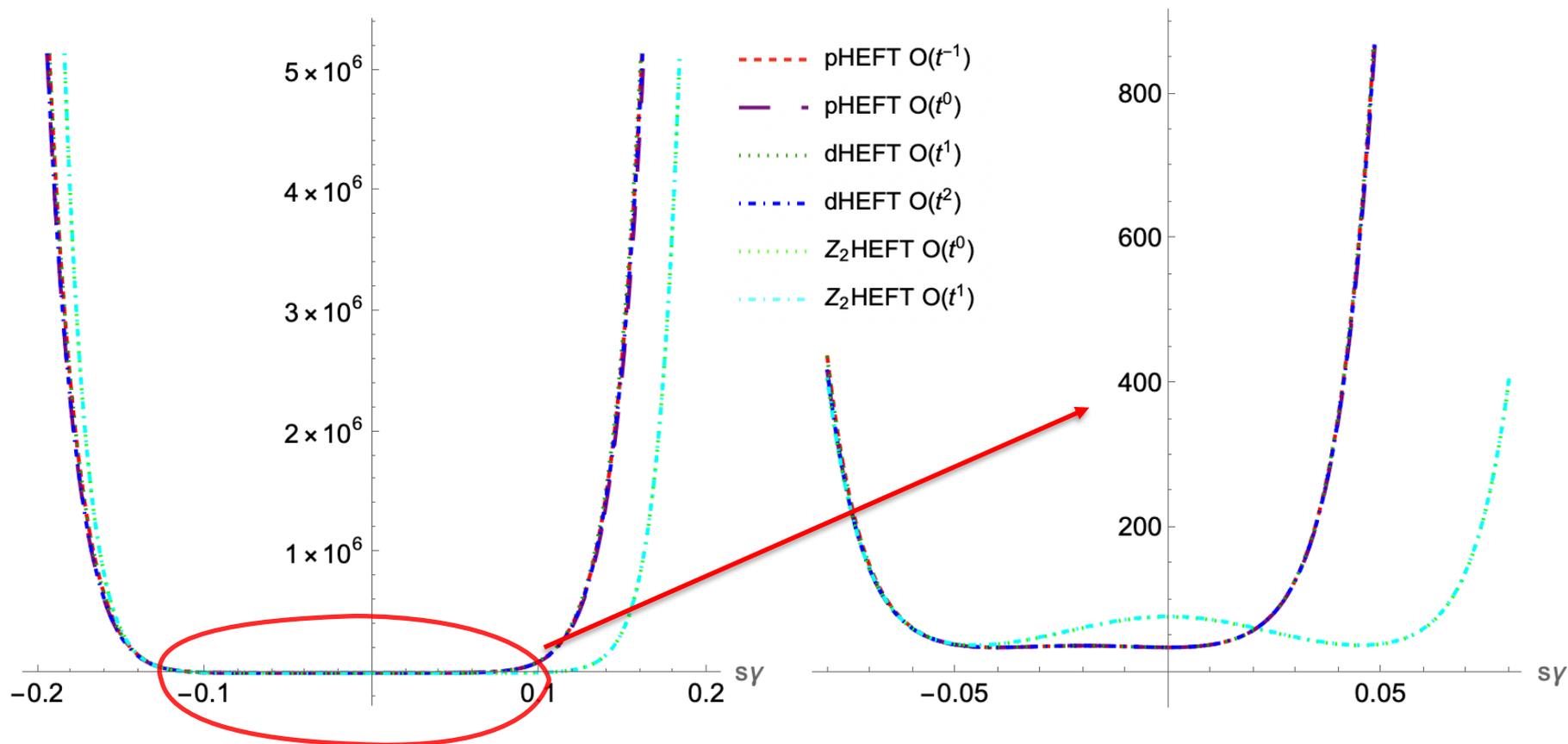
The SMEFT's regular part is consistent with the HEFT's

$$Y_2^2 = -Z_3 v_H^2/2 + m_{\phi^\pm}^2 + \mathcal{O}(\xi)$$

Amplitudes can be parametrized by different forms of operators (SMEFT and HEFT operators)

HEFT Matching for real scalar triplet

$\xi = 0.02, m_K = 800\,\text{GeV}, m_{\phi^\pm} = 799\,\text{GeV}, \theta = \pi/4, \sqrt{s} = 300\,\text{GeV}$

Summary

- We build a non-linear representation of general scalar extensions of the SM, in which the would-be Goldstones are explicitly factored out.

- Such a representation is suitable for HEFT matching.

- We firstly perform the tree-level matching between the real Higgs triplet model and HEFT beyond the leading order. (One-loop results and HEFT matchings for other scalar models are under checking and will be published soon.)

- We study the $hh, WW, ZZ \rightarrow hh$ scattering in the real Higgs triplet model within the framework of the HEFT and SMEFT, and re-discover the failure of the SMEFT due to the existence of further contributions to the EW symmetry breaking.

Thanks for your attention!

HEFT Matching

Can we do more?

- Recall 2HDM

$$\boldsymbol{\Phi}_n \equiv (\Phi_n^{\mathrm{c}}, \Phi_n) = \frac{1}{\sqrt{2}}\big[(v_n + \eta_n)\mathbb{1} + 2\mathrm{i}\boldsymbol{\phi}_n\big], \qquad \boldsymbol{\phi}_n \equiv \frac{\phi_{nj}\sigma_j}{2}$$

non-linearize

$$\boldsymbol{\Phi}_n = U(\boldsymbol{\zeta})\,\boldsymbol{\Phi}_n^{(\mathrm{u})}, \qquad \boldsymbol{\Phi}_n^{(\mathrm{u})} = \frac{1}{\sqrt{2}}\big[(v_n + h_n)\mathbb{1} + \mathrm{i}c_{nj}\sigma_j\rho_j\big]$$ ansatz

Apart from the physical CP-even neutral fields h_n some dependence on three further real physical Higgs fields ρ_j (j = 1, 2, 3) remains, which correspond to one CP-odd neutral and two charged Higgs bosons.

The coefficients c_{nj} (n = 1, 2; j = 1, 2, 3) are real constants, which can be determined upon requiring canonical field normalization (no mixing between the free scalar fields ζ_j, ρ_j, h_n).

$$c_{1j} = -\frac{v_2}{v} \equiv -\sin\beta, \qquad c_{2j} = \frac{v_1}{v} \equiv \cos\beta, \qquad v \equiv \sqrt{v_1^2 + v_2^2}.$$

HEFT Matching

Can we do more?

- Recall 2HDM

This is clearer in the following parametrization

$$\Phi_{12} = \begin{pmatrix} \bar{\phi}_1 & \phi_2 \end{pmatrix} \quad \Phi_{21} = \begin{pmatrix} \bar{\phi}_2 & \phi_1 \end{pmatrix} \quad \Phi_{21} = \tau_2 \Phi_{12}^* \tau_2$$

Under a $SU_L(2) \times U(1)$ transformation, $\Phi_{ij} \rightarrow \exp[i\bar{\tau} \cdot \bar{\alpha}]\Phi_{ij}\exp[-i\tau_3 \cdot \beta_3]$.
the larger group $SU_L(2) \times SU_R(2)$, $\Phi_{ij} \rightarrow \exp[i\bar{\tau} \cdot \bar{\alpha}]\Phi_{ij}\exp[-i\bar{\tau} \cdot \bar{\beta}]$

Though one can not decompose the Goldstone matrix in the form of

$$\Phi_{12} = \mathcal{U}H_{12} \quad \mathcal{U} = \exp(i\theta/v)\exp(i\bar{\tau}\bar{G}/v) \quad H_{12} = \sigma\mathbb{I} + \vec{\tau} \cdot \vec{\gamma}$$

$$\Phi_{12} = UM_{12} \quad M_{12} \text{ is not necessarily hermitian}$$

$$\delta_\epsilon = 1 + i\bar{T}^L \cdot \bar{\epsilon}^L + iT^R\epsilon^R = 1 + iT_+^L G_+ + iT_-^L G_- + i\frac{T_3^L - T_3^R}{2}G_0$$

HEFT Matching

Can we do more?

- Recall 2HDM

Making an infinitesimal gauge transformation specialized to the broken generators

$$\delta_\epsilon = 1 + i\bar{T}^L \cdot \bar{\epsilon}^L + iT^R \epsilon^R = 1 + iT^L_+ G_+ + iT^L_- G_- + i\frac{T^L_3 - T^R_3}{2}G_0$$

Acting with such a transformation on the vacuum configuration

$$\delta_\epsilon \, \Phi_{12} = \begin{pmatrix} v_1 + iG_0\frac{v_1}{v} & i\sqrt{2}G_+\frac{v_2}{v} \\ i\sqrt{2}G_-\frac{v_1}{v} & v_2 - iG_0\frac{v_2}{v} \end{pmatrix}$$

Goldstone bosons and massive excitations must be orthonormal for the kinetic terms to be diagonal.

$$\Phi_{12} = \begin{pmatrix} Re[\alpha_o] + i(G_0\frac{v_1}{v} + A_0\frac{v_2}{v}) & \sqrt{2}(H_+\frac{v_1}{v} + iG_+\frac{v_2}{v}) \\ \sqrt{2}(H_-\frac{v_2}{v} + iG_-\frac{v_1}{v}) & Re[\beta_0] + i(-G_0\frac{v_2}{v} + A_0\frac{v_1}{v}) \end{pmatrix}$$

$$\Phi_{12} = \exp[i\frac{\bar{G}\cdot\bar{\tau}}{v}] \begin{pmatrix} Re[\alpha_0] + iA_0\frac{v_2}{v} & \sqrt{2}H_+\frac{v_1}{v} \\ \sqrt{2}H_-\frac{v_2}{v} & Re[\beta_0] + iA_0\frac{v_1}{v} \end{pmatrix}$$

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HEFT Matching

Can we do more?

- Recall 2HDM

Making an infinitesimal gauge transformation specialized to the broken generators

$$\delta_\epsilon = 1 + i\bar{T}^L \cdot \bar{\epsilon}^L + iT^R\epsilon^R = 1 + iT_+^L G_+ + iT_-^L G_- + i\frac{T_3^L - T_3^R}{2}G_0$$

Acting with such a transformation on the vacuum configuration

$$\delta_\epsilon \, \Phi_{12} = \begin{pmatrix} v_1 + iG_0\frac{v_1}{v} & i\sqrt{2}G_+\frac{v_2}{v} \\ i\sqrt{2}G_-\frac{v_1}{v} & v_2 - iG_0\frac{v_2}{v} \end{pmatrix}$$

Goldstone bosons and massive excitations must be orthonormal for the kinetic terms to be diagonal.

$$\Phi_{12} = \exp[i\frac{\bar{G} \cdot \bar{\tau}}{v}] \begin{pmatrix} Re[\alpha_0] + iA_0\frac{v_2}{v} & \sqrt{2}H_+\frac{v_1}{v} \\ \sqrt{2}H_-\frac{v_2}{v} & Re[\beta_0] + iA_0\frac{v_1}{v} \end{pmatrix}$$

$$T = \frac{1}{4}\text{Tr}[D_\mu(UM_{12})(D_\mu(UM_{12}))^\dagger]$$

$$\frac{1}{v^2}\partial_\mu\bar{G}\partial^\mu\bar{G}\text{Tr}(\langle M_{12}\rangle\langle M_{12}\rangle^\dagger) + \text{Tr}(\partial_\mu M_{12}\partial^\mu M_{12}^\dagger)$$

$$\frac{i}{v}\text{Tr}(\bar{\tau}\partial_\mu\bar{G}(\langle M_{12}\rangle\partial^\mu M_{12}^\dagger - \partial^\mu M_{12}\langle M_{12}\rangle^\dagger))$$

$$= 2\frac{v_1v_2}{v^2}\text{Tr}(\partial_\mu\bar{G}\bar{\tau}\partial^\mu A_0) = 0$$

HEFT Matching

Scalar triplet

$$\mathcal{L}(\xi^0) = \frac{1}{2}D_\mu h D^\mu h + \frac{1}{4}v^4 Z_1 - h^2 v^2 Z_1 - h^3 v Z_1 - \frac{1}{4}h^4 Z_1 - \frac{1}{4}\left(v^2 + 2hv + h^2\right)\langle V_\mu V^\mu\rangle$$

$$\mathcal{L}(\xi^1) = \frac{\xi Y_3}{4v}\left(-v^4 + 4h^2 v^2 + 4h^3 v + h^4\right) \tag{4.6}$$

$$\mathcal{L}(\xi^2) = \frac{\xi^2}{4v^2}\left\{8h^2 D_\mu h D^\mu h + v^6 Z_3 + 8h^2 v^4(2Z_1 - Z_3) + 8h^3 v^3(5Z_1 - 2Z_3)\right.$$

$$+2h^4 v^2(16Z_1 - 7Z_3) + 2h^5 v(4Z_1 - 3Z_3) - h^6 Z_3$$

$$-4\left(v^4 + 3hv^3 + 4h^2 v^2 + 3h^3 v + h^4\right)\langle V_\mu V^\mu\rangle$$

$$\left.+2\left(v^4 + 4hv^3 + 6h^2 v^2 + 4h^3 v + h^4\right)\langle V_\mu\sigma_3\rangle\langle V^\mu\sigma_3\rangle\right\}$$

Operators	$P(h)/\left[\xi^3/(Y_3 v^3)\right]$
$\langle V_\mu V^\mu\rangle$	$-2hv^5(4Z_1 - Z_3) - 10h^2 v^4(4Z_1 - Z_3)$
$\langle V_\mu\sigma_3\rangle\langle V^\mu\sigma_3\rangle$	$2hv^5(4Z_1 - Z_3) + 9h^2 v^4(4Z_1 - Z_3)$
$D_\mu h D^\mu h$	$20h^2 v^2(4Z1 - Z3) + 24h^3 v(4Z1 - Z3)$
1	$h^2 v^4\left[v^2(4Z_1 - Z_3)^2 - 4Y_3^2\right]$
	$+h^3 v^3\left[v^2(4Z_1 - Z_3)(16Z_1 - 5Z_3) - 10Y_3^2\right]$
	$+\frac{1}{4}h^4 v^2\left[v^2(4Z_1 - Z_3)(60Z_1 - 41Z_3) - 32Y_3^2\right]$
	$-h^5 v\left[v^2(4Z_1 - Z_3)(4Z_1 + 11Z_3) + 2Y_3^2\right]$
	$-\frac{1}{2}h^6\left[v^2(4Z_1 - Z_3)(24Z_1 + 13Z_3)\right]$

Operator	$P(h)/\left[\xi^3/(Y_3 v^3)\right]$
$\langle V_\mu V^\mu\rangle\langle V_\nu V^\nu\rangle$	$(h + v)^4$
$\langle V_\mu\sigma_3\rangle\langle V^\mu\sigma_3\rangle\langle V_\nu V^\nu\rangle$	$-2(h + v)^4$
$\langle V_\mu\sigma_3\rangle\langle V_\nu\sigma_3\rangle\langle V^\mu V^\nu\rangle$	$2(h + v)^4$
$\langle V_\mu V_\nu\sigma_3\rangle\langle V^\mu\sigma_3\rangle D^\nu h$	$-4(h + v)^3$
$\langle V_\mu V_\nu\sigma_3\rangle\langle V^\nu\sigma_3\rangle D^\mu h$	$4(h + v)^3$
$\langle V_\mu V^\mu\rangle D_\nu h D^\nu h$	$4(h + v)^2$
$\langle V_\mu\sigma_3\rangle\langle V^\mu\sigma_3\rangle D_\nu h D^\nu h$	$-4(h + v)^2$
$\langle V_\mu V_\nu\rangle D^\mu h D^\nu h$	$-8(h + v)^2$
$\langle V_\mu\sigma_3\rangle\langle V_\nu\sigma_3\rangle D^\mu h D^\nu h$	$4(h + v)^2$
$D_\mu h D^\mu h D_\nu h D^\nu h$	4