

Interact or Twist: Cosmological Correlators from Field Redefinitions Revisited

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Why Cosmological Correlators?

Cosmological structures are not distributed randomly. There are correlations between them if we take a look at our universe from a larger scale.

If inflation do takes place as part of our cosmic history, the correlations we observe in the Large scale structure and the CMB all originate from correlations on the reheating surface, which is the boundary of a quasi de Sitter inflationary spacetime.

Inside the Bulk: Shwinger–Keldysh Path Integral

From the bulk perspective, the cosmological correlators can be generated by the so called Shwinger–Keldysh partition function

$$Z[J_+, J_-] = \int \mathcal{D}\phi_+ \mathcal{D}\phi_- \exp \{ \mathcal{L}[\phi_+] - \mathcal{L}[\phi_-] + J_+ \phi_+ - J_- \phi_- \} \\ \delta(\phi_+(\eta_0, \mathbf{x}) - \phi_-(\eta_0, \mathbf{x})) \quad (1)$$

On the Boundary: Cosmological Bootstrap

In the absence of secular time growth, the de Sitter isometries induce the $SO(4,1)$ conformal symmetry on the future boundary:

1. three translations
2. three rotations
3. the dilation $D = \mathbf{k} \cdot \partial_{\mathbf{k}} + 3 - \Delta$
4. the SCTs $\mathbf{K} = \mathbf{k}(\partial_{\mathbf{k}} \cdot \partial_{\mathbf{k}}) - 2(\mathbf{k} \cdot \partial_{\mathbf{k}} + 3 - \Delta)\partial_{\mathbf{k}}$

writing out all the conformal Ward identities, one can see that the three-point functions are fully determined by the conformal symmetries up to the conformal data [Arkani-Hamed et al. \(2020\)](#); [Baumann et al. \(2020\)](#).

Cosmological Correlators and Field Redefinition

- ▶ S-matrices are field–redefinition invariant.
- ▶ Cosmological correlators are not: they are defined on a space–like boundary of the inflationary spacetime, typically the reheating surface, and thus they are affected by field–redefinitions and boundary terms.
- ▶ For field redefinitions, within single field scenarios, correlators are highly constrained by scale invariance and locality [Maldacena \(2003\)](#); [Pajer \(2021\)](#). As only massless scalars are involved, these early studies normally led to the local shape in cosmological correlators.

Meanwhile...

More possibilities when we allow more than one degree of freedom!

The Toy Model

The toy model consists of one massless scalar field ϕ which enjoys a shift symmetry and another massive field σ ,

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m^2\sigma^2 + \frac{1}{\Lambda}\sigma\partial_\mu\sigma\partial^\mu\phi, \quad (2)$$

where the interaction term can be removed via a Integration-by-Parts (IBP) or the following field redefinition,

$$\phi = \tilde{\phi} + \frac{1}{2\Lambda}\sigma^2. \quad (3)$$

Field Redefinition Calculations

It can be noticed that the bispectrum of ϕ is completely determined by the field redefinition,

$$\begin{aligned}\langle \phi(\mathbf{x}_1)\phi(\mathbf{x}_2)\phi(\mathbf{x}_3) \rangle &= \frac{1}{8\Lambda^3} \langle \sigma(\mathbf{x}_1)^2 \sigma(\mathbf{x}_2)^2 \sigma(\mathbf{x}_3)^2 \rangle \\ &= \frac{C_\sigma^3 \eta_0^{6\Delta_\sigma} / 8\Lambda^3}{x_{12}^{2\Delta_\sigma} x_{23}^{2\Delta_\sigma} x_{31}^{2\Delta_\sigma}}.\end{aligned}\quad (4)$$

It is interesting to notice that this boundary correlator of a massless scalar takes the same form as the three-point function $\langle \mathcal{O}_\Delta(\mathbf{x}_1)\mathcal{O}_\Delta(\mathbf{x}_2)\mathcal{O}_\Delta(\mathbf{x}_3) \rangle_{\text{CFT}}$ in a 3d Euclidean CFT with $\Delta = 2\Delta_\sigma$.

Analyticity in the momentum space

With the "not-that-standard" Schwinger–Keldysh Feynman rule,
The diagram in the momentum space is

$$\langle \phi_{\mathbf{k}_1} \phi_{\mathbf{k}_2} \phi_{\mathbf{k}_3} \rangle' = \left(-\frac{\eta_0}{2}\right)^{9-6\nu} \frac{H^6 \Gamma(\nu)^6}{\pi^3 \Lambda^3} I_\nu(k_1, k_2, k_3), \quad (5)$$

$$\text{with } I_\nu \equiv \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \frac{1}{|\mathbf{q}|^{2\nu} |\mathbf{q} + \mathbf{k}_2|^{2\nu} |\mathbf{q} - \mathbf{k}_3|^{2\nu}}, \quad (6)$$

and

$$I_\nu = \frac{\pi^{-\frac{3}{2}} 2^{-\frac{1}{2}}}{\Gamma\left(\frac{3\Delta-3}{2}\right) \Gamma\left(\frac{3-\Delta}{2}\right)^3} (k_1 k_2 k_3)^{\Delta-\frac{3}{2}} \int_{-\infty}^0 d\eta (-\eta)^{1/2} \\ \times K_{\Delta-3/2}(-k_1 \eta) K_{\Delta-3/2}(-k_2 \eta) K_{\Delta-3/2}(-k_3 \eta), \quad (7)$$

where $K_{\Delta-3/2}$ is the Bessel K function [Bzowski et al. \(2014\)](#);
[Coriano et al. \(2013\)](#); [Bzowski et al. \(2016\)](#).

Analyticity in the momentum space

For general masses, this triple-K integral is hard to compute, here we provide the results for two particular cases:

$$I_{\frac{3}{2}} = \frac{\left(\frac{4}{3} - \gamma_E - \log k_T/\mu\right) \sum_{i=1}^3 k_i^3 + \sum_{i \neq j} k_i^2 k_j - k_1 k_2 k_3}{2\pi^2 k_1^3 k_2^3 k_3^3},$$

$$I_{\frac{1}{2}} = \frac{1}{2\pi^2} (-\gamma_E - \log k_T/\mu).$$

Analyticity in the momentum space

For general masses, we always find a total energy singularity in the $k_T = k_1 + k_2 + k_3 \rightarrow 0$ limit ,

$$\lim_{k_T \rightarrow 0} I_\nu = -\frac{(k_1 k_2 k_3)^{\Delta-2}}{4\Gamma\left(\frac{3\Delta-3}{2}\right)\Gamma\left(\frac{3-\Delta}{2}\right)^3} \log k_T. \quad (8)$$

It is normally expected that, for cosmological correlators, this total energy singularity is generated by bulk interactions, as the residue of the pole corresponds to flat-space amplitudes [Maldacena and Pimentel \(2011\)](#); [Raju \(2012\)](#), while field redefinitions can only affect parts of the correlators that are regular in the $k_T \rightarrow 0$ limit. The computation here serves as a counterexample showing that this singularity can also be a consequence of field redefinitions with composite operators.

Slide with a beautiful figure

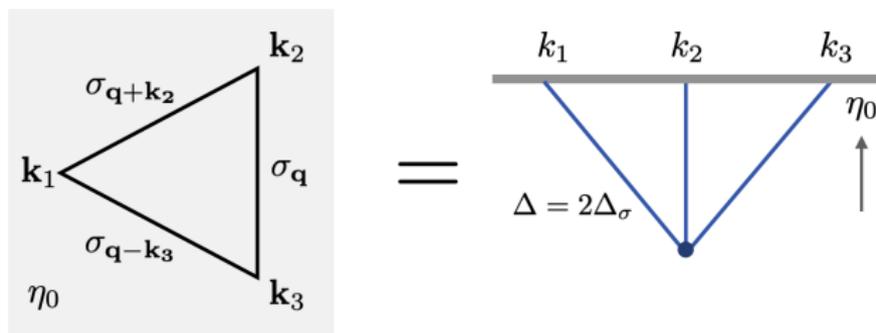


Figure 1: The boundary “one-loop” triangle graph from field redefinition can be re-expressed as a contact interaction of fields with a different mass in the bulk [Wang et al. \(2025\)](#).

Same Story Told in the IBP Way

by performing Integration-by-Part (IBP). The cubic mixing there becomes

$$\frac{1}{\Lambda} \sigma \partial_\mu \sigma \partial^\mu \phi = -\frac{1}{2\Lambda} \sigma^2 \square \phi + \frac{1}{2\Lambda} \nabla_\mu (\sigma^2 \partial^\mu \phi). \quad (9)$$

The Feynman rules lead to the following three-point function

$$\langle \phi_{\mathbf{k}_1} \phi_{\mathbf{k}_2} \phi_{\mathbf{k}_3} \rangle' = \frac{1}{8\Lambda^3} \sum_{a_1, a_2, a_3 = \pm} \int_{-\infty}^{\eta_0} \prod_{i=1}^3 \frac{d\eta_i}{(-H\eta_i)^2} \quad (10)$$

$$\begin{aligned} & \left[a_i \partial_{\eta_i} K_{a_i}(k_i, \eta_i) \delta(\eta_i - \eta_0) \right] \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \prod_{i < j} G_{a_i a_j}(\mathbf{q}_{ij}; \eta_i, \eta_j) \\ &= \frac{1}{8\Lambda^3} \int \frac{d^3 \mathbf{q}}{(2\pi)^3} |\sigma_{\mathbf{q}}(\eta_0)|^2 |\sigma_{|\mathbf{q}+\mathbf{k}_2|}(\eta_0)|^2 |\sigma_{|\mathbf{q}-\mathbf{k}_3|}(\eta_0)|^2, \end{aligned} \quad (11)$$

For Every Field Redef There is a IBP Boundary Term

When a field redefinition $\phi \rightarrow \phi + \delta\phi(\phi, \sigma)$ is used to eliminate a term proportional to the Equation of motion, a corresponding boundary term is generated:

$$\begin{aligned}\delta S &= \int dt \left[\partial_t \left(\frac{\partial L}{\partial \dot{\phi}} \delta\phi \right) - \left(\partial_t \frac{\partial L}{\partial \dot{\phi}} - \frac{\partial L}{\partial \phi} \right) \delta\phi \right] \\ &= (\text{boundary terms})\delta\phi - \int \text{EoM} \delta\phi.\end{aligned}\tag{12}$$

Modulated Reheating

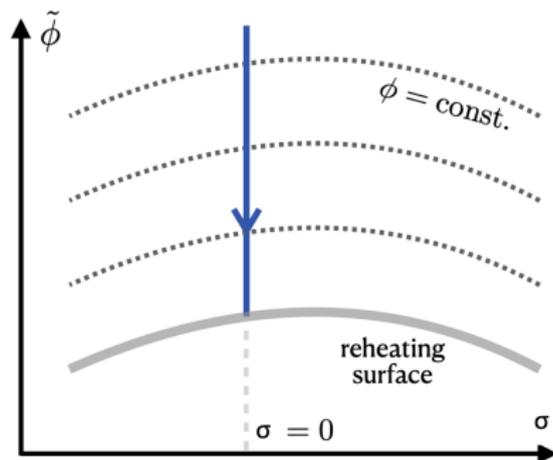


Figure 2: Field redefinitions twist the field space and lead to a nontrivial reheating surface in the new $(\tilde{\phi}, \sigma)$ basis. For our example, the dotted lines are constant ϕ surfaces, while the inflaton trajectory (blue) is along $\sigma = 0$ in both field bases [Wang et al. \(2025\)](#).

The Boundary Effective Field Theory

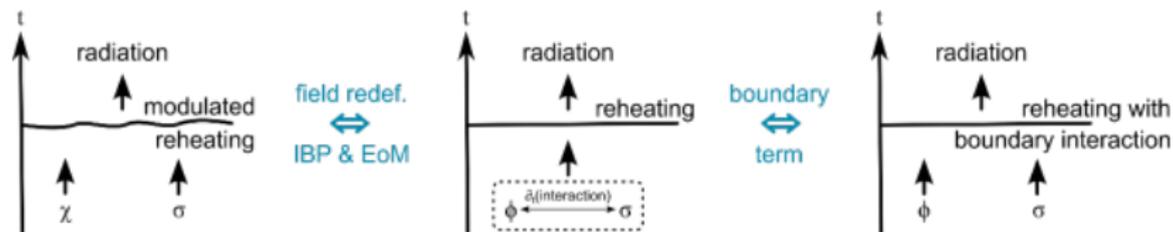


Figure 3: A model of modulated reheating can be described by a term in a BEFT

Summary

1. Boundary contributions from field redefinitions can produce non-Gaussianities similar to bulk interactions, with a total energy pole.
2. For each field redefinition that removes an EoM term, there is a corresponding boundary term.
3. The field redefinition can be interpreted as a modulation of the reheating surface.
4. A much larger set of theories with boundary terms, the so-called boundary EFTs, may be useful in studying physics happening within a few Hubble times, e.g. the reheating.

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