

Probing Ultra-Dense Substructures with Fast Radio Bursts

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YH, Weiyang Wang, Chen Zhang, Yi-Ming Zhong, arxiv 260X.XXXXX

OutLine

- Challenges to CDM models and small-scale structures
- Fast Radio Burst and Cosmology
- Gravitational lensing of ultra-dense structures
- Lensing Rate and time-delay distribution of FRB
- Test of SIDM core-collapsed halo with FRB lensing
- Summary and Outlooks

Challenges to CDM models and small-scale structures

- Cold dark matter successfully explains formation of structures
- Simulation shows the CDM halo follows **NFW profile** with inner density $\rho \propto r^{-1}$

Problems of CDM:

Core-Cusp;

Diversity;

Ultra-Dense structure



Challenges to CDM models and small-scale structures

- Ultra-Dense structures observed in strong gravitational lensing

Jackpot Event SDSS J0946+1006,

$$\sim 10^9 M_{\odot}, r^{-2.3}$$

Vegetti+ '10, Minor+ '21, Nightingale+ '22, Despali+ '24,
Enzi+ '25...

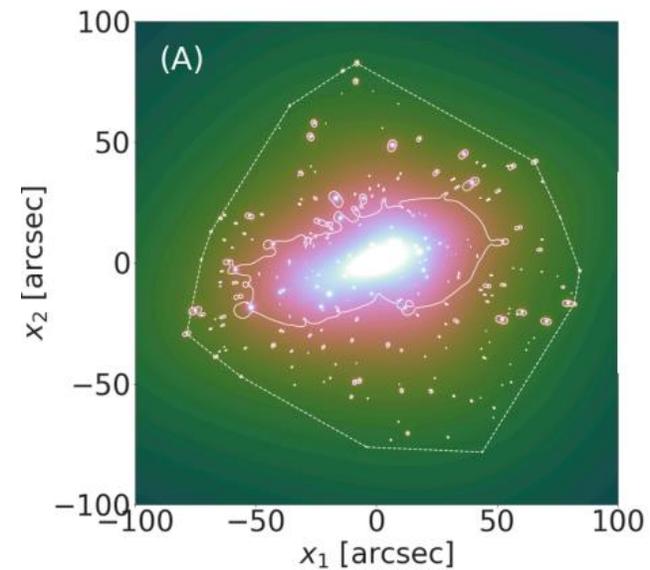
Galaxy-galaxy strong lensing (GGSL),

$$\sim 10^{10\sim 12} M_{\odot}, r < -2.5$$

Dutra, Natarajan & Gilman '24

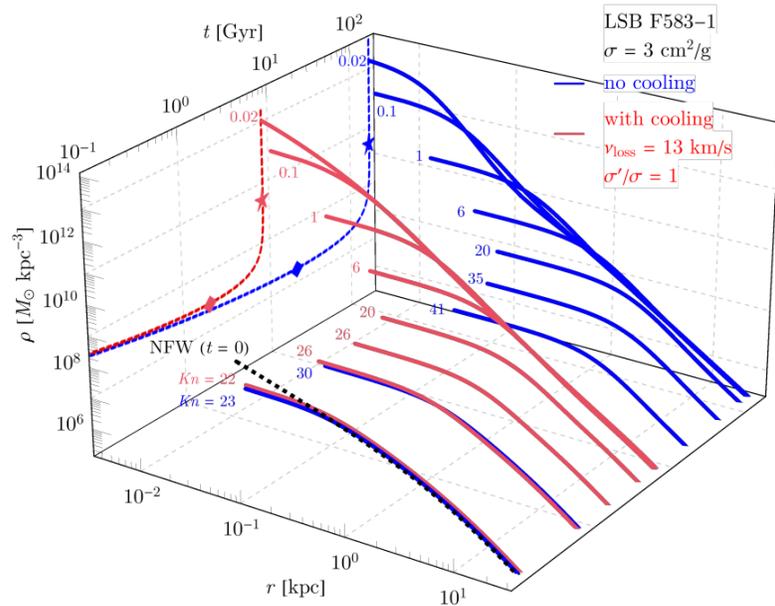
B1938+666 ...

Tajalli et al.'25 Powell et al.'25

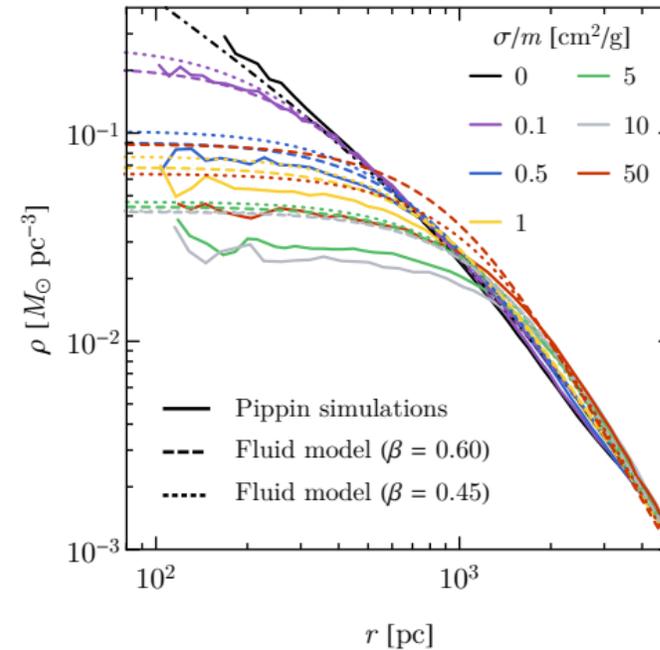


Challenges to CDM models and small-scale structures

- Self-interacting Dark Matter can possibly explain
- SIDM halo may possibly core-collapse



Essig et.al. 2018

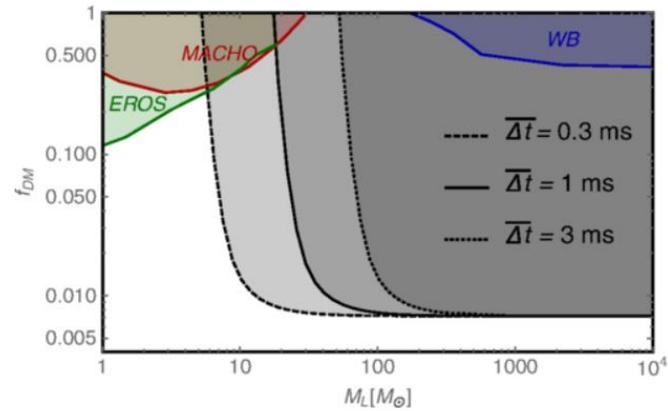


- Might be tested by strong lensing, weak lensing

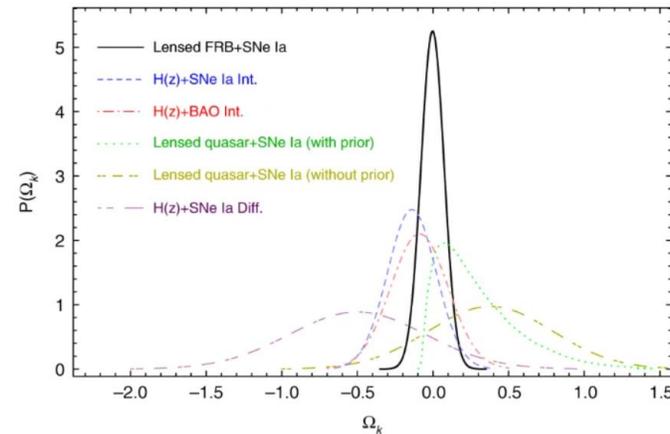
Gilman et.al. 2022 Adhikari et al. 2024

Fast Radio Burst and Cosmology

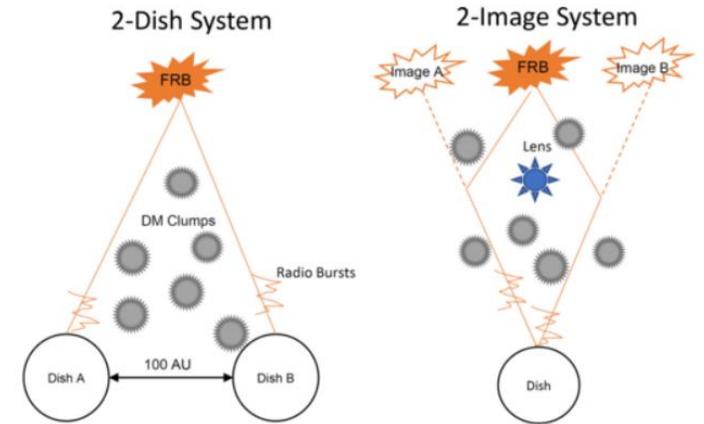
- Fast Radio Burst (FRB) becomes a powerful tools for testing fundamental problems



Muñoz et al. '16



Li et al. '18



Xiao et al. '24

For more, see Prof. Bing Zhang's talk

Gravitational lensing of ultra-dense structure

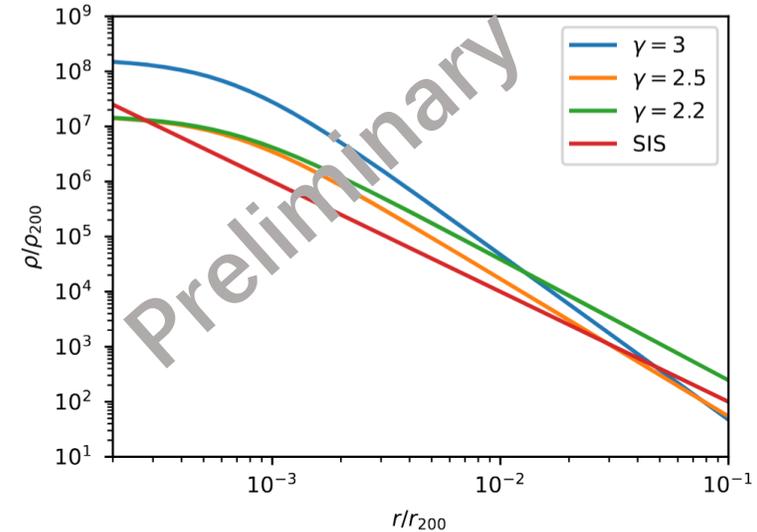
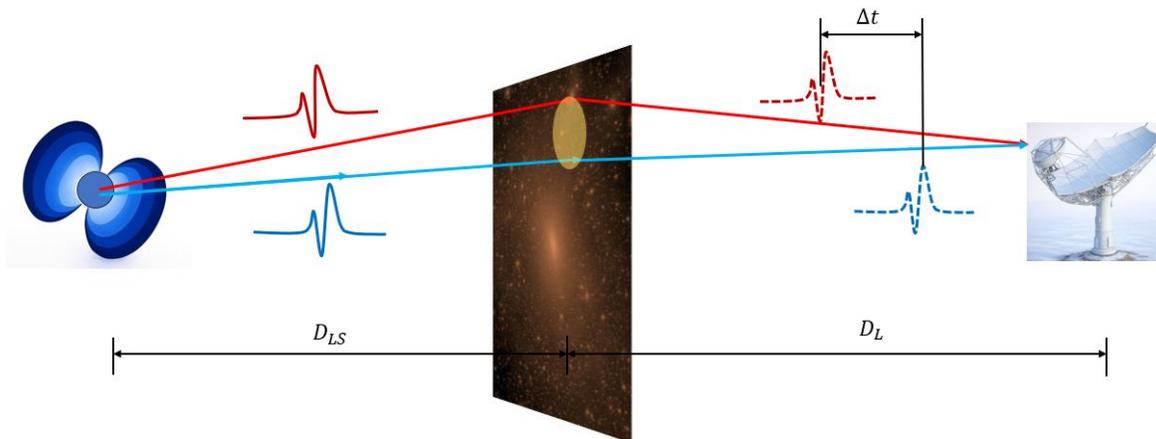
- Instead of the NFW profile, the simplified **cored-power-law** profile used

$$\rho(r) = \frac{\rho_0}{\left(1 + \frac{r^2}{r_c^2}\right)^{\frac{\gamma}{2}}}$$

$$2 \lesssim \gamma \lesssim 3 \quad r_c = 0.01 r_s \quad M(r_{200}) = M_{200}$$

Ludlow · 2016

- Simplify the system with isolating lenses



$$\psi(\theta) = \frac{4GD_{SL}D_L}{c^2D_S} \int d^2\theta' \Sigma(\theta' D_L) \ln |\theta - \theta'|$$

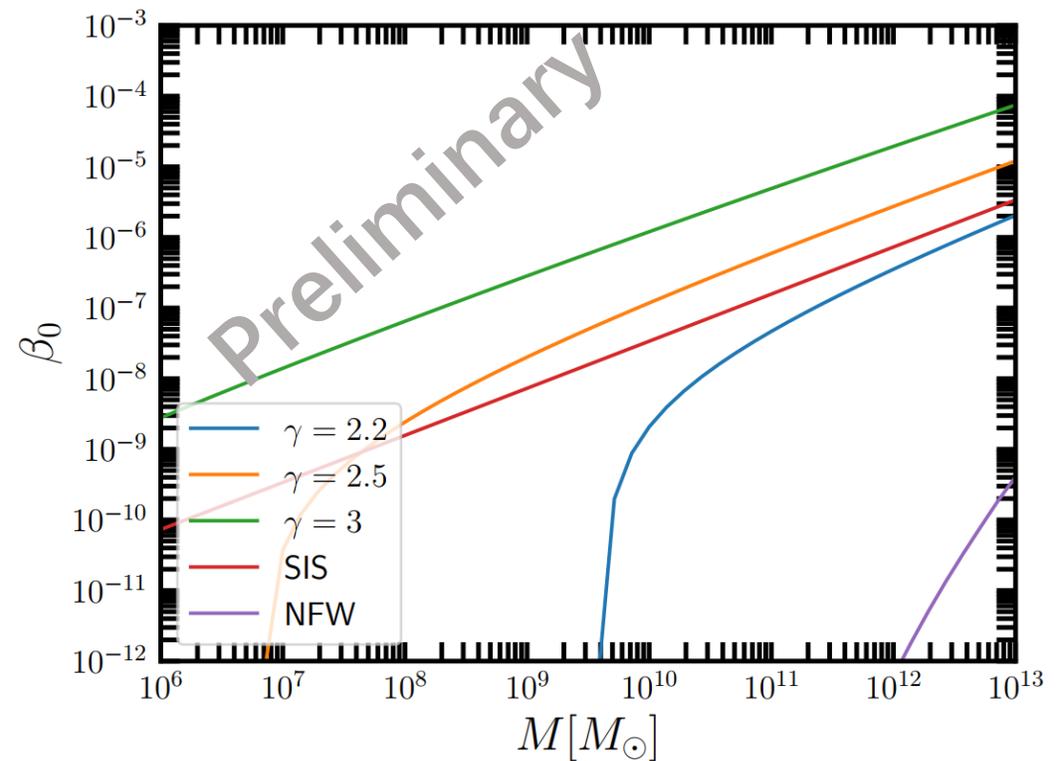
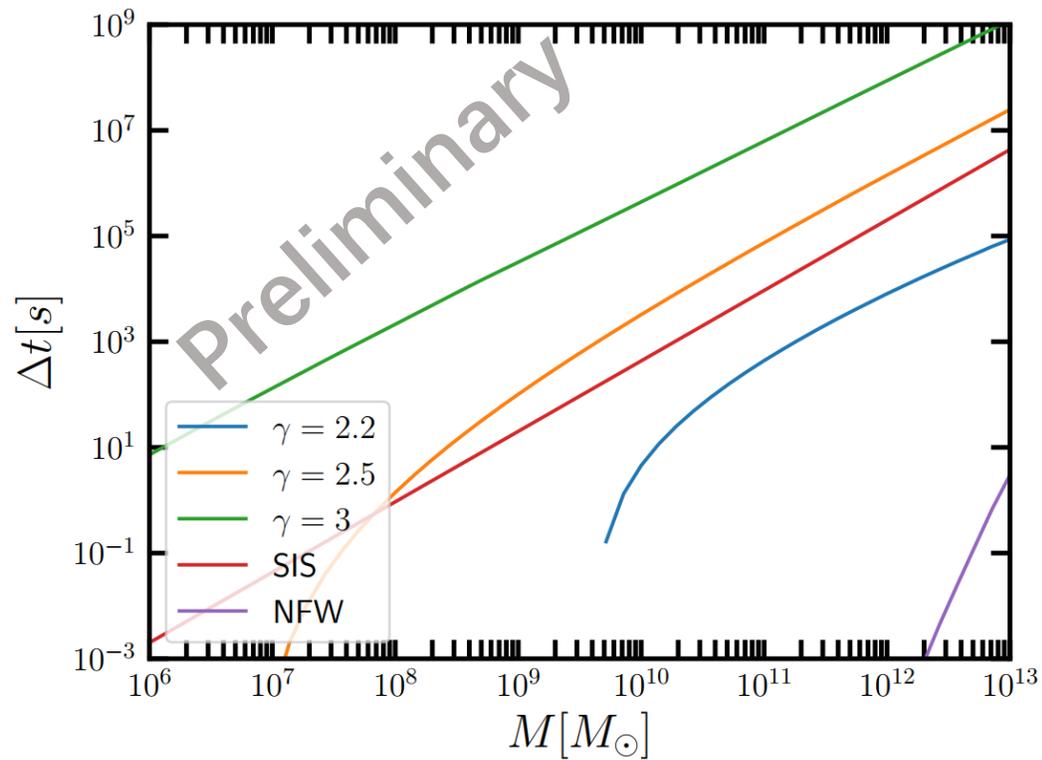
$$\Phi(\theta, \beta) = \frac{1}{2} |\theta - \beta|^2 - \psi(\theta)$$

$$\beta = \theta - \nabla_{\theta} \psi(\theta)$$

$$\Delta t_I = (1 + z_l) \frac{cD_{LS}D_L}{D_S} \Phi(\theta_i, \beta)$$

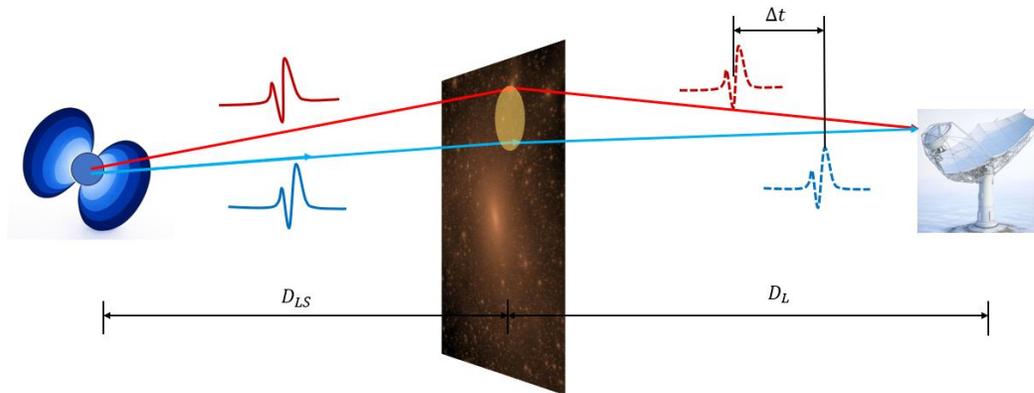
Gravitational lensing of ultra-dense structure

- For impact angle $\beta < \beta_0$, three separate images are formed.
- We focused on the **brightest two images**



Lensing Rate and time-delay distribution of FRB

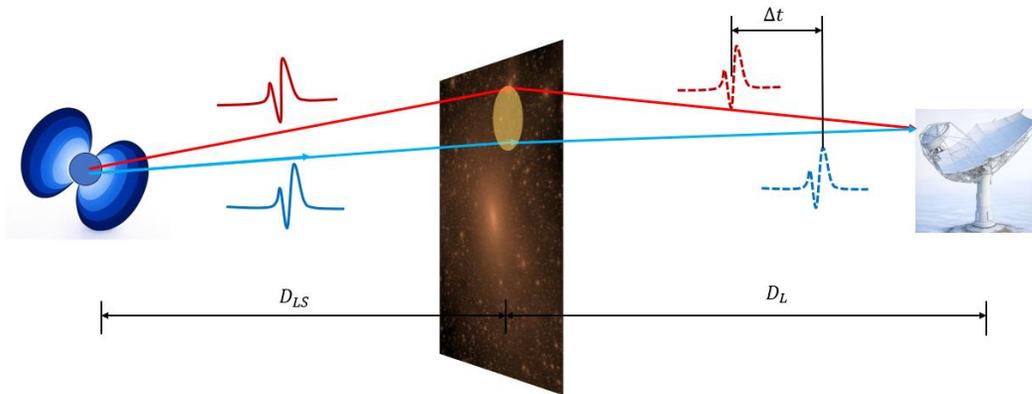
- Combining the isolated lens to estimate the total rate and time delay distribution



$$N_{\text{total}} = \int dz_s dL_s (1 - e^{-\tau(z_s, L_s)})$$

Lensing Rate and time-delay distribution of FRB

- Combining the isolated lens to estimate the total rate and time delay distribution

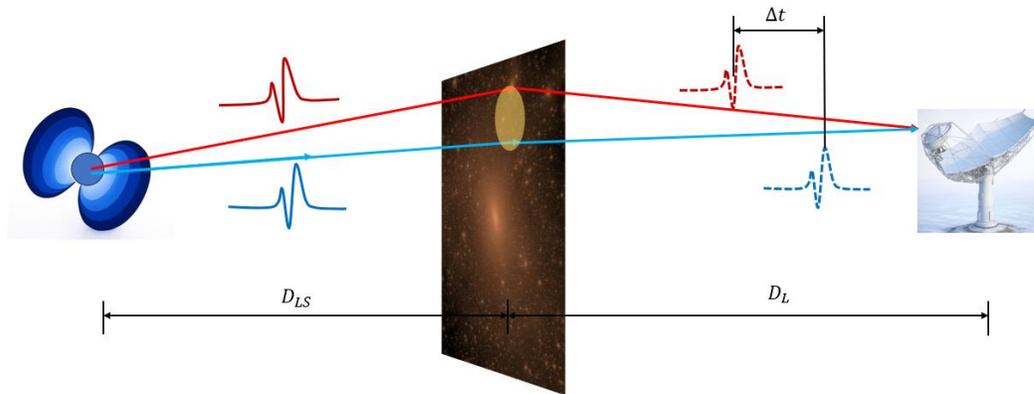


$$N_{\text{total}} = \int dz_s dL_s (1 - e^{-\tau(z_s, L_s)})$$

- $$\tau(z_s, L_s) = \int 2\pi\beta d\beta dM_l dz_l \frac{dN}{dz_l dz_s dM_l dL_s} \epsilon(\Delta t) \theta(\min(\mu_i 4\pi D_S^2 (1+z_s)^4 L_s) - S_{\text{min}})$$

Lensing Rate and time-delay distribution of FRB

- Combining the isolated lens to estimate the total rate and time delay distribution



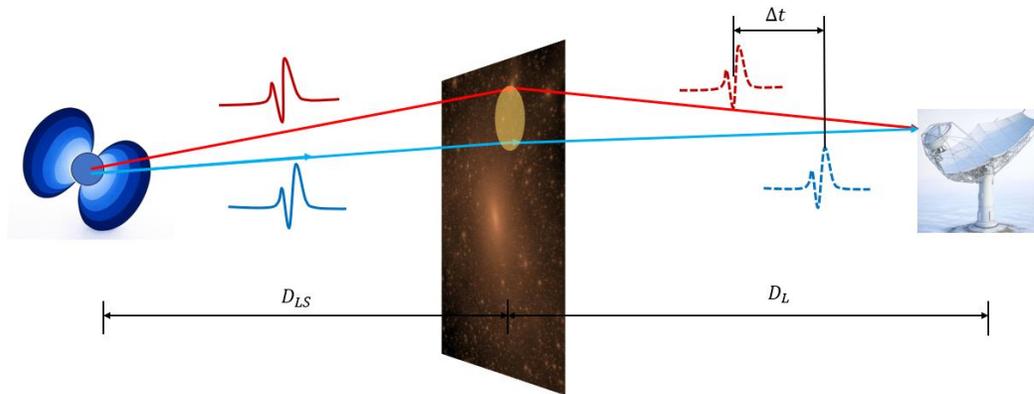
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$$\frac{dn_l}{dz_l dM_l} \frac{dn_s}{dz_s dL_s}$$

Lensing Rate and time-delay distribution of FRB

- Combining the isolated lens to estimate the total rate and time delay distribution



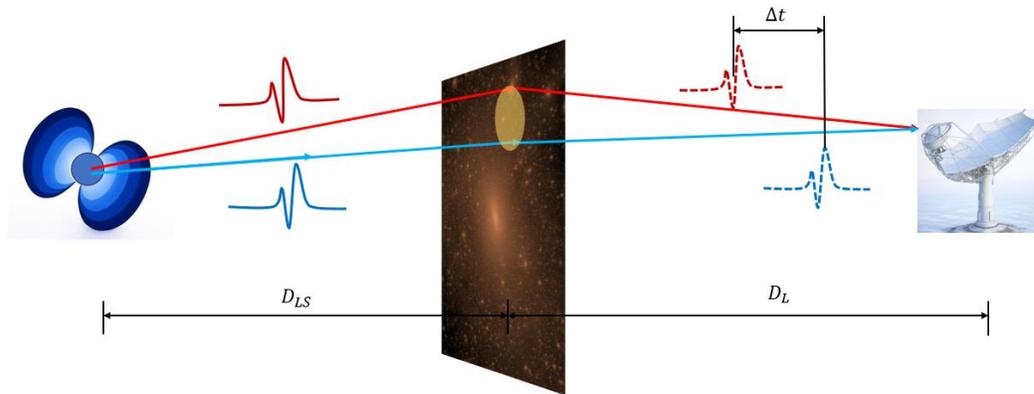
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$$\frac{dn_l}{dz_l dM_l} = \frac{dn_{\text{host}}}{dM_{\text{host}} dV} (M_{\text{host}}, z_l) \frac{dV}{dz_l} + \int dM_{\text{host}} \frac{dn_{\text{sub}}}{dM_{\text{sub}} dA} \frac{dn_{\text{host}}}{dM_{\text{host}} dV} (M_{\text{host}}, z_l) \frac{dV}{dz_l} \times A$$

Lensing Rate and time-delay distribution of FRB

- Combining the isolated lens to estimate the total rate and time delay distribution



$$N_{\text{total}} = \int dz_s dL_s (1 - e^{-\tau(z_s, L_s)})$$

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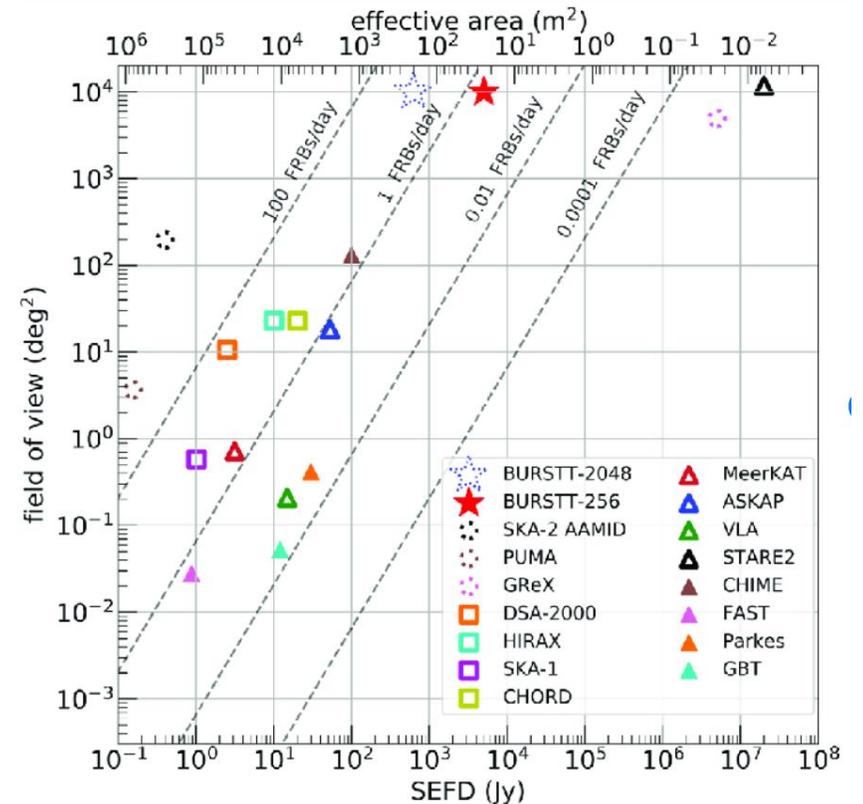
$$\frac{dn_s}{dz_s dL_s} = n_0 (1+z_s) \frac{dV}{dz_s} L^{-\alpha+1} \quad \begin{matrix} 5 \times 10^{37} \text{ erg/s} > L > 0.5 \times 10^{36} \text{ erg/s} \\ \alpha = -1.85 \end{matrix}$$

Lensing Rate and time-delay distribution of FRB

$$\tau(z_s, L_s) = \int 2\pi\beta d\beta dM_l dz_l \frac{dN}{dz_l dz_s dM_l dL_s} \epsilon(\Delta t) \theta(\min(\mu_i 4\pi D_S^2 (1+z_s)^4 L_s) - S_{\min})$$

Observatory	S_{\min} [Jy]	FoV [deg ²]	Expected Events
BURSTT	0.94	5000	10^5
SKA2-Low	0.00042	144	10^9
SKA2-Mid	0.00013	144	10^9

$$S_{\min} = \text{SEFD} / \sqrt{\Delta T \Delta f}$$

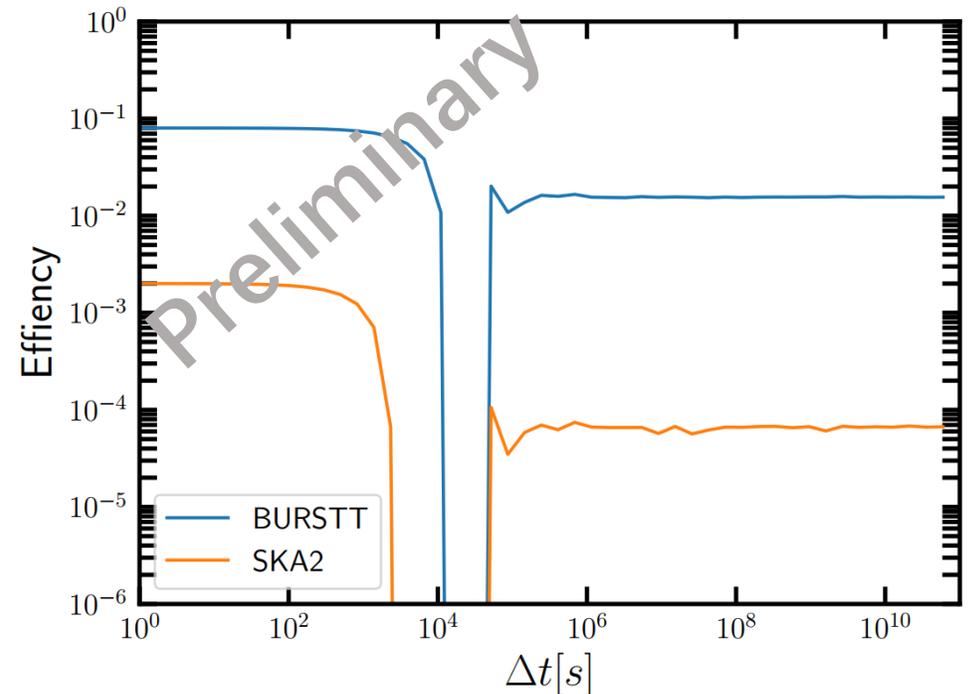


Lensing Rate and time-delay distribution of FRB

- $$\tau(z_s, L_s) = \int 2\pi\beta d\beta dM_l dz_l \frac{dN}{dz_l dz_s dM_l dL_s} \epsilon(\Delta t) \theta(\min(\mu_i 4\pi D_S^2 (1+z_s)^4 L_s) - S_{\min})$$

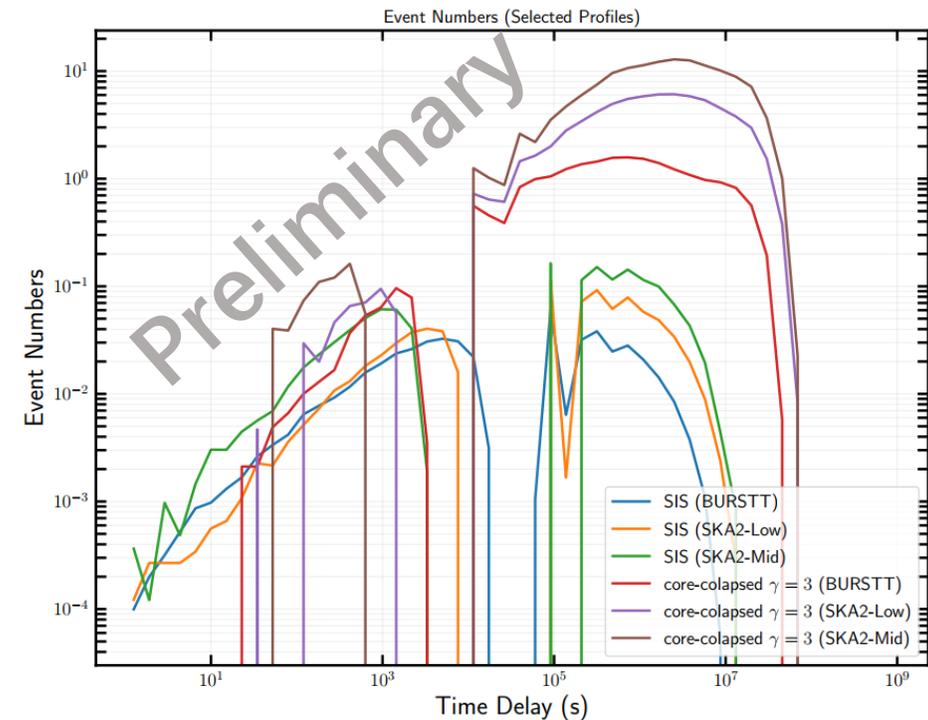
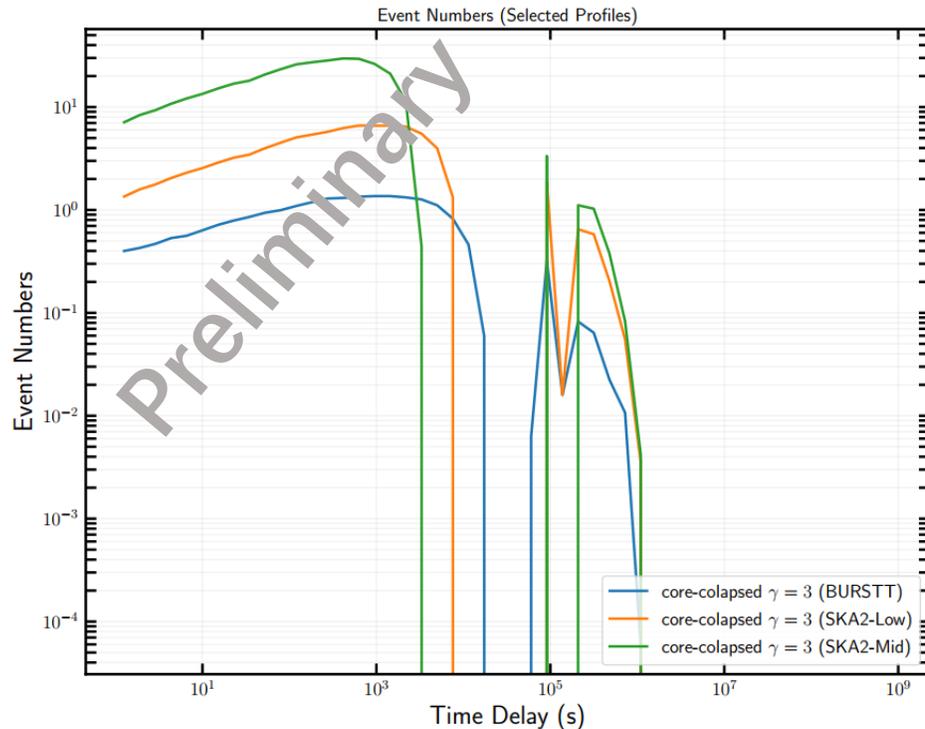
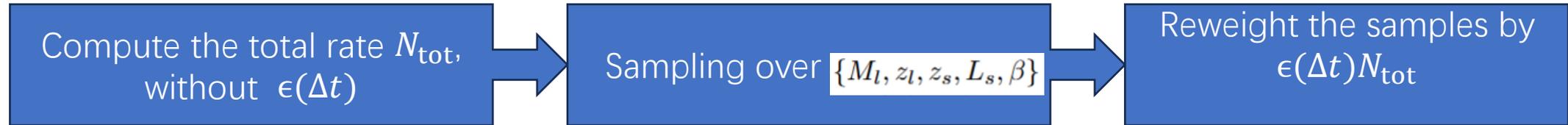
$$\epsilon(\Delta t) = \left(\frac{\Omega_{\text{FoV}}}{4\pi} \right) \frac{1}{\Delta T} \int_0^{\Delta T} dt \sum_{n=0}^N \theta(t + \Delta t - nT) \theta(nT + \Delta T - (t + \Delta t)),$$

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Lensing Rate and time-delay distribution of FRB

- Estimation of the lensing distribution



Test of SIDM core-collapsed halo with FRB lensing

- Simplified Core-collapsing SIDM model

$$P_c(m, z, \sigma_{\text{SI}}) = \frac{1}{2} \left[1 + \tanh \left(\frac{t_{\text{evo}}(z) - t_{\text{sub}}(m, z, \sigma_{\text{SI}})}{2s_{\text{sub}}} \right) \right]$$

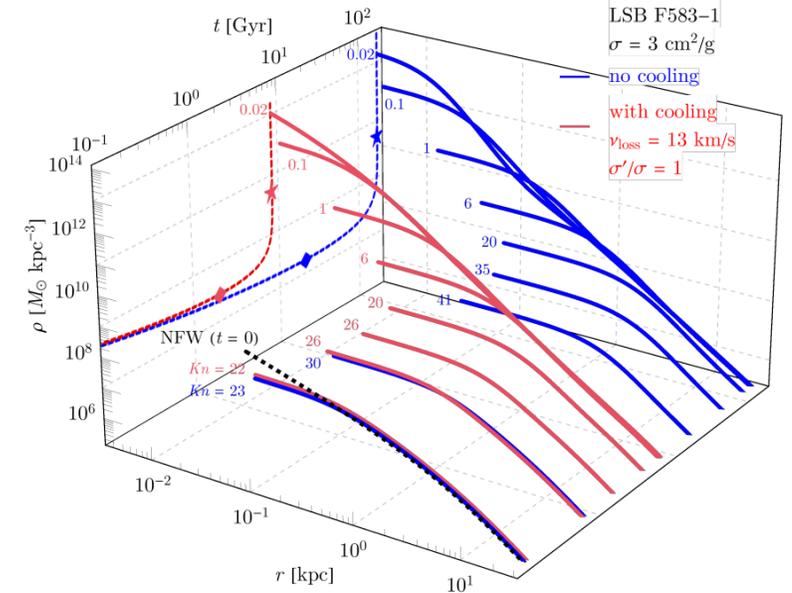
$$t_0(m, z, \sigma_V) = \left(\frac{1 \text{ cm}^2 \text{ g}^{-1}}{\sigma_{\text{SI}}} \right) \left(\frac{100 \text{ km s}^{-1}}{v_{\text{max}}} \right) \times \left(\frac{10^7 M_{\odot} \text{ kpc}^{-3}}{\rho_s} \right) \text{ Gyr},$$

$$t_{\text{field}} \equiv \lambda_{\text{field}} t_0;$$

$$t_{\text{sub}} \equiv \lambda_{\text{sub}} \lambda_{\text{field}} t_0;$$

$$\lambda_{\text{field}} = 264.1 \text{ and } \lambda_{\text{sub}}$$

$$v_{\text{max}} = 1.65 \sqrt{G \rho_s r_s^2};$$



Test of SIDM core-collapsed halo with FRB lensing

- Simplified Core-collapsing SIDM model

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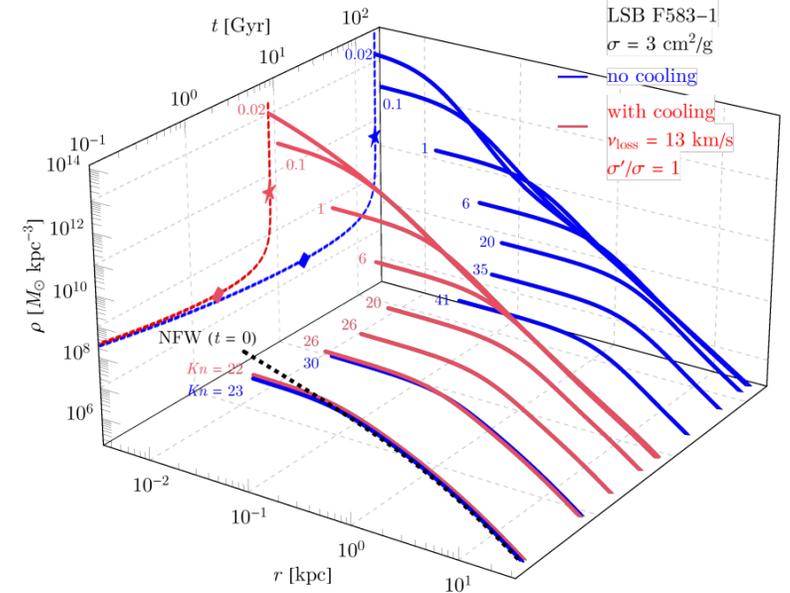
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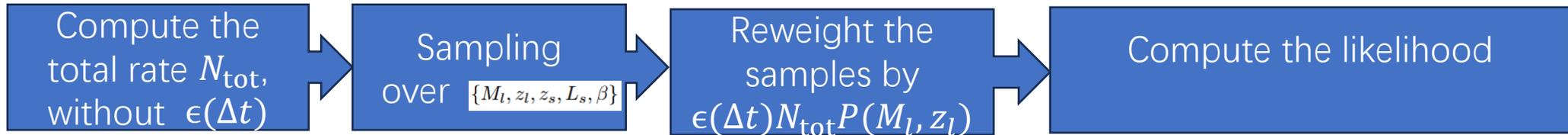
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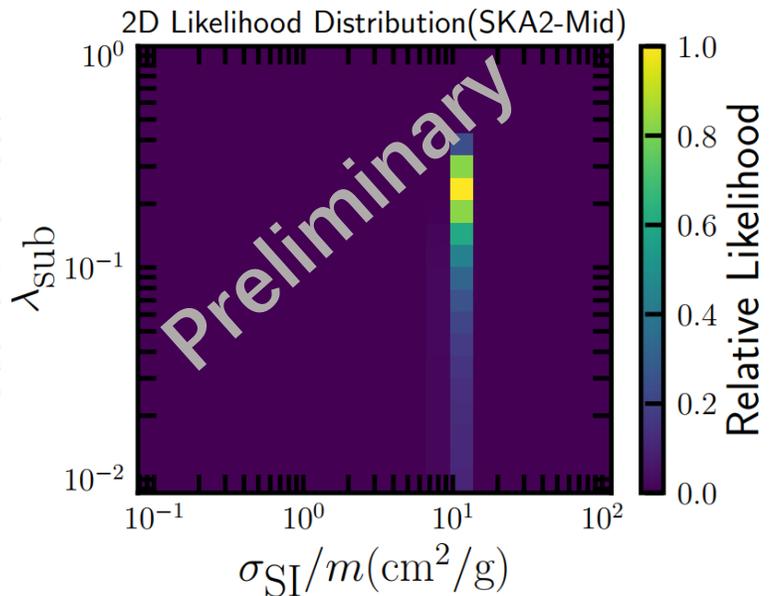
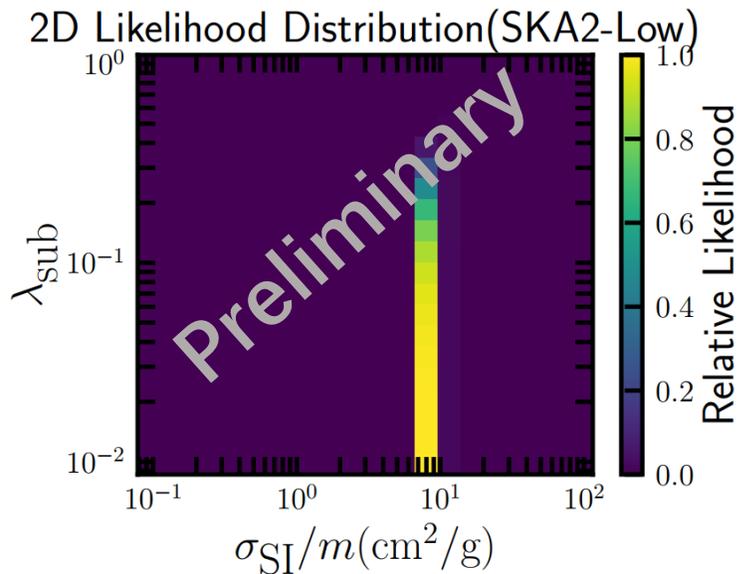
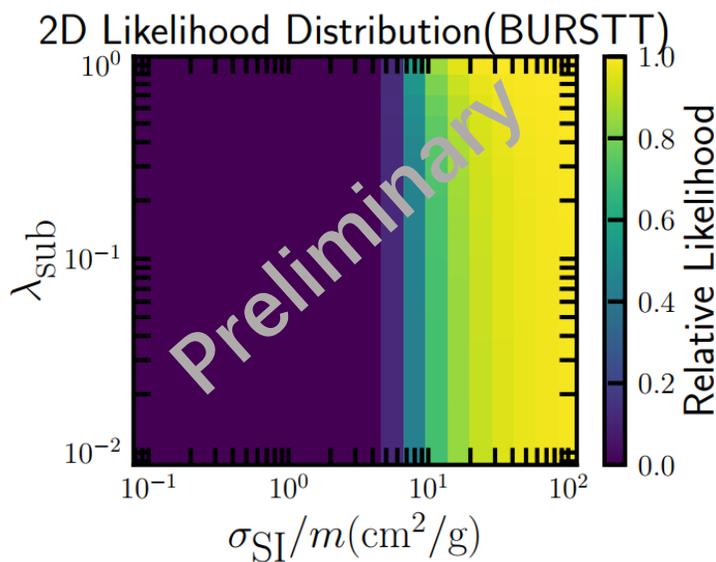


Test of SIDM core-collapsed halo with FRB lensing

- Compute the Likelihood of the parameters

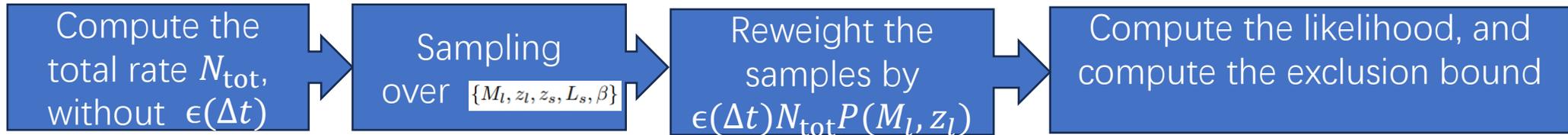


$$\mathcal{L}(\lambda_{\text{sub}}, \sigma_{\text{SI}}) = \prod_i \frac{e^{-n_i(\lambda_{\text{sub}}, \sigma_{\text{SI}})}}{N_i!}$$

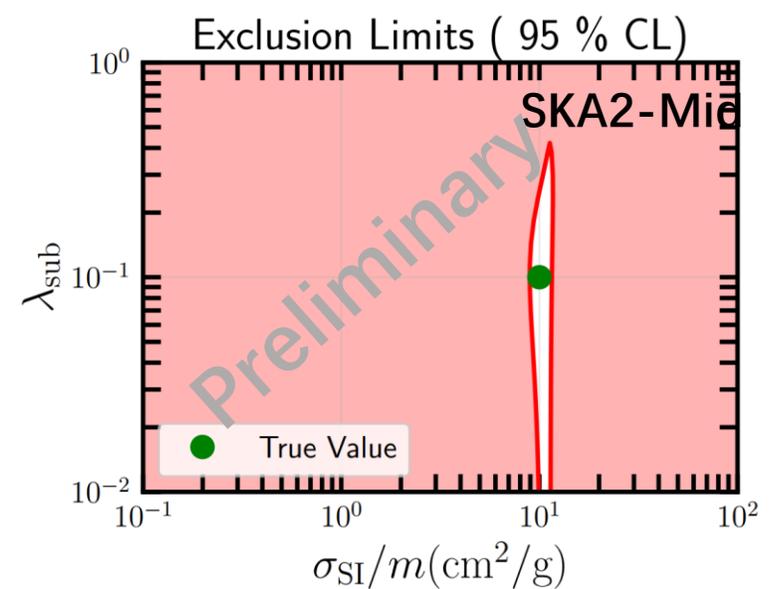
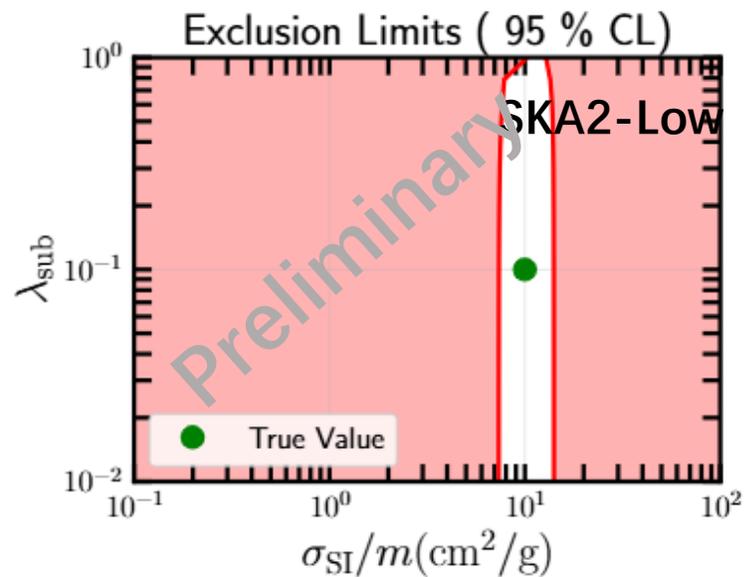
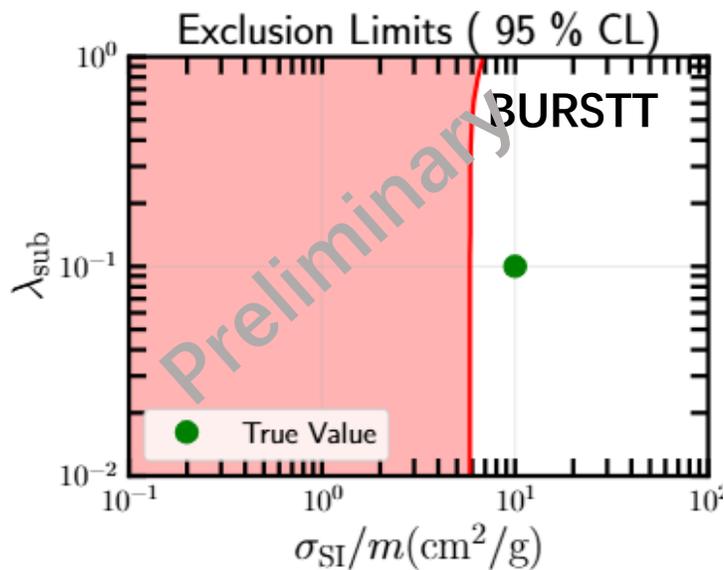


Test of SIDM core-collapsed halo with FRB lensing

- Exclusion of the SIDM parameters



$$\mathcal{L}(\lambda_{\text{sub}}, \sigma_{\text{SI}}) = \prod_i \frac{e^{-n_i(\lambda_{\text{sub}}, \sigma_{\text{SI}})}}{N_i!}$$



Summary and Outlooks

- The ultra-dense structures can increase the gravitational lensed FRB signals that **very likely** be observed by future telescopes
- The future FRB observatories can potentially test specific SIDM models that produce ultra-dense structure via core-collapsing
- Further developments of the detailed modeling and data analysis are needed

Backups

$$\frac{dn_{\text{host}}}{d \ln \sigma_v dV}(\sigma_v, z) = n_*(z) \left(\frac{\sigma_v}{\sigma_{v*}(z)} \right)^a \exp \left[- \left(\frac{\sigma_v}{\sigma_{v*}(z)} \right)^b \right]$$

$$n_* = 6.92 \times 10^{-3} (1 + z_d)^{-1.18} \text{Mpc}^{-3}, \quad \sigma_{v*} = 172.2 \times (1 + z_d)^{0.18} \text{km /s}, \quad a = -0.15 \text{ and } b=2.35.$$

$$\frac{dn_{\text{sub}}}{dM_{\text{sub}} dA} = \frac{\Sigma_{\text{sub}}}{m_0} \left(\frac{M_{\text{sub}}}{m_0} \right)^\alpha \mathcal{F}(M_{\text{host}}, z)$$

$$\log_{10} \mathcal{F} = k_1 \log_{10} \left(\frac{M_{\text{host}}}{10^{13} M_\odot} \right) + k_2 \log_{10}(z + 0.5)$$

Backups

$$\nabla_{\boldsymbol{\theta}}\psi(\boldsymbol{\theta}) = \kappa_0 \frac{\boldsymbol{\theta}}{|\boldsymbol{\theta}|} + \kappa_1 \frac{\boldsymbol{\theta} - \boldsymbol{\theta}_1}{|\boldsymbol{\theta} - \boldsymbol{\theta}_1|} = \kappa_0 \frac{\boldsymbol{\theta}}{|\boldsymbol{\theta}|} + \frac{\kappa_1}{|\boldsymbol{\theta}_1|^3} (|\boldsymbol{\theta}_1|^2 \boldsymbol{\theta} - (\boldsymbol{\theta}_1 \cdot \boldsymbol{\theta}) \boldsymbol{\theta}_1) - \kappa_1 \frac{\boldsymbol{\theta}_1}{|\boldsymbol{\theta}_1|} + \text{higher order terms.}$$

$$\nabla_{\boldsymbol{\theta}}\psi(\boldsymbol{\theta}) = \kappa_0 \frac{\boldsymbol{\theta}}{|\boldsymbol{\theta}|} + \hat{\Gamma} \boldsymbol{\theta}, \quad \beta_c \sim \frac{\kappa_1 \kappa_0}{|\boldsymbol{\theta}_1|} = \beta_0 \theta_{1E} / |\boldsymbol{\theta}_1|$$

$$\hat{\Gamma} = \begin{pmatrix} -\kappa_1/|\boldsymbol{\theta}_1| & 0 \\ 0 & 0 \end{pmatrix} \quad \langle \kappa_1/|\boldsymbol{\theta}_1| \rangle \approx \langle \theta_E \rangle / \langle d \rangle D_L \lesssim 3 \times 10^{-4} \left(\frac{M}{10^{12} M_{\odot}} \right)^{0.24}$$