

Quantum tomography and entanglement in $H \rightarrow ZZ \rightarrow 4l$

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Introduction

- Quantum entanglement (QE) is simply the existence of a non-separable quantum state
- A violation of Bell's inequality (BI) is a stricter condition that implies that no local hidden variable theory can describe the system
- $H \rightarrow ZZ \rightarrow 4l$ is an ideal system for studying QE and BI because final state is fully reconstructed and density matrix has simplified form
- [Talk by Leyun Gao on Tuesday](#) discussed the closely related $H \rightarrow ZZ \rightarrow 4l$ polarization fraction (i.e. f_{LL} versus f_{TT}) measurement

A brief history of Bell's inequality

- 1935: EPR paradox
 - Quantum mechanics cannot be consistent with completely predictive theory
- 1965: original Bell inequality
 - Inequality that must be satisfied in local hidden variable theory
 - $|P(a,b) - P(a,c)| < 1 + P(b,c)$
- 1969: CHSH inequality
 - Improved version of the inequality more resistant to noise
 - $|P(a,b) - P(a,c)| < 2 - P(b',b) - P(b',c)$
- 2002: CGLMP inequality
 - Generalizes the inequality to spin $> 1/2$ particles
 - $I_3 = P(A_1=B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) - P(A_1 = B_1 - 1) - P(B_1 = A_2) - P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1) < 2$, where P means probability and mod 3 is implied

A brief history of Bell's inequality experiments

- 1972: Clauser and Freedman perform experimental tests with photons used fixed polarizers
- 1982: Aspect, Dalibart, and Roger perform experimental tests of Bell's inequality with photons using polarizers with variable polarization axes
- 2015: new experiments in Delft, Vienna, and Boulder close all possible “loopholes” keeping local hidden variable theories alive
- 2022: Aspect, Clauser, and Zeilinger awarded Nobel Prize in Physics for experimental tests of Bell's inequality

Related quantum mechanical concepts

- We focus on Bell nonlocality and entanglement here, but there are actually additional related concepts that should be mentioned
- **Discord**: measurement on system changes state of *that* system
- **Steering**: measurement on system changes state of *other* system
- Hierarchy is **Bell nonlocality \supset steering \supset entanglement \supset discord**
- Any of these (even discord) imply classical physics is not correct
- Steering is the concept that is demonstrated by the EPR paradox

Comparison to table-top experiments

- Consider this 12 event example, if we could measure A_i, B_j directly
 - A_1, B_1 measurements: $A_1=0, B_1=0; A_1=1, B_1=1; A_1=2, B_1=2;$
 - A_1, B_2 measurements: $A_1=0, B_2=2; A_1=1, B_2=2; A_1=2, B_2=0;$
 - A_2, B_1 measurements: $A_2=0, B_1=1; A_2=1, B_1=2; A_2=2, B_1=0;$
 - A_2, B_2 measurements: $A_2=0, B_2=0; A_2=1, B_2=1; A_2=2, B_2=2;$
- $P(A_1=B_1)=3/3, P(B_1=A_2+1)=3/3, P(A_2=B_2)=3/3, P(B_2=A_1)=0/3,$
 $P(A_1=B_1-1)=0/3, P(B_1=A_2)=0/3, P(A_2=B_2-1)=0/3, P(B_2=A_1-1)=1/3$
- $I_3 = 3/3 + 3/3 + 3/3 + 0/3 - 0/3 - 0/3 - 0/3 - 1/3 = 8/3$
- **We cannot measure A_i, B_j directly at a collider, so instead we reconstruct the spin density matrix** and use it to predict the outcome of an experiment where we could measure A_i, B_j directly

Definition of $H \rightarrow ZZ$ spin density matrix

- For a quantum system with discrete states ψ_i , the density operator ρ is defined as $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$, where p_i is the probability of state ψ_i , and the density matrix ρ_{ij} is defined as $\rho_{ij} = \langle\psi_i|\rho|\psi_j\rangle$
- Here we are interested in spin states, meaning the quantization of the z component of the spin angular momentum
- A spin-1 massive particle like a Z boson has $2 \times (1) + 1 = 3$ spin states, so its spin density matrix is a 3 by 3 matrix, and for two Z bosons the product spin density matrix is a 9 by 9 matrix
- Observables can be calculated by taking the trace of the product of ρO , where O is the operator corresponding to the observable

H \rightarrow ZZ spin density matrix

- ZZ spin density matrix is greatly simplified with spin-0 assumption:

$$\rho = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & ? & 0 & ? & 0 & ? & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & ? & 0 & ? & 0 & ? & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & ? & 0 & ? & 0 & ? & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Optimal Bell operator construction

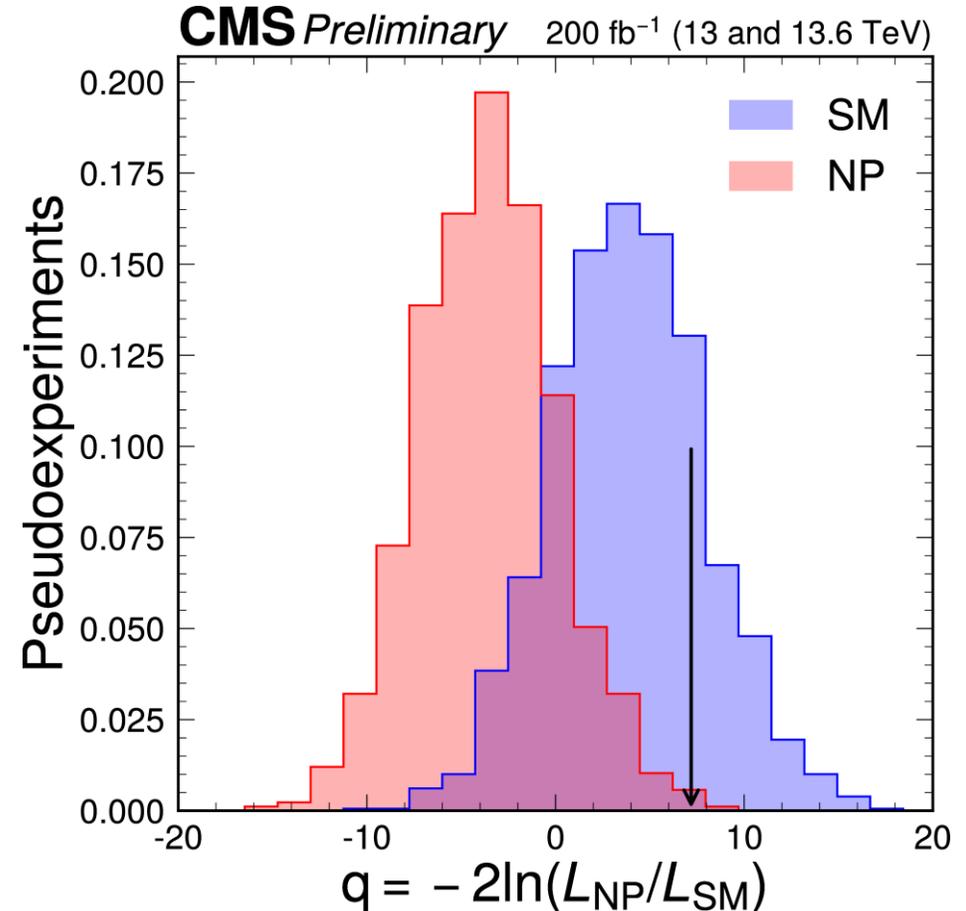
- The Bell operator represents the measurement $I_3 = P(A_1=B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) - P(A_1 = B_1 - 1) - P(B_1 = A_2) - P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1)$
- We are free to choose A_1, A_2, B_1, B_2 to obtain highest expected I_3
- Expression for optimal Bell operator: $\left(\frac{2}{3\sqrt{3}} (T_1^1 \otimes T_1^1 - T_0^1 \otimes T_0^1 + T_1^1 \otimes T_{-1}^1) + \frac{1}{12} (T_2^2 \otimes T_2^2 - T_2^2 \otimes T_{-2}^2) + \frac{2}{2\sqrt{6}} (T_2^2 \otimes T_0^2 + T_0^2 \otimes T_2^2) - \frac{1}{3} (T_1^2 \otimes T_1^2 + T_1^2 \otimes T_{-1}^2) + \frac{1}{4} T_0^2 \otimes T_0^2 \right) + \text{Hermitian conjugate}$
- T are 3 by 3 matrices defined in the backup slide

Summary of recent CMS PAS [HIG-25-011](#)

- Based on Run II + 2022 + 2023 dataset (200 fb⁻¹)
- Five results based on similar multi-dimensional template fits: EFT, polarization, density matrix, identical leptons, entanglement
- Signal samples generated with standard POWHEG+JHUGen setup
- Discriminants between SM and non-SM hypotheses are obtained from MELA package and provide one of the variables in the fit
- Binning optimized to increase sensitivity with reasonable computational complexity and statistical power → ~300 bins

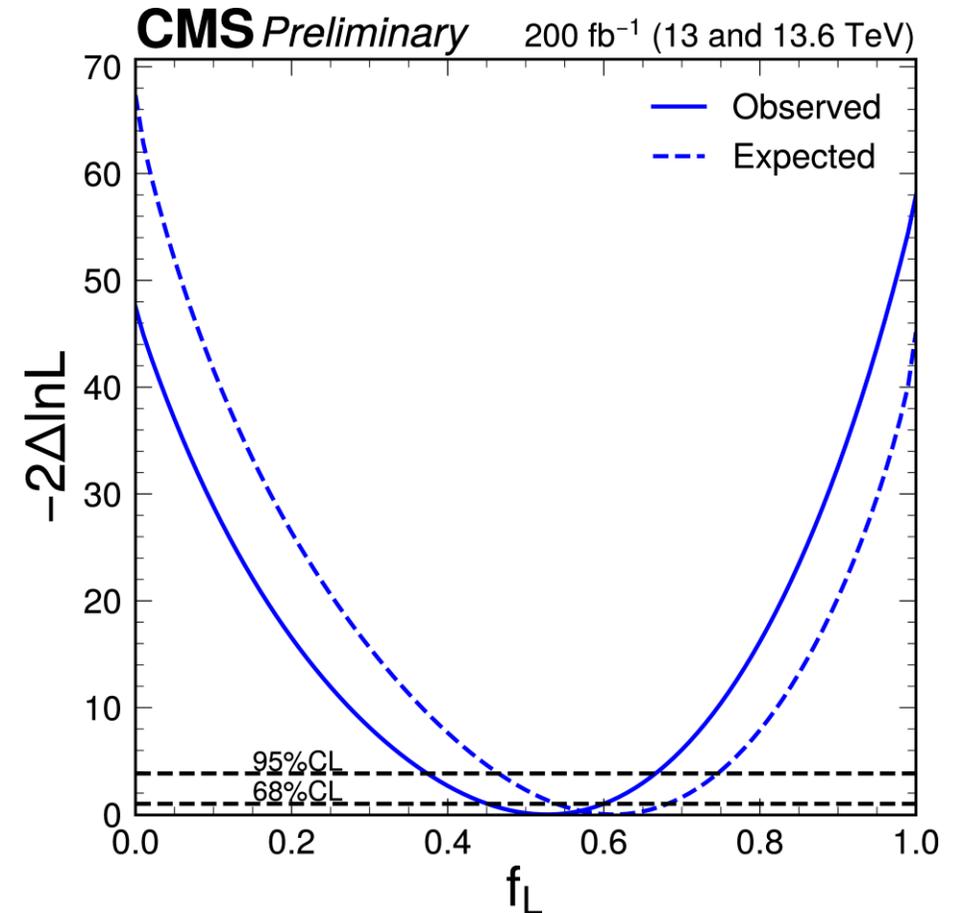
CMS $H \rightarrow ZZ \rightarrow 4l$ identical lepton results

- Model without identical lepton permutations (NP) disfavored at 2.7σ compared to the SM



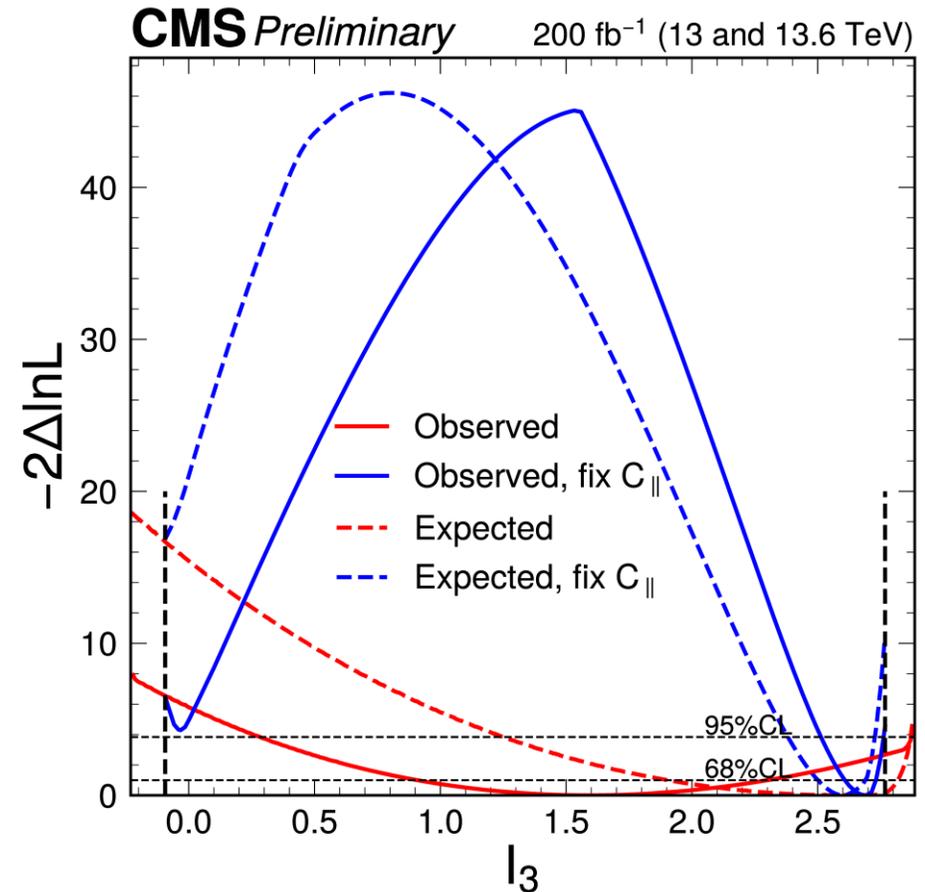
CMS $H \rightarrow ZZ \rightarrow 4l$ entanglement results

- The hypothesis that the ZZ spin system is not entangled, which corresponds to $f_L = 1$, is excluded at $> 5 \sigma$



CMS $H \rightarrow ZZ \rightarrow 4l$ Bell inequality results

- Results presented for two scenarios, depending on number of free parameters
- In scenario with more stringent assumptions, called “fix C_{\parallel} ”, the hypothesis that Bell’s inequality is violated ($I_2 < 2$) is established at 95% CL
- Caveats to interpretation discussed on next slide



Caveats to Bell's inequality interpretation

- Can Bell's inequality be truly studied at colliders (and should it)?
- An [arXiv paper by Matthew Low](#) analyzes this question extensively
 - “We showed that *it is not possible to exclude local realism at a collider because the observables used, rest-frame decay-product angles, are not sufficiently correlated with the underlying particle spins.*”
 - “Despite this conclusion, Bell nonlocality, as a quantum correlation, remains an interesting and informative threshold for correlations at colliders.”
- Our tests are clearly not as robust in terms of loopholes as the table-top experiments involving photons, however they are in a much higher energy regime and with different particle content

Conclusions

- The ZZ system in $H \rightarrow ZZ \rightarrow 4l$ events is ideal for studying quantum entanglement due to simplified form of the spin density matrix
- Recently released CMS results
 - Based on Run II + 2022 + 2023 dataset (200 fb^{-1})
 - Entanglement (rejecting the pure LL hypothesis) established at $> 5 \sigma$
 - Bell inequality results presented for two scenarios (depending on number of floating parameters), with 95% CL violation for stricter scenario
 - Many caveats to interpretation of Bell inequality results
- Spin density matrix calculation feature [recently added](#) to MadGraph5_aMC@NLO, opening up many new possibilities

Backup

T matrix definitions

- $T_{\pm 1}^1 = \mp \frac{\sqrt{3}}{2} (S_x \pm iS_y)$
- $T_{\pm 2}^2 = \frac{2}{\sqrt{3}} T_{\pm 1}^1$
- $T_0^2 = \frac{\sqrt{2}}{3} (T_1^1 T_{-1}^1 + T_{-1}^1 T_1^1)$
- $T_0^1 = \frac{\sqrt{3}}{2} S_z$
- $T_{\pm 1}^2 = \frac{\sqrt{2}}{3} (T_{\pm 1}^1 T_0^1 + T_0^1 T_{\pm 1}^1)$