Generalize to arbitrary 2-complex [0909.0939]

spinfoam amplitude can be formulated beyond simplicial complex

for any 4d 2-complex K not necessarily dual to simplicial complex, we construct spinfoam amplitude

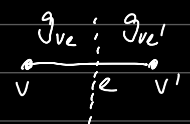
(1) 2-complex K consists of faces f, edges $e \subset \partial f$, vertices $v \in \partial e$

↳ internal face : h

boundary face : b

(2) we color every face $f \subset K$: $SU(2)$ spin j_f

we color every edge $e \subset K$: a pair of $g_{ve}, g_{v'e} \in SL(2,\mathbb{C})$



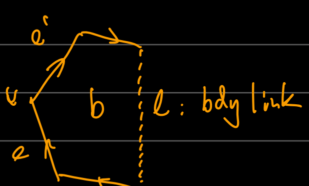
(3) partion function of internal face h [1304.5626]

$$\tau^{(h)}_{j_h}(\underset{\uparrow}{g_h}) = \dim(j_h)\, \mathrm{Tr}_{(2\gamma j_h, 2j_h)}\left[\overrightarrow{\prod_{v\in\partial h}} P_{j_h}\, g_{ve}^{-1} g_{ve'}\, P_{j_h}\right]$$

$\{g_{ve}\}_{e\in\partial h}$

$P_j : \mathcal{H}_{(p,n)} \to \mathcal{H}_j$ projection to sol. of simplicity constraint.

(4) partition function of boundary face b



$$\tau^{(b)}_{j_b}(\underset{\uparrow}{g_b}, H_\ell) = \dim(j_b)\, \mathrm{Tr}_{(2\gamma j_b, 2j_b)}\left[\left(\overrightarrow{\prod_{v\in\partial b}} P_{j_b}\, g_{ve}^{-1} g_{ve}\, P_{j_b}\right) H_\ell\right]$$

$\{g_{ve}\}_{e\in\partial b}$

H_ℓ : $SU(2)$ holonomy along ℓ

(5) spinfoam amplitude on K :

(holonomy rep)

$$A(K) = \sum_{\{j_f\}} \int \prod_{(v,e)} dg_{ve} \prod_h \tau^{(h)}_{j_h}(g_h) \prod_b \tau^{(b)}_{j_b}(g_b, H_\ell)$$

$\prod_v \delta(g_{ve_0})$ for gauge fixing spin-network rep. $\quad A(K) = \sum_{\{j_h, i_e\}} \prod_h \dim(j_h) \prod_v A_v(\vec{j}, \vec{i})$

gluing vertex amplitudes with summing over intermediate states

(6) path integral expression of coherent state rep.

$$A(K) = \sum_{\{j_h\}} \int \prod_{(v,e)} dg_{ve} \prod_{(v,f)} dz_{vf} \; e^{S[j_h, j_b, \underset{SL(2,\mathbb{C})}{g_{ve}}, \underset{\mathbb{CP}^1\text{-spinor}}{z_{vf}}, \overset{\text{normalized spinor}}{\xi_{eb}}]}$$

$$S = \sum_h j_h F(g, z) + \sum_b j_b F(g, z, \xi) \qquad \text{linear in } j_h\ j_b$$

[1304.5626, 2301.02930]

(7) $A(K)$ is lattice dependent.

summing over lattices with fixed bdy. : complete amplitude $\mathcal{A} = \sum_{K,\ \partial K = \Gamma} c_K A(K)$

arbitrary coefficients $c_K \in \mathbb{C}$

<u>properties of spinfoam amplitudes</u>

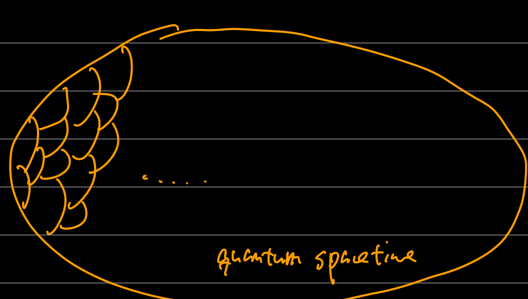
- Bdy data of spinfoam amplitude : $SU(2)$ spin-networks

  $\Rightarrow$ spinfoam amplitude is transition amplitude on $\mathcal{H}_{LQG}$

- spinfoams = histories of spin-networks complete amplitude $\mathcal{A} = \sum_K c_K$ quantum spacetime geometry

  all DOFs of quantum spacetime are supported on 2d surfaces & 1d edges

  $\rightarrow$ quantum is foam-like



Some promising aspects

- simple formulation but encode rich physics, e.g. gravitational waves, quantum black holes, quantum cosmology [2404.02796, 2402.07984, 1003.3483 ...]

  (quantum version of $G_{\mu\nu} = 8\pi T_{\mu\nu}$)

- It is likely a UV-complete theory of quantum gravity [2602.18665]

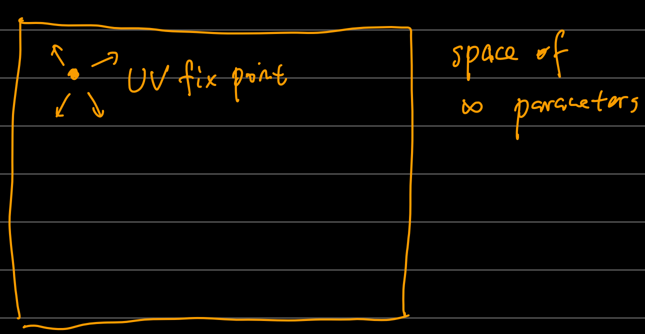
UV fix point of spinfoam theory

spinfoam theory dep. on ∞ many parameters :

$$\text{complete amplitude } \mathcal{A} = \sum_{\substack{K, \\ \partial K = \Gamma}} C_K A(K)$$

arbitrary coefficients $C_K \in \mathbb{C}$

It relates to non-renormalizability of QG : ∞ many higher curvature couplings



idea of RG : near fix point, non renormalizable theory becomes simple :

∞ many parameters $\longrightarrow$ finitely many parameters.

There exists a UV fix point of spinfoam theory [2602.18665] the fix-point theory : (1) topological theory, triangulation indep.

(2) No bulk local DOFs, physical DOFs only on bdy

(3) finitely many parameters

What is UV in LQG ? UV = small j

- in standard QFT, RG scale = length scale is an external parameter because the spacetime geometry is external for QFT
- for LQG, the spacetime geometry is dynamical
  $\Rightarrow$ the length scale is dynamical
- spin j relates to scale because $A = 8\pi \ell_p^2 \gamma \sqrt{j(j+1)}$ defines the spacing of lattice

small j : $A \sim \ell_p^2$ UV regime

large j : $A \gg \ell_p^2$ IR regime

j is dynamical : $A(K) = \sum_{\{j\}} \cdots$,

$\langle j \rangle$ is scale.

The fix point theory is defined in small-$\langle j \rangle$ regime :

Fix bdy graph Γ, finite dim vector space V_Γ spanned by "bdy blocks" $B_\mathfrak{z}(\vec{H})$

each $B_\mathfrak{z}(\vec{H})$ is a linear functional over the space of spin-networks

[2602.18665]

complete spinfoam amplitude at fix point :

$$A = \sum_{\mathfrak{z}} b_\mathfrak{z} \, B_\mathfrak{z}(\vec{H}) \qquad \text{finite sum} : \mathfrak{z} \in \mathbb{Z}_3^{|E(\Gamma)|}$$

$\uparrow$ finitely many coefficients

Turn on perturbations drive the theory away from the fix point.

$$\mathcal{A} = \sum_{3} b_3 B_3(\vec{H}) \left[1 + \sum_{s=1}^{\infty} \left(\frac{1}{N}\right)^s w_s \right]$$

- coupling const. $\left(\frac{1}{N}\right)$
- introduce defects to topological theory
- make bulk DOFs propagating.

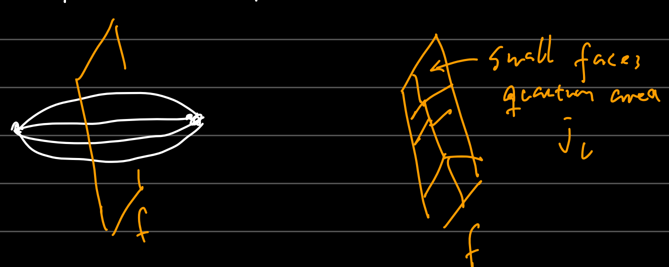
$\mathcal{A}$ recover the standard form of spinfoam amplitude

$$\mathcal{A} = \sum_{\substack{K \\ \partial K = \Gamma}} c_K A(K)$$

interpretation of the coupling const

N : number of stacked faces f sharing the same ∂f.

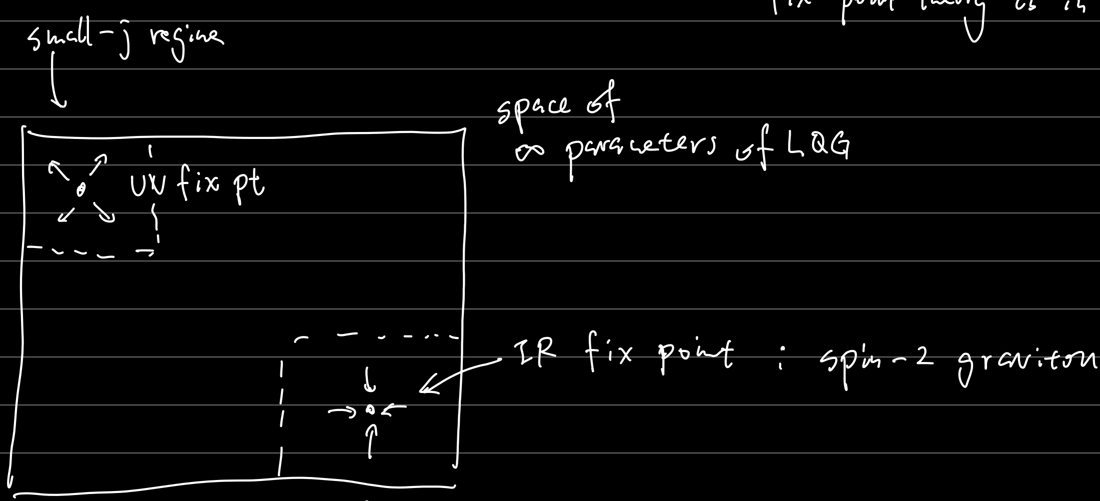
the dual of stacked faces



$$N \sim \frac{\text{maximal area of } f}{\langle j_l \rangle} \gg 1$$

for small $\langle j_l \rangle$

fix point theory is in small $\langle j \rangle$ regime



$\uparrow$
Large-j regime

Large-$\langle j \rangle$ $\rightarrow$ N small $\rightarrow$ $\frac{1}{N}$ large , strongly coupled
perturbation expansion fails
$\rightarrow$ we need a different scheme of effective theory

Large-j effective theory [2301.02930]

$$A = \sum_{\{j_h\}} \int \prod_{v,e} dg_{ve} \prod_{v,f} dz_{vf}\, e^{S[j_h,\, j_b,\, g_{ve}\, z_{vf},\, \xi_{eb}]}$$

bdy data

$S \sim j_h, j_b$

Large-j regime : $j \rightarrow \lambda j$ $\lambda \rightarrow \infty$, $S \rightarrow \lambda S$

like $\frac{1}{\hbar}$ in path integral

$$A \sim e^{S[\text{class. trajectory}]} \left[1 + O\left(\frac{1}{\lambda}\right) \right]$$

scheme : $\sum_j \rightsquigarrow \int dj$

$\underline{j} \sim$ triangle areas $\qquad \underline{j} = \underline{j(\ell)} + \underline{t}$

areas determined by length $\underline{\ell}$

Regge geometry

$$S[j, x, \cancel{\text{bdy}}] = S[\underline{j(\ell)}, \underline{t}, \underline{x}]$$

(g, z)

$$A \sim \int dj \int dx\, e^{\lambda S[j\, x]} \sim \int d\ell \underbrace{\int \overbrace{dt\, J(\ell)}^{dx}\, e^{\lambda S[j(\ell)\, t\, x]}}$$

$$\sim \int d\ell\, J(\ell)\, Z(\ell)$$

$$Z(\ell) = e^{\lambda I_{eff}(\ell)}$$

$$e^{\lambda I_{eff}(\ell)} = \int \underline{dt\, dx}\, e^{\lambda \underline{S[j(\ell), t, x]}} \qquad \lambda \to \infty$$

- $\delta S = 0 \to$ complex critical point
- numerical computation
- $$I_{eff}(\ell) = i S_{Regge}(\ell) + \mathcal{O}(\underline{\gamma \varepsilon}^2) + \mathcal{O}(\tfrac{1}{\lambda})$$

S_{Regge}: discrete E-H action

ε: deficit angle (dicrete Riemann tensor)

$\mathcal{O}(\gamma\varepsilon^2)$: higher curvature correction

- perturbation on flat spacetime $\ell = \mathring{\ell} + \delta \ell$

$\mathring{\ell}$: flat geometry

$$I_{eff}(\ell) = i \sum_{r,s} \left[\sum_h \frac{\partial A_h}{\partial \ell_s} \frac{\partial \varepsilon_h}{\partial \ell_h} \right] \delta \ell_s \delta \ell_r + \mathcal{O}(\gamma \varepsilon^2) + \mathcal{O}(\tfrac{1}{\lambda})$$

$\delta I_{eff}(\ell) = 0 \Rightarrow$ linear Regge eqn $+ \mathcal{O}(\gamma\varepsilon^2)$

small γ, long wavelength limit : $\mathring{\ell} \ll$ curvature radius

$\Rightarrow$ linear spin-2 graviton (IR fix-pt theory)

[William 1986, Höhn 2014]