



Challenge Gravitational Wave Measurement with a Novel Mössbauer Spectrometer

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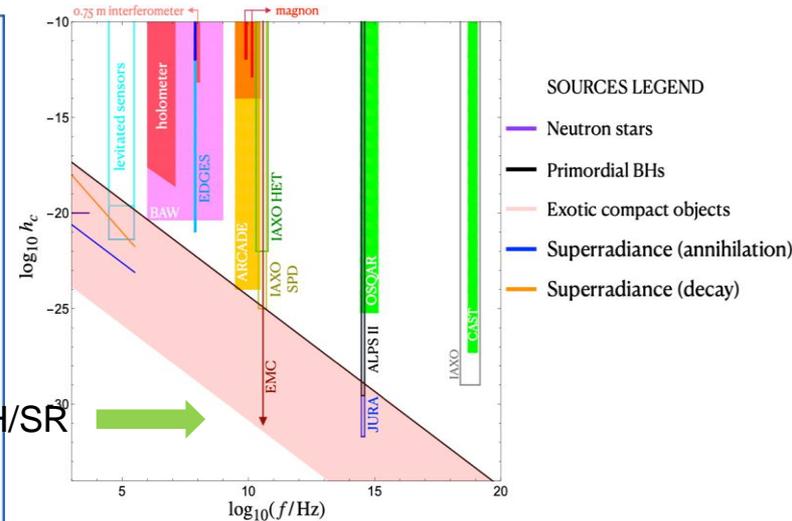
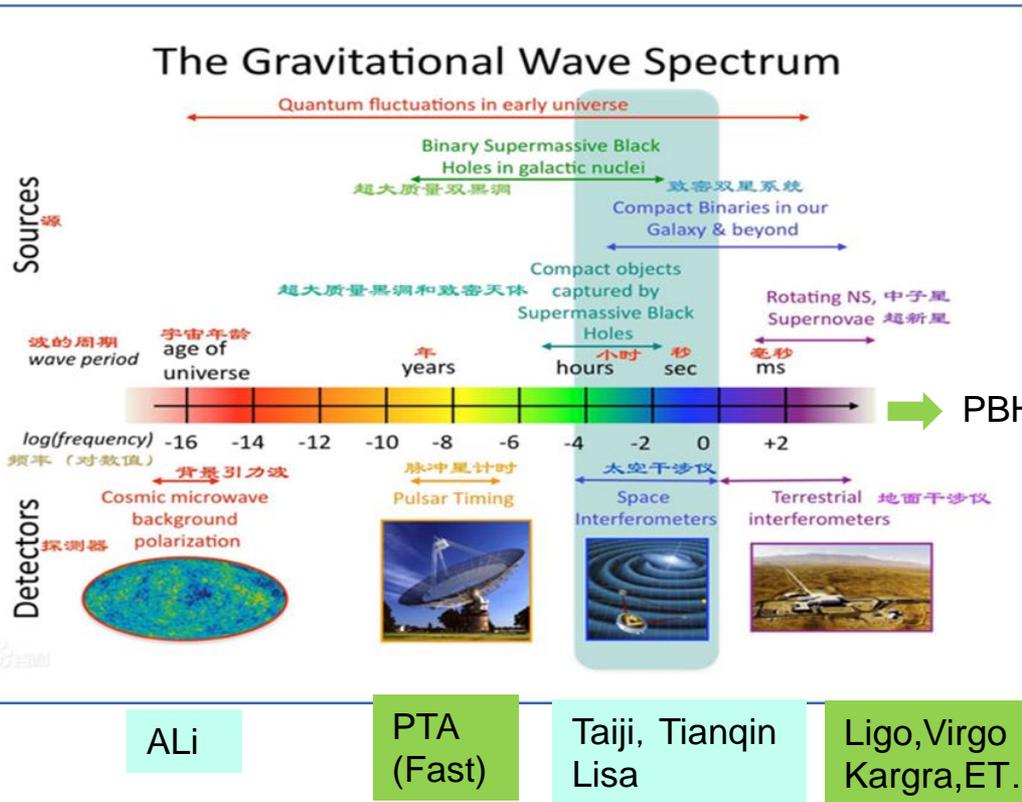
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Outline

- Introduction
- How to achieve precision and speed needed for gravitational wave use novel Mossbauer Spectrometer
- Setup for Multi-band gravitational wave detection use this novel Mossbauer Spectrometer
- Summary and proposals

Gravitational Wave

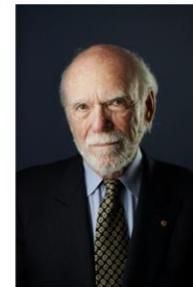
GW: a new way to explore our universe



The Nobel Prize in Physics 2017



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Accuracy needed at the order of thickness of a paper over the diameter of solar system

Mössbauer effect

- Recoil-less emission and resonant absorption of nuclear transition photons
 - Mössbauer active nucleus bound to solid state lattice
 - Thermal vibration ($\sim 10^{13}$ Hz) much faster than nuclear transitions (lifetime $\sim > 10^{-7}$ s)
 - Effect averages to zero, no 1st Doppler broadening
 - Recoil to the whole crystallite ($> 10^{14}$ atom)
 - Without phonon excitations \rightarrow recoil free

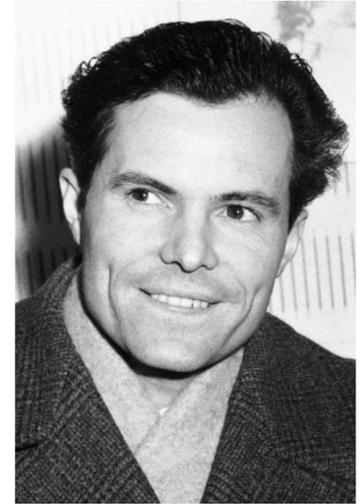
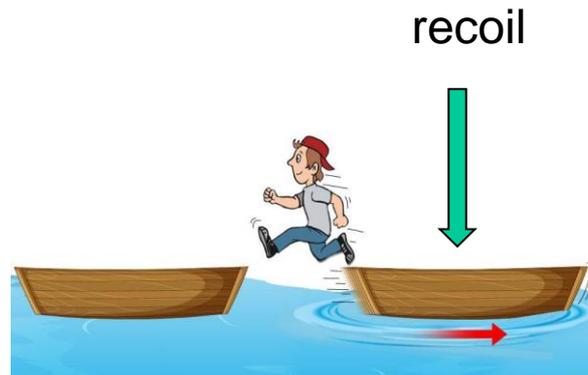
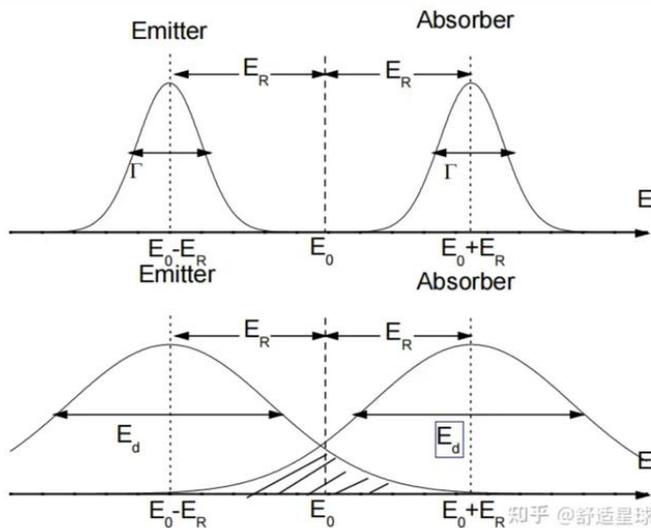
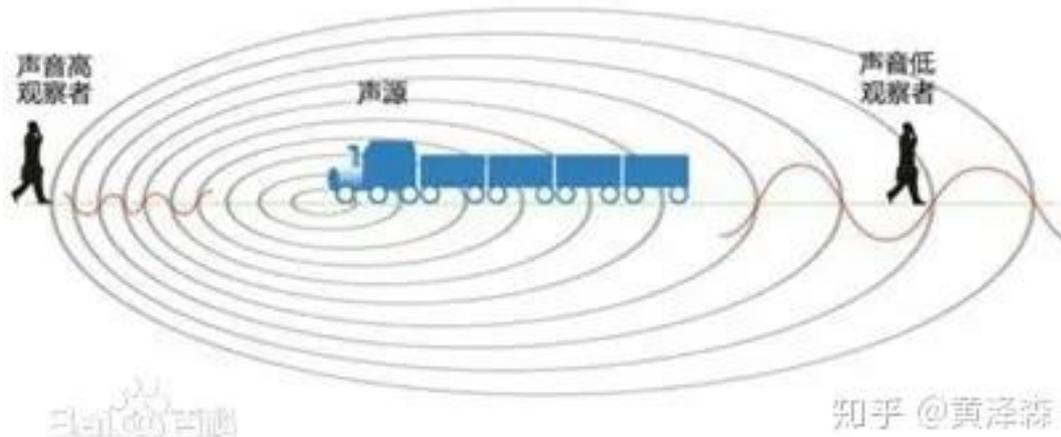


Photo from the Nobel Foundation archive.
Rudolf Ludwig Mössbauer
Prize share: 1/2

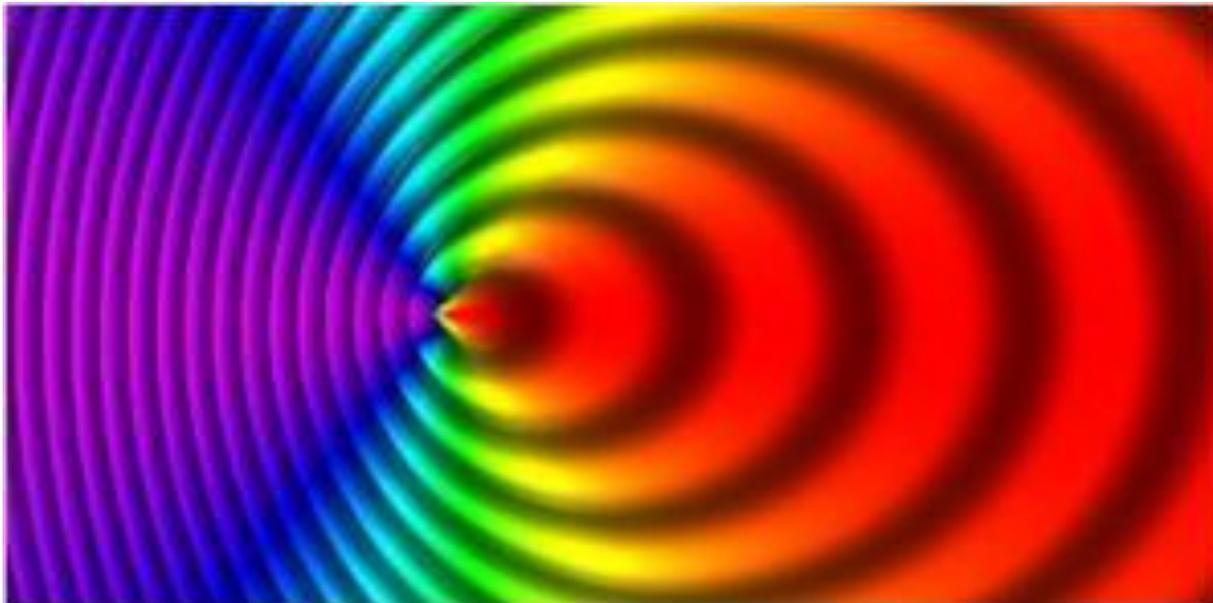


Doppler shift

- Sound

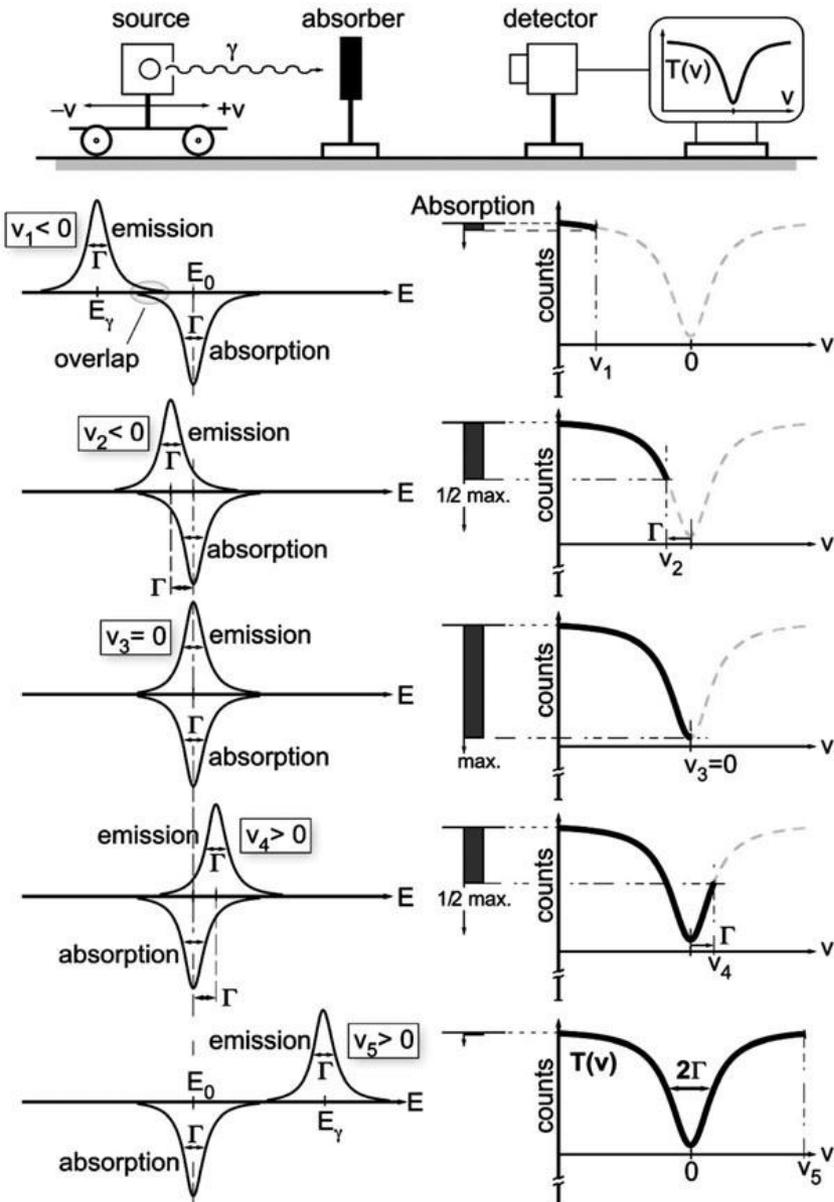


- Photon: $E=h\nu$



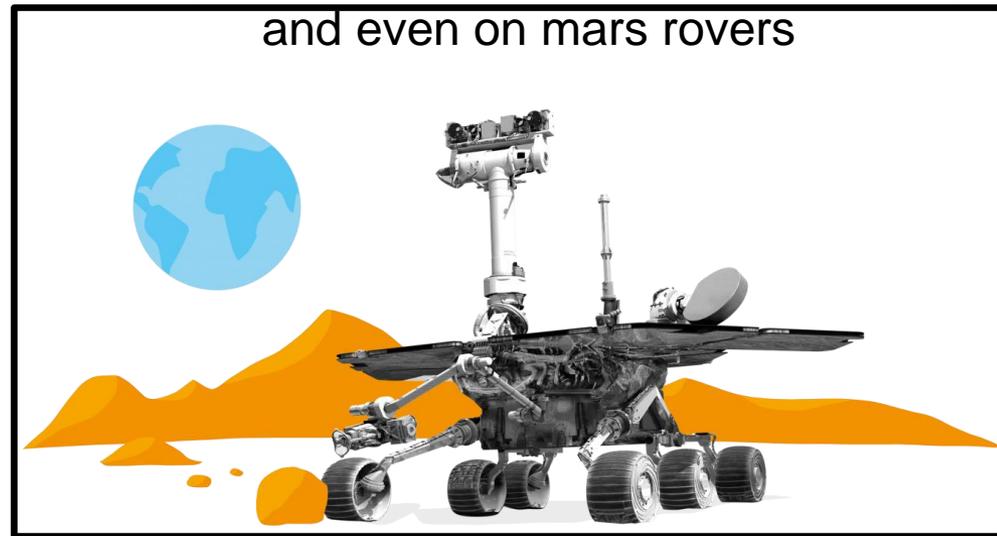
Red shift

“Traditional” Mössbauer Spectrometer



- Change the recoil-less emission and absorption overlap with doppler effects
- Strength of recoilless nuclear resonant absorption as determined by the “overlap” of emission and absorption lines when the emission line is shifted by Doppler modulation
- High accuracy of the resonant absorption width w.r.t. photon energy

Widely used in material science, chemistry, and even on mars rovers



Relativity test with Mössbauer effect

- Gravitational redshift test with Mössbauer effect



Jefferson laboratory at Harvard University. The experiment occurred in the left "tower". The attic was later extended in 2004.

Pound, Rebka & Snyder (1960-1965)

Harvard Tower Experiment

In just 22.6 meters, the fractional gravitational red shift given by

$$\nu = \nu_0 \left[1 + \frac{gh}{c^2} \right]$$

is just 4.92×10^{-15} , but the Mössbauer effect with the 14.4 keV gamma ray from iron-57 has a high enough resolution to detect that difference. In the early 60's physicists Pound, Rebka, and Snyder at the Jefferson Physical Laboratory at Harvard measured the shift to within 1% of the predicted shift.

Observation of a height-induced frequency shift

$$ghc^{-2} \sim 4.92 \times 10^{-15}$$

Mössbauer for Gravitational Wave

- Idea since 1970
 - Photon frequency varies when it propagates in an un-even space-time background.
 - No experimental proposal till our work
- Gave way to clock-based experiments in later tests of “static” gravity.
 - See [K. Hentschel, *Annals of Science* 53, 269–295 \(1996\)](#)

Published: 11 July 1970

Redshift Fluctuations arising from Gravitational Waves

[WILLIAM J. KAUFMANN](#)

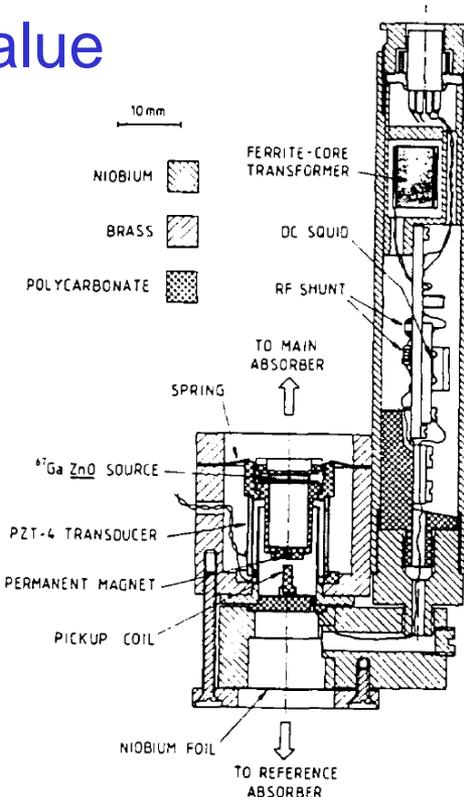
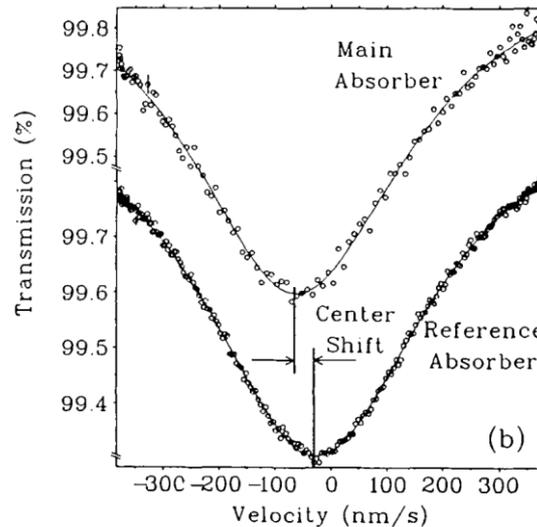
[Nature](#) 227, 157–158 (1970) | [Cite this article](#)

350 Accesses | 34 Citations | 3 Altmetric | [Metrics](#)

It should be noted that the gravitational waves which Weber^{4,5} claims to have observed at 1,660 Hz are too weak to be detected by the method suggested in this paper. A gravitational radiation flux of 10^4 ergs $\text{cm}^{-2} \text{s}^{-1}$ determined by Weber corresponds to the $h_{\mu\nu}$ being several orders of magnitude below the present limits of detectability of the Mössbauer effect ($h_{\mu\nu} \sim 10^{-18}$). Nevertheless, we might expect that refined techniques using the Mössbauer effect will one day become important tools in the detection of gravitational radiation.

Issues in traditional Mössbauer Gravity tests

- A cryogenic ^{65}Zn measurement of the local g -value (Potzel et.al. 1992)
- Differential measurement of the resonance with a sinusoidal oscillator
 - Resonance is achieved
 - g -value is off, possibly due to various line-shifts
 - Take hours-days to achieve high acc.



“such solid-state effects might be difficult there might exist two exceptions. The first are null redshift experiments, in particular measurements with stationary source and absorber”.

Can be improved with a stationary scheme:
Quoted from the paper

Idea of the stationary measurement

- Doppler shift

$$1 + z = \frac{1 + v \cos(\theta)/c}{\sqrt{1 - v^2/c^2}}$$

$$V=1\text{mm/s} \rightarrow \sim 10^{-12}$$

==>

- Gravitational shift

$$1 + z = \frac{1}{\sqrt{1 - \frac{2GM}{rc^2}}}$$

$$\Delta h = 1\text{mm} \rightarrow \sim 10^{-19}$$

$\Delta h \ll r$



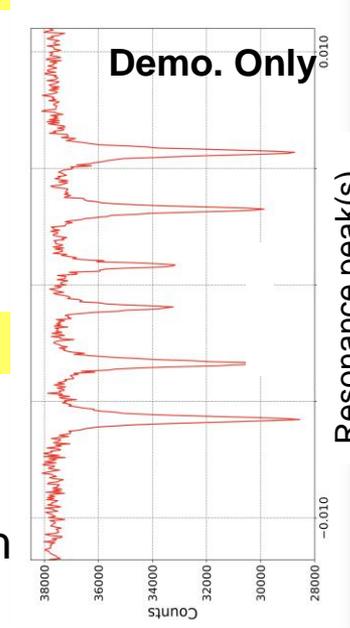
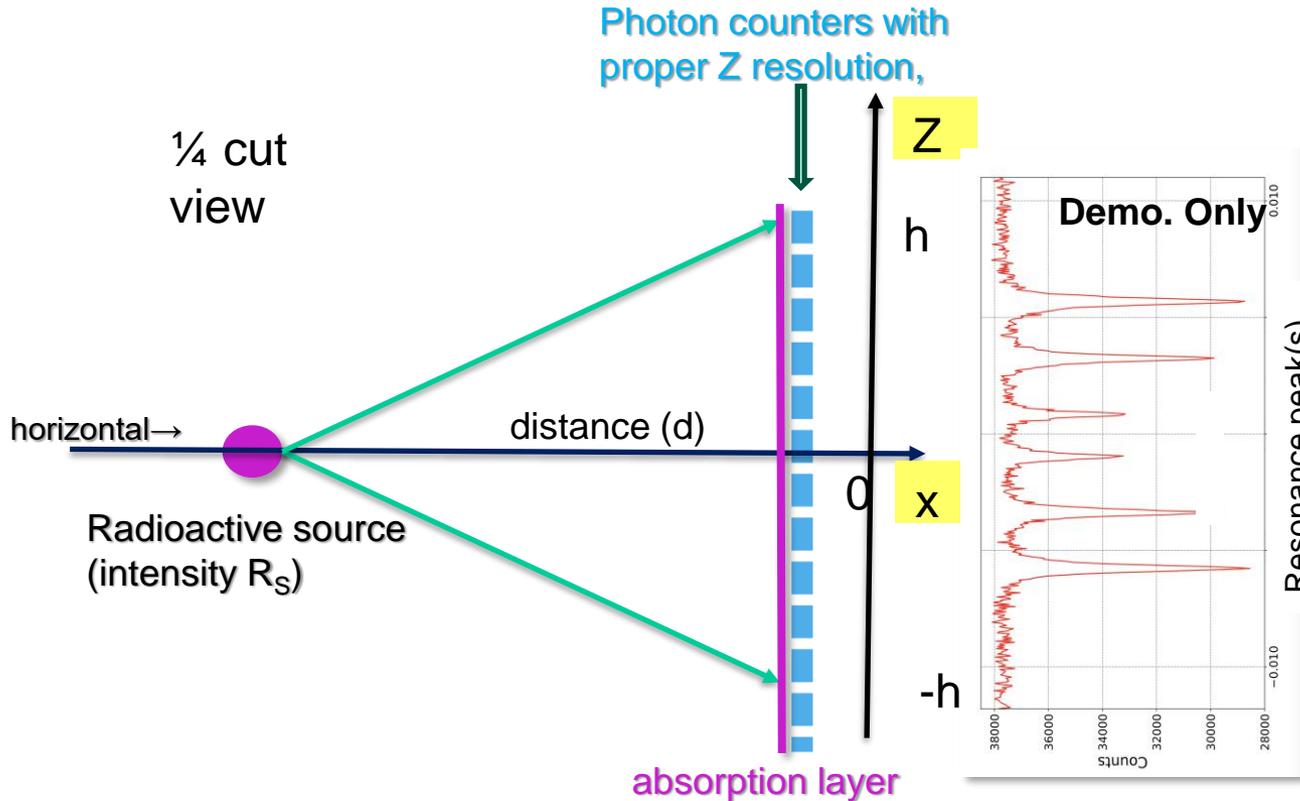
$$\frac{\delta E}{E} = z = \frac{g}{c^2} * \Delta h$$

Improve on sensitivity to energy shift



Design of a gravitational shift Mössbauer Spectrometer

- Replace Doppler shift with gravitational shift
- Parallel readout the resonant absorption shape
- Time dependent resonance's **height-shift** (not absolute height)
 - Frequency of resonance peak height shift → GW frequency: **multi-band**



Energy loss E_R is compensated by a slight height difference between absorber and source: *can be calibrated in advance.*

--- the absolute height of resonance (Z_0) is affected by large systematics: 2nd Doppler, chemical composition, etc.

--- but its time-dependent shift under GW is *not* affected.

The choice of isotope source: ^{109}Ag

^{109}Ag Isotope Properties

Isotopic abundance 48.161(5)%

Ground state properties:

? = -0.130563(23) nm

Excited state properties:

E = 88.0341(11) keV

$E_R = 4.3544(9) \cdot 10^{-2}$ eV

? = 4.400(6) nm

Q = 1.02(12)b

$T_{1/2} = 39.6(2)$ s

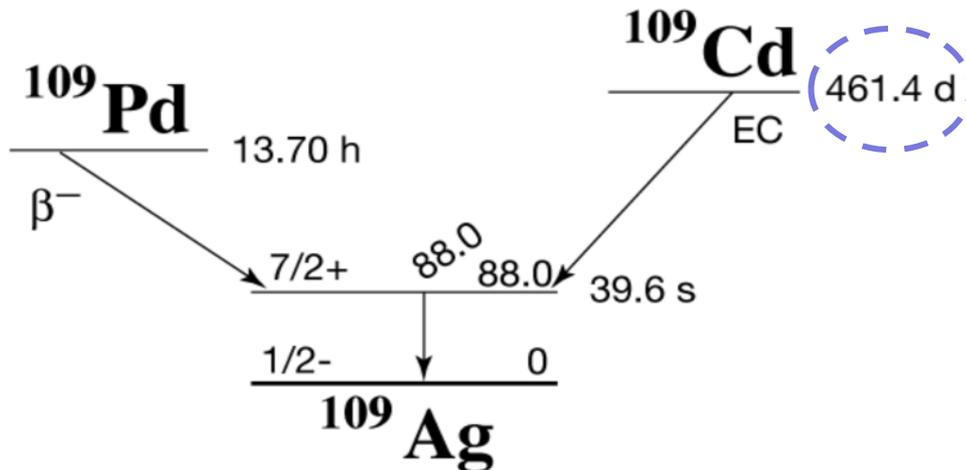
W = $7.9(2) \cdot 10^{-11}$ mm/s

Unit Conversion:

1mm/s = 71.0043(9) MHz

1mm/s = $2.9365(4) \cdot 10^{-7}$ eV

Decay Diagram



- Narrow 88 keV linewidth: $O(10^{-22})$ sensitivity
- Workable $\Delta Z \sim 10\mu\text{m}$ under terrestrial (1 g) gravity field for $\Gamma_{exp} = 4.1\Gamma$
- Long parent nuclei lifetime: 461 days allow for sufficient operation time
- IC ~ 26 , small fraction of gamma emitted

The quest of the ^{109}Ag resonance:

$\Gamma_{exp} \sim 30\Gamma$, ([W. Wildner and U. Gonser, 1979](#))

Improved resonance resolution, w broadening factors down to 16 (US)

R. D. Taylor and G. R. Hoy, SPIE **875**, 126 (1988).

S.RezaieSerej, G. R. Hoy, and R. D. Taylor, Laser Phys. **5**, 240 (1995).

Russian group: improvements with Grav. Effects

V. G. Alpatov, et.al. Laser Physics **17**, 1067–1072 (2007).

Yu. D. Bayukov, et.al. JETP Letters **90**, 499–503 (2009).

Mössbauer [database](#) (DICP, CAS)

The transmission integral

- Photon emission at source

- Fractional recoil-less : f_s ,
- Fractional recoiled: $1-f_s$

$$f(T) = \exp \left[\frac{-3E_\lambda^2}{k_B \Theta_D M c^2} \left\{ \frac{1}{4} + \left(\frac{T}{\Theta_D} \right)^2 \int_0^{\Theta/T} \frac{x}{e^x - 1} dx \right\} \right]$$

- Photon absorption at absorber

- Mass attenuation effect: all photons
- Resonant absorption: only resonant emitted photons

- The transmission integral (textbook)

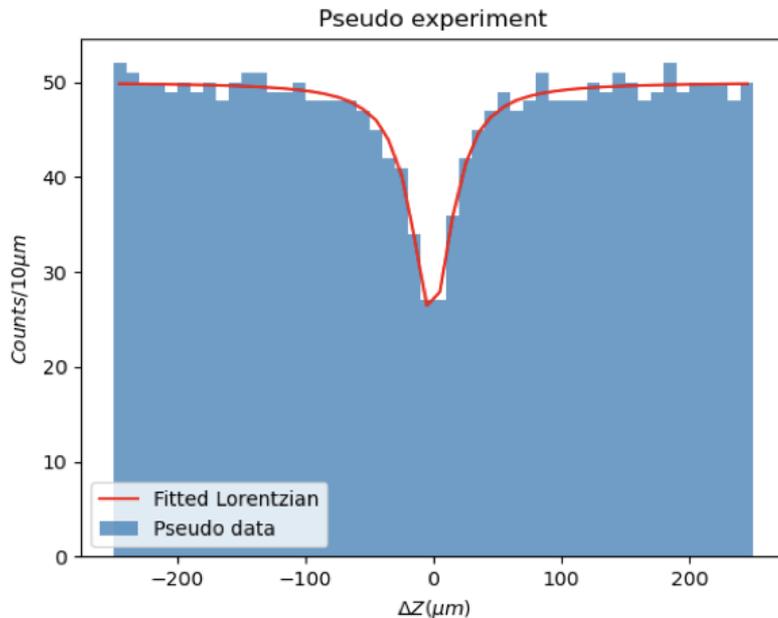
- Replaced the doppler shift with gravitational shift

$$C(Z) = \dot{N}_0 e^{-\mu_e t'} \cdot \left[(1 - f_s) + \int_{-\infty}^{\infty} f_s \xi(Z_S, E_0) \cdot e^{-t\xi(Z, E_0 + \Delta E_0)\Gamma/2\pi} dE \right]$$

$$\xi(Z, E_0) \equiv \frac{\Gamma/2\pi}{[E - g(Z - Z_S)E - E_0]^2 + (\Gamma/2)^2},$$

Accuracy from Pseudo experiments

- Pseudo experiment simulation with reasonable parameter
 - Ex: vertical z-position resolution $10\ \mu\text{m}$, detection efficiency $\sim 100\%$
 - Readout frequency $\sim 10 f_{\text{GW}}$, Energy resolution exclude non-88 keV BG
- Sensitivity extracted by fitting the resonance absorption peak



Recoil free fraction	C_∞			
	50	500	5000	50000
f_S				
0.05*	-	-	-	1.2e-22
0.10	-	-	1.3e-22	3.8e-23
0.20	-	1.3e-22	4.5e-23	1.4e-23
0.30	-	7.9e-23	1.9e-23	7.0e-24
0.40	-	4.8e-23	1.5e-23	4.5e-24
0.50	-	3.3e-23	9.4e-24	2.9e-24
0.60	7.3e-23	2.2e-23	7.2e-24	2.1e-24
0.70	5.0e-23	1.5e-23	5.0e-24	1.5e-24
0.80	4.1e-23	1.2e-23	4.0e-24	
0.90	3.7e-23	9.5e-24	3.1e-24	

* for metallic silver

Silver alloy/compound with higher T_{debye} helps improve f_S
 e.g. AgB_2 has a higher T_{debye} and $f_S = 0.2$ (@ 4K)

Accuracy compatible to the accuracy needed for GW strain

Parallel measure of resonance absorption shape: fast enough for GW

The GW signal: photon energy shifts

Energy shift due to different space-time matrix for photon emission and absorption

Consider a plain-wave strain perturbation

$$h(\mathbf{x}, t) = h_0 e^{i(\omega t - \mathbf{k} \cdot \mathbf{x})}$$

$$ds^2 = c^2 dt^2 - [1 + h] dx^2 - [1 - h] dy^2 - dz^2$$

A particle's 4-momentum response to GW strain after one-way propagation:

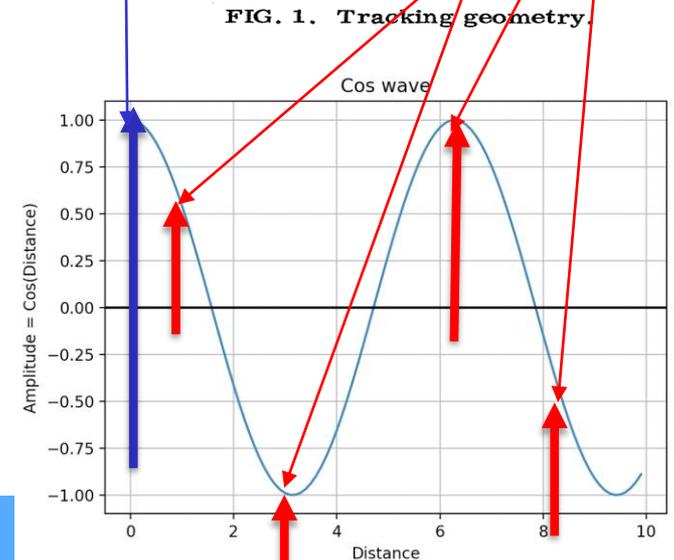
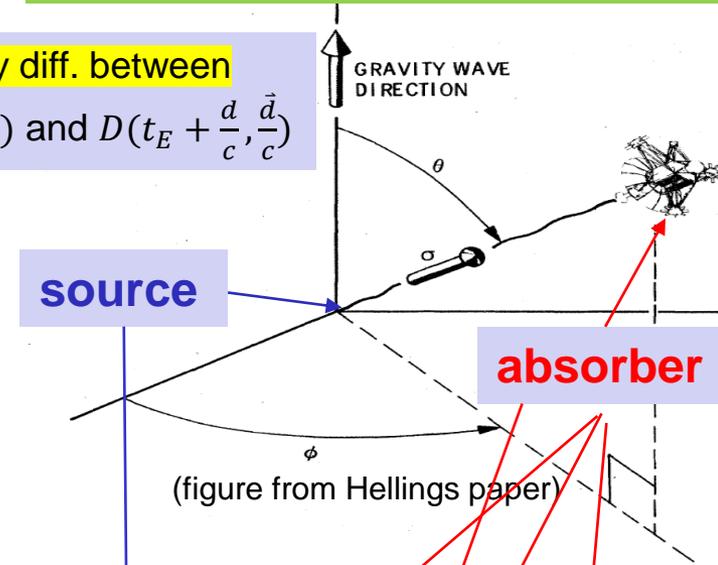
$$\frac{\Delta f}{f_\gamma} = \frac{\ell^\mu \ell^\nu}{1 - \cos \theta} [h_{\mu\nu}^D - h_{\mu\nu}^E]$$

$$\ell^\mu = f_\gamma (1, \sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$$

$$\frac{\Delta f}{f_\gamma} = 2h_0 \cos^2 \frac{\theta}{2} \cos 2\phi \sin \left(\omega d \sin^2 \frac{\theta}{2} \right) \cdot \sin \left(\omega t - \omega d \cos^2 \frac{\theta}{2} \right),$$

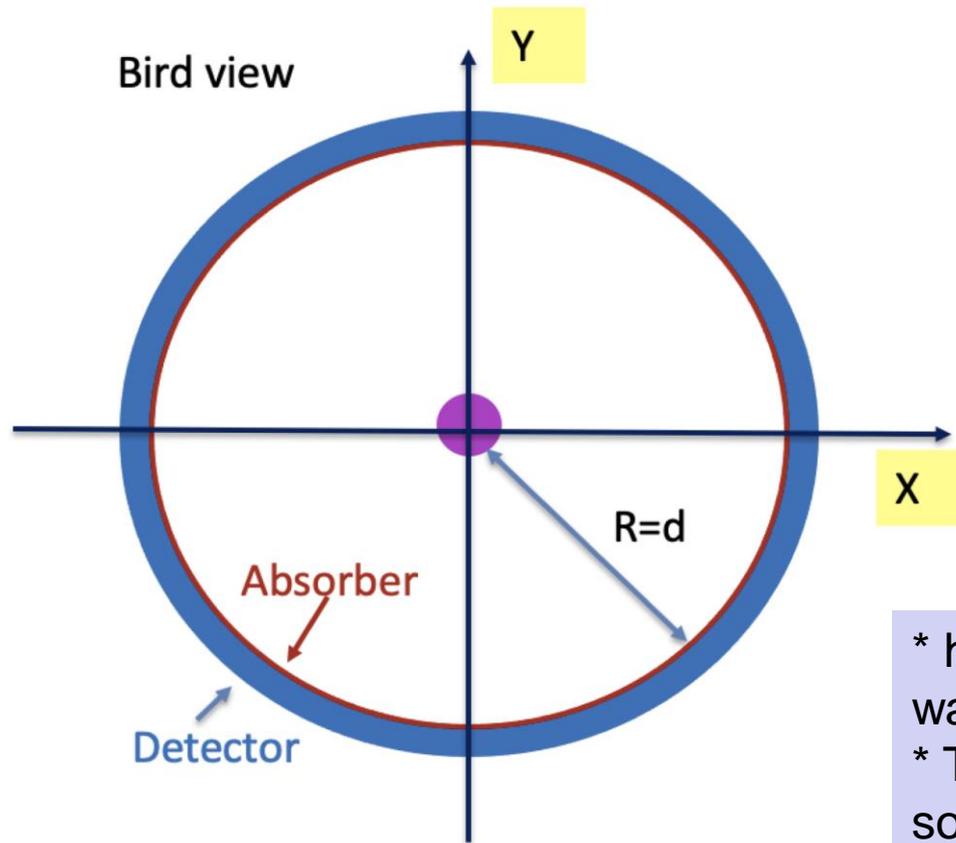
Estabrook and Wahlquist, Gen. Relat. Gravit. 6, 439–447 (1975); Hellings, Phys. Rev. D 23, 832–843 (1981).

Energy diff. between $E(t_E, \vec{0})$ and $D(t_E + \frac{d}{c}, \frac{\vec{d}}{c})$



Stationary Mössbauer GW setup

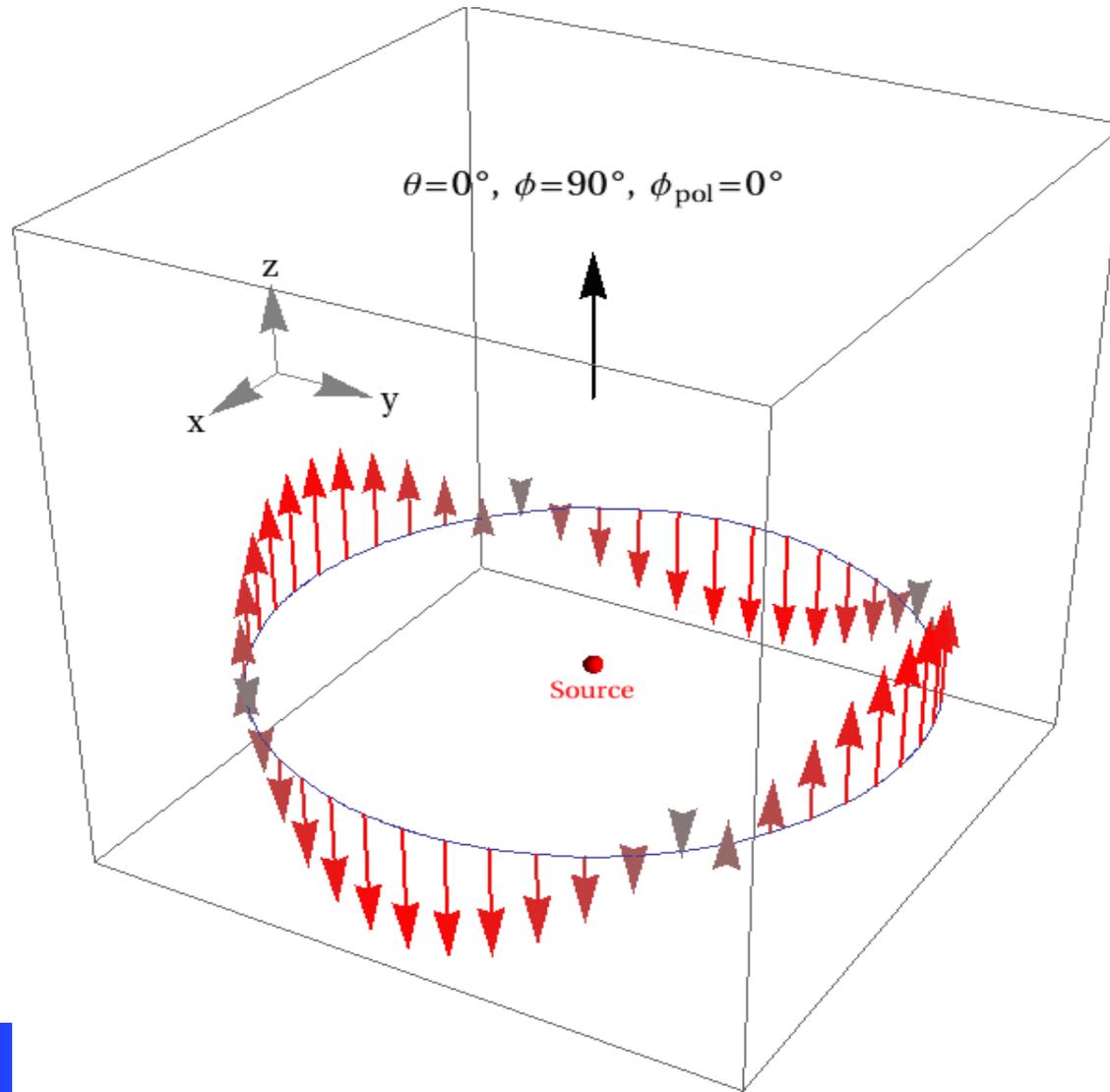
- Absorber+detectors are in the resonance plane around source
 - At least two perpendicular directions relative to any GW incident θ angle.
 - Enhance acceptance of resonances photons



* highest sensitive to GW wavelength $\sim d/2$
* The distance d between source and absorber can be tabletop size

Hypothetical GW signal “seen” by our experiment

- Modulation of resonance peak shifts in the ring
 - Allow GW direction and polarization reconstruction



Expected sensitive on GWs

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A^C: 10⁶ periods: table top

B: single period

Max Freq. Reach:
cutoffs at:

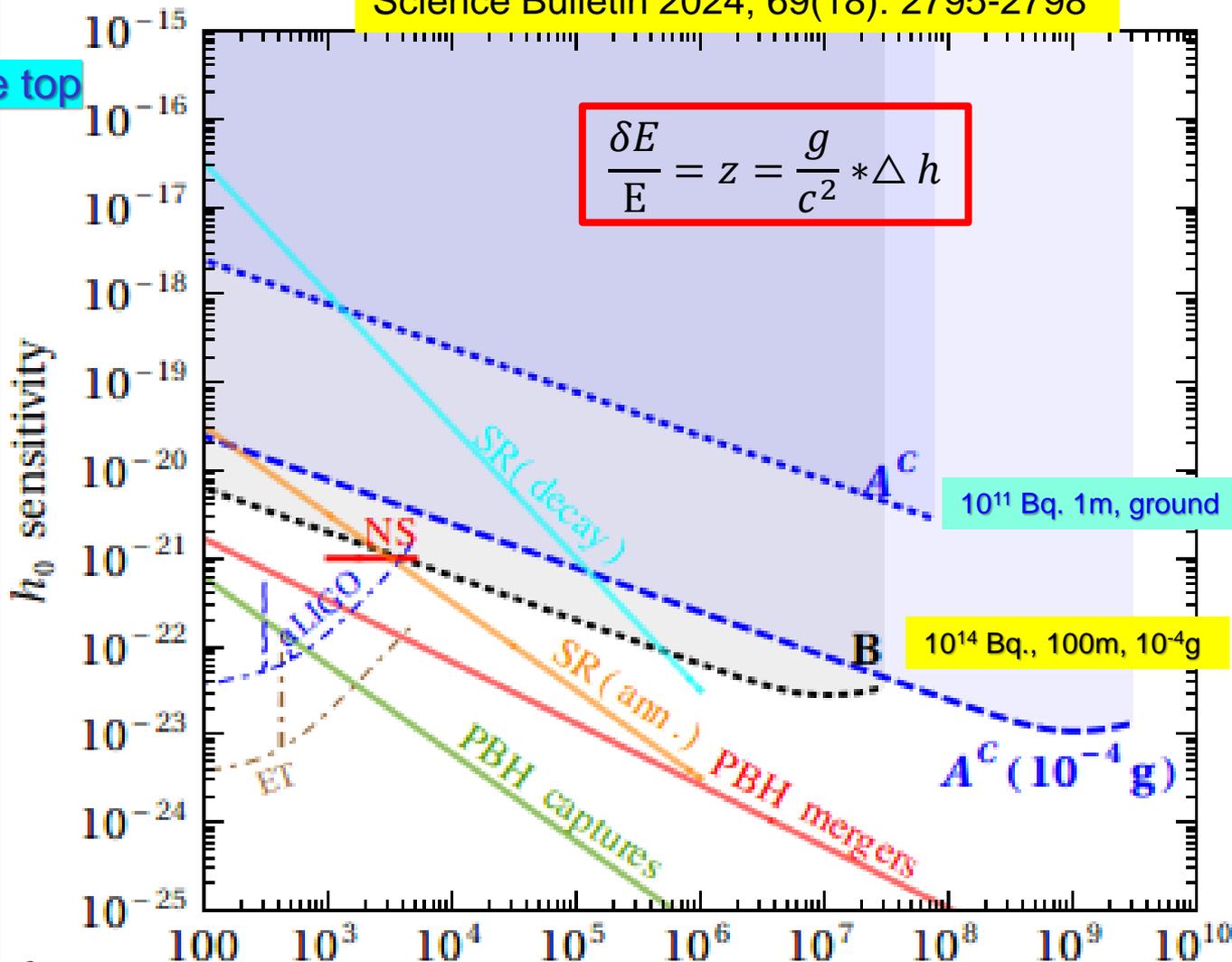
$$*3\sigma \text{ peak with } C_\infty,$$

$$C_\infty > (\sigma)^2 / (fs\epsilon)^2$$

$$* 2\pi fd < O(10)$$

Coherent GWs from
NS, SR, PBHs, see
[N.Aggarwal et al.,
Living Rev. Rel. 24, 4 \(2021\)](#)

Inverse Gentsenshtein:
 $\Delta F = F_0 h * (\text{form factor} \sim \omega d^{n,z2})$
also see: [2305.00877](#)



Region of interests

Limited by photon
counter and readout

Summary

- Conceptual design GW experiment use novel gravitational redshift Mössbauer spectrometer:
 - Measure the spatial resonance absorption **peak shift** under GW.
 - Allow direction and **polarization** reconstruction.
 - Relatively small-scale setup (meter – 100 meter for your preference)
 - ^{109}Ag source gives 10^{-22} strain sensitivity, long lived mother particle.
 - **Multiband** sensitivity (*Sensitive to $f_{\text{GW}} > \text{kHz}$*)
- Possible extension on sensitivity
 - low-g, focusing of photons...
- Larger source-absorber distance:
 - Aiming for lower frequency GWs



Phase 0: Tabletop exotic/Axion(arXiv:2511.14851, arXiv:2512.18960) physics searches

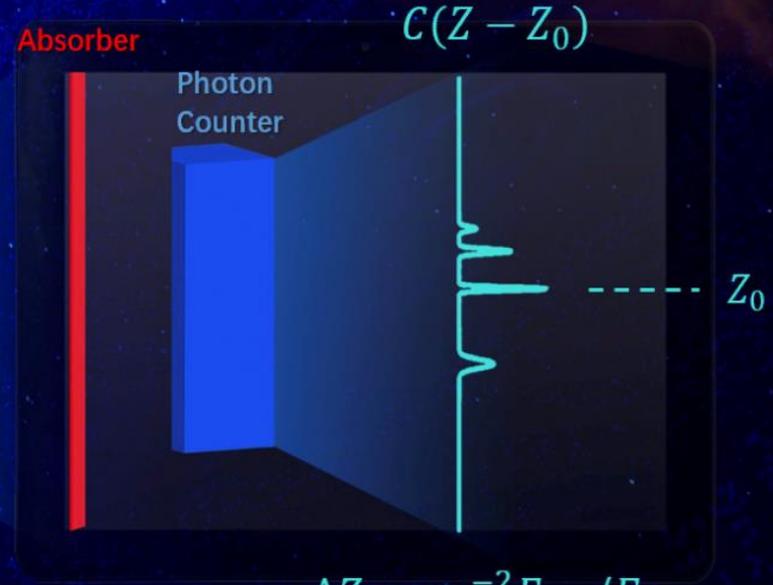
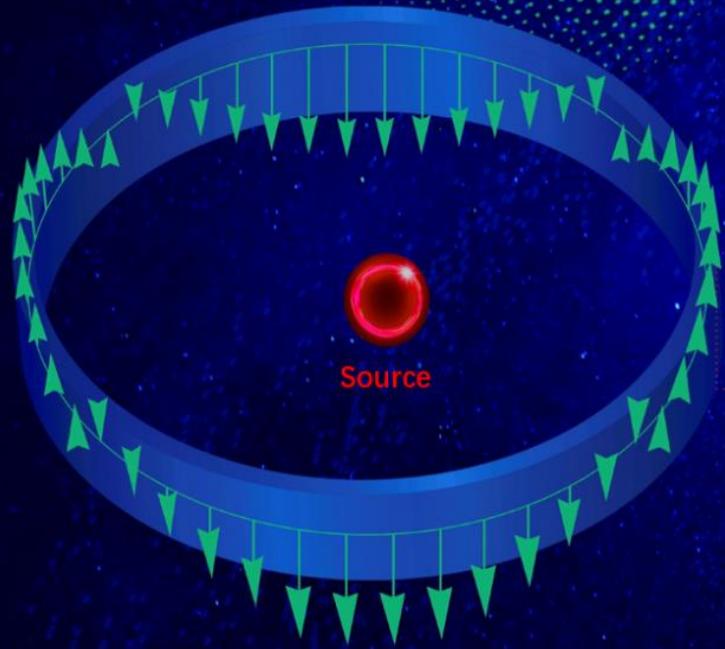
Phase 1: Tabletop Ground based GW experiment

Phase 2: low-g GW experiment (higher sensitivity)

Space based experiment

Moon/Asteroid based experiment

saddle/Lagrangian point based experiment ?



$$\Delta Z = gc^{-2}\Gamma_{exp}/E_0$$