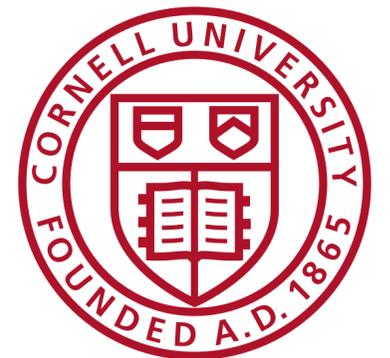# Holographic Origins of the QCD Vacuum Angle and Axion

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[arXiv:2601.XXXXX]

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Holographic Origin of θ angle
10D AdS/CFT

Witten (1998) used Type IIA string theory on $\mathbf{R}^4 \times \mathbf{S}^4 \times D$

$$ds^2 = \frac{8\pi}{3}\eta\lambda^3 \sum_{i=1}^{4}(dx^i)^2 + \frac{8}{27}\eta\lambda\pi\left(\lambda^2 - \frac{1}{\lambda^4}\right)d\psi^2 + \frac{8\pi}{3}\eta\lambda\frac{d\lambda^2}{\lambda^2 - \frac{1}{\lambda^4}} + \frac{2\pi}{3}\eta\lambda d\Omega_4^2$$

λ, ψ are polar coordinates on D ($1 \leq \lambda \leq \infty$, $0 \leq \psi \leq 2\pi$).

A gauge field a is added in the bulk to get θ angle

On the boundary, $f = 0$ but there is a non-vanishing Wilson loop around the boundary of the disc

$$\int_{\mathbf{S}^1} a = \theta_a = \theta$$

Holographic Origin of θ angle
5D AdS/CFT

We would like to reproduce this in a simple 5D model

First we see that $\mathbf{S}^4$ played no role — just look at simple AdS$_6$ on a circle with coordinate y

$$ds_6^2 = \frac{L^2}{z^2}(dx^2 - dz^2 - dy^2)$$

A gauge field A is added in the bulk to get θ angle

Witten had a DISC, not a circle. For most cases it will not make a difference whether disc or cylinder. However, the BC for θ on the IR brane will be different
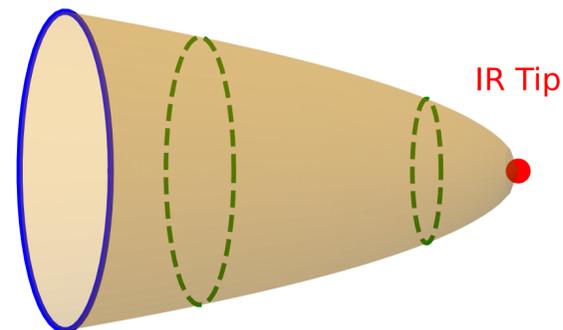
Holographic Origin of θ angle

5D AdS/CFT

For theory on disc, the origin corresponds to the IR brane — it is like the origin of the polar coordinates. Angular variables must vanish at the origin, so the geometry implies $\theta(z_{\mathrm{IR}}) = 0$
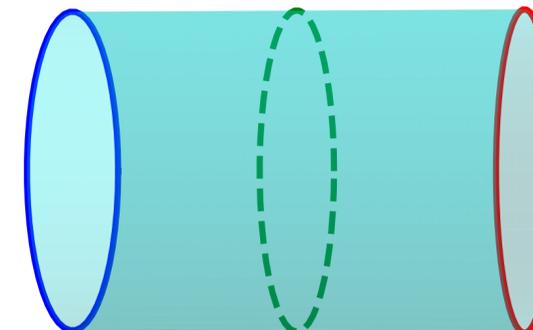

WSS Model
(Cigar)

UV Boundary

IR Tip

θ, ϕ loops

Geometric Regularity:
Circle shrinks to point
$\theta \to 0, \phi \to 0$

Hard Wall Model
(Cylinder)

UV Boundary   IR Wall

θ, ϕ loops

Boundary Condition:
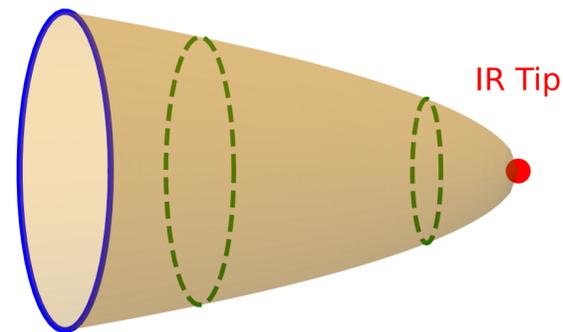$\theta - N_f \phi = 0$
(Manual)

Holographic Origin of θ angle

5D AdS/CFT

This condition is NOT automatically present in the theory on the circle. To reproduce the effects of the true QCD dynamics, we simply impose $\theta(z_{\text{IR}}) = 0$ at the IR brane.

WSS Model
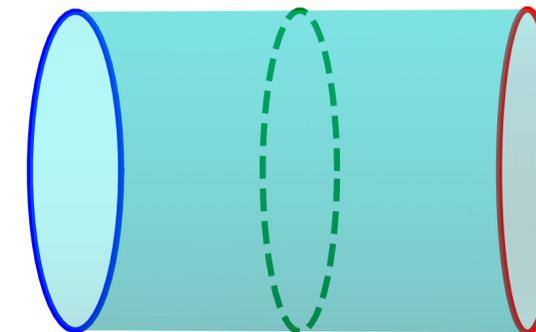(Cigar)

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Hard Wall Model
(Cylinder)

UV Boundary

IR Wall

θ, ϕ loops

Boundary Condition:
$\theta - N_f \phi = 0$
(Manual)

Holographic Origin of θ angle

5D AdS/CFT

Using the disc-like BC

$$A_y(0) = A_0 \text{ and } A_y(z_{\text{IR}}) = 0$$

Background solution of Maxwell equation gives

$$A_y(z) = A_0 \left(1 - \frac{z^3}{z_{\text{IR}}^3} \right)$$

Wilson line

$$W_y = \exp\left(i \oint_{S^1} A_y \mathrm{d}y \right) = \exp\left(iA_y^{(0)} \cdot 2\pi R \right) = e^{i\theta}$$

The θ field

$$\theta(x, z) \equiv 2\pi R A_y^{(0)}(x, z)$$

Holographic Origin of θ angle

5D AdS/CFT

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$$\theta(x, z) \equiv 2\pi R A_y^{(0)}(x, z)$$

$$\theta(x, z) \to G\tilde{G}$$

$$\theta_{\text{UV}} \to \theta_{\text{QCD}}$$

Holographic Origin of θ angle

5D AdS/CFT

The vacuum energy density

$$\mathcal{E} = \frac{3g^2 L^2}{2z_{\mathrm{IR}}^3} \theta_{UV}^2$$

Wilson line gives $2\pi n$ periodicity from large gauge transformation

$$E_{\mathrm{phys}}(\theta) = \min_n C(\theta + 2\pi n)^2$$

If we shrink the 6th dimension to 0, the effective 5D Lagrangian for θ will be

$$S_{\mathrm{5D}}[\theta] = \frac{g^2}{2} \int \frac{z}{L} d\theta \wedge *d\theta$$

Holographic Origin of Anomaly

Dualize

The holographic pure QCD theory does't have much structure: only glueballs

We now introduce flavors and see the effects of anomaly

It is useful to dualize the θ field (Hodge dual) $F_1 = d\theta$

Treat F_1 as an independent field, and impose $dF_1 = 0$ via Lagrange multiplier C_3 3-form gauge field $H = dC$

$$S_{\text{parent}} = \int \left(\frac{g^2}{2} \frac{z}{L} F_1 \wedge *F_1 - H \wedge F_1 \right)$$

Holographic Origin of Anomaly

Dualize

Integrating out F_1, we obtain the equivalent action

$$S_{\text{dual}}[H] = \frac{1}{2} \int \frac{L}{g^2 z} H \wedge *H$$

Dualize back, but in the presence of the bulk Chern-Simons term

$$S_{\text{parent}} = \int \left(\frac{1}{2} \frac{L}{g^2 z} H \wedge *H + \kappa \, A \wedge H + \theta \, dH \right)$$

A is the $U(1)_A$ gauge field

Now θ is the Lagrange multiplier enforcing the Bianchi identity on H

Holographic Origin of Anomaly

Shift Symmetry

$$S_{\text{parent}} = \int \left(\frac{1}{2} \frac{L}{g^2 z} H \wedge *H + \kappa \, A \wedge H + \theta \, dH \right)$$

The action must be bulk gauge invariant under $A \to A + d\lambda$

If $H = dC_3$, the Chern-Simons term is bulk gauge invariant. However, H is unconstrained with the Lagrange multiplier θ

$A \to A + d\lambda$ is only cancelled out with $\theta \to \theta + \kappa \, \lambda$

θ picks up a shift symmetry under $U(1)_A$

Holographic Origin of Anomaly

Dualize Back

After integration by parts

$$S_{\text{parent}} = \int \left(\frac{1}{2} \frac{L}{g^2 z} H \wedge *H - (d\theta - \kappa A) \wedge H \right)$$

Solving the EOM for H gives

$$H = -\frac{g^2 z}{L} * (d\theta - \kappa A)$$

The resulting action is

$$\frac{g^2}{2} \int \frac{z}{L} (d\theta - \kappa A) \wedge *(d\theta - \kappa A)$$

Holographic dual of anomaly is just 5D Stückelberg term!

Holographic QCD Axion

Setup

Using the lessons from the 6D/5D model, we can build a simple 5D model of axion with quark condensates

$$X = \rho e^{i\phi}: \langle \bar{q}q \rangle$$

$A_M: U(1)_A$ gauge boson

$$Y = \zeta e^{ia}: \text{PQ breaking}$$

$B_M: U(1)_{\mathsf{PQ}}$ gauge boson

Holographic QCD Axion

Setup

Using the lessons from the 6D/5D model, we can build a simple 5D model of axion with quark condensates

$$S_{5D} \supset \int \frac{1}{2g^2} H \wedge *H + \kappa\, H \wedge A + \lambda H \wedge B + \theta dH + DX^\dagger \wedge *DX + DY^\dagger \wedge *DY$$

$$\to \int \mathrm{d}^5 \sqrt{-g} \left[\frac{g^2}{2}(\theta' - \kappa A_z - \lambda B_z)^2 + \rho^2(\phi' - qA_z)^2 + \zeta^2(a' - pB_z)^2 \right].$$

$X = \rho e^{i\phi}$: $\langle \bar{q}q \rangle$

$Y = \zeta e^{ia}$: PQ breaking

A_M: $U(1)_A$ gauge boson

B_M: $U(1)_{\mathsf{PQ}}$ gauge boson

Holographic QCD Axion

Dynamics

Gauge invariant combinations

$$\Theta = \theta' - \kappa A_z - \lambda B_z,$$

$$\Phi = \phi' - q A_z, \quad \mathcal{A} = a' - p B_z,$$

Their corresponding momenta

$$\Pi_\theta = -\frac{L^3}{z^3} g^2 \Theta, \quad \Pi_\phi = -2\frac{L^3}{z^3} \rho^2 \, \Phi \quad \Pi_a = -2\frac{L^3}{z^3} \zeta^2 \, \mathcal{A}.$$

Background EOMs

$$\partial_z \left(\frac{L^3}{z^3} g^2 \Theta \right) = 0, \quad \partial_z \left(\frac{L^3}{z^3} 2\rho^2 \Phi \right) = 0, \quad \partial_z \left(\frac{L^3}{z^3} 2\zeta^2 \mathcal{A} \right) = 0$$

Holographic QCD Axion

Dynamics

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Background EOMs

$$\Pi_{\theta,\phi,a} \text{ are const.}$$

$$\partial_z \left(\frac{L^3}{z^3} g^2 \Theta \right) = 0, \quad \partial_z \left(\frac{L^3}{z^3} 2\rho^2 \Phi \right) = 0, \quad \partial_z \left(\frac{L^3}{z^3} 2\zeta^2 \mathcal{A} \right) = 0$$

Holographic QCD Axion

Holographic PQ mechanism

Background energy of the system is written as boundary terms

$$\mathcal{E} \supset -V(X_{\mathrm{UV}}) - U(Y_{\mathrm{UV}}) + \frac{1}{2}\left|\Pi_\theta\,\theta + \Pi_\phi\,\phi + \Pi_a\,a\right|_{\mathrm{UV}}^{\mathrm{PQ}} + \frac{1}{2}\left|\Pi_\theta\,\theta + \Pi_\phi\,\phi\right|_{\mathrm{PQ}}^{\mathrm{IR}}$$

A_z, B_z equations imply

$$\kappa\Pi_\theta + q\Pi_\phi = 0 \quad \text{and} \quad \lambda\Pi_\theta + p\Pi_a = 0.$$

UV BCs

$$\Pi_\phi\big|_{\mathrm{UV}} = -V_\phi, \qquad \Pi_a\big|_{\mathrm{UV}} = -U_a, \qquad \delta\theta\big|_{\mathrm{UV}} = 0.$$

V, U: $U(1)_A$, $U(1)_{\mathsf{PQ}}$ breaking UV potential

$$V \supset \frac{L^4}{z^4} m_q^\dagger L^{-\frac{3}{2}} X$$

Holographic QCD Axion

Holographic PQ mechanism

$$\mathcal{E} \supset -V(X_{\mathrm{UV}}) - U(Y_{\mathrm{UV}}) + \frac{1}{2}\left|\Pi_\theta\,\theta + \Pi_\phi\,\phi + \Pi_a\,a\right|_{\mathrm{UV}}^{\mathrm{PQ}} + \frac{1}{2}\left|\Pi_\theta\,\theta + \Pi_\phi\,\phi\right|_{\mathrm{PQ}}^{\mathrm{IR}}$$

$$\kappa\Pi_\theta + q\Pi_\phi = 0 \quad \text{and} \quad \lambda\Pi_\theta + p\Pi_a = 0.$$

$$\Pi_\phi|_{\mathrm{UV}} = -V_\phi, \qquad \Pi_a|_{\mathrm{UV}} = -U_a, \qquad \delta\theta|_{\mathrm{UV}} = 0.$$

V, U: $U(1)_A$, $U(1)_{\mathrm{PQ}}$ breaking UV potential

$$U_a = 0 \implies \Pi_a = 0 \implies \Pi_{\theta,\phi} = 0 \implies V_\phi = 0$$

All θ dependence in energy is vanishing

topological susceptibility is 0 → no strong CP problem

Holographic QCD Axion

Holographic PQ mechanism

$$\mathcal{E} \supset -V(X_{\mathrm{UV}}) - U(Y_{\mathrm{UV}}) + \frac{1}{2}\left|\Pi_\theta\,\theta + \Pi_\phi\,\phi + \Pi_a\,a\right|_{\mathrm{UV}}^{\mathrm{PQ}} + \frac{1}{2}\left|\Pi_\theta\,\theta + \Pi_\phi\,\phi\right|_{\mathrm{PQ}}^{\mathrm{IR}}$$

$$\kappa\Pi_\theta + q\Pi_\phi = 0 \quad \text{and} \quad \lambda\Pi_\theta + p\Pi_a = 0.$$

$$\Pi_\phi|_{\mathrm{UV}} = -V_\phi, \qquad \Pi_a|_{\mathrm{UV}} = -U_a, \qquad \delta\theta|_{\mathrm{UV}} = 0.$$

V, U: $U(1)_A$, $U(1)_{\mathsf{PQ}}$ breaking UV potential
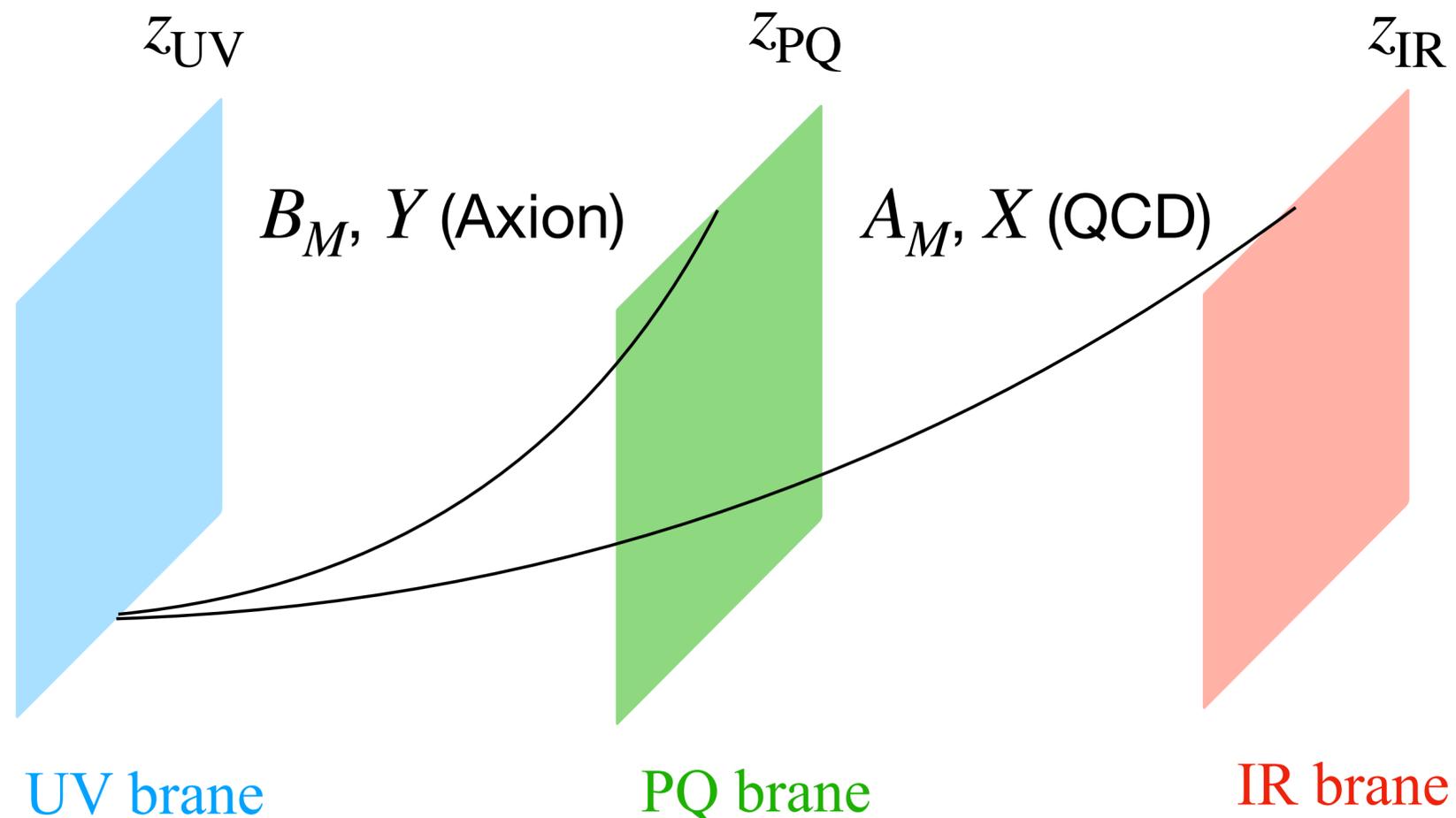
The value of the axion will slide to satisfy equation

$$\theta_{\mathrm{UV}} - \frac{\kappa}{q}\phi_{\mathrm{UV}} - \frac{\lambda}{p}a_{\mathrm{UV}} = 0$$

Holographic QCD Axion

Axion Quality Problem

If $U_a \neq 0$ then we will have the axion quality problem

The magnitude of the quality problem strongly depends on the wave function of the PQ breaking scalar $Y = \zeta e^{ia}$

z_{UV} z_{PQ} z_{IR}

B_M, Y (Axion) A_M, X (QCD)

UV brane   PQ brane   IR brane

$X = \rho e^{i\phi}$: $\langle \bar{q}q \rangle$

A_M: $U(1)_A$ gauge boson

$Y = \zeta e^{ia}$: PQ breaking

B_M: $U(1)_{\text{PQ}}$ gauge boson

Holographic QCD Axion

Axion Quality Problem

If $U_a \neq 0$ then we will have the axion quality problem

The magnitude of the quality problem strongly depends on the wave function of the PQ breaking scalar $Y = \zeta e^{ia}$

If strongly peaked on the PQ brane - the effect of U will be tiny — no quality problem

If peaked on the UV brane - the effect of U will be big - large quality problem

The PQ wave function
$$\zeta = L^{-\frac{3}{2}} \left[v_Y z_{\mathrm{PQ}} \left(\frac{z}{z_{\mathrm{PQ}}} \right)^{\Delta_Y} + s_Y z_{\mathrm{UV}} \left(\frac{z}{z_{\mathrm{UV}}} \right)^{4-\Delta_Y} \right]$$

Δ_Y is dimension of PQ breaking operator: the bigger Δ_Y, the more peaked on the PQ brane

Holographic QCD Axion

$$V \supset \frac{L^4}{z^4} m_q^\dagger L^{-\frac{3}{2}} X$$

Axion Quality Problem

Assume a PQ breaking potential on the UV brane

$$U(Y) = \epsilon_{\cancel{PQ}} \Lambda_{\mathrm{UV}}^3 \left(L^{\frac{3}{2}} Y \right)^n$$

n: effect of some discrete symmetry

Using the wave function profile, this roughly contribute to the potential

$$U(Y_{\mathrm{UV}}) \sim \epsilon_{\cancel{PQ}} \Lambda_{\mathrm{UV}}^3 \left(\frac{z_{\mathrm{UV}}}{z_{\mathrm{PQ}}} \right)^{n\Delta_Y}$$

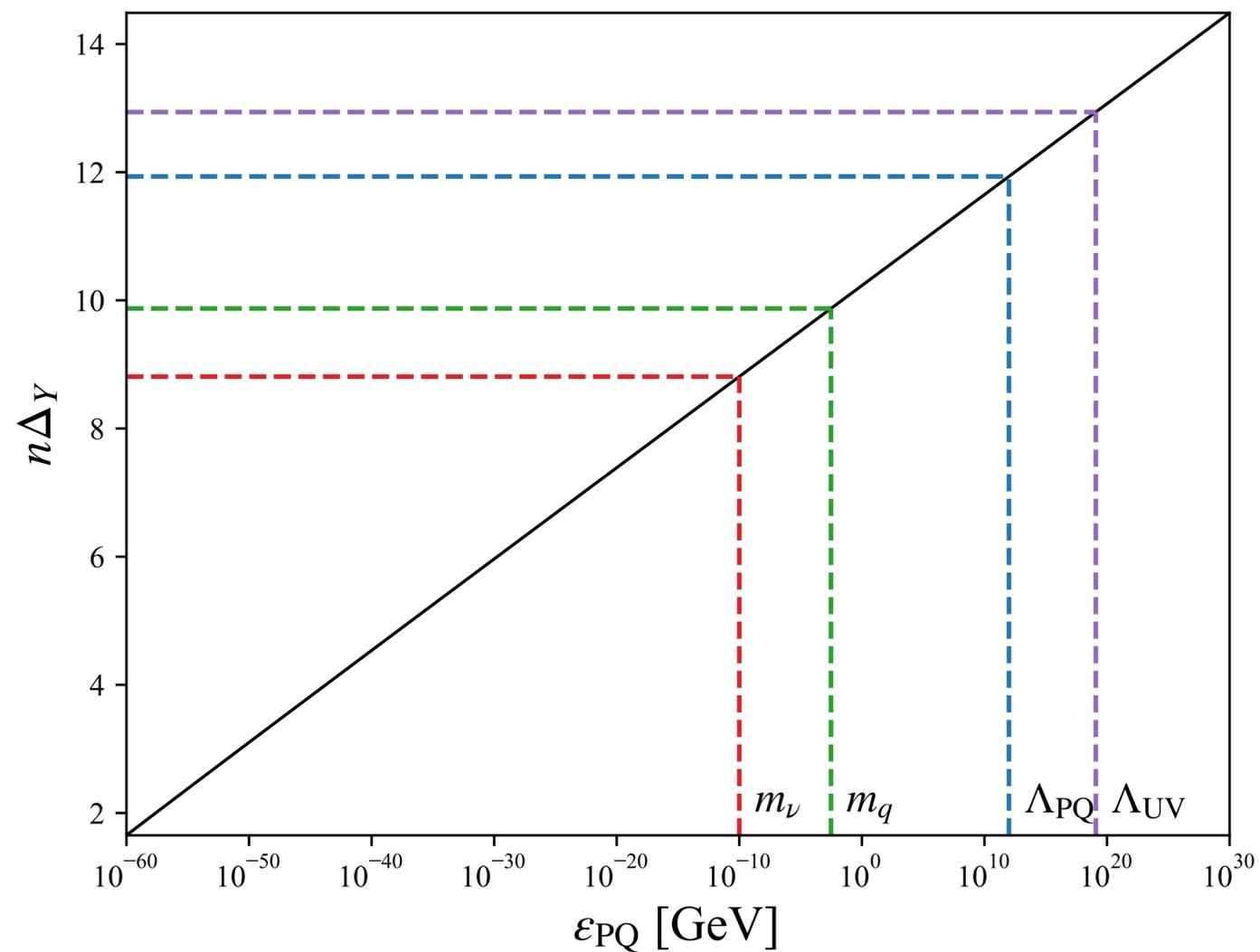This needs to be sub-leading to quark mass term with phase of order 10^{-10}

$$\Delta\bar{\theta}\, m_q v z_{\mathrm{IR}}^{-2} > \epsilon_{\cancel{PQ}} \Lambda_{\mathrm{UV}}^3 \left(\frac{z_{\mathrm{UV}}}{z_{\mathrm{PQ}}} \right)^{n\Delta_Y}$$

Holographic QCD Axion

Axion Quality Problem

$$U(Y) = \epsilon_{\cancel{\text{PQ}}} \Lambda_{\text{UV}}^3 \left(L^{\frac{3}{2}} \zeta \right)^n$$

$$n\Delta_Y \gtrsim 13 \qquad (\epsilon_{\cancel{\text{PQ}}} = z_{\text{UV}}^{-1}),$$

$$n\Delta_Y \gtrsim 12 \qquad (\epsilon_{\cancel{\text{PQ}}} = z_{\text{PQ}}^{-1}),$$

$$n\Delta_Y \gtrsim 10 \qquad (\epsilon_{\cancel{\text{PQ}}} = m_q),$$

$$n\Delta_Y \gtrsim 9 \qquad (\epsilon_{\cancel{\text{PQ}}} = 0.1 \text{ eV}).$$

Same findings as Cox, Gherghetta, Nguyen (2019)

$$m_q = 3 \text{ MeV}, \ v = 0.27 \text{ GeV}, \ z_{\text{PQ}}^{-1} = 10^{12} \text{ GeV and } \Lambda_{\text{UV}} = z_{\text{UV}}^{-1} = M_{pl}.$$

Holographic QCD Axion

Mass Spectrum

We recover the Witten-Veneziano relation for η' and axion masses

$$m_{\eta'}^2 = \frac{2N_f}{f_\pi^2}\chi_{\mathrm{YM}} + \mathcal{O}(m_q) + \cdots$$

$$m_a^2 = \frac{\lambda}{f_a^2}\chi_{\mathrm{QCD}} + \mathcal{O}(\epsilon_{\mathrm{PQ}}) + \cdots$$

Conclusion

Summary and Outlook

QCD θ can be understood as a Wilson of disc / circle

Anomalies are realized via Stückelberg term(s) with shift symmetries

We found fully holographic **QCD** axion - double Stückelberg term reflecting the two anomalies

The axion quality problem is severe unless $n\Delta_Y \gtrsim 10$

    Large operator dimensions or high discrete symmetries

Open a window to so many applications: finite T effects for axion, phase transition, axion string / domain wall, etc.