Spinfoam quantization

1. overview of LQG
2. overview of quantum geometry
3. covariant dynamics of LQG : spinfoam theory
4. Main recent results

Reviews: Rovelli & Vidotto "Covariant LQG" Perez, 1205.2019

Overview of LQG

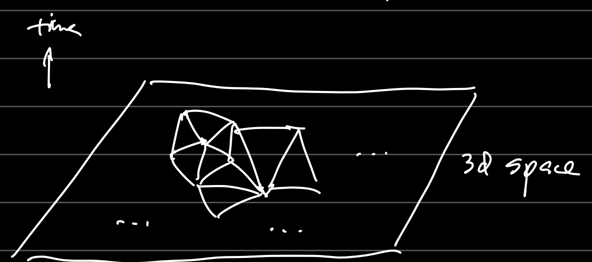
GR : Gravity = Curved geometry

LQG : Quantum Gravity = Quantum Geometry

Features of quantum gravitational field :

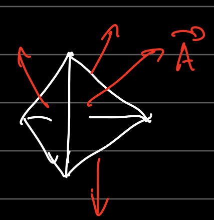
- Gravitational field / spacetime geometry is formed by "quanta"
- "quanta of geometry" = elementary building blocks of quantum geometry
- Quantum geometry built by blocks relates to fundamental discreteness of quantum geometry

if we zoom in Planck scale, our 3d space looks like :



Overview of Quantum Geometry

Example of building block : Quantum tetrahedron.



classical tetrahedron : • large 3d geometry can be triangulated build block is tetrahedron

triangulation is refined ; the terahedron is flat

- $\underline{\text{6 DOF}}$: 6 edge lengths fix geometry

Equivalent description, 4 faces → 4 oriented area vectors $\vec{A}_{i=1..4}$

$\underline{|\vec{A}_i| = A_i}$ face area , $\frac{\vec{A}}{|\vec{A}|}$ = normal of face

DOFs : $4 \times 3 = 12$ more than 6 there is redundency

(1) closure condition $\sum_{i=1}^{4} \vec{A}_i = 0$ (−3) DOFs

(2) "gauge transf" : 3d rotations (−3) DOFs

$12 - 3 - 3 = 6$

Quantum geometry from LQG ; $\vec{A}_i \rightsquigarrow \hat{\vec{A}}_{i=1..4}$ "flux operator"

commutation relation ; $\underline{[\hat{A}_i^a, \hat{A}_j^b]} = i\gamma \ell_p^2 \,\underset{\sim}{\delta_{ij}}\, \varepsilon^{abc} \hat{A}_i^c$

$\uparrow$ B-I parameter

$a = 1, 2, 3$

$i = 1, \cdots, 4$

$$[\hat{J}^a, \hat{J}^b] = i\hbar \varepsilon^{abc} \hat{J}^c$$

The quantum tetrahedron carries quantum number of area

area operator ; $\hat{A}_i = \sqrt{\hat{A}_i^a \hat{A}_i^a} \sim \underset{\sim}{\sqrt{J^2}}$ $i = 1..4$

$[\hat{A}_i, \hat{A}_j] = 0$ $\boxed{\text{area quantum number}}$

discrete spectrum : $A_i = 8\pi\gamma \ell_p^2 \underline{\sqrt{j(j+1)}}$ spin $j \in \mathbb{N}/2$

Hilbert space (fixed quantum area) $\mathcal{H}_{kin} = \underline{\mathcal{H}_{j_1} \otimes \mathcal{H}_{j_2} \otimes \mathcal{H}_{j_3} \otimes \mathcal{H}_{j_4}}$

$(\mathcal{H}_{j_i}, \pi^{j_i})$: SU(2) irrep with spin j_i

$\hat{\vec{A}}_i$ acts on $\mathcal{H}_{j_i}$ as angular momentum $\vec{J}$

quantum closure condition :

$$\sum_{i=1}^{4} \vec{A}_i = 0 \rightsquigarrow \boxed{\sum_{i=1}^{4} \hat{\vec{J}}_i |\psi\rangle = 0}$$

solution of quantum closure constraint

$$\mathcal{H}_{phys} = \underline{\mathrm{Inv}_{SU(2)}(\mathcal{H}_{j_1} \otimes \mathcal{H}_{j_2} \otimes \mathcal{H}_{j_3} \otimes \mathcal{H}_{j_4})}$$

- the space of zero total angular momentum states
- the space of intertwiners / invariant tensors

$\forall g \in SU(2)$ $\underline{\bigotimes_{i=1}^{4} \pi^{j_i}(g) |\psi\rangle = |\psi\rangle} \Leftrightarrow \sum_{i=1}^{4} \vec{J} |\psi\rangle = 0$

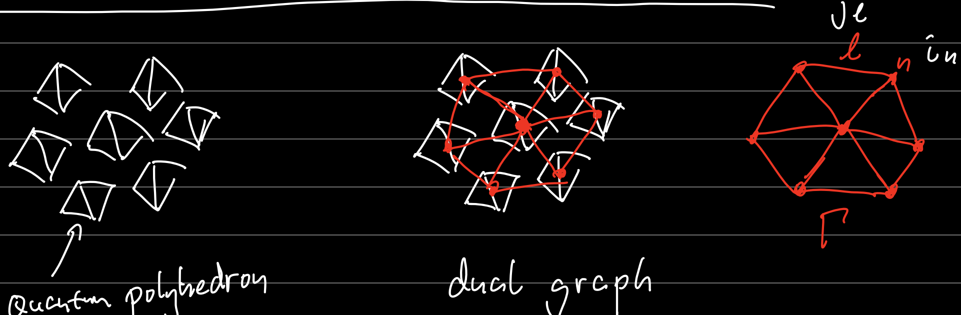
$g = e^{\vec{\alpha} \cdot \vec{J}}$

orthonormal basis in $\mathcal{H}_{phys}$, $|i\rangle$ $i = 1 \cdots \dim(\mathcal{H}_{phys})$

↑ intertwiners

Easy to generalize to quantum polyhedron,

Build quantum geometry by gluing building blocks



Γ: a graph of links ℓ and nodes n

links ℓ ⇕ face shared by a pair of polyhedron

nodes n ⇕ polyhedron

link ℓ: carry quantum area; quantum number j_ℓ

node n: carry quantum tetrahedron / polyhedron; quantum number i_n

a triple $|\Gamma, \{j_\ell\}, \{i_n\}\rangle$ spin-network state

↑ spins on all links ; ↑ intertwiners on all nodes

Consider all graphs Γ, all spin $j_\ell \neq 0$, all intertwiners i_n

$\to \mathcal{H}_{LQG}$: LQG Hilbert space is spanned by all spin-network states

spin-network state form a complete orthonormal basis

$$\langle \Gamma, \{j_\ell\}, \{i_n\} | \Gamma', \{j_\ell'\}, \{i_n'\}\rangle$$

$$= \delta_{\Gamma,\Gamma'}\, \delta_{\{j_\ell\}\{j_\ell'\}}\, \delta_{\{i_n\}\{i_n'\}}$$

Remarks: (1) the theory of quantum geometry is background indep,

i.e. the quantum states are not on spacetime but rather quantum states of $\underline{I}$ spacetime

Spacetimes are emergent from quantum states

(2) geometrical quantities (e.g. area, volume, normals) are operators on the Hilbert space $\mathcal{H}_{LQG}$

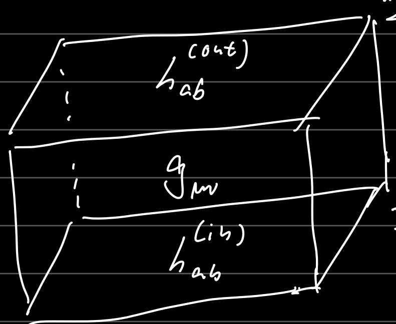
Spectra of area and volume are discrete

in general, geometrical operator are non-commutative

$\rightarrow$ Heisenberg uncertainty of quantum geometry

(quantum fluctuation)

<u>Covariant dynamics of LQG</u>



Σ^{out} 3d final surface

M_4 : 4d manifold

Σ^{in} : 3d initial surface

3d metrics $h_{ab}^{(in)}$: initial 3-metric

$h_{ab}^{(out)}$: final 3-metric

4d metric $g_{\mu\nu}$: evolution of 3-geometry from $h_{ab}^{(in)}$ to $h_{ab}^{(out)}$

"history of 3-geometries"

path integral of 4d gravity : $Z[h_{ab}^{(out)}, h_{ab}^{(in)}]$

$$= \int_{g_{\mu\nu}|_{\Sigma^{(in)}} = h_{ab}^{(in)}}^{g_{\mu\nu}|_{\Sigma^{out}} = h_{ab}^{(out)}} Dg_{\mu\nu} \; e^{\frac{i}{\ell_p^2}\int R + \cdots}$$

sum over 4d metric w/ bdy condition $g_{\mu\nu}|_{\Sigma^{in}} = h_{ab}^{(in)}$

$g_{\mu\nu}|_{\Sigma^{out}} = h_{ab}^{(out)}$

$\parallel$

sum over histories of 3d geometries

LQG : 3d geometry is quantized

3d quantum geometry = spin-network states $|\Gamma, \vec{j}, \vec{i}\rangle$

$$\vec{j} = \{j_e\}, \quad \vec{i} = \{i_n\}$$

4d quantum geometry = history of 3d quantum geometries

= history of spin-networks

Simple examples

trivial history

• evolution of link ℓ → spinfoam face f

spin j colors ℓ → spin j colors f

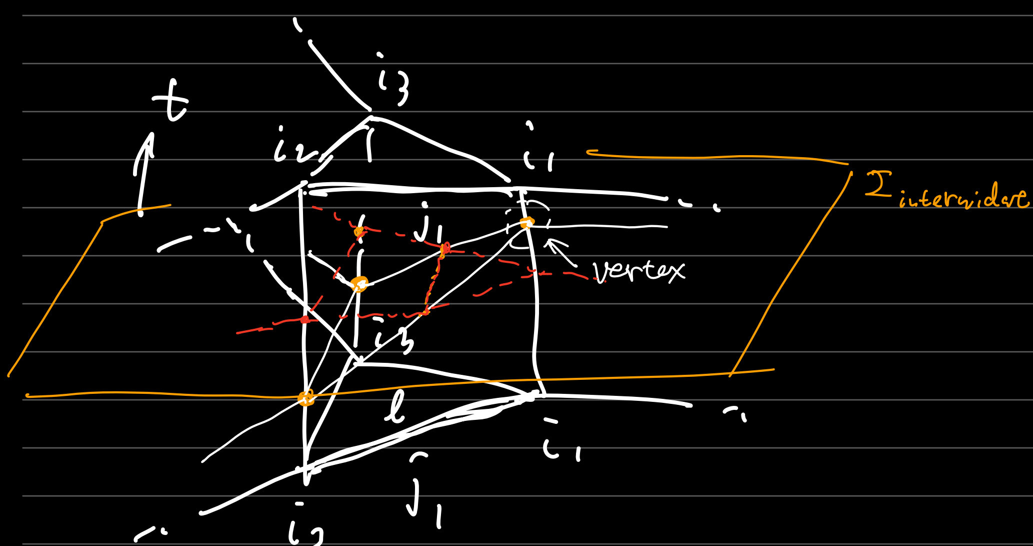
j_ℓ → j_f

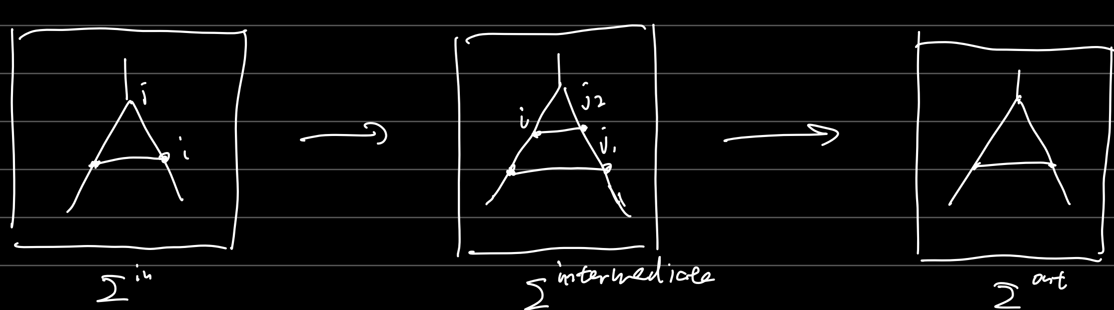
• evolution of node n → spinfoam edge e

intertwiner i colors n → intertwiner i colors e

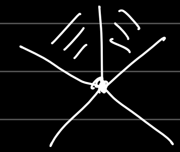
i_n → i_e

nontrivial history: spinfoam vertices





nontrivial process: spinfoam vertex



interaction among edges and faces
colored by i_e ... colored by j_f

spinfoam model of BF theory

classical BF theory : G : matrix Lie group , M_D : D-manifold

$$S_{BF} = \int_{M_D} \mathrm{tr}\left(B \wedge F(A) \right)$$

B: $\mathfrak{g}$-valued $(D-2)$-form

$F(A)$: curvature 2-form of a $\mathfrak{g}$-connection A

BF theory is a topological field theory : $\delta S_{BF} = 0 \Rightarrow d_A B = 0$, $F(A) = 0$ ← flat conn.

gauge inv. $\begin{cases} B \to g B g^{-1} \\ A \to g A g^{-1} + g^{-1} dg \end{cases}$ $\quad \begin{cases} B \to B + d_A f \\ A \to A \end{cases}$ (f: $(D-3)$-form)

$g(x) \in G$

for topological trivial manifold e.g. $M_D = \mathbb{R}^D$, $F(A) = 0 \Rightarrow \underline{A = 0}$ up to gauge

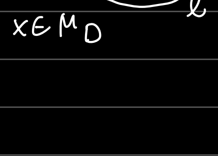
$0 = d_A B = dB$: B is closed $\overset{M_D = \mathbb{R}^D}{\Longrightarrow}$ B is exact $\boxed{B = df}$

$\Rightarrow \underline{B = 0}$ up to gauge transf.

- there is no local propagating DOF in BF theory
- only DOFs are topologies of M_D

quantization : $Z = \int DB\, DA\, e^{i \int_{M_D} \mathrm{tr}(B \wedge F)} = \underline{\int DA\, \delta(F(A))}$

consider loop holonomy of A



l : loop in M_D

S : 2-surface , $\partial S = l$

$$h_l(A) = P e^{\oint_l A} = P_S\, e^{\int_S g F g^{-1}}$$

(F: curvature; P_S: nonabelian Stokes thm.)

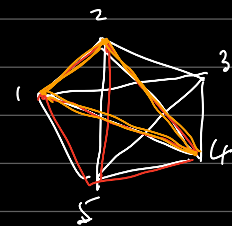
$F(A) = 0 \iff h_l(A) = \mathbb{1} \;\; \forall l$

$$Z = \int \mathcal{D}A \prod_{\ell} \delta_G(h_\ell(A))$$

$D=4$, $G = SU(2)$ "Oguri model" Oguri 1992

spinfoam model to make sense BF path integral

simplicial complex in 4d (triangulation of 4-manifold)

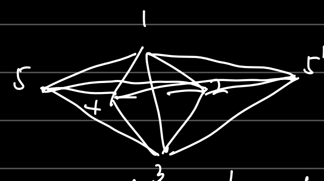


elementary cell in 4d triangulation : 4-simplex

bdy of 4-simplex : 5 tetrahedra, $C_5^4 = 5$

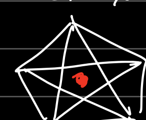
10 triangles. $C_5^3 = 10$

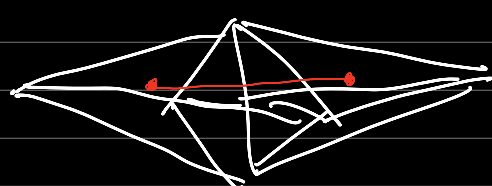
4d simplicial complex :



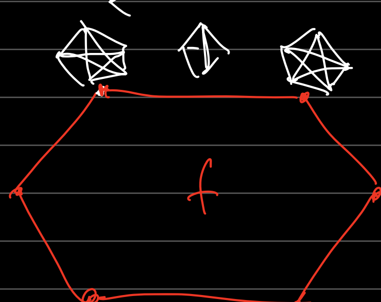
glue 4-simplices by sharing tetrahedra

Any simpicial complex associates a unique dual 2-complex

simpicial complex		dual 2-complex
4d - 0d: 4-simplex 	$\longleftrightarrow$	vertex v 
3d - 1d: tetrahedron	$\longleftrightarrow$	edge e



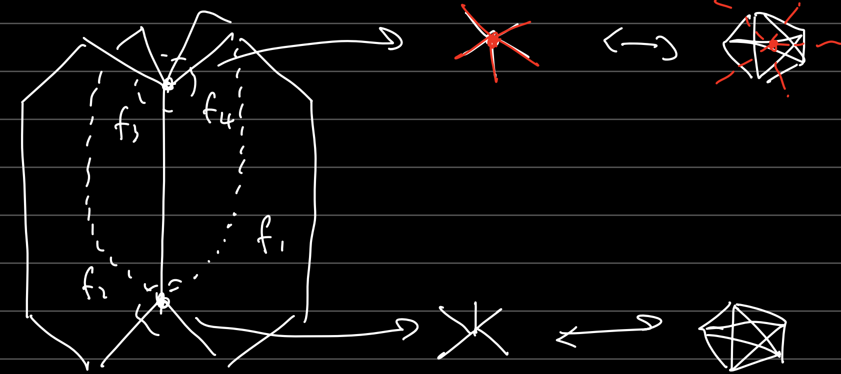
2d - 2d: triangle $\longleftrightarrow$ face f



triangle shared by N 4-simplices

face with N vertices along ∂f

Example :



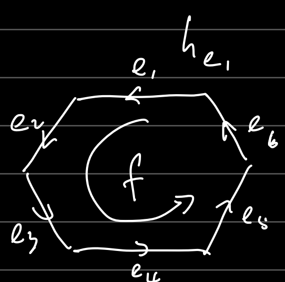
1 edge connecting 2 vertices $\longleftrightarrow$ a tetrahedron shared by 2 4-simplices

4 faces connection to the edge $\longleftrightarrow$ 4 triangle on the body of the tetrahedron

Discrete BF path integral : $Z = \int DA \prod_{e} \delta_G (h_e(A))$

$$\rightarrow \int DA \prod_{f} \delta_G (h_{\partial f}(A))$$

faces in 2-complex



$$DA = \prod_{e} d\mu_H (h_e (A))$$

edges in 2-complex

character of irrep j

for this face, $\delta_G(h_{\partial f}) = \sum_{j=0}^{\infty} \dim(j) \, \chi_j(h_{\partial f})$

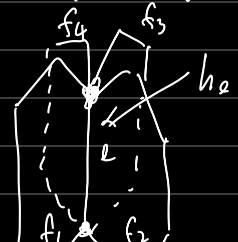
$2j+1$

$$\chi_j(h_{\partial f}) = tr_j(h_{\partial f}) = \pi^j_{mk}(h_{e_n}) \, \pi^j_{kl}(h_{e_{n-1}}) \cdots \pi^j_{pm}(h_{e_1})$$

$$\pi^j_{mn}(h) = \langle j\, m | h | j\, n \rangle$$

$$Z = \int \prod_e d\mu_H(h_e) \prod_f \sum_{j_f=0}^{\infty} \dim(j_f) \, \pi^{j_f}_{m_f k_f}(h_{e_n}) \cdots \pi^{j_f}_{p_f m_f}(h_{e_1})$$

a single edge



the integral to this edge

$$\int d\mu_H(h_e) \; \pi^{j_1}_{m_1 n_1}(h_e) \cdots \pi^{j_4}_{m_4 n_4}(h_e)$$

$$= P^{j_1 \cdots j_4}_{m_1 \cdots m_4,\, n_1 \cdots n_4} = \sum_i i^*_{m_1 \cdots m_4}\, i_{n_1 \cdots n_4} = \sum_i |i\rangle\langle i|$$

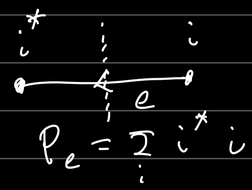
projection on $\mathrm{Inv}_{SU(2)}(\mathcal{H}_{j_1} \otimes \cdots \mathcal{H}_{j_4})$; basis of intertwiners

$\Rightarrow$ we associate to each edge a projection $\left(P_e^{j_1 \cdots j_4}\right)_{\underline{\vec{m}, \vec{n}}}$

$$Z = \sum_{\{j_f\}} \prod_f \dim(j_f)\ \underset{\sim}{\mathrm{tr}}\left(\prod_e P_e\right)$$

Contraction of indices according to faces

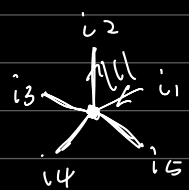
How indices are contracted :



associate i^* to the end point

- - - - - i to the begin point

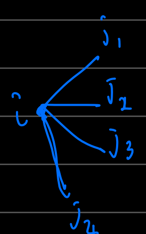
$P_e = \sum_i i^* i$



contract a pair of i_1, i_2 according to the orientation of face

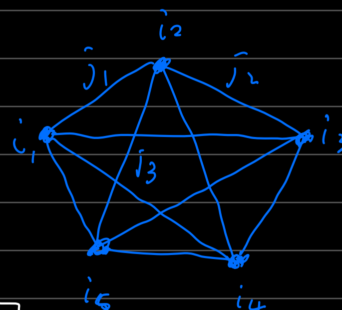
a pair of edges $\to$ a face

5 edges $\to$ 10 faces $\to$ 10 contractions



$\equiv i^{j_1 \cdots j_4}$

the contraction



10 j's

5 i's

$\Rightarrow \{15j\}$

$$\boxed{Z = \sum_{\{j_f, i_e\}} \prod_f \dim(j_f) \prod_v \{15j\}}$$

face amplitude ; vertex amplitude

EPRL model of 3+1 d LQG

Engle - Pereira - Rovelli - Livine 2007

classical gravity : Plebanski formulation

$$S_{Pl} = \int_{M_4} \left(B^{IJ} + \frac{1}{2\gamma} B^{KL} \varepsilon^{IJ}{}_{KL}\right) \wedge F_{IJ}(A) + \varphi^{IJKL} B_{IJ} \wedge B_{KL}$$

B-I parameter

Lagrangian multiplier

$SL(2,\mathbb{C})$ valued 2-form

$SL(2,\mathbb{C})$ curvature

φ^{IJKL} anti symm in IJ, KL

$\varphi^{IJKL} = \varphi^{KLIJ}$, traceless $\varepsilon_{IJKL}\varphi^{IJKL} = 0$

$\delta S_{PL} = 0 \Rightarrow$ Einstein equ

$$Z_{PL} = \int DADBD\varphi\, e^{\frac{i}{\ell_P^2} S_{PL}}$$

$$= \int DADB\, \delta(\text{simplicity constraint})\, e^{\frac{i}{\ell_P^2}\int_{M_4}(B+\frac{1}{\gamma}*B)^{IJ}\wedge F_{IJ}}$$

"gravity is a constrained BF theory"

$$\varepsilon^{\mu\nu\rho\sigma} B^{IJ}_{\mu\nu} B^{KL}_{\rho\sigma} = V\varepsilon^{IJKL}, \quad V = \frac{1}{4!}\varepsilon_{IJKL} B^{IJ}_{\mu\nu} B^{KL}_{\rho\sigma}\varepsilon^{\mu\nu\rho\sigma}$$

classical sol: $\exists\, e^I_\mu$ s.t. $B^{IJ} = \pm e^I \wedge e^J$ or $B^{IJ} = \pm(*e\wedge e)^{IJ}$

or $V = 0$ (deg)

Spinfoam quantization on a simplicial complex in 4d

outline: (1) we define Z_{PL} on a single 4-simplex

the result is the spinfoam vertex amplitude

(2) first quantize BF theory (ignore constraint)

$$\int DADB\, e^{\frac{i}{\ell_P}\int \underline{(B+\frac{1}{\gamma}*B)^{IJ}}\wedge F_{IJ}}$$

gauge group is $SL(2,\mathbb{C})$

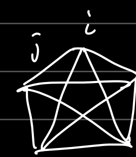
$$= Z_{BF}(\text{bdy data})$$

spans bdy Hilbert space

(3) quantize the simplicity constraint & impose to BF bdy Hilbert space

$\rightarrow$ restriction of bdy data

(4) vertex amplitude: $A_v = Z_{BF}(\text{bdy data}\big|_{\text{simplicity}})$

- SU(2) BF theory in a 4-simplex $= \{i_a, j\} =$ 

bdy data : $i_{a=1\cdots5}$ on 5 tetrahedra

j_{ab} $a<b$ on 10 triangle

bdy Hilbert space $\simeq L^2(SU(2)^{\times 10}) / \text{gauge}$ ← spanned by spin-networks $|j_{ab}, i_a\rangle$

- SL(2, ℂ) BF theory : bdy Hilbert space $\simeq L^2(SL(2,\mathbb{C})^{\times 10}) / \text{gauge}$

bdy data : SL(2, ℂ) intertwiner on 5 tetrahedra

SL(2, ℂ) unitary irrep on 10 triangles

quantization of B-field : $B_{\mu\nu}^{IJ} \rightsquigarrow \hat{B}_f^{IJ}$ operator on $L^2(SL(2,\mathbb{C}))$

one for each triangle of 4-simplex

$$\hat{J}_f^{IJ} = \left(\hat{B}_f + \frac{1}{\gamma} * \hat{B}_f\right)^{IJ} \qquad I, J = 0, 1, 2, 3$$

is the right inv. vector field on SL(2, ℂ)

$\hat{J}_f^{0i} \equiv K_f^i$ boost generator $\quad \hat{J}_f^{ij}\varepsilon_{ijk} \equiv L_f^k$ rotation generator

linear simplicity constraint : For each tetrahedron, ∃ internal timelike normal N^I

(discretization of $B = *(e\wedge e)$) s.t. ∀ f triangle of the tetrahedron

$$\boxed{N_I * B_f^{IJ} = 0}$$

time gauge : $N_I = (1, 0, 0, 0)$ $\qquad \underline{*B_f^{0i} = 0}$ ∀ f ⊂ tetrahedron

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$*B_f^{IJ} \sim$ oriented area of f in 4d

linear simplicity $\rightsquigarrow$ f is spacelike, tetrahedron is spacelike

quantum simplicity constraint : " $\underbrace{\left(\hat{L}_f^i + \frac{1}{\gamma}\hat{K}_f\right)}$ $\Psi = 0$ "

However $\hat{C}_f^i$ contain 2nd class constraint $[\hat{C}_f^i, \hat{C}_f^j] =$ nonvanishing on constraint surface

$\Rightarrow$ we have to impose $\hat{C}_f^i$ weakly

EPRL's proposal:

$$\begin{cases} \underline{\underline{C_{ff}}} = *B_f \cdot B_f = \underline{{}^*J_f \cdot J_f}(1-\frac{1}{\gamma^2}) + \frac{2}{\gamma}\underline{J_f \cdot J_f} \\ \quad \text{diagonal simplicity : 1st class.} \\ M_f = \underline{(\vec{L}_f + \frac{1}{\gamma}\vec{K})^2} \quad \text{for weakly impose the 2nd class constraint.} \end{cases}$$

$$X \cdot Y = X^{IJ} Y_{IJ}$$

express the constraints in terms of $SL(2,\mathbb{C})$ quadratic casimir

$$\hat{C}_1 = \hat{J} \cdot \hat{J} = 2(\hat{L}^2 - \hat{K}^2) \qquad \hat{C}_2 = *J \cdot J = -4\hat{L}\cdot\hat{K}$$

equivalent constraint

$$\begin{cases} \underline{\hat{C}_2(1-\frac{1}{\gamma^2}) + \frac{2}{\gamma}\hat{C}_1} = 0 + \text{quantum corrections} \quad (1) \leftarrow \\ \underline{\hat{C}_2 - 4\gamma \hat{L}^2} = 0 + \text{quantum corrections} \quad (2) \leftarrow \end{cases}$$

<u>principal series unitary irrep of $SL(2,\mathbb{C})$</u>

$SL(2,\mathbb{C})$ non compact group $\rightarrow$ unitary irrep is carried by infinite-dim Hilbert space

principal series irrep is labelled by (ρ, n) $\rho \in \mathbb{R}$, $n \in \mathbb{N}_0$

$\mathcal{H}_{(\rho,n)}$: ∞-dim Hilbert space

$SU(2) \subset SL(2,\mathbb{C}) \rightarrow \mathcal{H}_{(\rho,n)}$ is reducible rep of $SU(2)$

i.e. $\underline{\underline{\mathcal{H}_{(\rho,n)}}} = \bigoplus_{j=\frac{n}{2},\frac{n}{2}+1,\dots}^{\infty} \mathcal{H}_j$ ← SU(2) irrep, finite-dim

canonical basis in $\mathcal{H}_{(\rho,n)}$: $\{ |(\rho,n), j, m\rangle \}$, $j = \frac{n}{2}, \frac{n}{2}+1 \dots$, $m = -j, -j+1, \dots, j$

quadratic casimirs:

$$C_1 = \frac{1}{2}(n^2 - \rho^2 - 4)\,\mathbb{1}$$
$$C_2 = n\rho\,\mathbb{1}$$

on $\mathcal{H}_{(\rho,n)}$

insert in (1) and (2) and use $\hat{L}^2 = j(j+1)$ when acting on $|(\rho,n)\, j\, m\rangle$

$$\Rightarrow \boxed{\rho = 2\gamma j\,, \quad n = 2j}$$ up to $O(\frac{1}{j})$ corrections

semiclassical limit = Large-j limit.

State satisfying constraints

$$|(\rho,n)\, j\, m\rangle = |(2\gamma j, 2j),\, j, m\rangle \in \mathcal{H}_{j=\frac{n}{2}}$$ lowest level in $\mathcal{H}_{(\rho,n)} = \bigoplus_{j=\frac{n}{2}}^{\infty} \mathcal{H}_j$

As a result:

(1) restrict principal series irrep $(\rho, n) = (2\gamma j, 2j)$ for every face

(2) Y-map: $Y: \mathcal{H}_j \hookrightarrow \mathcal{H}_{(2\gamma j, j)} = \bigoplus_{k=j}^{\infty} \mathcal{H}_k$

by identifying the lowest level

$$Y: |j\, m\rangle \longmapsto |(2\gamma j, \gamma j), j, m\rangle$$

the image of Y-map is solution of quantum simplicity constraint

(3) EPRL intertwiner: Given SU(2) intertwiner (quantum tetrahedron)

$$i \in \mathrm{Inv}_{SU(2)}(\mathcal{H}_{j_1} \cdots \mathcal{H}_{j_4})$$

$Y^{\otimes 4} \circ i$ is $SL(2,\mathbb{C})$ tensor

$$\boxed{I(i) = P \circ Y^{\otimes 4} \circ i}$$ is EPRL intertwiner (boosted quantum tetrahedron)

projection onto $SL(2,\mathbb{C})$ inv. tensors.

Explicitly, use $SL(2,\mathbb{C})$ wigner D-matrix

$$\forall g \in SL(2,\mathbb{C}) \qquad D^{(\rho,n)}_{j'm', jm}(g) = \langle (\rho,n)\, j'\, m' |\, g\, | (\rho,n)\, j\, m\rangle$$

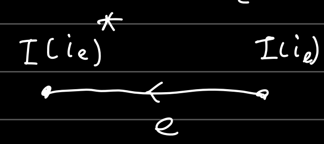
$$\rightarrow I(i) = \int_{SL(2,\mathbb{C})} dg \prod_{i=1}^{4} D^{(2\gamma j_i, 2j_i)}_{j_i' m_i',\, j_i m_i}(g)\; i^{j_1 \cdots j_4}_{m_1 \cdots m_4}$$

Haar measure

contraction (Y-map)

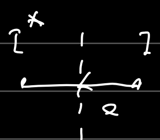
group average projection.

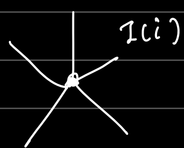
"Edge operator" : $P_e = \sum_{i_e} I(i)^* \, I(i)$ $\leftarrow$



For any 2-complex, EPRL spin foam amplitude

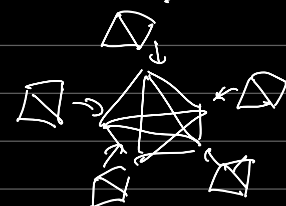
$$Z_{EPRL} = \sum_{\{j_f\}} \prod_f \dim(j_f) \, \mathrm{tr}\left(\prod_e \underline{\underline{P_e}} \right)$$





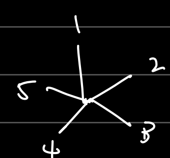
$$A_v = \mathrm{tr}\left(I(i_1) \otimes \cdots \otimes I(i_5) \right)$$

$\underline{\text{EPRL vertex amplitude}}$



$$Z_{EPRL} = \sum_{\{j_f, i_e\}} \prod_f \dim(j_f) \prod_v \underline{A_v(\vec{j}, \vec{i})}$$

Integral expression of A_v



10 face : j_{ab} $\quad a, b = 1 \cdots 5$

5 edges : $I(i_a)$

$$A_v = \int_{SL(2,\mathbb{C})} \prod_{a=1}^{5} dg_a \prod_{a<b} \langle j_{ab} \, m_{ab} | Y^\dagger g_a^{-1} g_b Y | j_{ab} \, m_{ba} \rangle$$

$\uparrow$

inner product of $\mathcal{H}_{(2\gamma j_{ab}, 2j_{ab})}$

$$= \int_{SL(2,\mathbb{C})} \prod_{a=1}^{5} dg_a \prod_{a<b} \langle\!\langle (2\gamma j_{ab}, 2j_{ab}) \, j_{ab} \, m_{ab} | g_a^{-1} g_b | (2\gamma j_{ab} \, 2j_{ab}) \, j_{ab} \, m_{ba} \rangle\!\rangle$$

$SL(2,\mathbb{C})$ gauge redundancy : $g_a \to x g_a \quad g_a, x \in SL(2,\mathbb{C})$

cause the divergence of A_v

gauge fixing : $\int \prod_{a=1}^{5} dg_a \cdots \to \int \prod_{a=1}^{5} dg\, \delta(g_5)$

resulting A_v is finite $\begin{bmatrix} 0805.4696 \\ 1010.5384 \end{bmatrix}$

Coherent state rep.

SU(2) irrep : $\mathcal{H}_j \circlearrowleft SU(2)$ high weight state $|j,j\rangle$

Coherent state : $\underbrace{|j\ \xi\rangle = g(\xi)|j,j\rangle}$
normalized spinor

$$\xi = \begin{pmatrix} \xi^0 \\ \xi^1 \end{pmatrix}$$

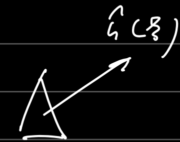
$$\langle \xi, \xi \rangle = \bar{\xi}^0 \xi^0 + \bar{\xi}^1 \xi^1 = 1$$

$$g(\xi) = \begin{pmatrix} \xi^0 & -\bar{\xi}^1 \\ \xi^1 & \bar{\xi}^0 \end{pmatrix} \in SU(2)$$

$|j,j\rangle$: $\uparrow \hat{z}$ Length $\sim j$

$|j,\xi\rangle$: $\hat{z}$ $\nearrow \vec{n}(\xi) = \langle \xi | \vec{\sigma} | \xi \rangle$

$\vec{n}(\xi)$ is the 3-normal of tetrahedron face



Coherent state rep. of A_v

$$A_v(\vec{j}, \vec{\xi}) = \int \prod_{a=1}^{5} dg_a\, \delta(g_5) \prod_{a<b} \langle j_{ab}\, \xi_{ab} | Y^\dagger g_a^{-1} g_b Y | j_{ab}\, \xi_{ba} \rangle$$

it has a path integral rep. $A_v(\vec{j}, \vec{\xi}) = \int D\mu\, e^{S}$ ← spinfoam action

$\mathcal{H}_{(\rho,n)}$ ⊂ space of homogeneous func. of $(z^0, z^1) \equiv z \in \mathbb{C}^2$

$\Downarrow$

$f(z,\bar{z})$, $f(\lambda z, \lambda \bar{z}) = \lambda^{-1+\frac{i\rho}{2}+\frac{n}{2}} \bar{\lambda}^{-1+\frac{i\rho}{2}-\frac{n}{2}} f(z,\bar{z})$

L^2-inner product on $\mathcal{H}_{(\rho,n)}$

$$\langle f, f' \rangle = \int_{\mathbb{CP}^1} \Omega_{z\bar{z}}\, \bar{f}(z,\bar{z})\, f'(z,\bar{z})$$

$$\Omega_{z\bar z} = \frac{i}{2}(z^0 dz^1 - z^1 dz^0)\wedge(\bar z^0 d\bar z^1 - \bar z^1 d\bar z^0)$$

$Y|j\,j\rangle = |(2\gamma j, 2j)\, j\, j\rangle \in \mathcal{H}_{(2\gamma j, 2j)}$:

$$\sqrt{\frac{\dim(j)}{\pi}}\,\langle \bar z, \bar z\rangle^{i\gamma j - 1 - j}\,(z^0)^{2j}$$

$Y|j\,\xi\rangle = |(2\gamma j, 2j)\, j\, \xi\rangle$:

$$\sqrt{\frac{\dim(j)}{\pi}}\,\langle \bar z, \bar z\rangle^{i\gamma j - 1 - j}\,\langle \bar z, \xi\rangle^{2j} \equiv f_\xi(z, \bar z)$$

$$\langle j_{ab}\,\xi_{ab}|Y^{\dagger} g_a^{-1} g_b Y|j_{ab}\,\xi_{ba}\rangle$$

$$= \int_{\mathbb{CP}^1} d\Omega_{z\bar z}\,\overline{f_{\xi_{ab}}(g_a^T z\ \ \overline{g_a^T z})}\,f_{\xi_{ba}}(g_b^T z\ \ \overline{g_b^T z})$$

$$= \frac{\dim(j_{ab})}{\pi}\int_{\mathbb{CP}^1} dz\ e^{S_{ab}}$$

$$dz = \frac{\Omega_{z\bar z}}{\langle Z_{ab}, Z_{ab}\rangle\langle Z_{ba}, Z_{ba}\rangle} \qquad Z_{ab} = g_a^{\dagger}\bar z \qquad Z_{ba} = g_b^{\dagger}\bar z$$

$$S_{ab} = j_{ab}\,F_{ab}(g, z, \xi)$$

$$F_{ab} = 2\ln\frac{\langle \xi_{ab}\, Z_{ab}\rangle\langle Z_{ba}\,\xi_{ba}\rangle}{\|Z_{ab}\|\,\|Z_{ba}\|} + 2i\gamma\ln\frac{\|Z_{ba}\|}{\|Z_{ab}\|}$$

$\langle\ ,\ \rangle$ Hermitian inner product on $\mathbb{C}^2$

$\|Z\| = \sqrt{\langle Z, Z\rangle}$

$$A_v = \frac{\prod_{a<b}\dim(j_{ab})}{\pi^{10}}\int \prod_a dg_a \prod_{a<b} dz_{ab}\,\delta(g_5)\,e^{S} \qquad S = \sum_{a<b} S_{ab}$$

key property : S is linear in j_{ab} , large j , $j_{ab}\to\infty$

stationary phase approx. to compute A_v

(semiclassical analysis)