

# Modified Theories of Gravity, Parity, and the Dark Sector



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HKUST IAS Program on Fundamental Physics,

15 Jan 2026



# Tests of general relativity

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Black hole observations

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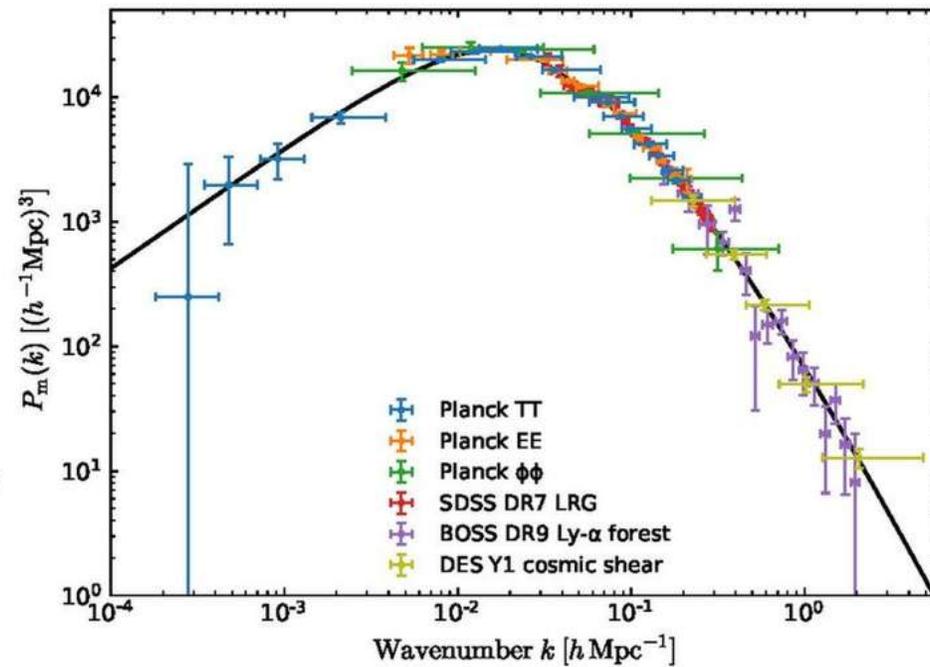
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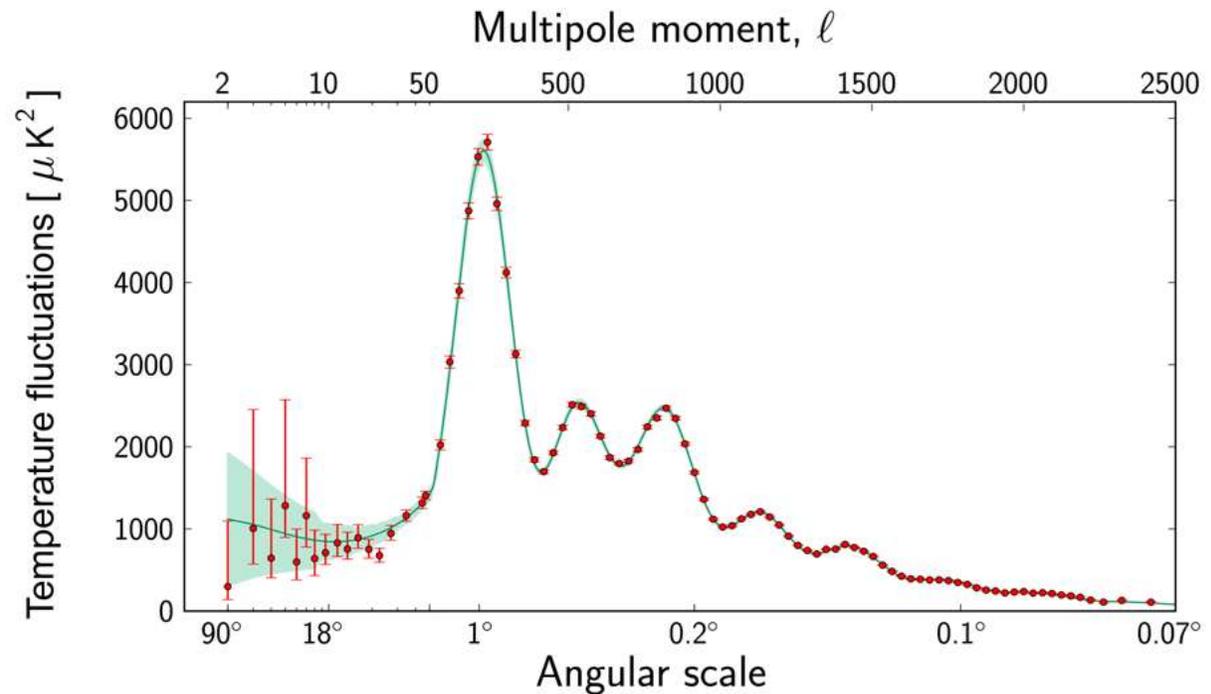
- EHT, X-rays from accretion disks

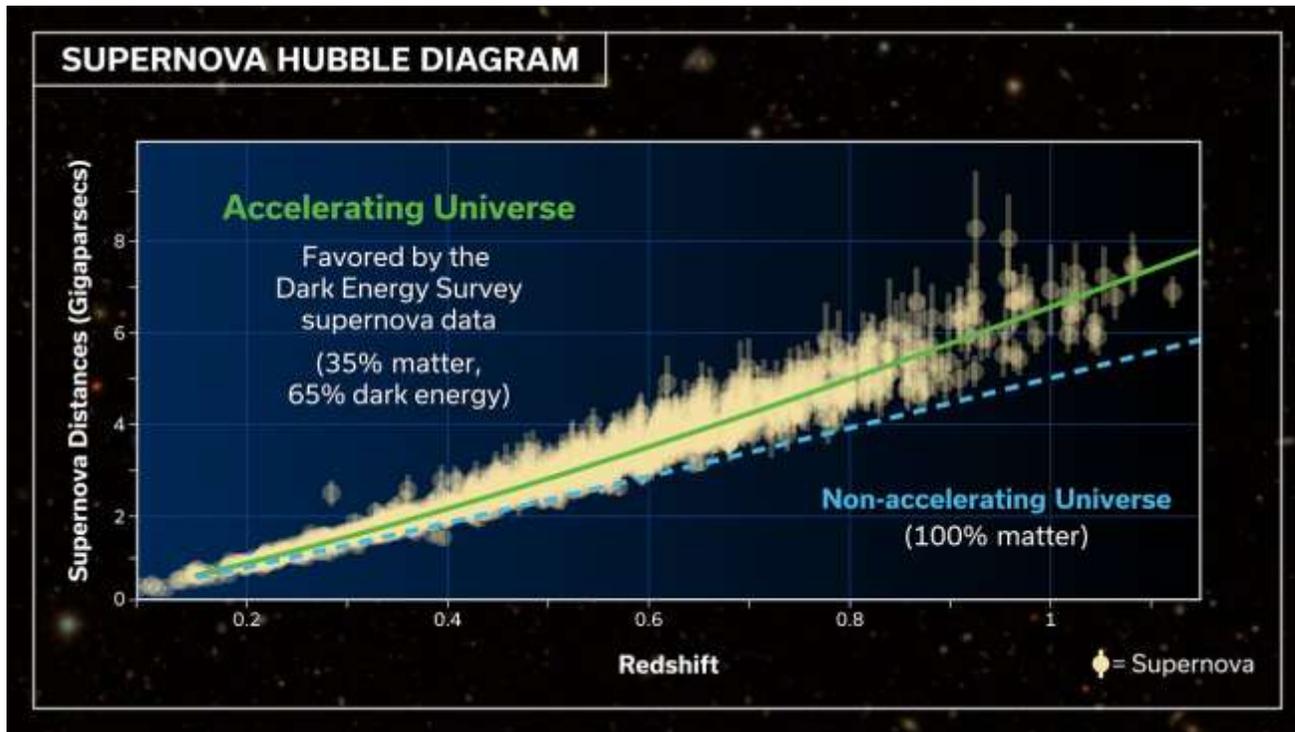
...but it  
doesn't  
explain  
everything

Credit:  
European  
Space  
Agency &  
Planck  
Collaboration

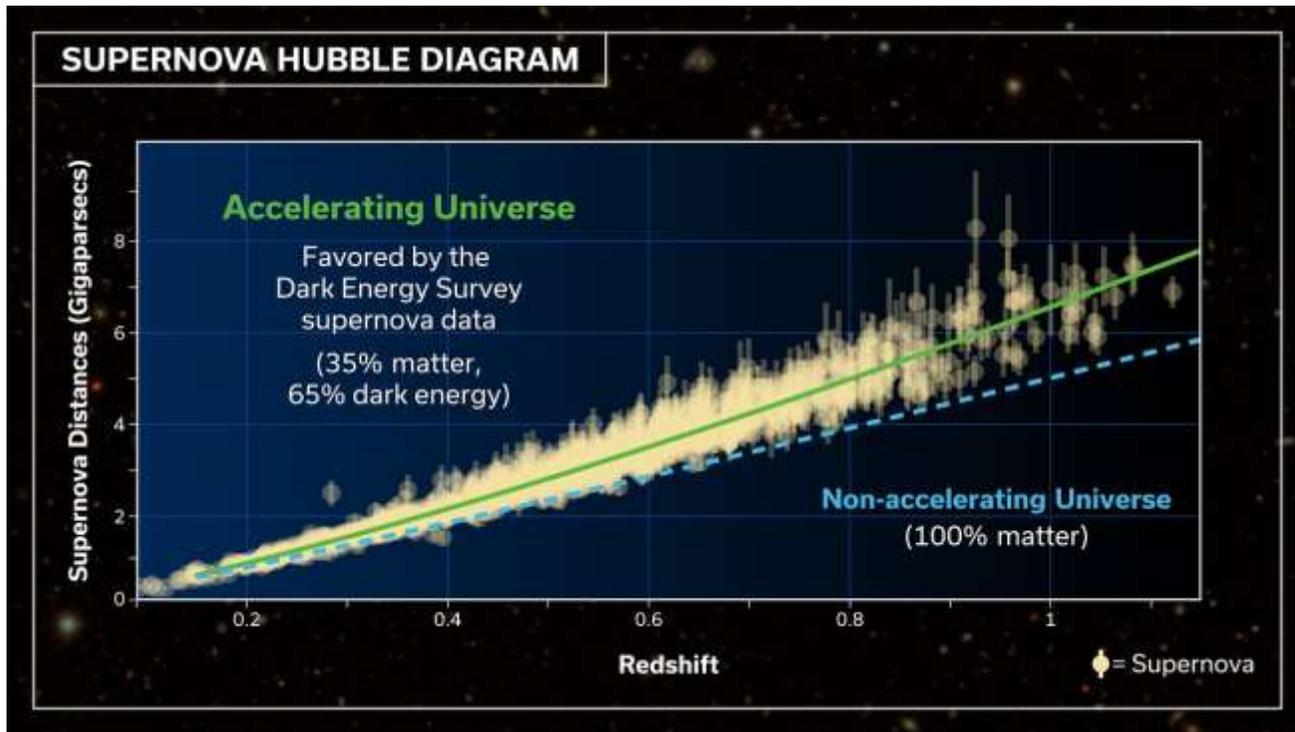


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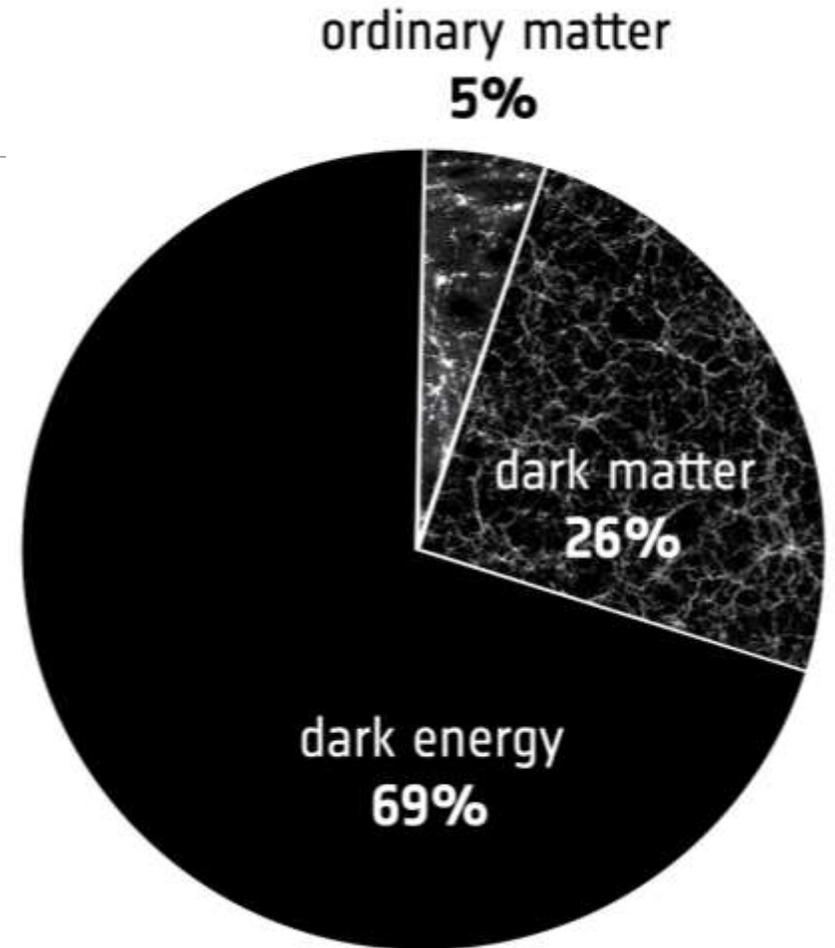




*Credit: DES Collaboration*



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*Credit: European Space Agency*

# What is dark matter and dark energy?

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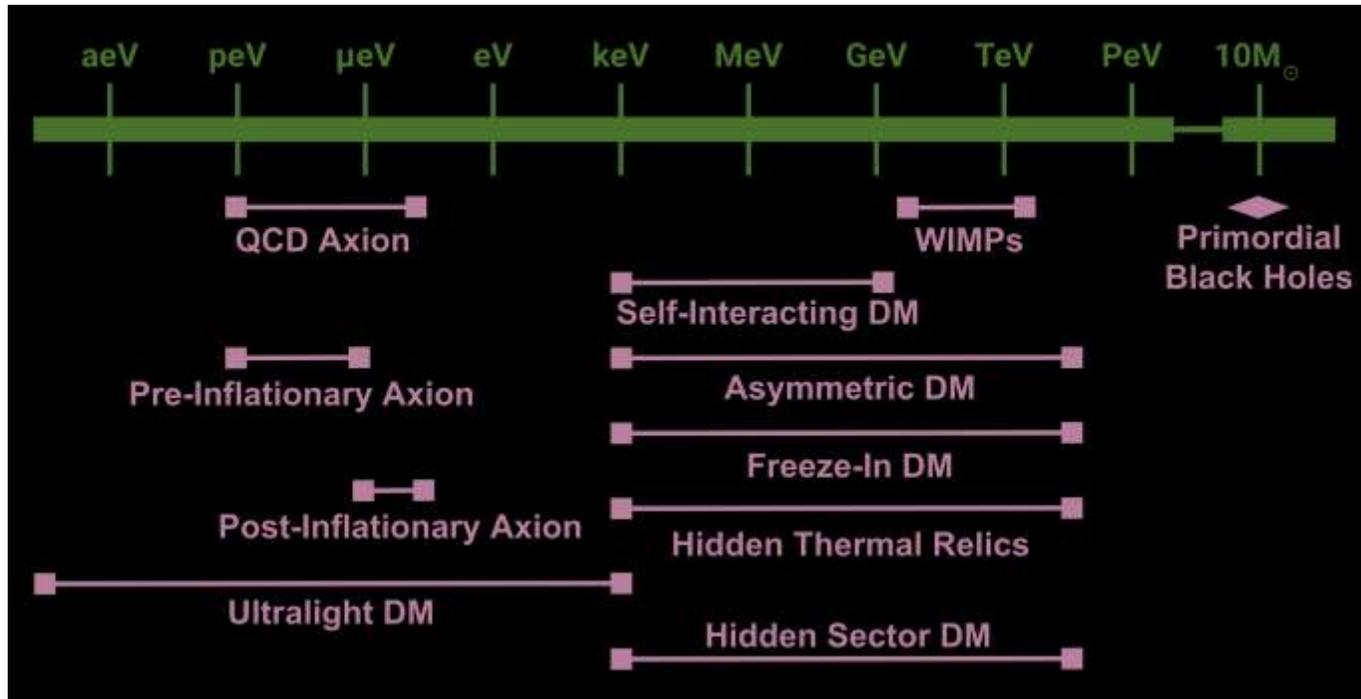
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Short answer: we don't know

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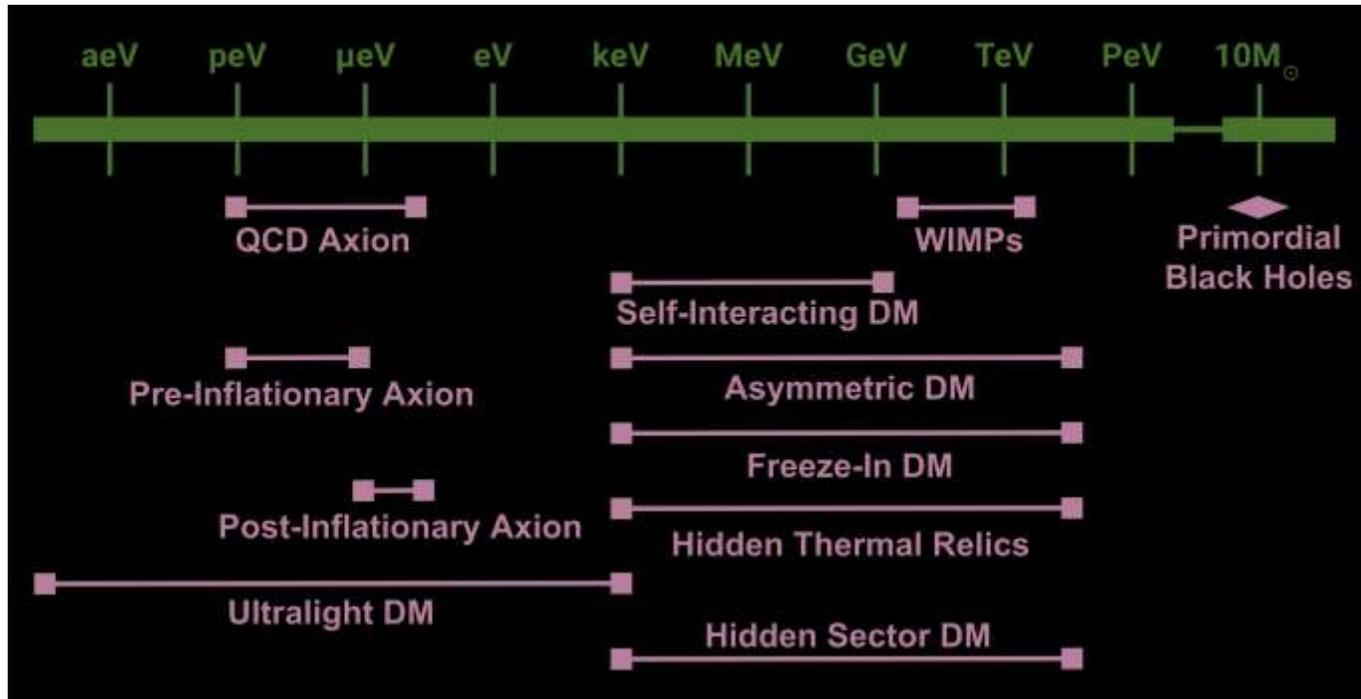
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*Credit: University College London, HEP Group*

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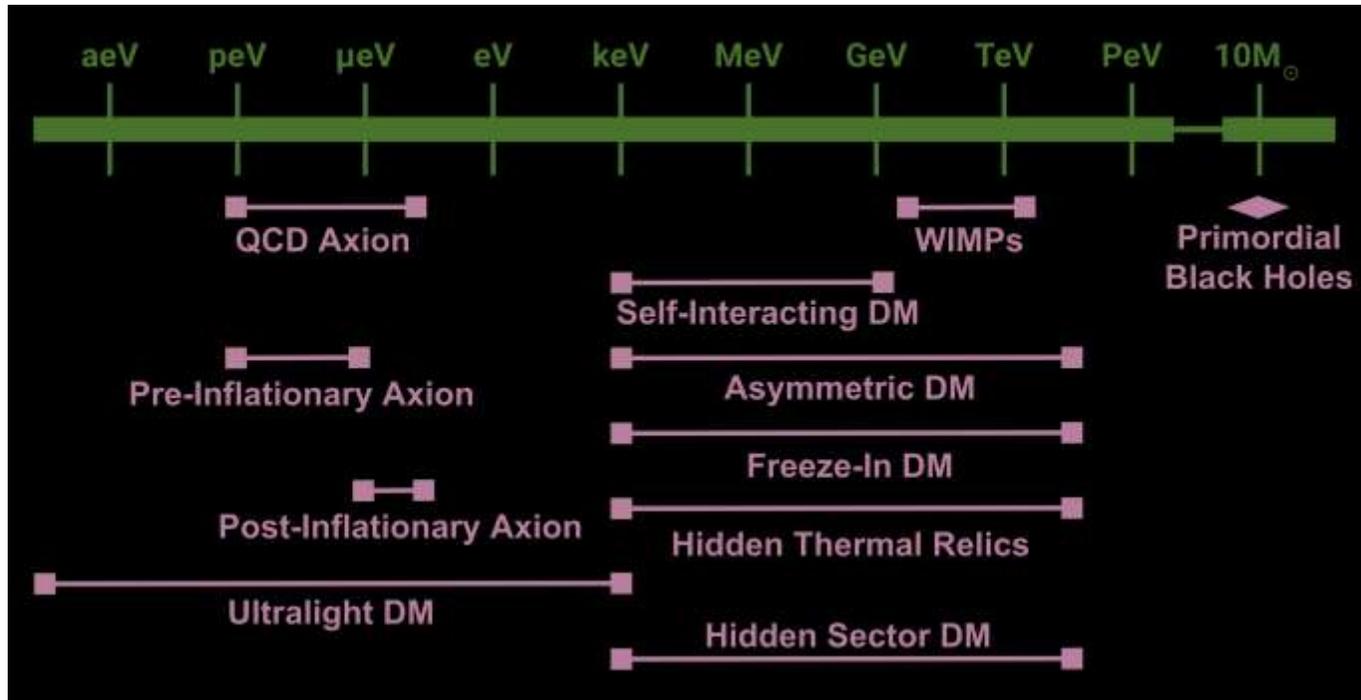
## DM: Axion

- Peccei & Quinn, 1977 PRD

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## DM: Axion

- Peccei & Quinn, 1977 PRD

## DE: Quintessence

- Peebles & Ratra, 1988 PRD
- Wetterich, 1988 Nuclear Physics B
- Caldwell, Dave & Steinhardt, 1998 PRL

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$$S = \frac{1}{16\pi} \int d^4x \sqrt{|g|} R$$

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 Dilaton       GB

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 Dilaton

 GB

$$\mathcal{X}_4 = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

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Dilaton

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↑
}
}

Dilaton
Axion
GB
CS

# A model with both DM and DE

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Dilaton                      Axion                      GB                      CS

**Chern-Simons-Gauss-Bonnet gravity**

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Chern-Simons: parity violation, baryogenesis

- Alexander, Peskin & Sheikh-Jabbari, 2006 PRL

# Observational Windows

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**Gravitational waves**

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## Gravitational waves

- GW generation: inspiral of compact objects, Bayesian parameter estimation

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## Neutron stars

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## Neutron stars

- TOV equations
- Mass-radius relation
- I-Love-Q relations (Yagi & Yunes, 2013 PRD)

# Model Agnostic Framework

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$$h''_{R,L} + \left\{ 2\mathcal{H} + \sum_{n=0}^{\infty} (\lambda_{R,L} k)^n \left[ \frac{\alpha_n(\eta)}{(\Lambda a)^n} \mathcal{H} + \frac{\beta_n(\eta)}{(\Lambda a)^{n-1}} \right] \right\} h'_{R,L} \\ + k^2 \left\{ 1 + \sum_{m=0}^{\infty} (\lambda_{R,L})^{m+1} k^{m-1} \left[ \frac{\gamma_m(\eta)}{(\Lambda a)^m} \mathcal{H} + \frac{\delta_m(\eta)}{(\Lambda a)^{m-1}} \right] \right\} h_{R,L} = 0$$

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$$\alpha_1 = -2\alpha' \tilde{\varphi}'$$

$$\gamma_1 = -\frac{1}{2}\alpha' \tilde{\phi}'$$

$$\beta_1 = 2\alpha' \phi' \varphi'$$

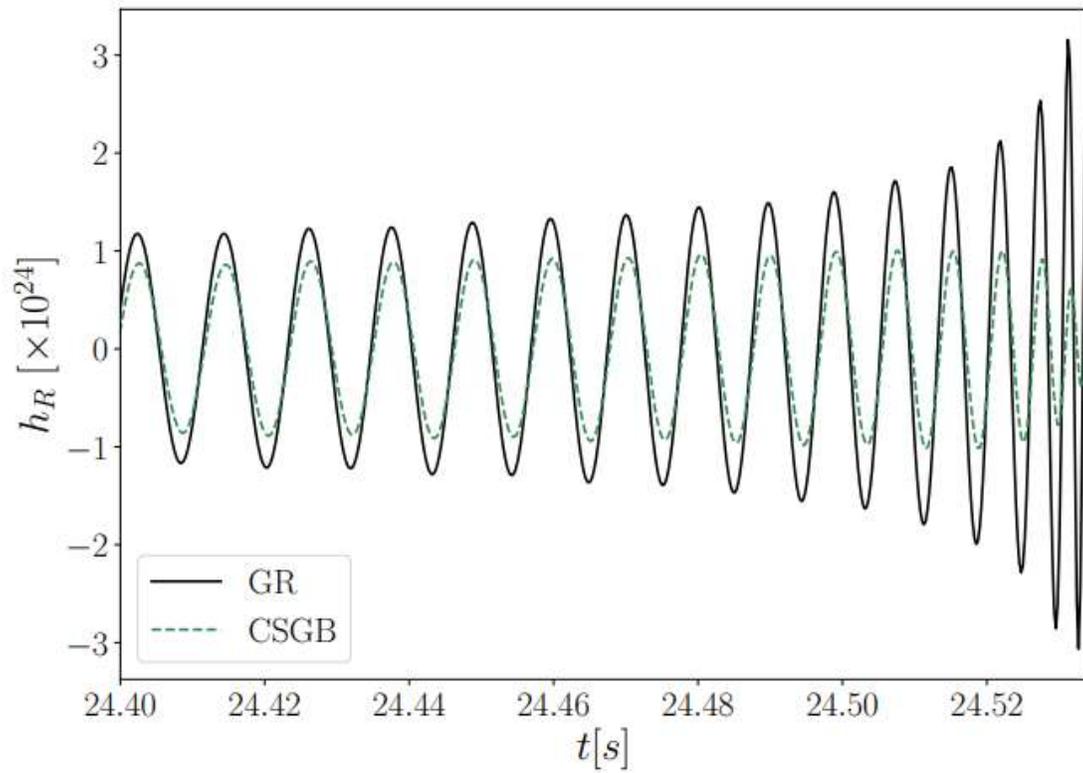
$$\delta_1 = -\frac{1}{2}\alpha' \varphi'^2$$

# CS-GB Waveforms

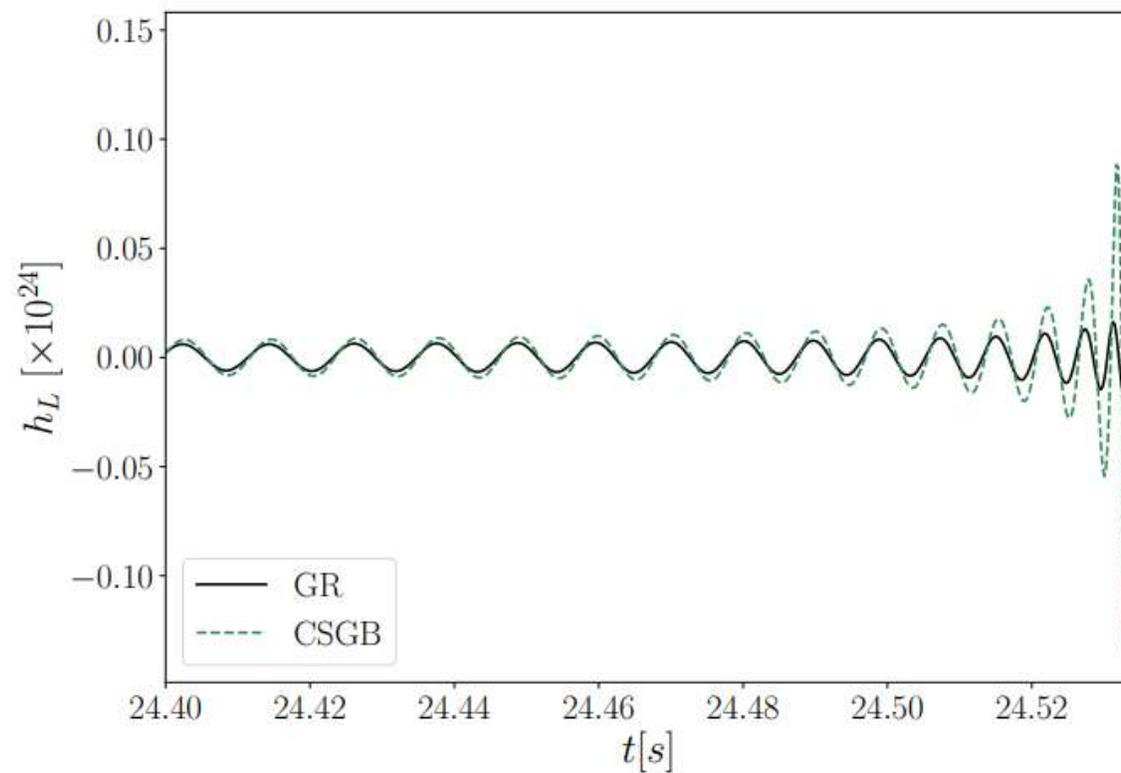
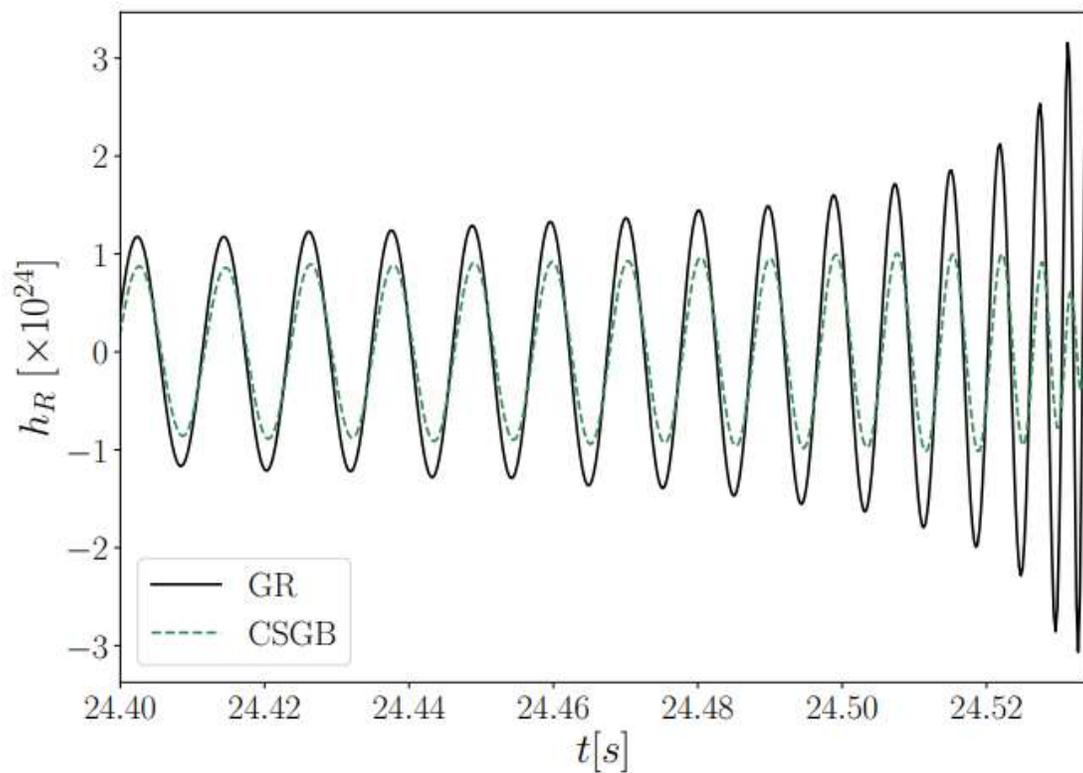
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# Constraints on Theory Parameters

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Velocity (GW170817)

$$-7 \times 10^{-16} < 1 - c_T < 3 \times 10^{-15}$$

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$$\begin{aligned} -7 \times 10^{-16} < 1 - c_T < 3 \times 10^{-15} \\ v_g^{R,L} = v_p^{R,L} = 1 - \frac{\alpha'}{4} \left( \frac{1}{a} \mathcal{H} \phi' + \phi'^2 \right) \end{aligned} \quad \left. \vphantom{\begin{aligned} -7 \times 10^{-16} < 1 - c_T < 3 \times 10^{-15} \\ v_g^{R,L} = v_p^{R,L} = 1 - \frac{\alpha'}{4} \left( \frac{1}{a} \mathcal{H} \phi' + \phi'^2 \right) \end{aligned}} \right\} \longrightarrow \tilde{\varphi}'_0 \lesssim 10^{-15} \text{ eV}^2$$

TD, Jenks & Alexander, 2024 PRD

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$$\tilde{\phi}'_0 \lesssim 10^{-22} \text{ eV}^2$$

# Extended Parameterization (Alexander, TD & Manton, 2025 [2510.25895])

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(Alexander, TD & Manton, 2025 [2510.25895])

$$h''_{\text{R,L}} + \left\{ 2\mathcal{H} + C_O^{(1)} + C_E^{(1)} \right\} h'_{\text{R,L}} + k^2 \left\{ 1 + C_O^{(0)} + C_E^{(0)} \right\} h_{\text{R,L}} = 0$$

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$$C_E^{(1)} = \sum_{n=0} k^{2n} \left[ \frac{\alpha_{2n}}{(\Lambda_E a)^{2n}} \mathcal{H} + \frac{\beta_{2n}}{(\Lambda_E a)^{2n-1}} + \frac{\mu_{2n} \mathcal{H}^2 + \rho_{2n} \mathcal{H}'}{(\Lambda_E a)^{2n+1}} \right]$$

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$$C_E^{(0)} = \sum_{m=0} k^{2m} \left[ \frac{\gamma_{2m+1}}{(\Lambda_E a)^{2m+1}} \mathcal{H} + \frac{1}{(\Lambda_E a)^{2m}} \left( \delta_{2m+1} + \frac{\nu_{2m+1} \mathcal{H}^2 + \sigma_{2m+1} \mathcal{H}'}{k^2} \right) \right]$$

# Kalb-Ramond Dark Matter

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# Kalb-Ramond Dark Matter

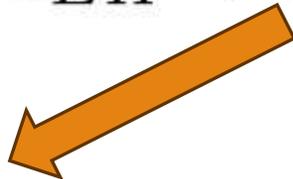
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$$\mathcal{L} = \mathcal{L}_{EH} + \mathcal{L}_B - \mathcal{L}_{int} + \mathcal{L}_m$$

# Kalb-Ramond Dark Matter

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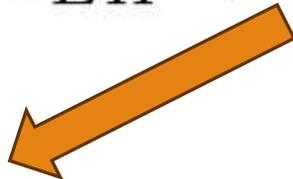
$$\mathcal{L} = \mathcal{L}_{EH} + \mathcal{L}_B - \mathcal{L}_{int} + \mathcal{L}_m$$


$$\mathcal{L}_B = \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} - V(B)$$

# Kalb-Ramond Dark Matter

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$$\mathcal{L} = \mathcal{L}_{EH} + \mathcal{L}_B - \mathcal{L}_{int} + \mathcal{L}_m$$

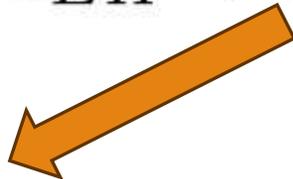

$$\mathcal{L}_B = \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} - V(B)$$

$$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$$

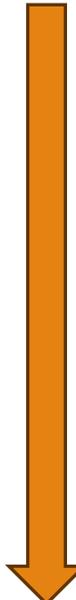
# Kalb-Ramond Dark Matter

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$$\mathcal{L} = \mathcal{L}_{EH} + \mathcal{L}_B - \mathcal{L}_{int} + \mathcal{L}_m$$


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$$\mathcal{L}_{int} = \tilde{\xi}_1 \tilde{R}^{\mu\nu\rho\sigma} B_{\mu\nu} B_{\rho\sigma} + \xi_1 R^{\mu\nu\rho\sigma} B_{\mu\nu} B_{\rho\sigma} + \xi_2 R^{\mu\rho} B_{\mu\lambda} B^\lambda_\rho + \xi_3 R B_{\mu\nu} B^{\mu\nu} + \xi_4 R B_{\mu\nu} \tilde{B}^{\mu\nu}$$

# Kalb-Ramond Dark Matter

$$\mathcal{L} = \mathcal{L}_{EH} + \mathcal{L}_B - \mathcal{L}_{int} + \mathcal{L}_m$$

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$$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$$

$$B_{\mu\nu}^{(0)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -\mathcal{V}_e a^2 & 0 & 0 \\ \mathcal{V}_e a^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{V}_m a^2 \\ 0 & 0 & -\mathcal{V}_m a^2 & 0 \end{pmatrix}$$

$$\mathcal{L}_{int} = \tilde{\xi}_1 \tilde{R}^{\mu\nu\rho\sigma} B_{\mu\nu} B_{\rho\sigma} + \xi_1 R^{\mu\nu\rho\sigma} B_{\mu\nu} B_{\rho\sigma} + \xi_2 R^{\mu\rho} B_{\mu\lambda} B^\lambda_\rho + \xi_3 R B_{\mu\nu} B^{\mu\nu} + \xi_4 R B_{\mu\nu} \tilde{B}^{\mu\nu}$$

# Kalb-Ramond Dark Matter (continued)

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# Kalb-Ramond Dark Matter (continued)

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$$h''_{R,L} + 2\mathcal{H}h'_{R,L} + k^2 \left( 1 + A + \lambda_{R,L} B \frac{\mathcal{H}}{k} + C \frac{(\mathcal{H}' + 2\mathcal{H}^2)}{k^2} \right) h_{R,L} = 0$$

# Kalb-Ramond Dark Matter (continued)

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$$h''_{R,L} + 2\mathcal{H}h'_{R,L} + k^2 \left( 1 + A + \lambda_{R,L} B \frac{\mathcal{H}}{k} + C \frac{(\mathcal{H}' + 2\mathcal{H}^2)}{k^2} \right) h_{R,L} = 0$$

---

$$A = (\tilde{\mathcal{V}}_m^2 + \tilde{\mathcal{V}}_e^2)(\xi_2 - 6\xi_1)$$

$$B = 2 \left( 2\tilde{\mathcal{V}}_e \tilde{\mathcal{V}}_m \xi_1 - \tilde{\xi}_1 (\tilde{\mathcal{V}}_m^2 - \tilde{\mathcal{V}}_e^2) \right)$$

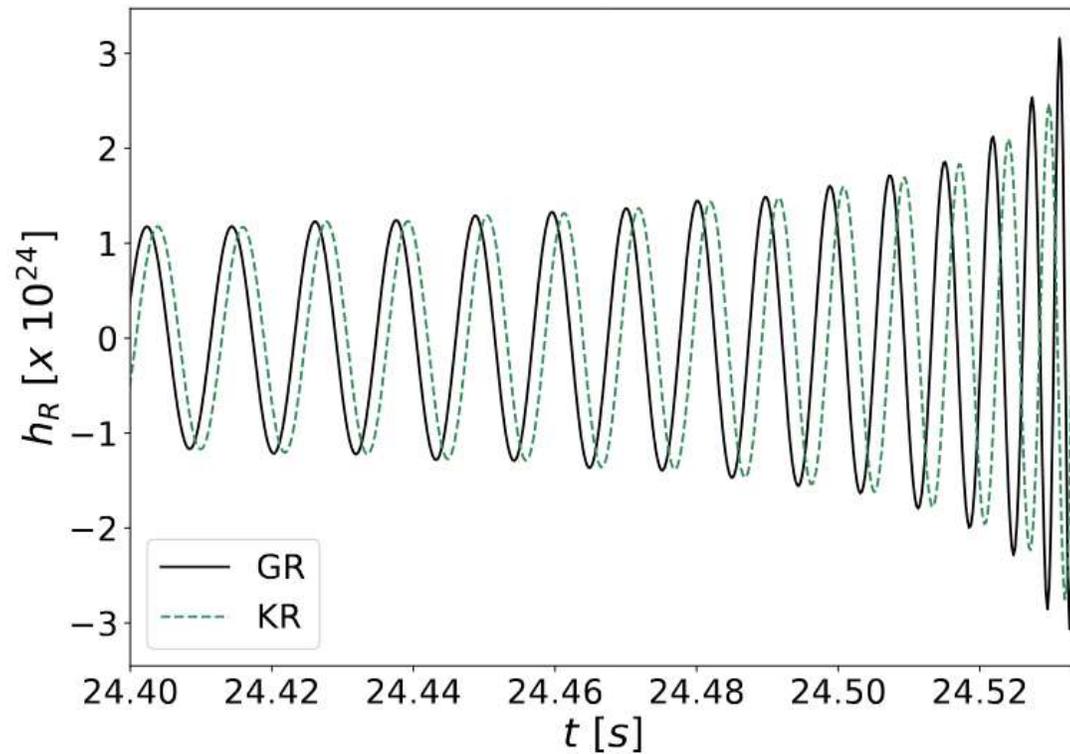
$$C = 2 \left( \tilde{\mathcal{V}}_e^2 (2\xi_1 - 2\xi_3) + \tilde{\mathcal{V}}_m^2 (2\xi_1 + 2\xi_3 - \xi_2) + 4\xi_4 \tilde{\mathcal{V}}_e \tilde{\mathcal{V}}_m \right)$$

Alexander,  
TD & Manton,  
2025 [2510.25895]

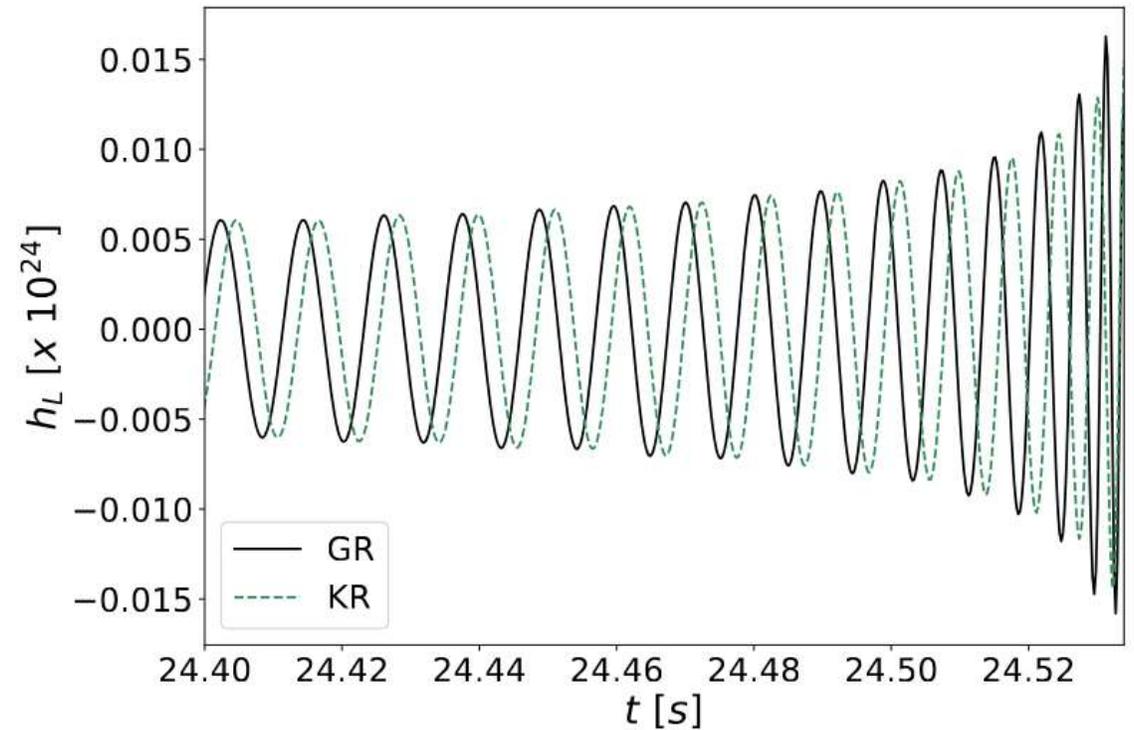
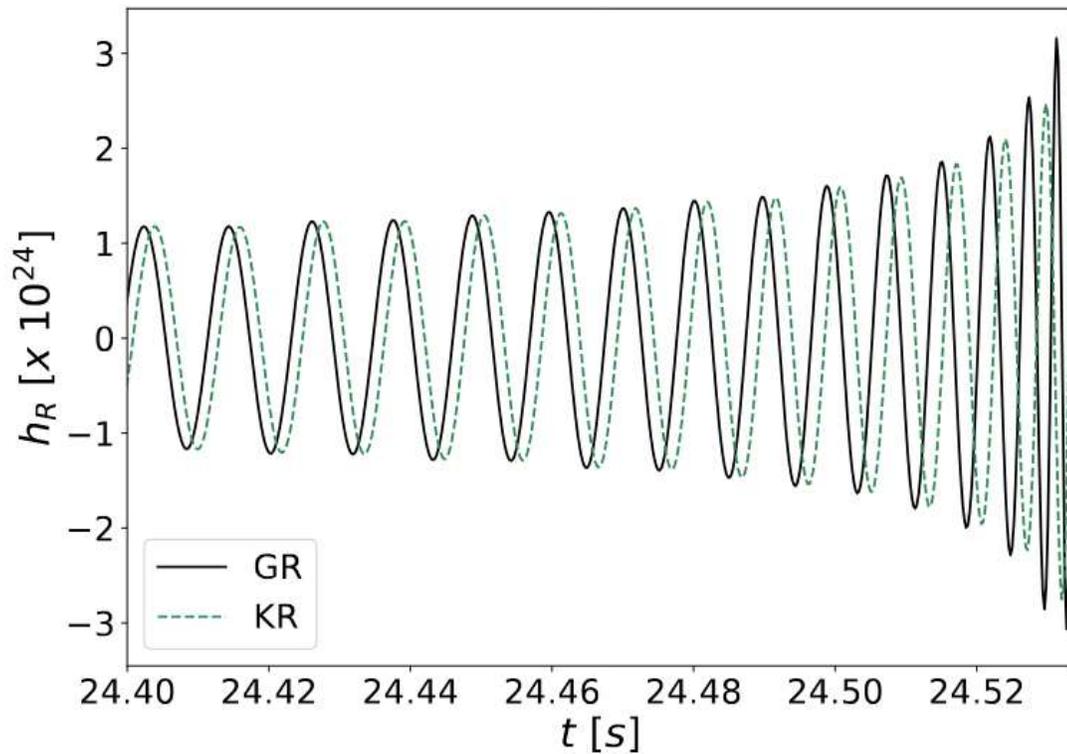
# Kalb-Ramond Dark Matter Waveforms

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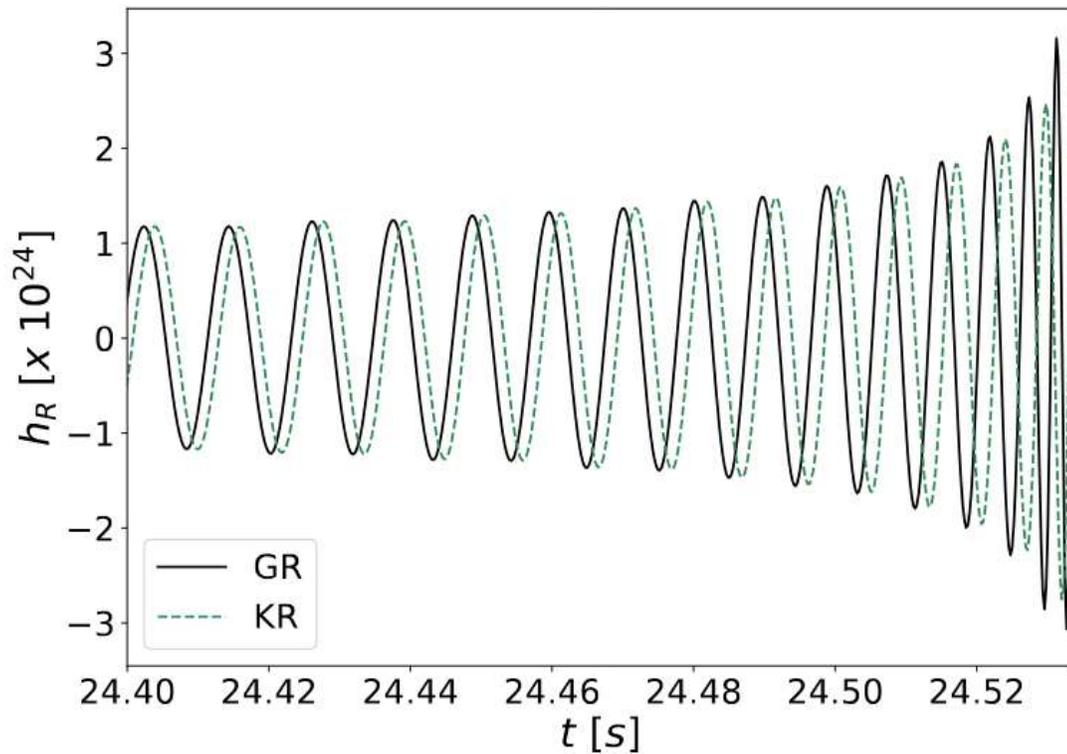
# Kalb-Ramond Dark Matter Waveforms



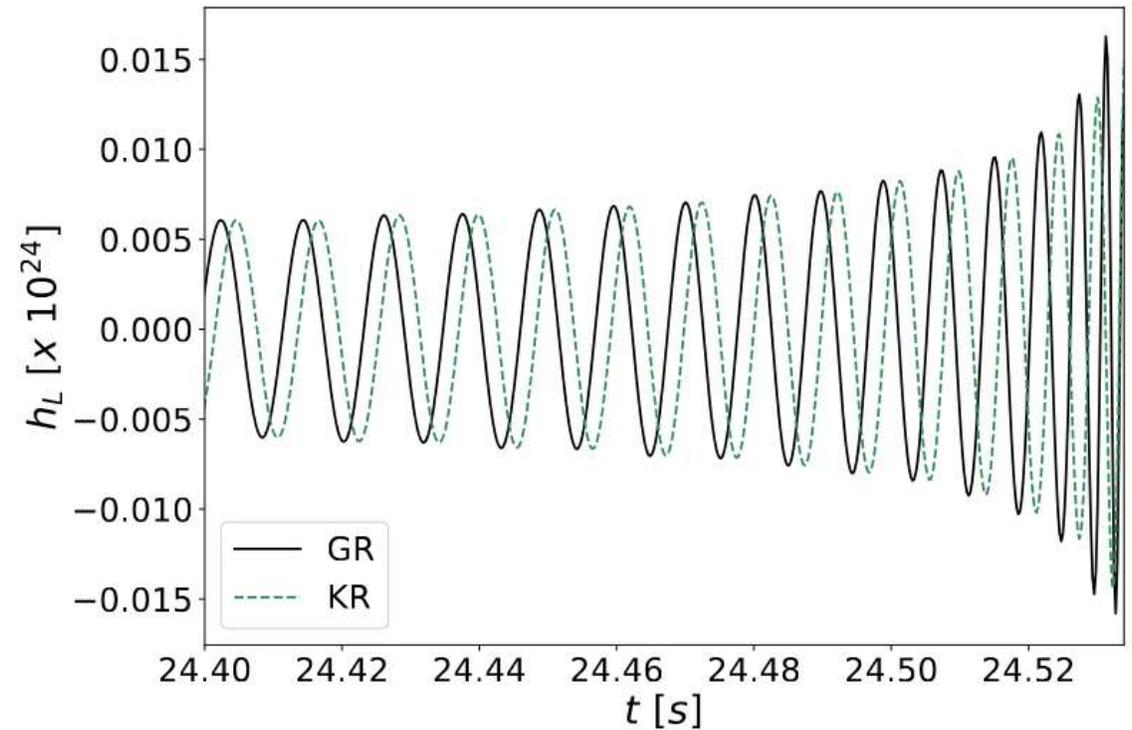
# Kalb-Ramond Dark Matter Waveforms



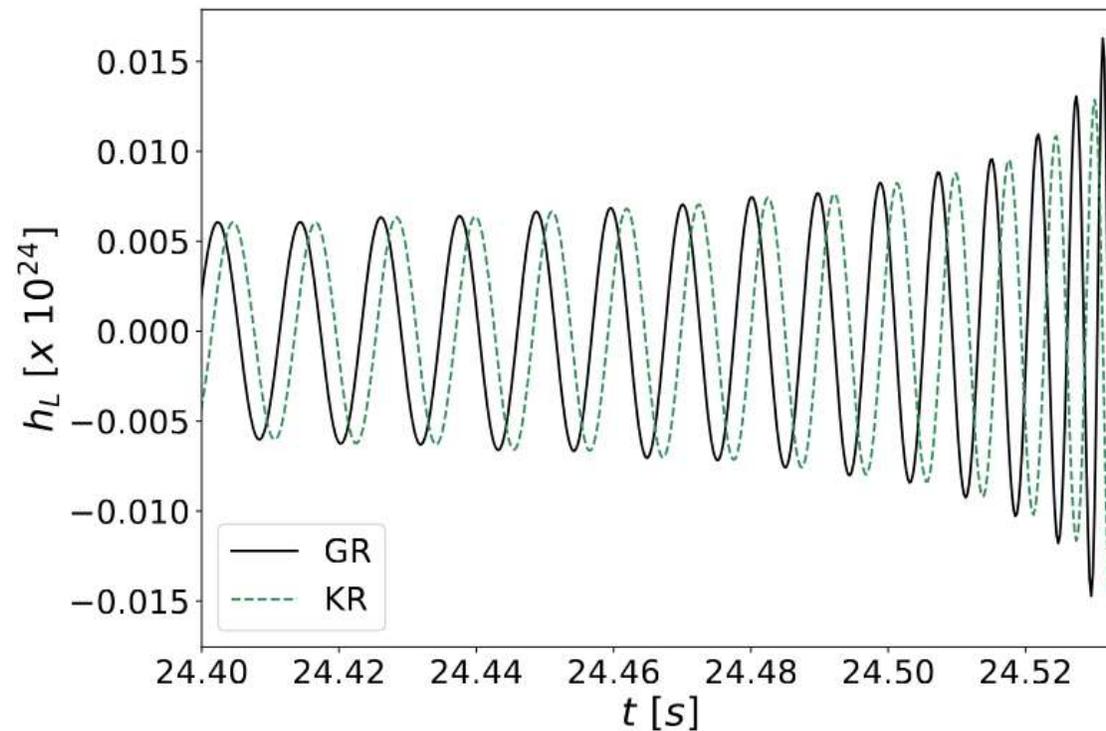
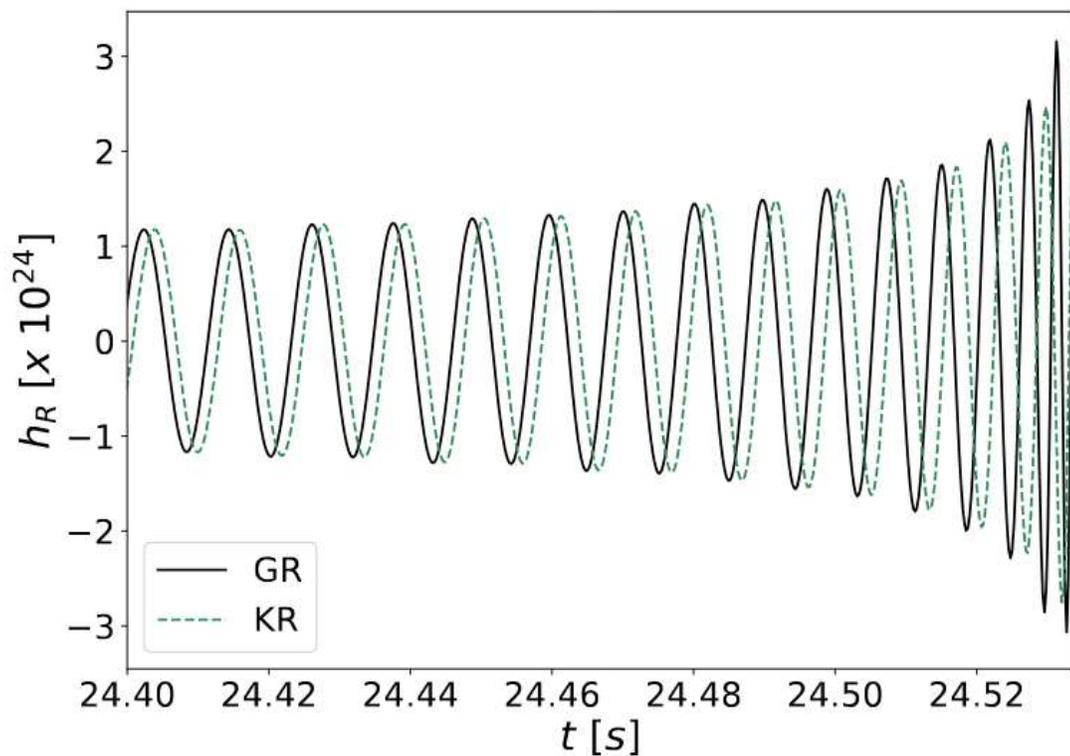
# Kalb-Ramond Dark Matter Waveforms



$$m_B \lesssim 10^{11} \text{ GeV}$$



# Kalb-Ramond Dark Matter Waveforms



$m_B \lesssim 10^{11}$  GeV    compare     $m_B \lesssim 10^6$  eV    Capanelli et al., 2024 JHEP

# U(1) fields

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$$\mathcal{L}_{em} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - A_{\mu}J^{\mu} - V(A)$$

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# U(1) fields

---

$$\mathcal{L}_{em} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - A_{\mu}J^{\mu} - V(A) \quad F_{\mu\nu} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -aB_y \\ 0 & 0 & 0 & aB_x \\ 0 & aB_y & -aB_x & 0 \end{pmatrix}$$

$$\mathcal{L}_{int} = \frac{\tilde{\xi}_1}{\Lambda_O^2} \tilde{R}^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + \frac{1}{\Lambda_E^2} \left( \xi_1 R^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + \xi_2 R^{\mu\rho} F_{\mu\lambda} F^{\lambda}_{\rho} + \xi_3 R F_{\mu\nu} F^{\mu\nu} + \xi_4 R F_{\mu\nu} \tilde{F}^{\mu\nu} \right)$$

# U(1) fields (continued)

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# U(1) fields (continued)

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$$h''_{\text{R,L}} + (2\mathcal{H} + A)h'_{\text{R,L}} + k^2(1 + B)h_{\text{R,L}} = 0$$

# U(1) fields (continued)

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$$h''_{R,L} + (2\mathcal{H} + A)h'_{R,L} + k^2(1 + B)h_{R,L} = 0$$

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$$A = -2\mathcal{H} \frac{(\xi_2 - 2\xi_3)}{(\Lambda_E a)^2} \mathcal{B}^2 + \frac{(\xi_2 - 2\xi_3)}{(\Lambda_E a)^2} \frac{d}{d\eta} \mathcal{B}^2,$$

Alexander,  
TD & Manton, 2025  
[2510.25895]

$$B = -\frac{(6\xi_1 - \xi_2)}{(\Lambda_E a)^2} \mathcal{B}^2 + \frac{1}{k^2} \frac{4(\xi_1 - \xi_2 + 3\xi_3)}{(\Lambda_E a)^2} \frac{d}{d\eta} (\mathcal{H} \mathcal{B}^2) - \lambda_{R,L} \frac{1}{k} \frac{\tilde{\xi}_1}{(\Lambda_O a)^2} \frac{d}{d\eta} \mathcal{B}^2$$

# U(1) fields (continued)

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$$h''_{R,L} + (2\mathcal{H} + A)h'_{R,L} + k^2(1 + B)h_{R,L} = 0$$

$$A = -2\mathcal{H} \frac{(\xi_2 - 2\xi_3)}{(\Lambda_E a)^2} \mathcal{B}^2 + \frac{(\xi_2 - 2\xi_3)}{(\Lambda_E a)^2} \frac{d}{d\eta} \mathcal{B}^2,$$

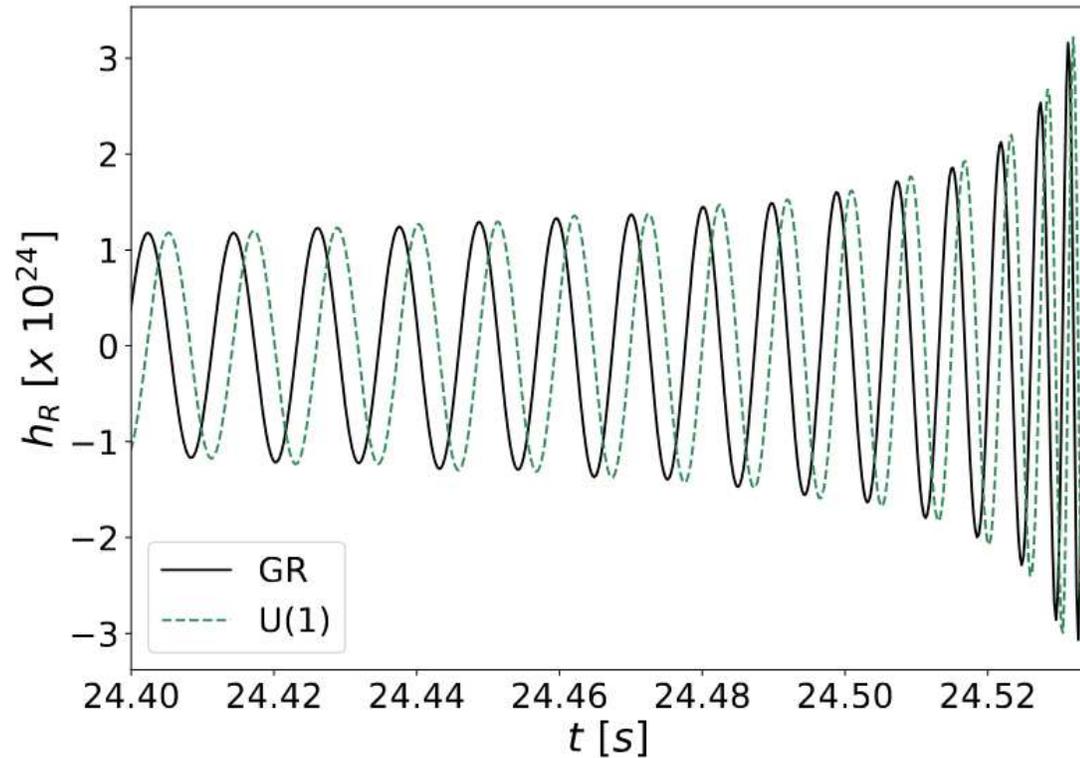
Alexander,  
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[2510.25895]

$$B = -\frac{(6\xi_1 - \xi_2)}{(\Lambda_E a)^2} \mathcal{B}^2 + \frac{1}{k^2} \frac{4(\xi_1 - \xi_2 + 3\xi_3)}{(\Lambda_E a)^2} \frac{d}{d\eta} (\mathcal{H} \mathcal{B}^2) - \lambda_{R,L} \frac{1}{k} \frac{\tilde{\xi}_1}{(\Lambda_O a)^2} \frac{d}{d\eta} \mathcal{B}^2$$

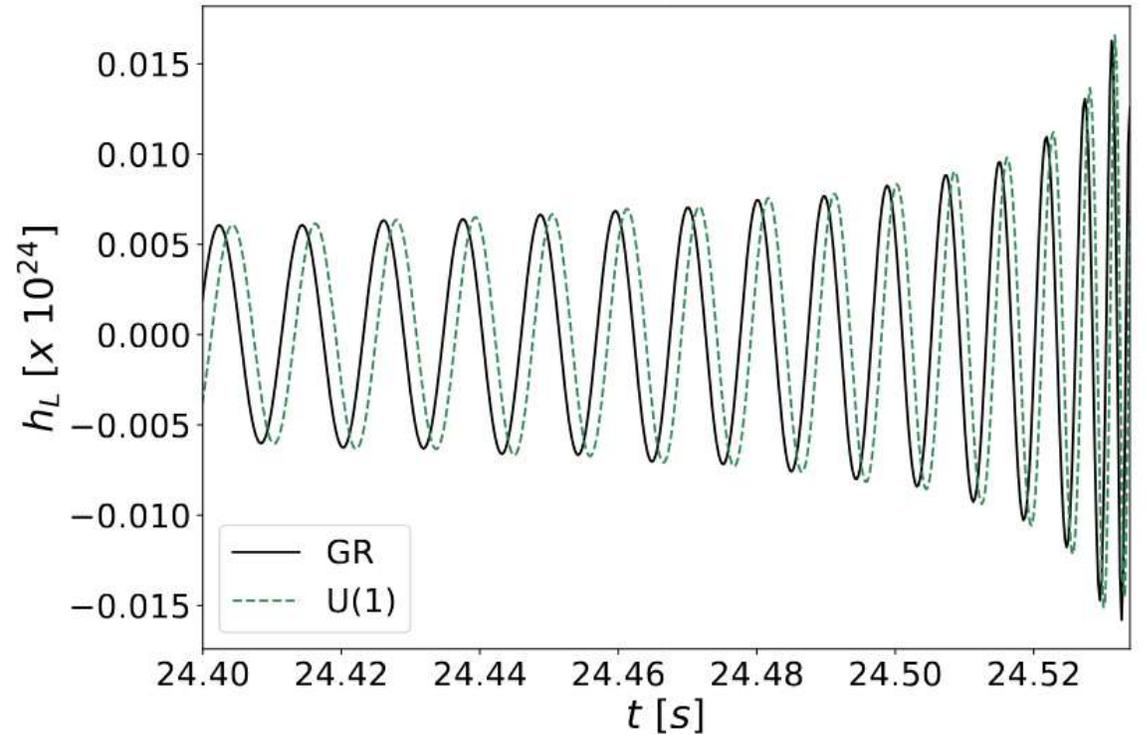
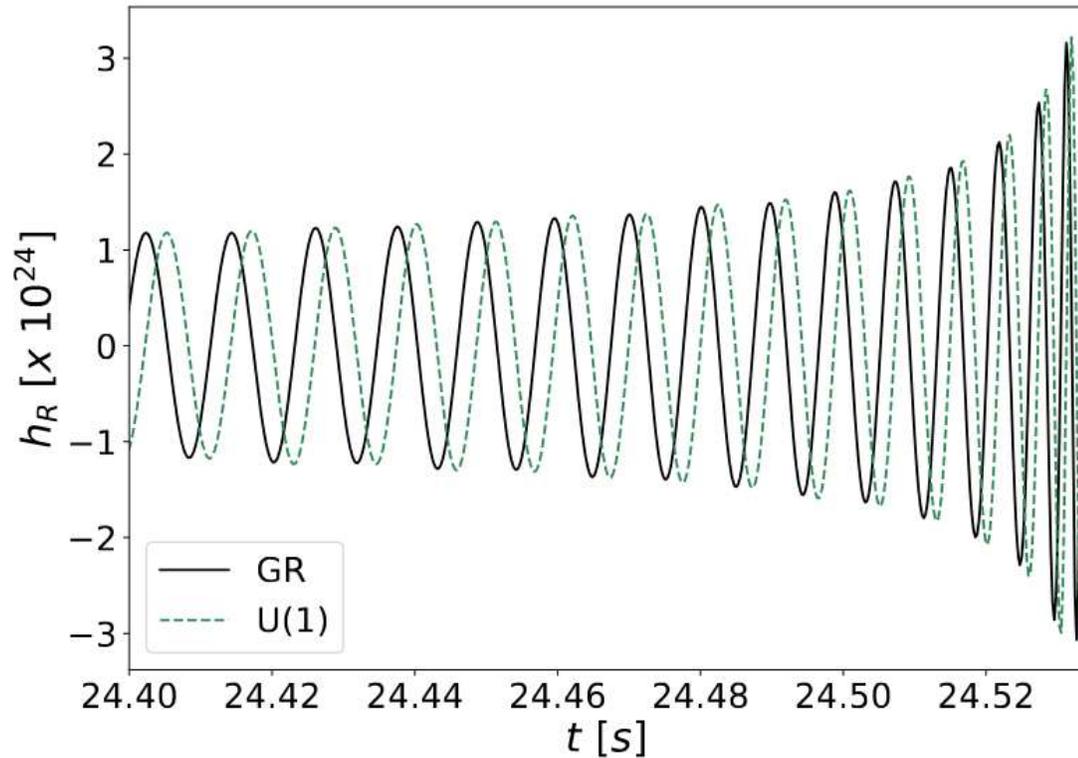
# Waveforms for U(1) fields

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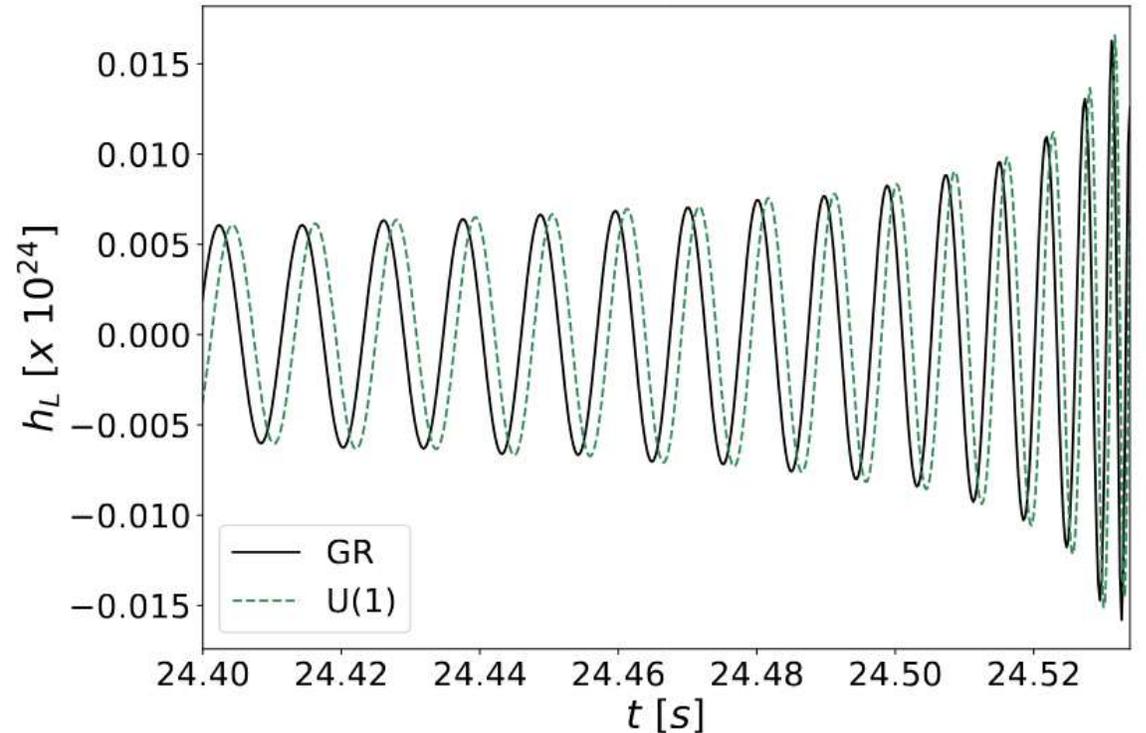
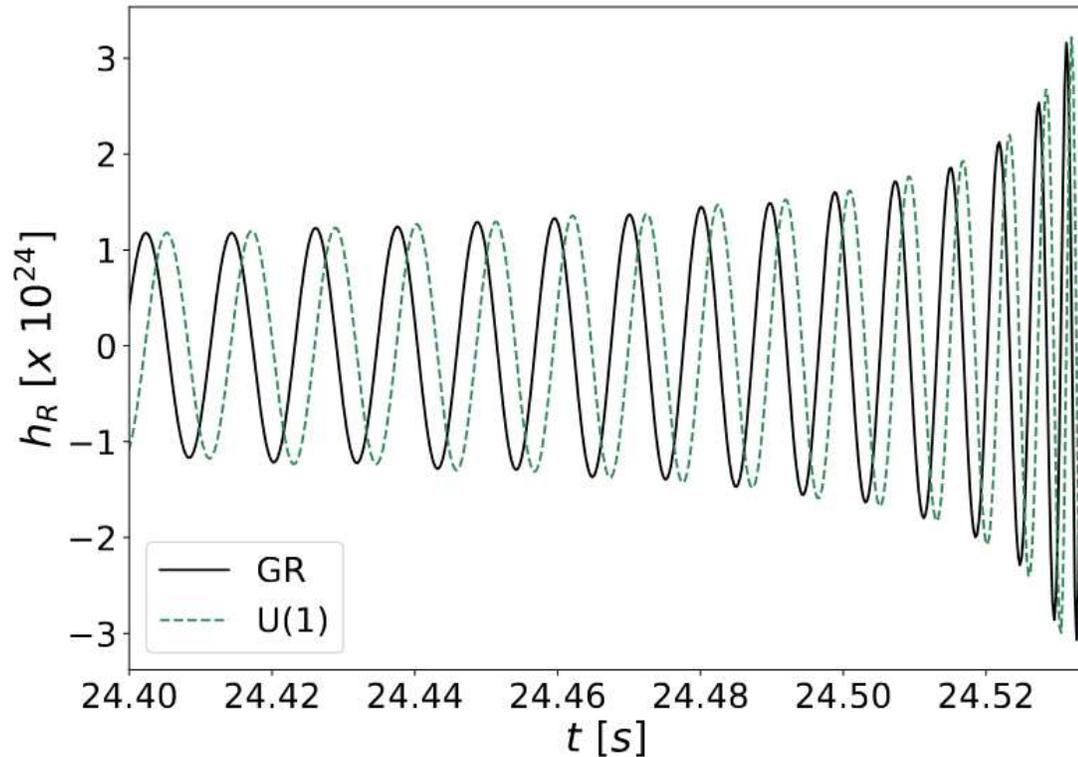
# Waveforms for U(1) fields



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# Waveforms for U(1) fields



$$\frac{|B|^2}{\Lambda^2} < 10^{23} \text{ (GeV)}^2$$

# Summary

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GR agrees well with observations, but challenges

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- Dark matter and dark energy

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Discussed model that gives both (CS-GB gravity)

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Extended framework to DM theories