Loop Quantum Gravity Summer School 2026

Quanutum Information and Quantum Entanglement

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Outlines

- Lecture I: Background： quantum gravity meets quantum information
- Lecture II: Basic concepts in quantum information
- Lecture III: Quantum entanglement in tensor networks
- Lecture IV: Quantum entanglement and the structure of spacetime

Acknowledgement to collaborators：

Peng Liu, Yu-Xuan Liu, Chao Niu, Jianpin Wu, Meng-he Wu, Zhuoyu Xian, Yikang Xiao

Lecture IV
Quantum entanglement and the structure of spacetime

- Entanglement creats geometric connection
- Tensor networks and AdS space
- Emergence of space from entanglement by spin networks

Entanglement creats geometric connection

Gedanken experiment

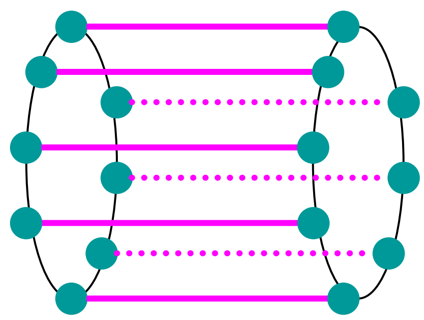
2013, Maldacena and Susskind ***Cool horizons for entangled black holes***

arXiv:1306.0533

EPR state

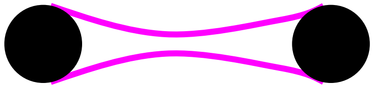
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow_A \downarrow_B\rangle + |\downarrow_A \uparrow_B\rangle)$$

N EPR pairs

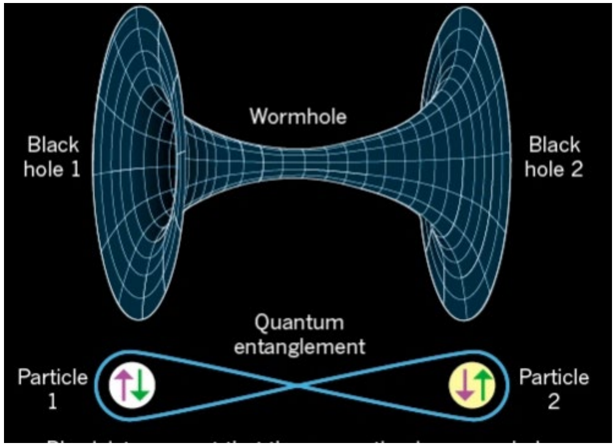


Increase N to form two black holes

ER bridge



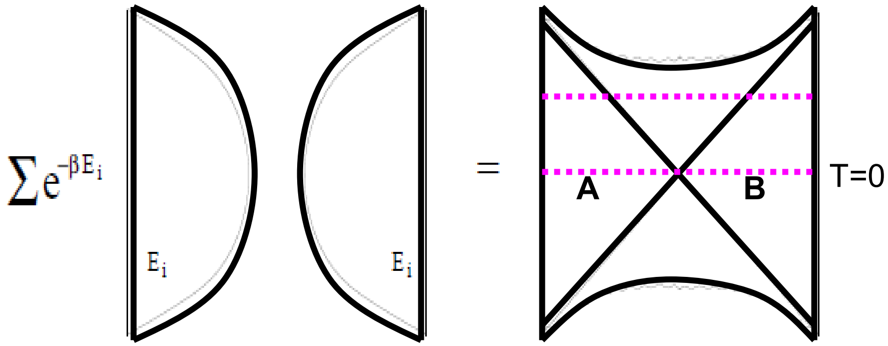
ER=EPR

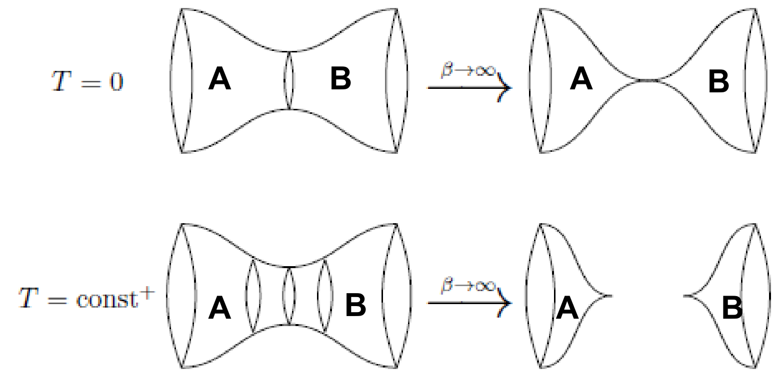


Entanglement creats geometric connection

2001， Maldecena *Eternal black holes in anti-de Sitter* arXiv:hep-th/0106112

2010, Van Raamsdonk *Building up spacetime with quantum entanglement* arXiv:1005.3035





- **TFD state**

$$|\psi\rangle = \frac{1}{\sqrt{Z}}\sum_i e^{-\frac{\beta E_i}{2}} |\psi_i\rangle_A \otimes |\psi_i\rangle_B$$

$$\langle\psi|\psi\rangle = 1 \qquad \rho = |\psi\rangle\langle\psi|$$

$$\rho_A = Tr_B \rho_{AB} = \langle\psi_i|\psi\rangle\langle\psi|\psi_i\rangle_B$$
$$= \frac{1}{Z}\sum_i e^{-\beta E_i} |\psi_i\rangle_A \langle\psi_i| = \rho_B$$

$$|\psi\rangle \xrightarrow{\beta\to\infty} |\Phi\rangle$$

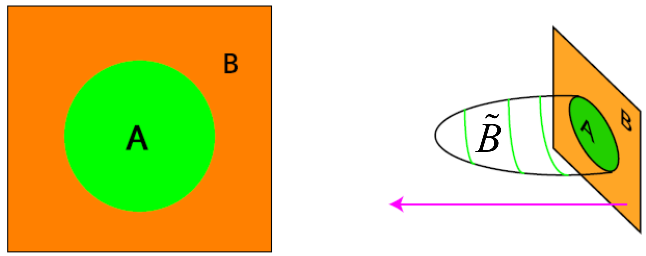
$$|\Phi\rangle = |\psi_0\rangle_A \otimes |\psi_0\rangle_B$$

Spacetime perhaps is just a geometric picture for the entanglement style of matter in a quantum system.

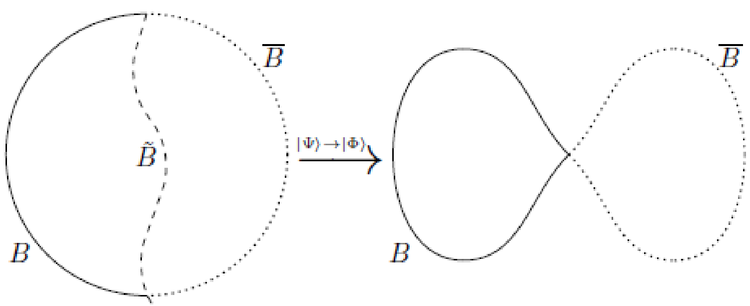
Entanglement creats geometric connection

2006, Ryu and Takayanagi *Holographic derivation of entanglement entropy from AdS/CFT* arXiv:hep-th/0603001

$$S_E = \frac{\text{Area}(\tilde{B})}{4G_N}$$



$S_A := -\text{Tr}_A\ [\hat{\rho}_A \ln(\hat{\rho}_A)]$

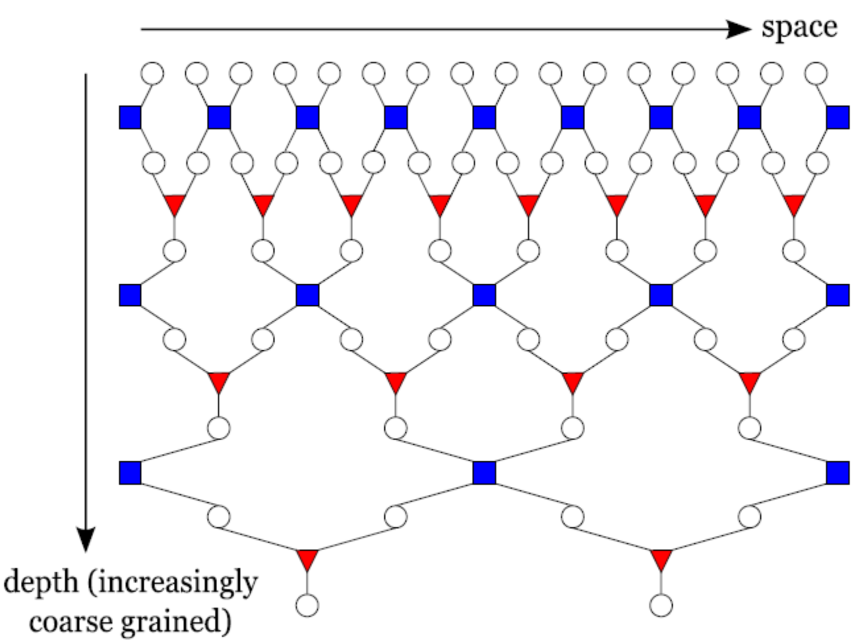


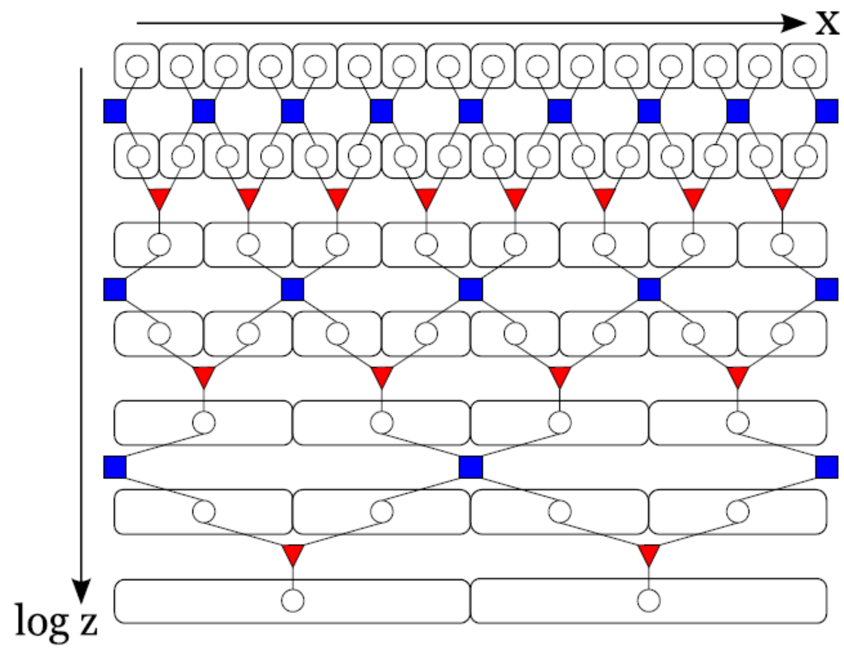
$$|\psi\rangle = \sum_{i,j} p_{i,j} \left|\phi_i^B\right\rangle \otimes \left|\phi_j^{\bar{B}}\right\rangle \longrightarrow |\Phi\rangle = \left(\sum_i c_i \left|\phi_i^B\right\rangle\right) \otimes \left(\sum_j d_j \left|\phi_j^{\bar{B}}\right\rangle\right)$$

arXiv:1711.10854

Entanglement creats geometric connection

2009,   Swingle   *Entanglement Renormalization and Holography*   arXiv:0905.1317



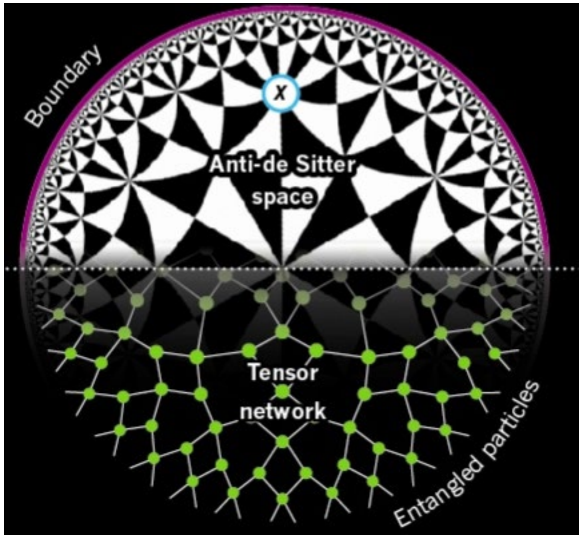


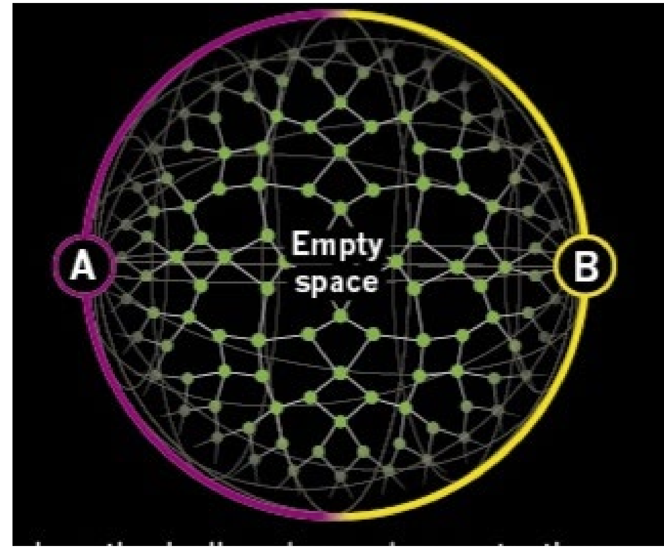
$$dS \sim \frac{L^{d_s-1}}{z^{d_s-1}}\frac{dz}{z}.$$

$$ds^2 = R^2\left(\frac{dz^2 + dx^2}{z^2}\right) = R^2\left(dw^2 + e^{-2w}dx^2\right)$$

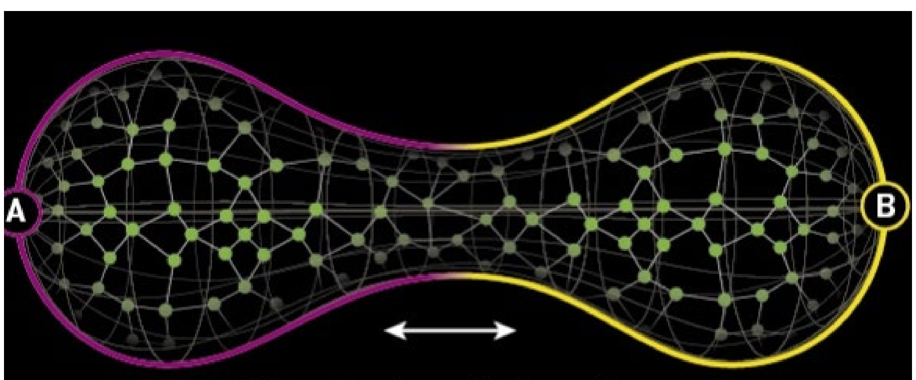
Entanglement creats geometric connection

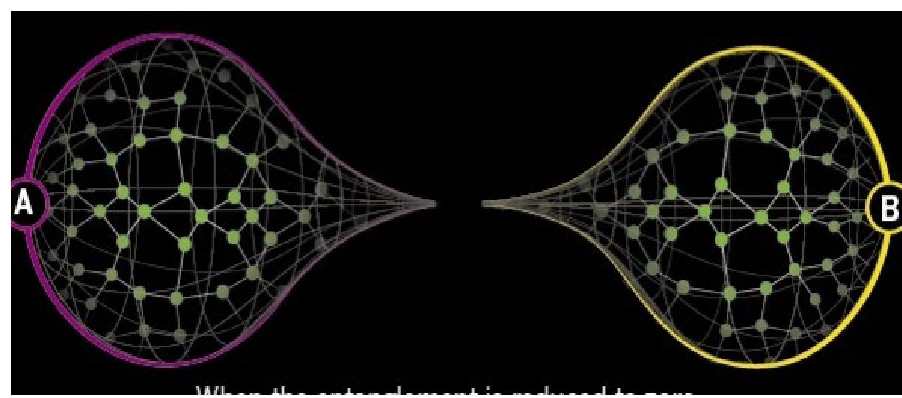
- **The microscopic structure of space time by tensor networks**





Space is precisely constructed through quantum entanglement via tensor networks





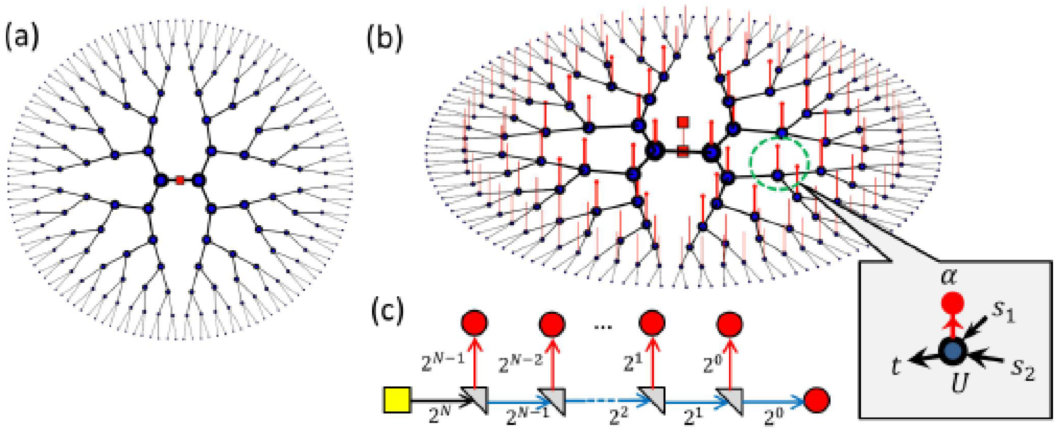
http://www.nature.com/news/the-quantum-source-of-space-time-1.18797

Entanglement creats geometric connection

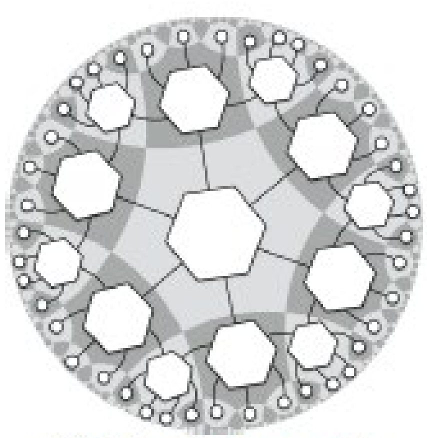
2013， X. Qi *Exact holographic mapping and emergent space-time geometry* arXiv:1309.6282

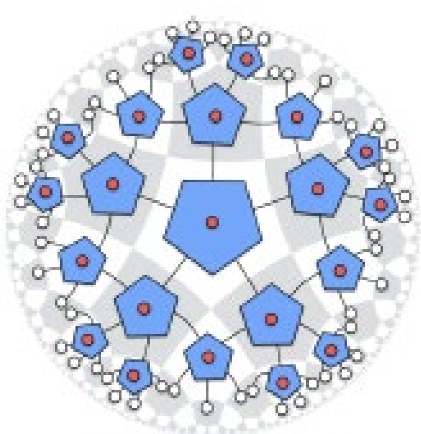
$$d_{(x,t_1)(y,t_2)} = -\xi \log \frac{\langle O_x(t_1) O_y(t_2) \rangle}{C_0}$$

$$d_{xy} = -\xi \log \frac{I_{xy}}{I_0}$$



2015， Harlow and Preskill *Holographic quantum error-correcting codes: Toy models for the bulk/boundary correspondence* arXiv:1503.06237


(a) Holographic hexagon state


(b) Holographic pentagon code

Holographic duality itself is a QEC

Perfect tensor （PT）

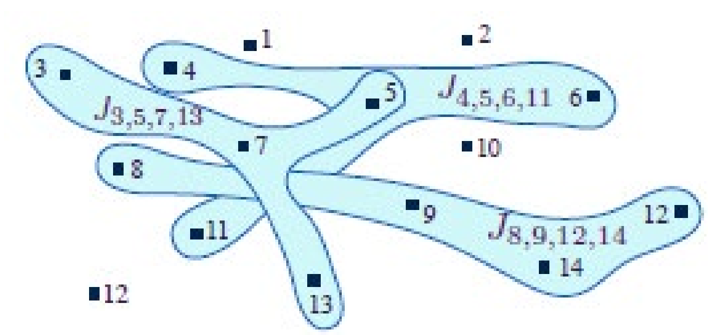
Entanglement creats geometric connection

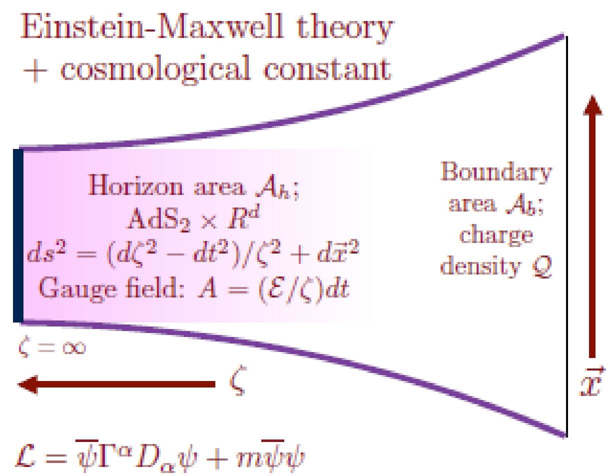
2015， Miyaji *et.al.* *cMERA as Surface/State Correspondence in AdS/CFT* arXiv:1506.01353

2015， Kitaev，Sachdev *A simple model of quantum holography - 2015* arXiv:1506.05111

Bekenstein-Hawking Entropy and Strange Metals

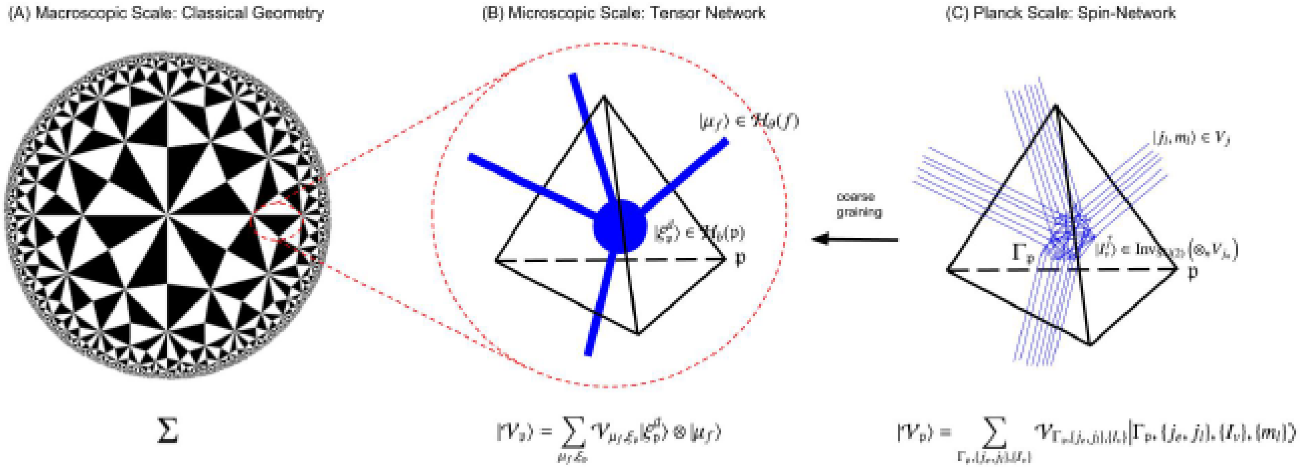
Sachdev-Ye-Kitaev Model





Entanglement creats geometric connection

2016, Han and Hung *Loop Quantum Gravity, Exact Holographic Mapping, and Holographic Entanglement Entropy* arXiv:1610.02134

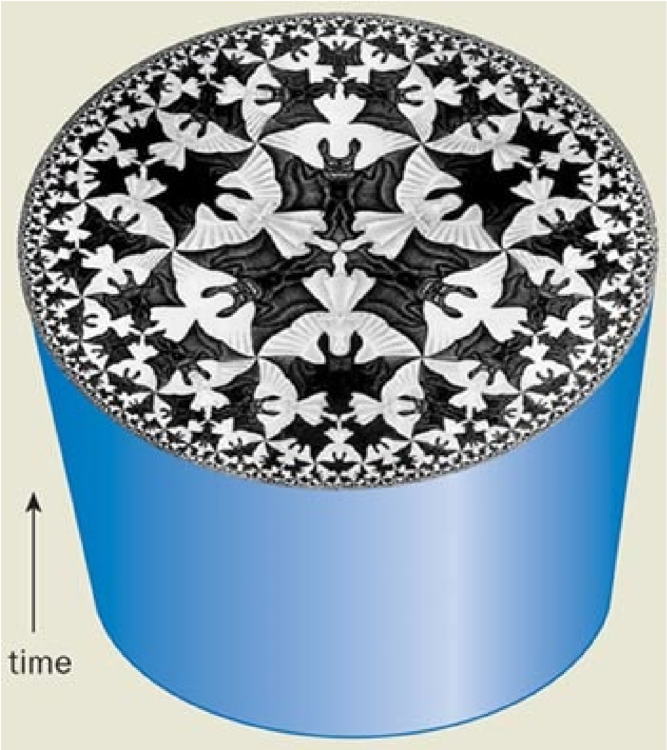


2019， A. Almheiri, N. Engelhardt, D. Marolf, H. Maxfield *The entropy of bulk quantum fields and the entanglement wedge of an evaporating black hole* arXiv: 1905.08762

2019， G. Penington *Entanglement Wedge Reconstruction and the Information Paradox* arXiv: 1905.08255

Tensor Networks and AdS space

- **Poincare patch of AdS space**



Angels & Devils by M.C. Escher

Hyperbolic structure of H_2 space

$$ds^2 = L^2 \left(\frac{dz^2 + dx^2}{z^2} \right)$$

$$\xi = x + iz \qquad L = 1$$

$$ds^2 = -\frac{d\xi d\xi^*}{(\xi - \xi^*)^2}$$

Isometry: $SL(2, R)$

$$\xi' = \frac{\alpha\xi + \beta}{\gamma\xi + \delta} \qquad \alpha\delta - \beta\gamma = 1$$

Tensor Networks and AdS space

- **Two remarkable holographic features of AdS space**

1. For the vacuum in AdS/CFT correspondence, Renyi entropy satisfies Cardy-Calabrese formula and a non-flat ES is inherent.

$$S_n = \left(\frac{n+1}{n}\right)\frac{A_{\min}}{8G}$$

$$S_n = \frac{1}{1-n}\ln\frac{Tr\rho_A{}^n}{(Tr\rho_A)^n}$$

Flat ES

$$\rho_A^2 = a\rho_A \quad \Rightarrow \quad S_n = \ln\frac{Tr\rho}{a}$$

2. A local operator in the bulk can be reconstructed in a subsystem A on the boundary. It can be be viewed as the accomplishment of QEC.

Tensor Networks and AdS space

What kind of tensor networks could capture the holographic features of AdS?

1. MERA breaks the isometry SL(2,R) and has a preferred direction, implying that QEC can not be realized along all directions.

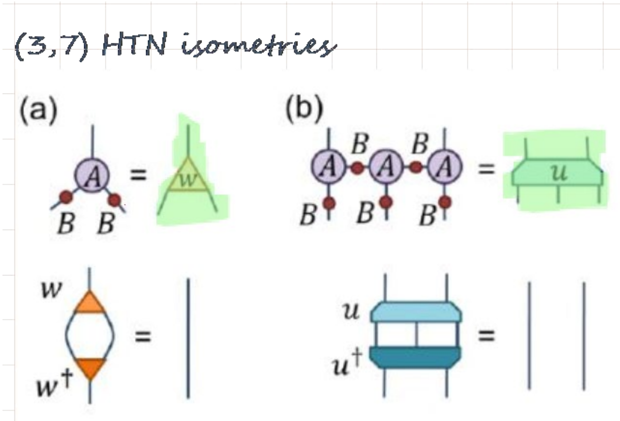
2. Perfect tensor networks has a flat ES and trivial connected correlation functions, which is not a reflection of the holographic property of AdS.

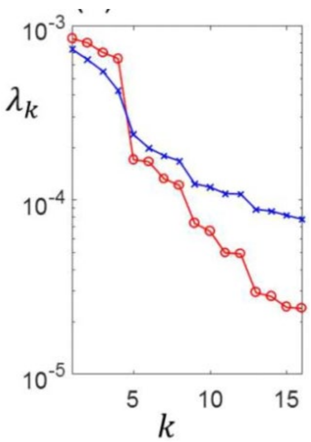
Tensor Networks and AdS space

■ Hyperinvariant Tensor Networks (HTN)：

arXiv:1704.04229

2017， Evenbly ***Hyperinvariant Tensor Networks and holography***





1. What kind of multi-tensor constraints could endow desirable features of AdS spacetime to a given tensor network?

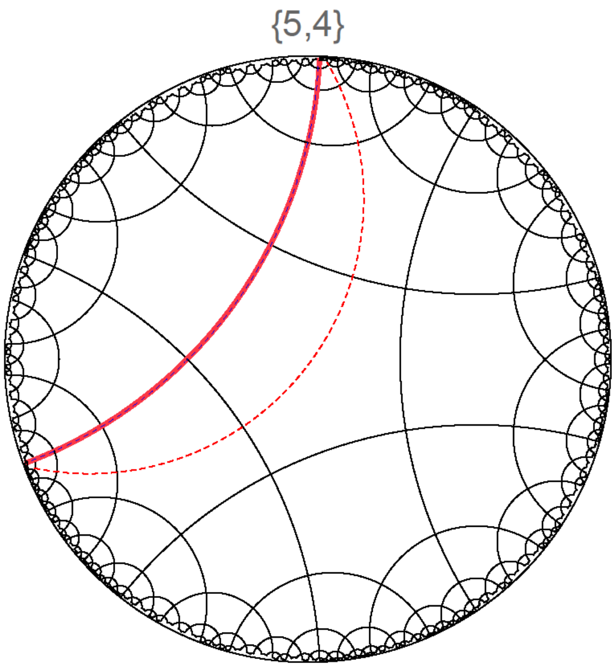
2. Is there any criteria to justify the ability of QEC and the non-flatness of ES for a tensor network with given constraints?

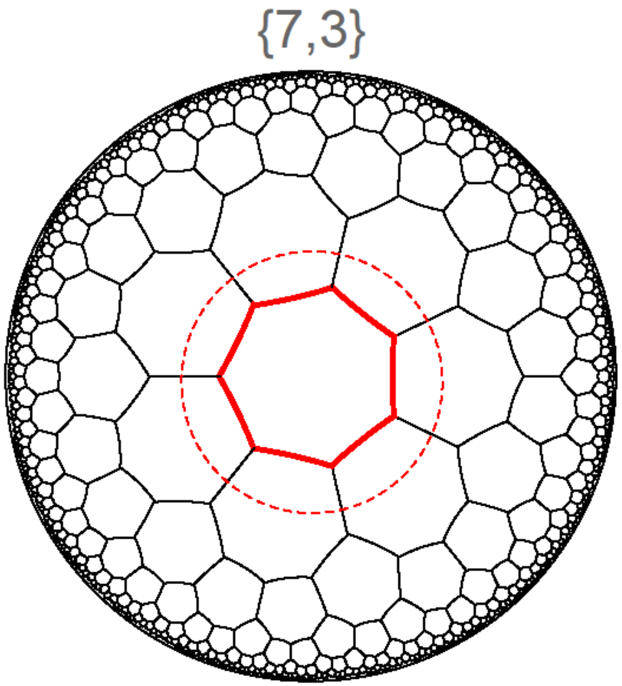
Y.Ling, et.al., arXiv:1806.05007

Tensor Networks and AdS space

- The (b,a) tiling of hyperbolic space

$$\frac{1}{a}+\frac{1}{b}<\frac{1}{2}$$





$$T_{ijkl}=T_{jkli}=T_{klij}=T_{lijk}$$

$$T_{ijk}=T_{jki}=T_{kij}$$

Tensor Networks and AdS space

Isometry

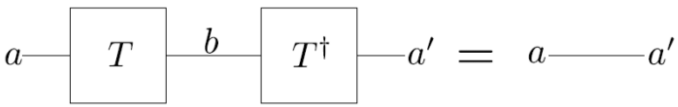


Figure 1. Diagrammatic tensor notation, here showing that T is an isometry.

$$\sum_b T_{ab} \, T^{\dagger}{}_{ba'} = \delta_{aa'}$$

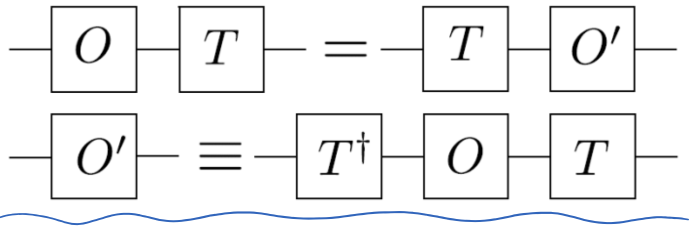
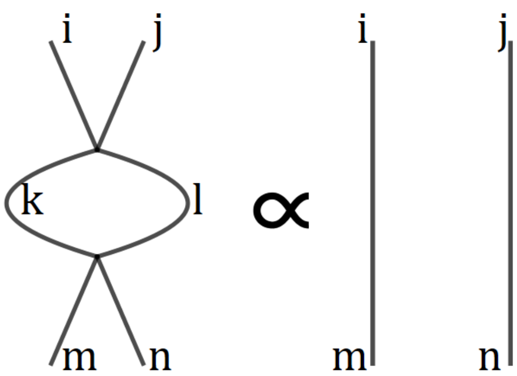


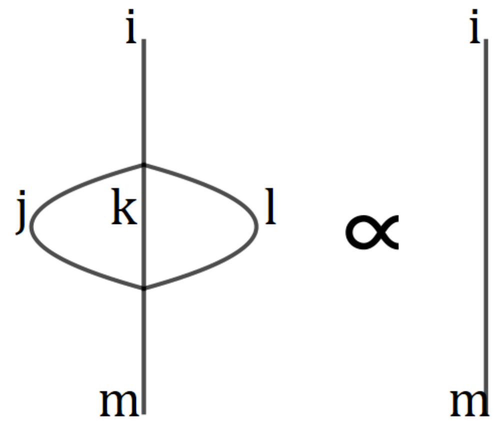
Figure 2. Operator pushing through an isometric tensor.

$$OT = TT^{\dagger}OT = T(T^{\dagger}OT) = TO'$$

Tensor constraints



$$\sum_{kl} T_{ijkl} \, T^{*}{}_{mnkl} \propto \delta_{im}\delta_{jn}$$

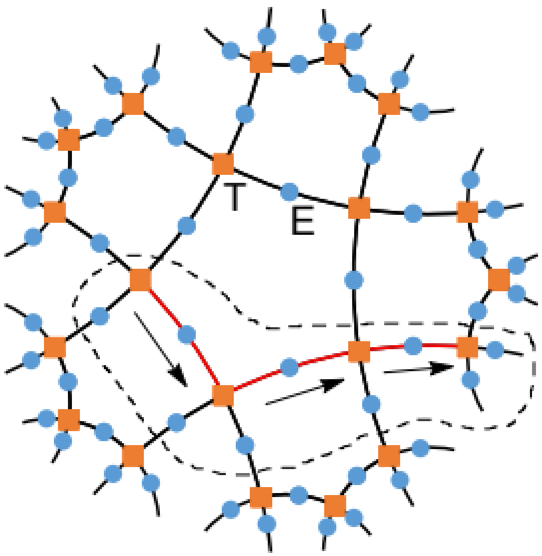


$$\sum_{jkl} T_{ijkl} \, T^{*}{}_{mjkl} \propto \delta_{im}$$

Tensor Networks and AdS space

- The (b,a) tiling of hyperbolic space

$$\frac{1}{a}+\frac{1}{b}<\frac{1}{2}$$

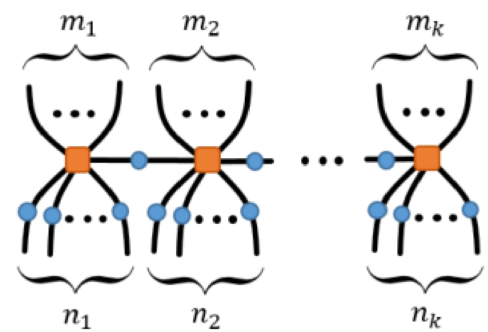


$$\{5,4\}$$

$$T_{ijkl}=T_{jkli}=T_{klij}=T_{lijk}$$

$$E_{ij}=E_{ji}$$

- Tensor Chains



$$m_i+n_i=a-2+\delta_{i1}+\delta_{ik}$$

$$k=4$$

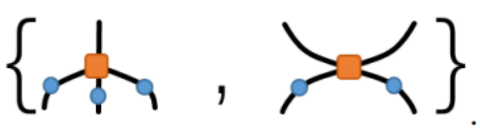
$$(m_1,m_2,m_3,m_4)=(2,0,1,1)$$

$$(n_1,n_2,n_3,n_4)=(1,2,1,2)$$

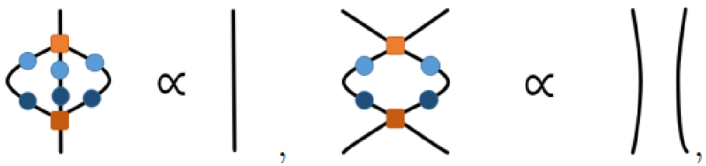
- Average induced interior angle

$$\kappa=\frac{1}{k}\left(\sum_{i=1}^{k}m_i+k-1\right)$$

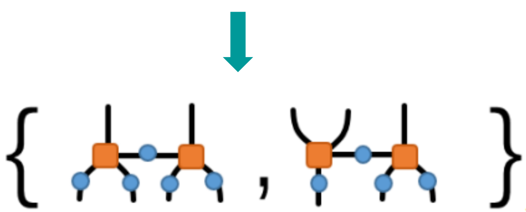
Tensor Networks and AdS space



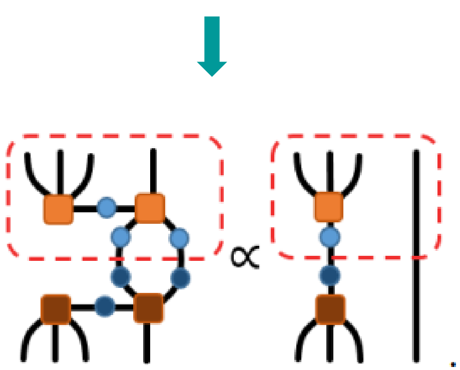
- Tensor constraints

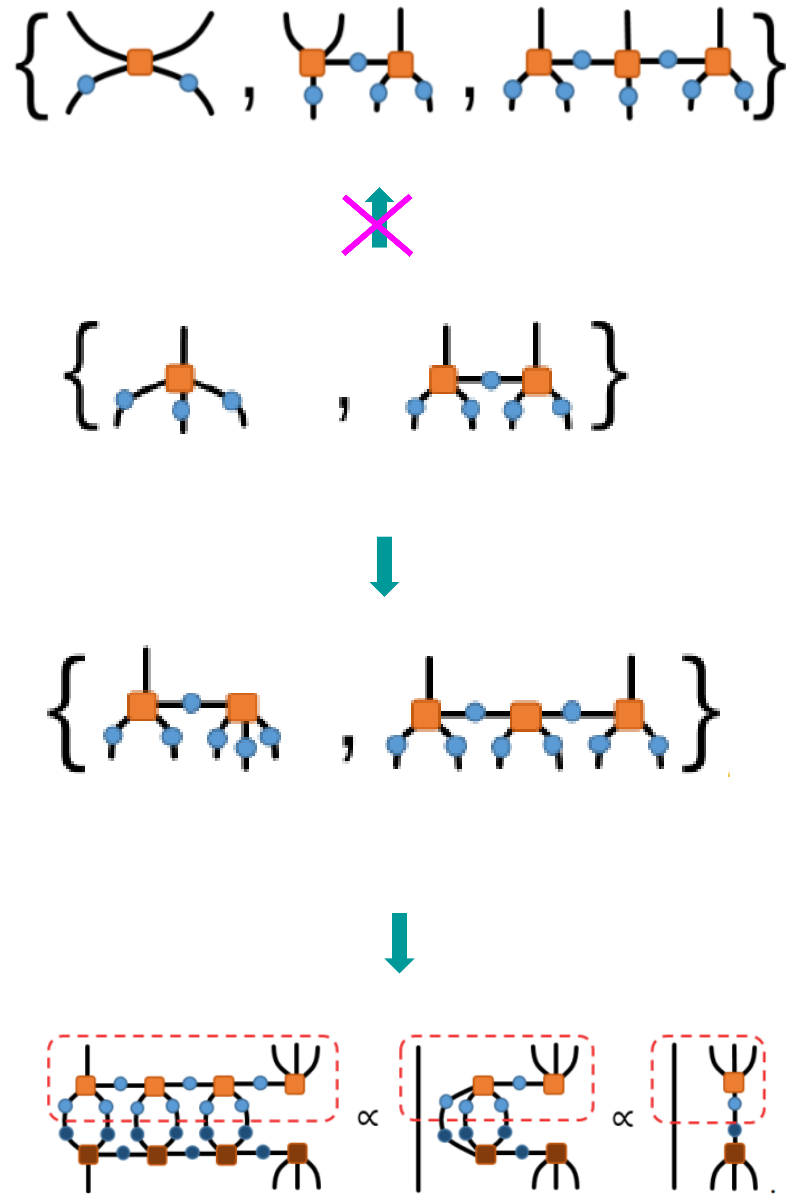


- A set of isometry



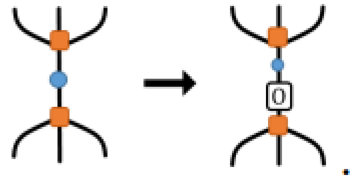
- Greedy algorithm



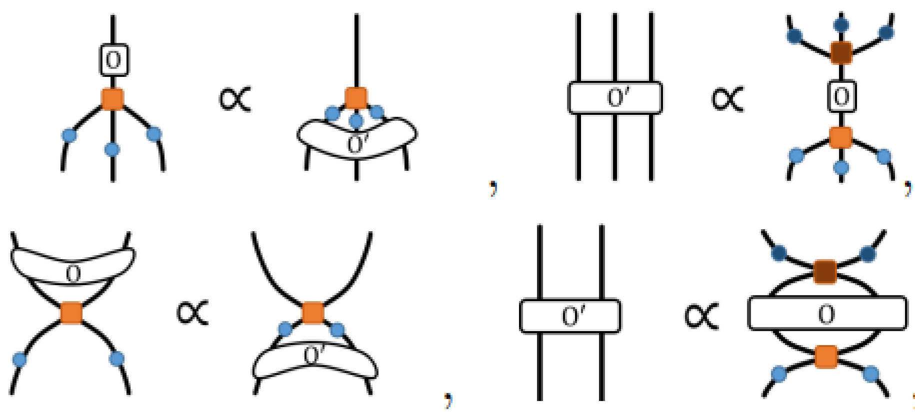


Tensor Networks and AdS space

- **QEC in a tensor netowork**

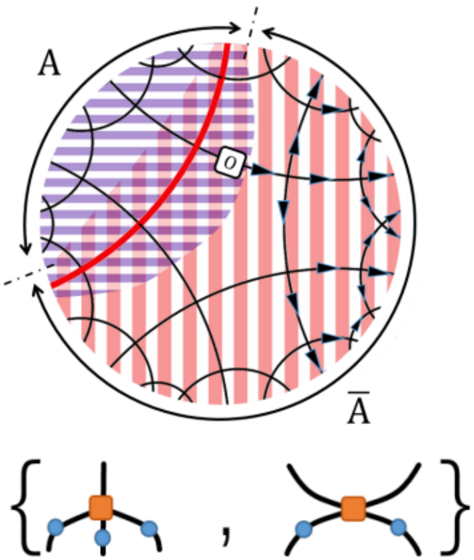


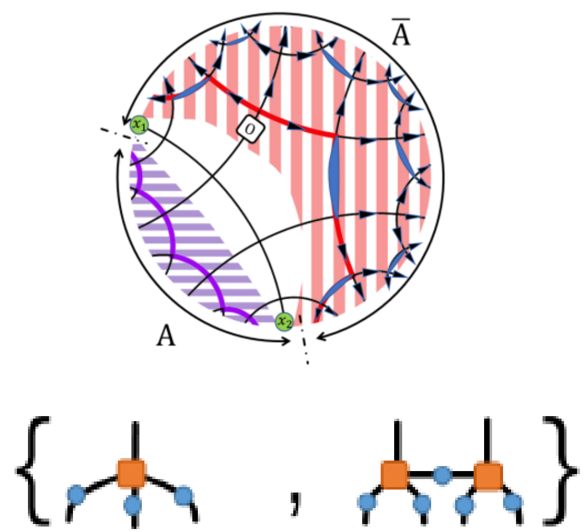
  - Tensor pushing



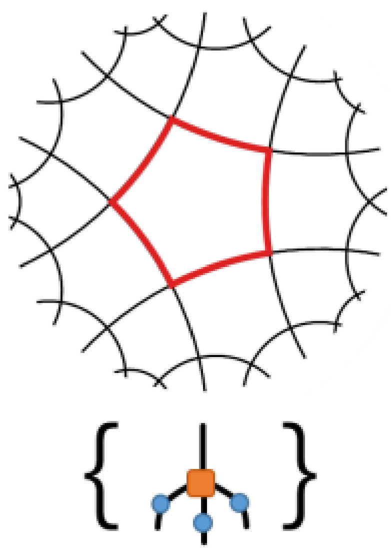
Tensor Networks and AdS space

- QEC in networks





- The failure of QEC in a network

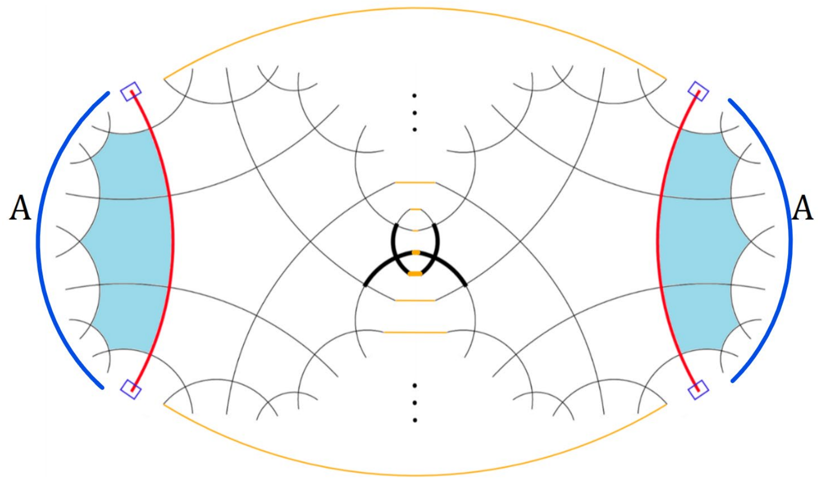


Tensor Networks and AdS space

The entanglement spectrum of a tensor netowork

- The reduced density metrix for a network

$$\rho_A = Tr_{\bar{A}}|\psi\rangle\langle\psi| = \psi\psi^\dagger$$



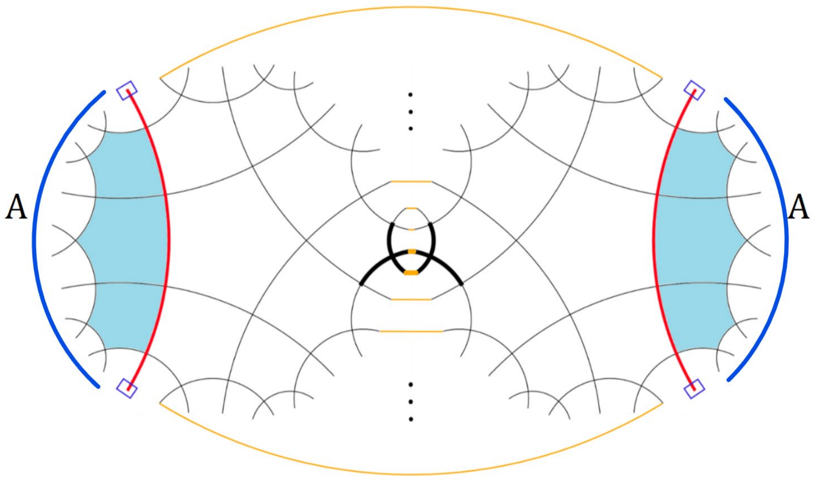
The flatness of ES is equivalent to

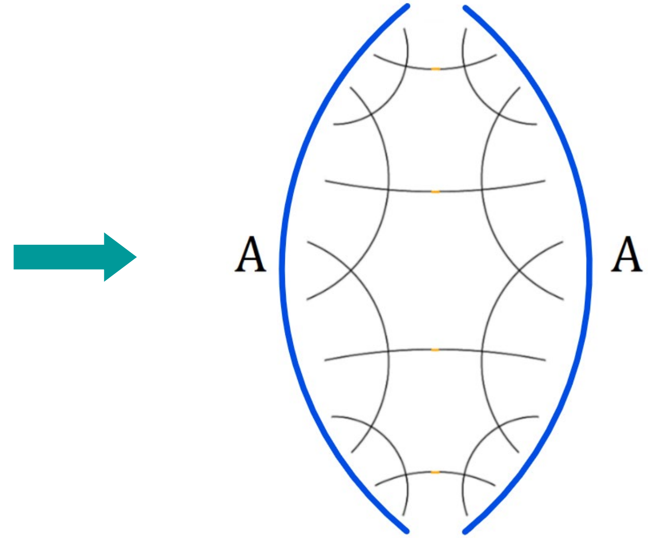
$$\boxed{\rho_A^2 = a\rho_A} \Rightarrow \rho_A\rho_A = \psi\psi^\dagger\psi\psi^\dagger \propto \psi\psi^\dagger = \rho_A$$

Tensor Networks and AdS space

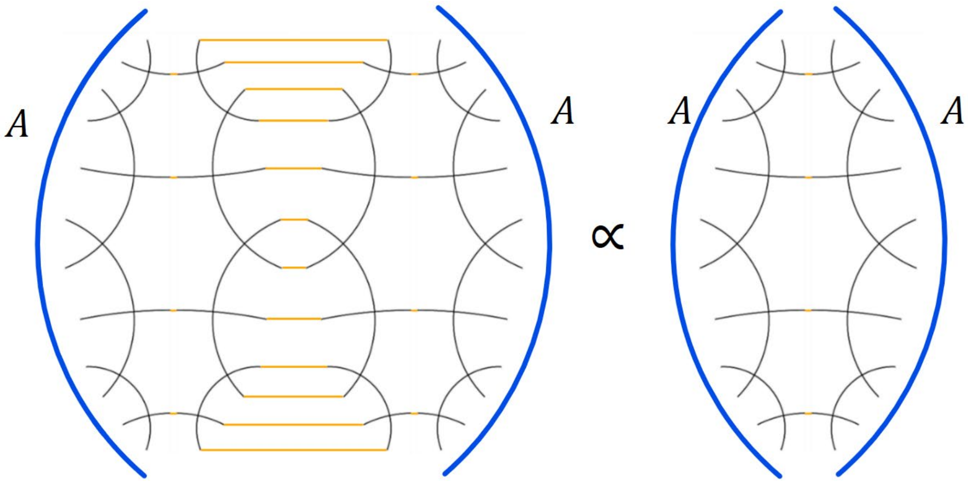
- Single tensor constraint leads to flat ES

ρ_A





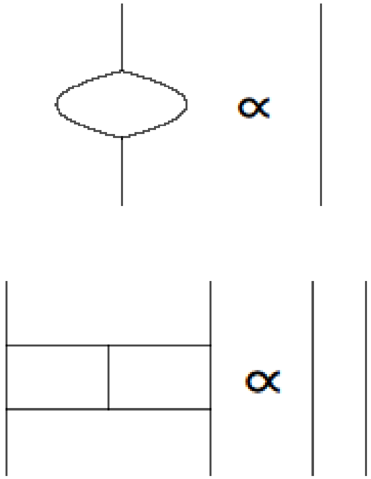
$\rho_A{}^2$

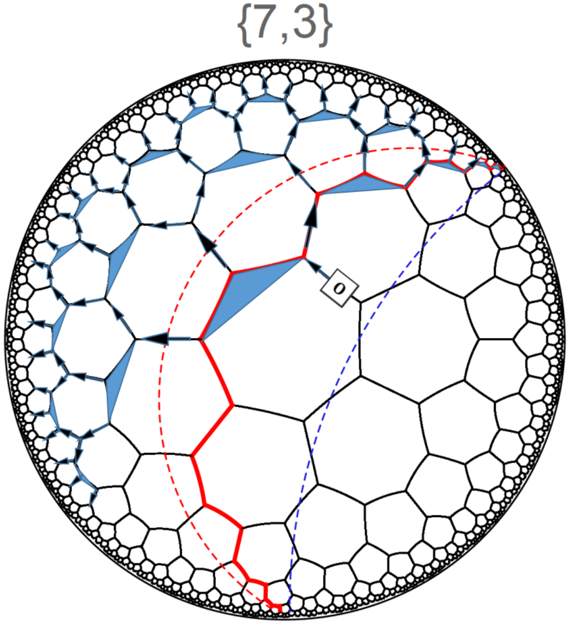


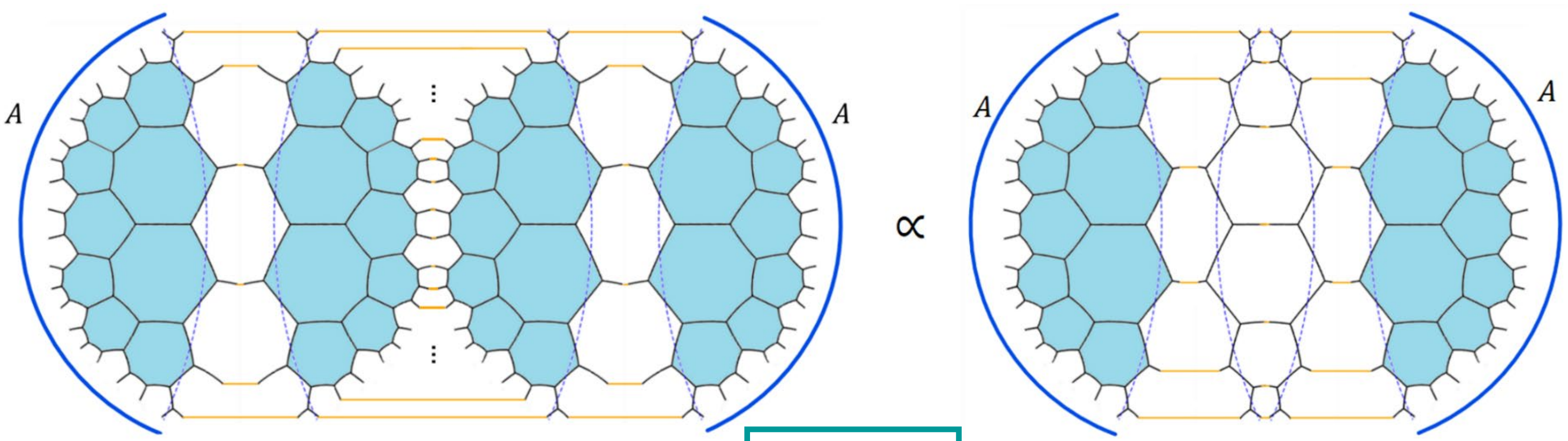
$$\rho_A{}^2 \propto \rho_A$$

Tensor Networks and AdS space

- Multi-tensor constraints and non-flat ES



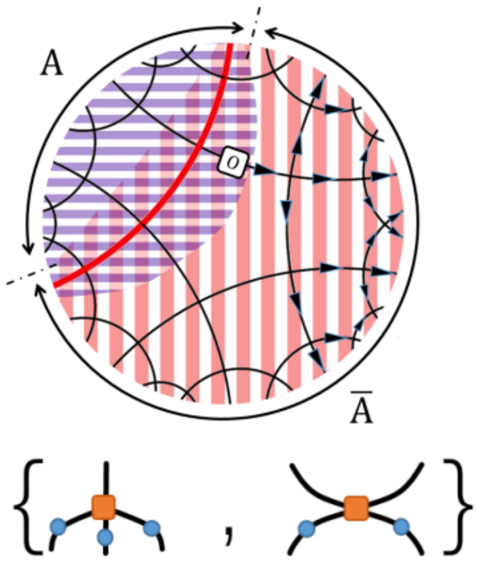




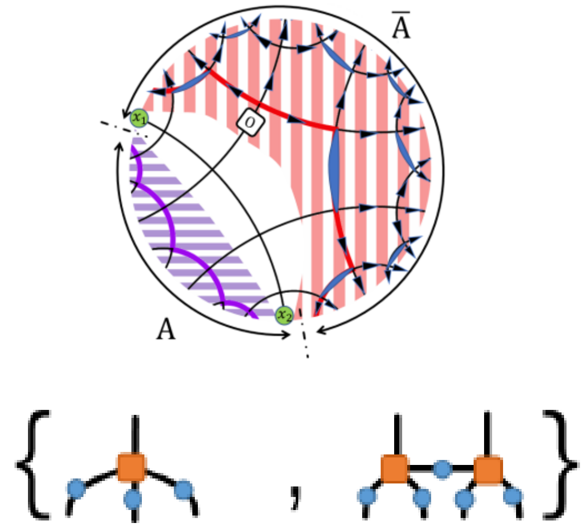
$$\rho_A^{\ 2} \neq \rho_A$$

Tensor Networks and AdS space

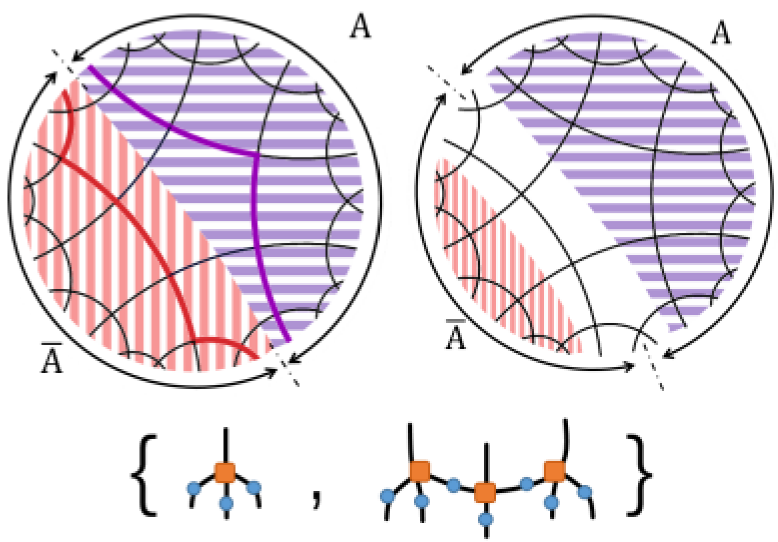
- A flat ES



- A non-flat ES



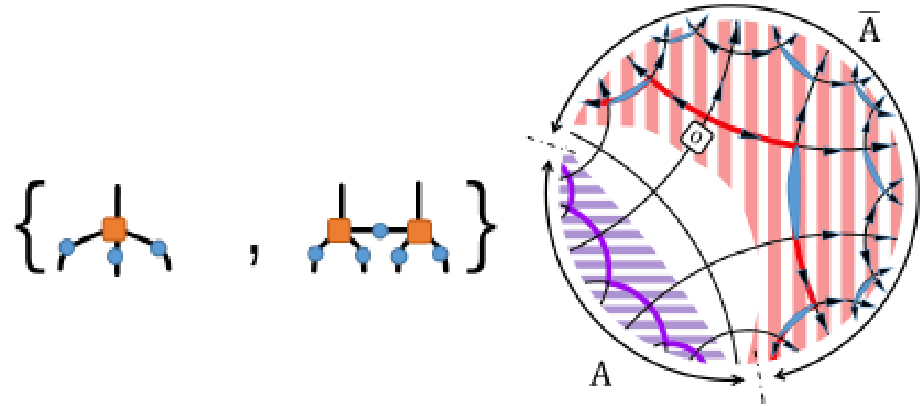
- A mixed ES



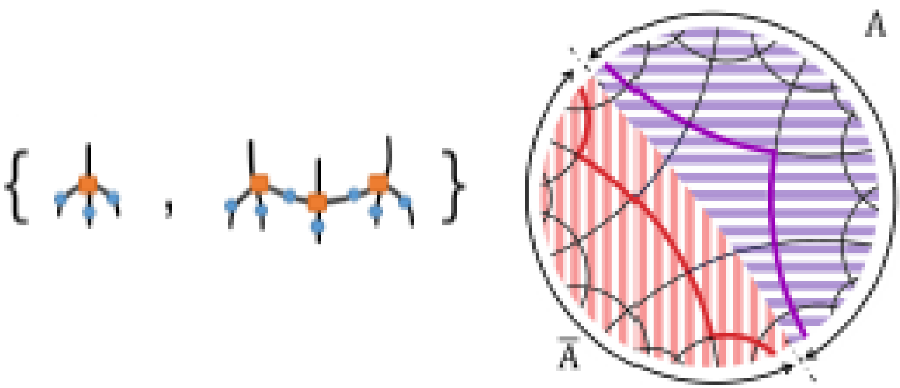
Tensor Networks and AdS space

- κ_c of CP tensor chain

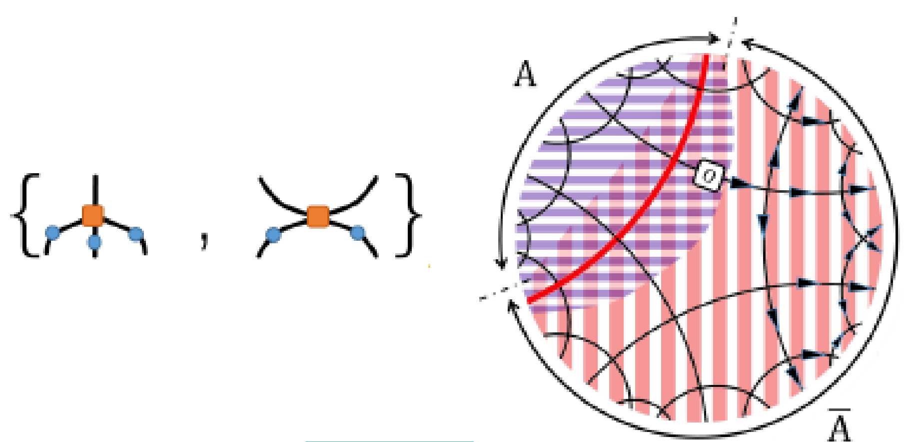
$\kappa_c = 1$



$\kappa_c = 3/2$



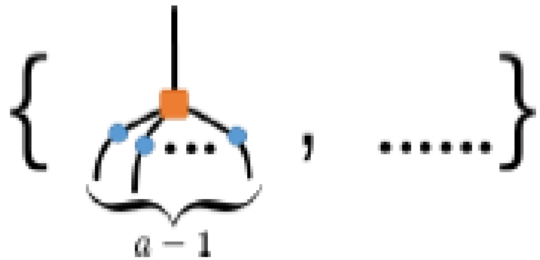
$\kappa_c = 5/3$



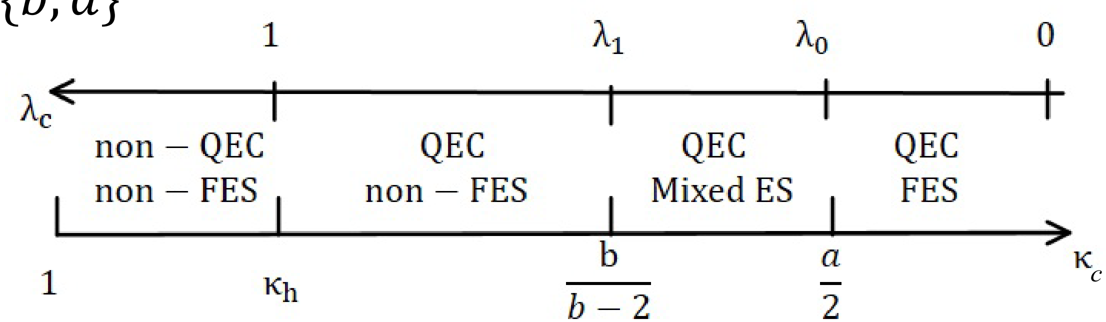
$\kappa_c = 2$

Tensor Networks and AdS space

- General constraints $\{b, a\}$



- Classification $\{b, a\}$

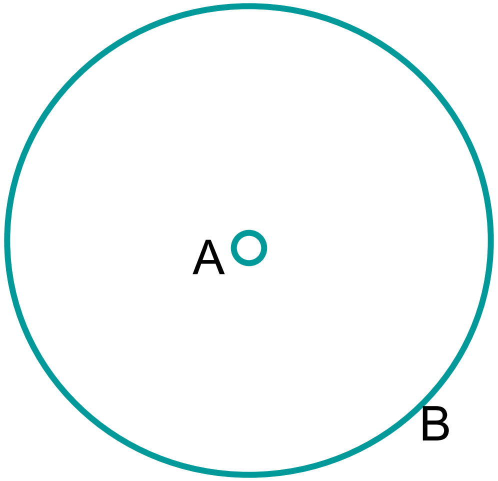


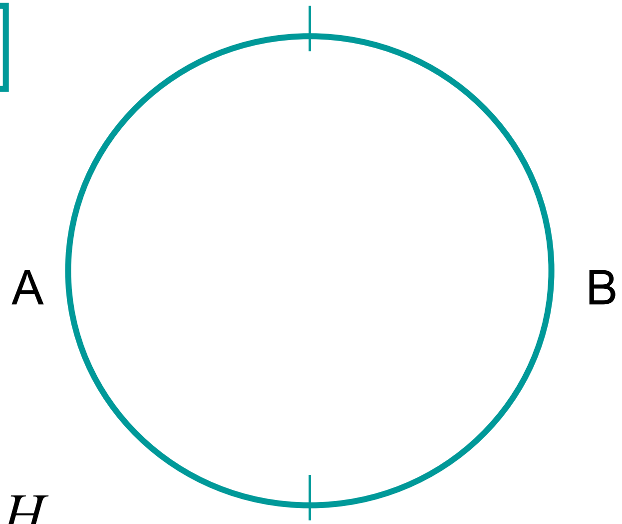
$$\kappa_h = \frac{a}{2} - \frac{a-2}{2}\sqrt{\frac{ab-2a-2b}{ab-2a-2b+4}}$$

$\{5,4\}$ $\kappa_h = 1.42,$ $\frac{b}{b-2} = \frac{5}{3},$ $\frac{a}{2} = 2$

Summary:

QEC vs. ES





$$M: \quad H_A \rightarrow H_B$$

QEC is better ⟷ ES more easily becomes flat!

Angles vs. Devils

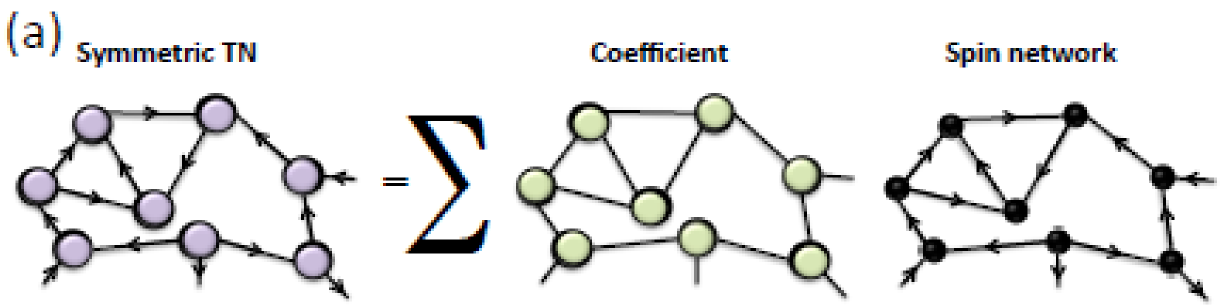
AdS is fantastic !

Emergence of space from entanglement by spin networks

Roger Penrose

Guifre Vidal

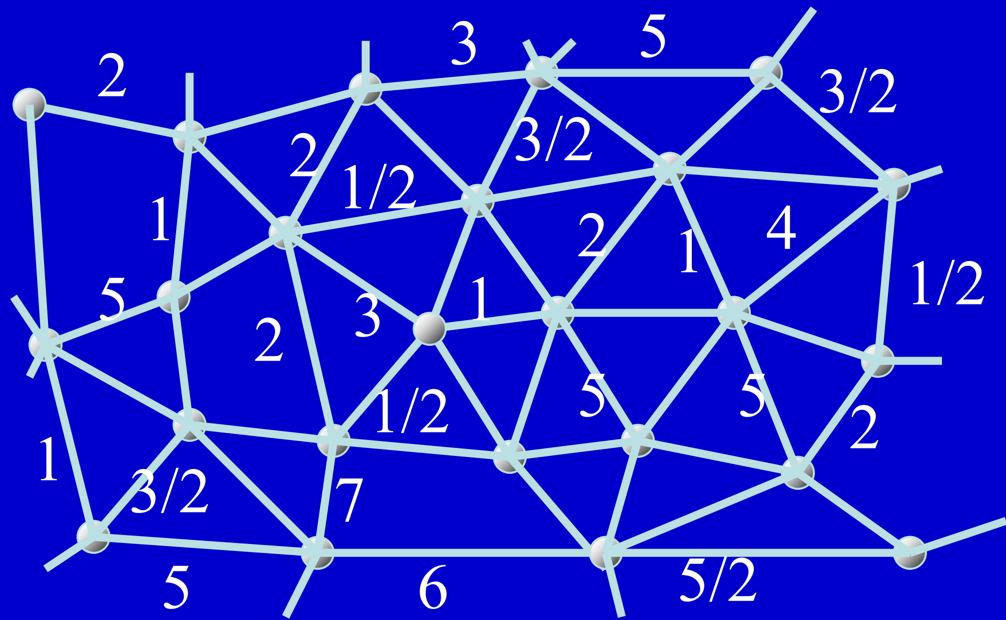
2014 Orus *Advances on Tensor Network Theory: Symmetries, Fermions, Entanglement, and holography* ArXiv:1407.6552



Spin networks and quantum geometry

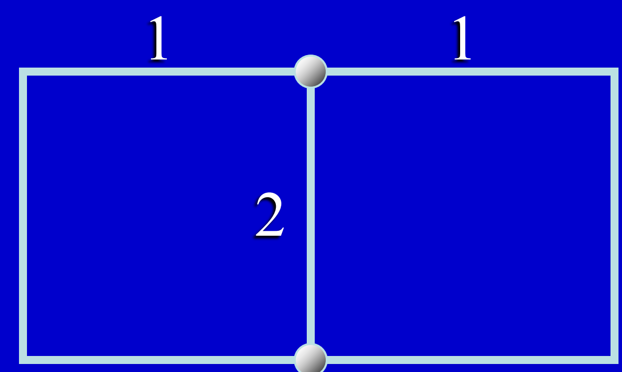
- **Spin network states**

$$\left| \Gamma, j_m, I_v \right\rangle$$



$$j = \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$$

Γ



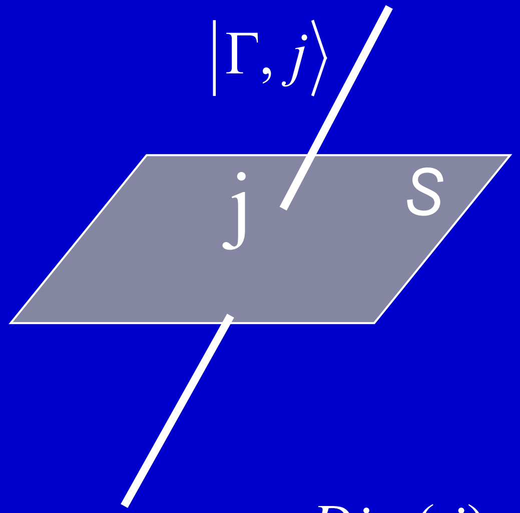
$$\left| n, l, m \right\rangle$$

Spin networks and quantum geometry

- **Area operator**

$$A(S) = \int_S d\sigma^2 \sqrt{\det({}^2h)}$$
$$= \int_S d\sigma^2 \sqrt{\hat{E}_a^i \hat{E}^{aj} n_i(\sigma) n_j(\sigma)}$$

$$\hat{A}(S)\left|\Gamma, j\right\rangle = 8\pi\gamma l_p^{\,2} \sqrt{j(j+1)}\left|\Gamma, j\right\rangle$$

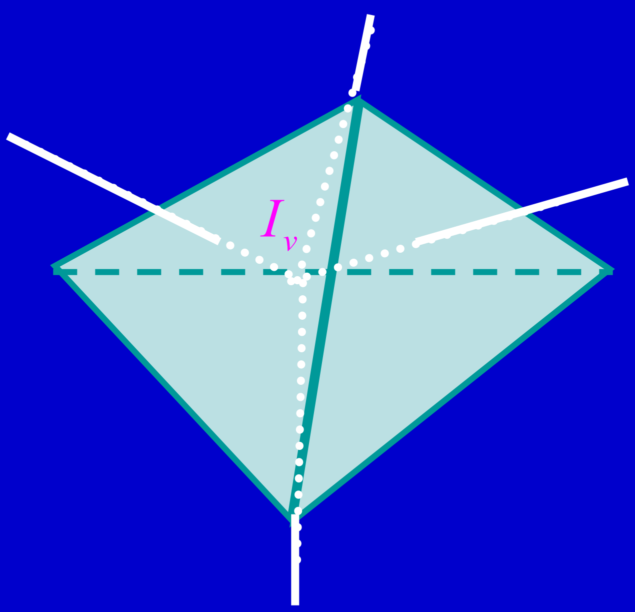


$$Dim(j) = 2j+1$$

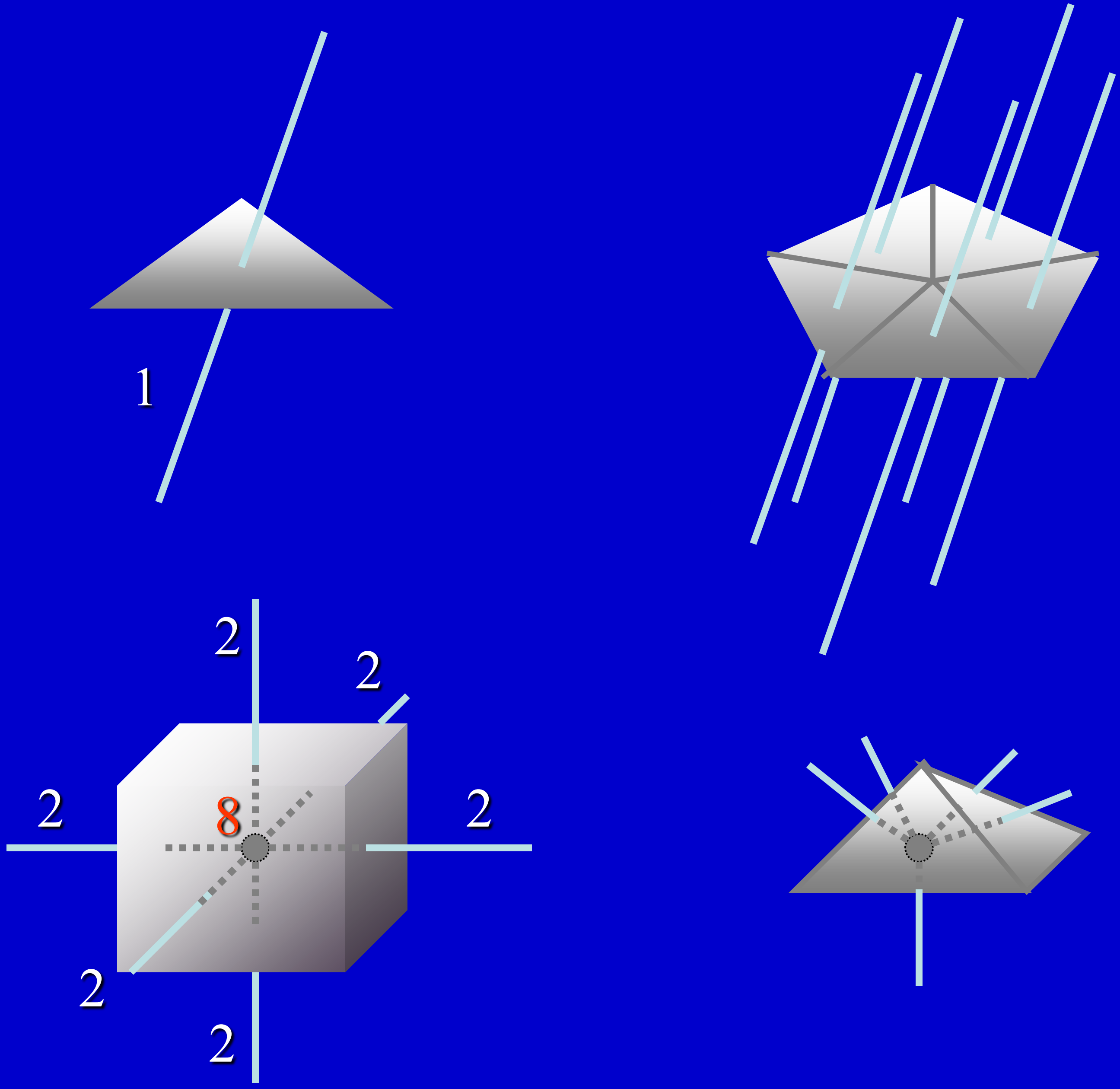
- **Volume operators**

$$V(R) = \int_R d^3x \sqrt{\det g}$$
$$= \int_R d^3x \sqrt{\frac{1}{3!}\left|\varepsilon_{abc}\varepsilon_{ijk}\hat{E}^{ai}\hat{E}^{bj}\hat{E}^{ck}\right|}$$

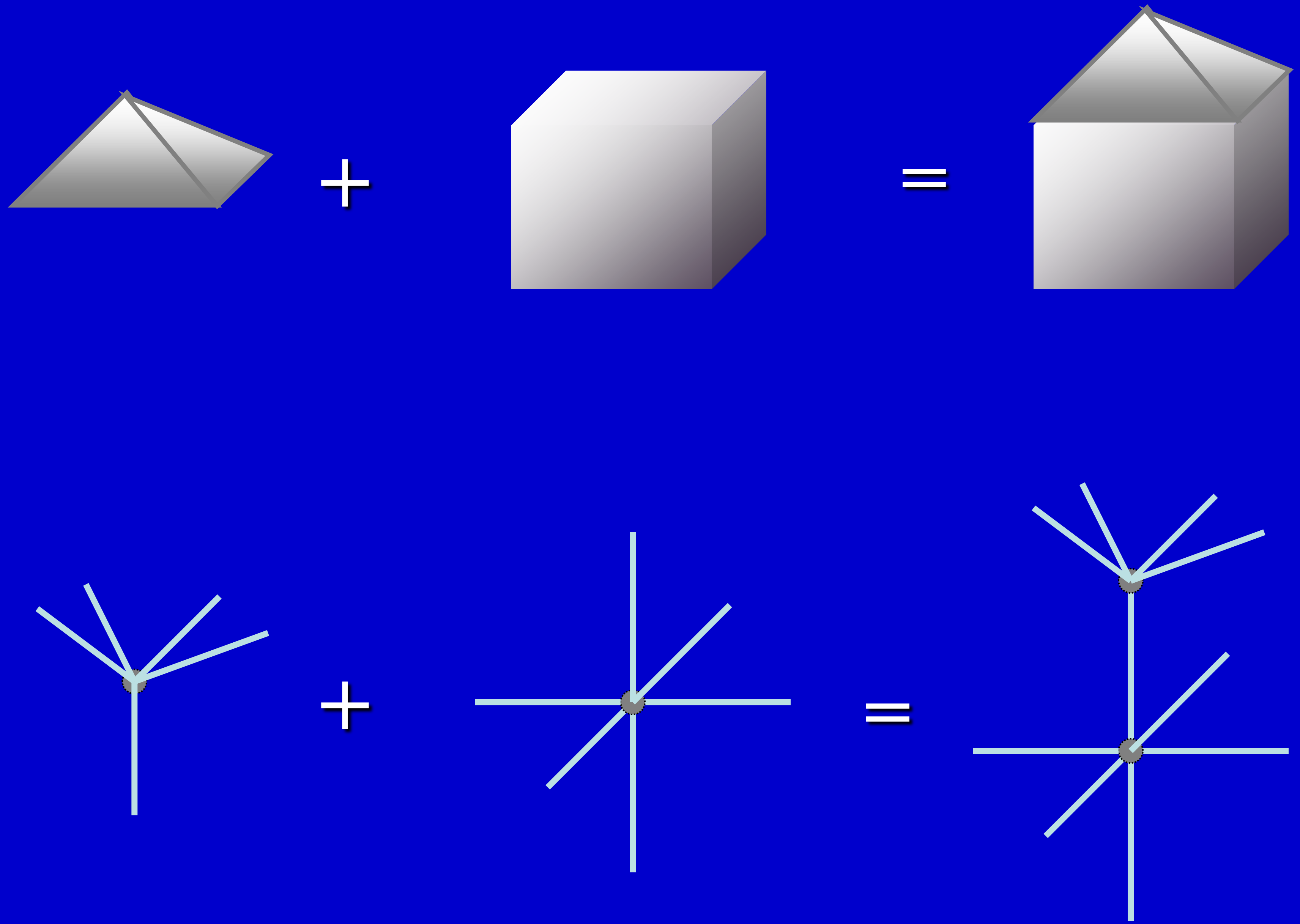
$$\hat{V}(R)\left|\Gamma, j_m, I_v\right\rangle = \sum V(I_v)\left|\Gamma, j_m, I_v\right\rangle$$



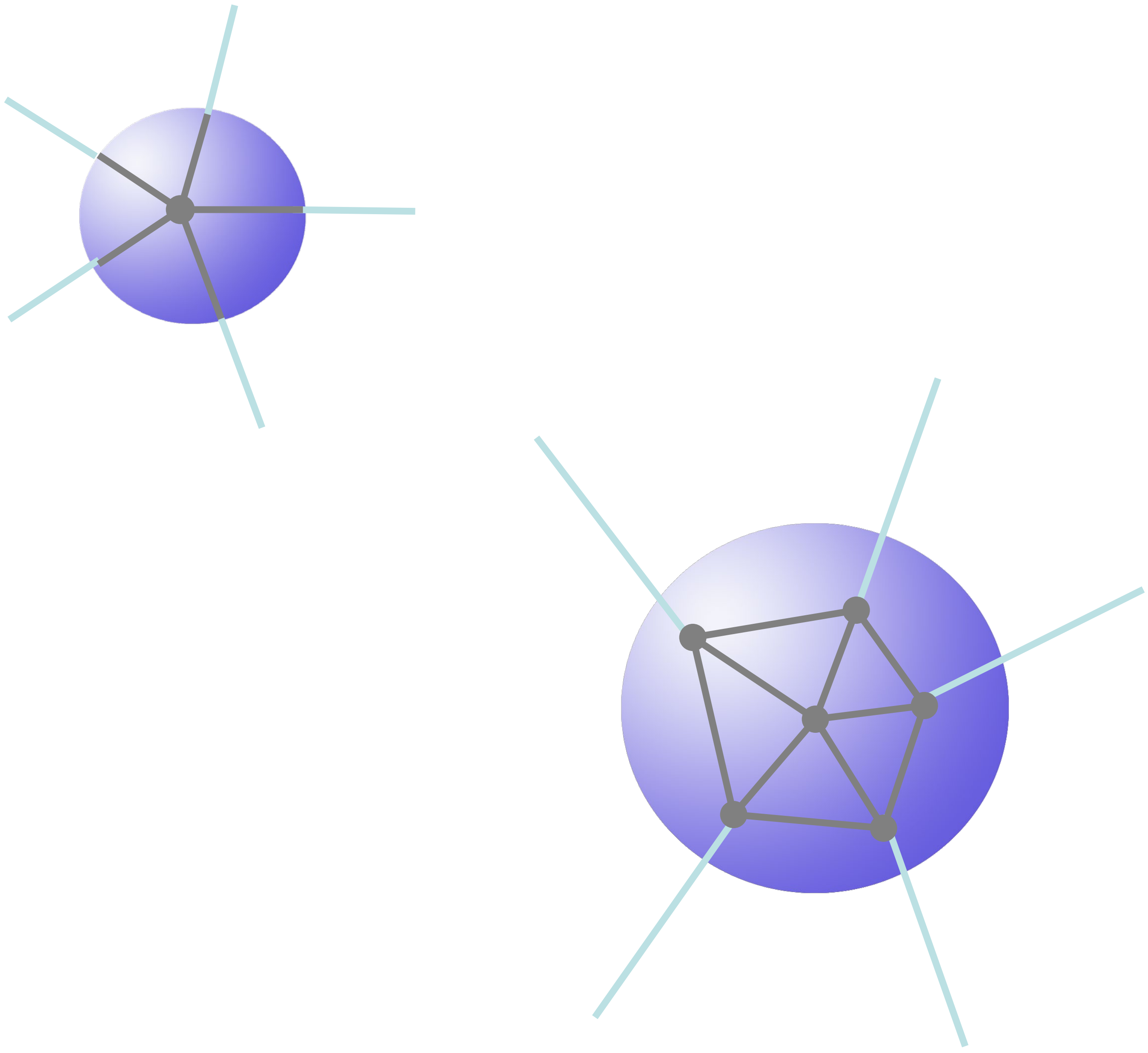
Spin networks and quantum geometry



Spin networks and quantum geometry



Spin networks and quantum geometry



Spin networks with boundary

- **Spin network states with boundary**

$$|\Gamma, \{j_e, j_o\}, \{I_v\}, \{M_o\}\rangle = \otimes \left| I_v{}^{\{j_e, j_o\}} \right\rangle \otimes |j_o, M_o\rangle \in H_\Gamma \otimes H^\partial$$

$$\langle \{h_e\}, \{h_o\} | \Gamma, \{j_e, j_o\}, \{I_v\}, \{M_o\}\rangle = \sum_{m_e, n_e, m_o} \prod_e U^{j_e}_{m_e n_e}(h_e) \prod_v (I_v)^{\{j_e, j_o\}}_{\{m_e, n_e, m_o\}} \prod_o U^{j_o}_{m_o n_o}(h_o)$$

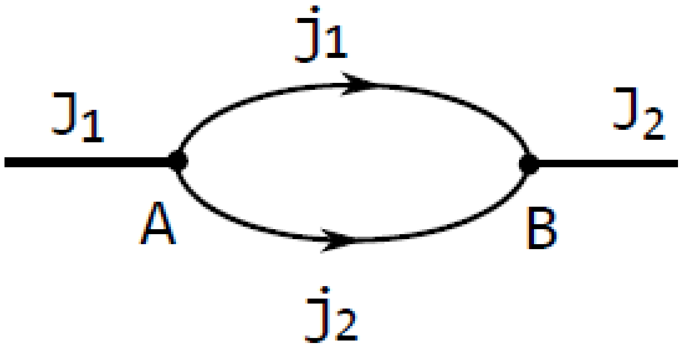
- **The boundary states**

$$\left| \tilde{\psi}[\Gamma, \{j_e, j_o\}, \{I_v\}, \{h_o\}] \right\rangle \in H^\partial$$

$$\otimes \left\langle j_o, M_o \middle| \tilde{\psi}[\Gamma, \{j_e, j_o\}, \{I_v\}, \{h_o\}] \right\rangle = \langle \Gamma, \{j_e, j_o\}, \{I_v\}, \{M_o\}, | \{h_e\}, \{h_o\}\rangle$$

Spin networks with boundary

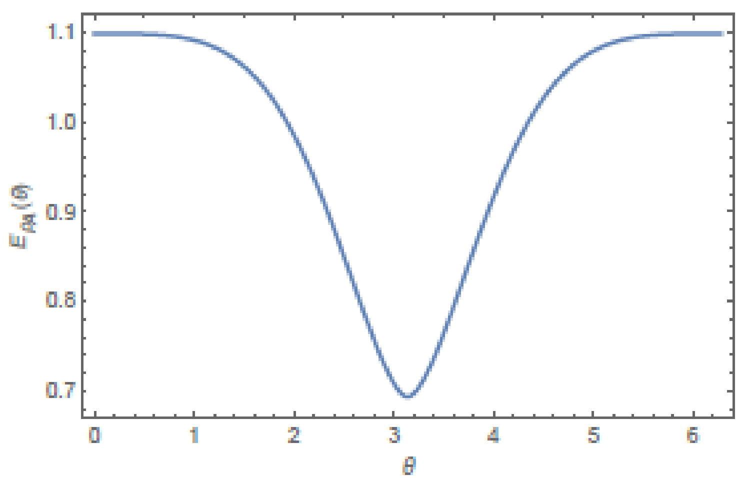
- The boundary states in simple SN



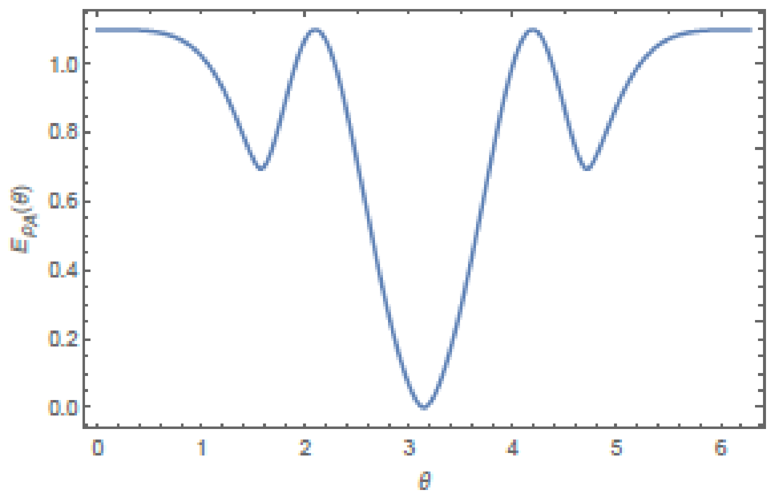
$J_1 = J_2 = 1$

$$\left|\psi_{J_1;J_2}\right\rangle = \sum_{M_1 M_2} N C^{J_1\, j_1\, j_2}_{M_1\, m_1\, m_2} D^{m_1 n_1} D^{m_2 n_2} U^{k_1 *}_{n_1}[g(\theta)] U^{k_2 *}_{n_2}[g(0)] C^{J_2\, j_1\, j_2}_{M_2\, k_1\, k_2} |M_1 M_2\rangle$$

$$C^{J\, j_1\, j_2}_{M\, m_1\, m_2} = \langle j_1\, m_1; j_2\, m_2 | J, M\rangle \qquad D^{mn} = (-1)^{j-m}\delta^{m,-n} \qquad U^{k_1}_{n_1}[g(\theta)] = e^{-ik_1\theta}\delta^{k_1}_{n_1}$$



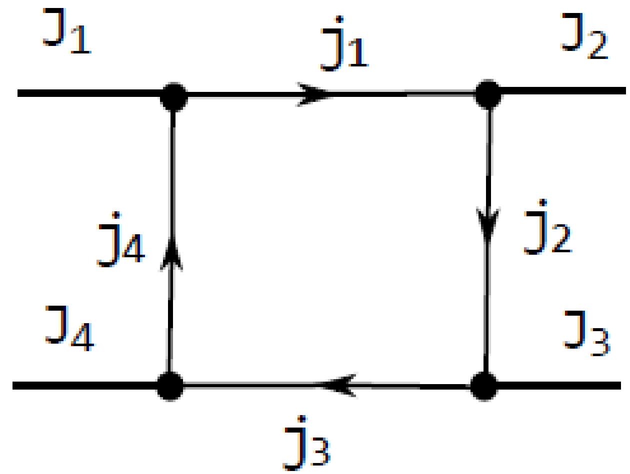
$j_1 = j_2 = 1/2$



$j_1 = j_2 = 1$

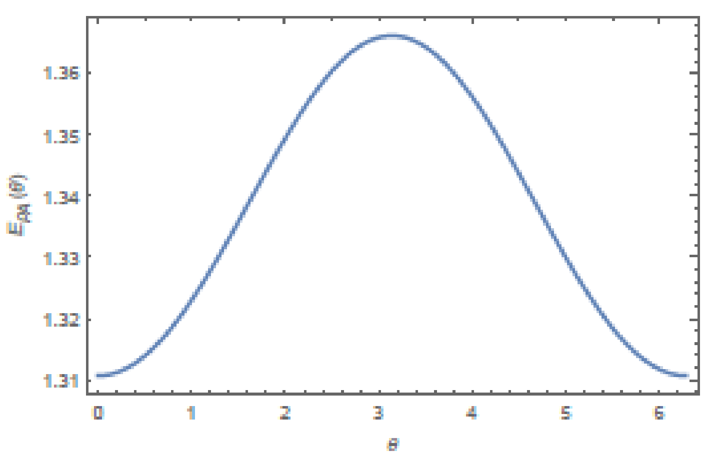
Spin networks with boundary

- The boundary states in simple SN

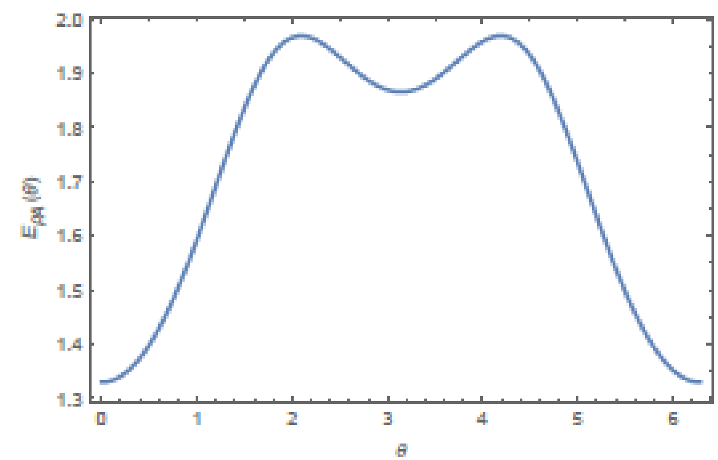


$$J_1 = J_2 = J_3 = J_4 = 1$$

$$\left|\psi_{J_1 J_4; J_2 J_3}\right\rangle = \sum_{M_l} N C^{J_1\, j_1\, j_4}_{M_1\, m_1\, k_4} D^{m_1 n_1} U^{k_1 *}_{n_1}[g(\theta)] C^{J_1\, j_2\, j_1}_{M_2\, m_2\, k_1} D^{m_2 n_2} U^{k_2 *}_{n_2}[g(0)]$$
$$\times C^{J_3\, j_3\, j_2}_{M_3\, m_3\, k_2} D^{m_3 n_3} U^{k_3 *}_{n_3}[g(0)] C^{J_4\, j_4\, j_3}_{M_4\, m_4\, k_3} D^{m_4 n_4} U^{k_4 *}_{n_4}[g(0)] |M_1 M_2 M_3 M_4\rangle$$



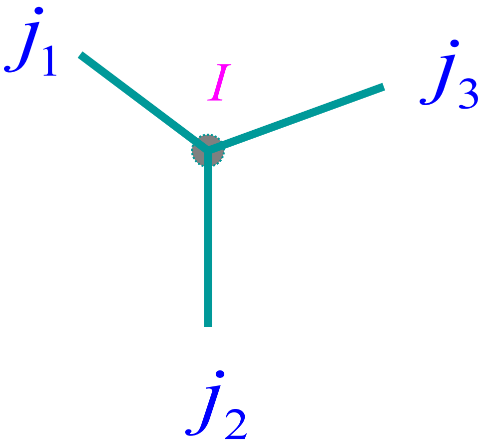
$$j_1 = j_2 = j_3 = j_4 = 1/2$$



$$j_1 = j_2 = j_3 = j_4 = 1$$

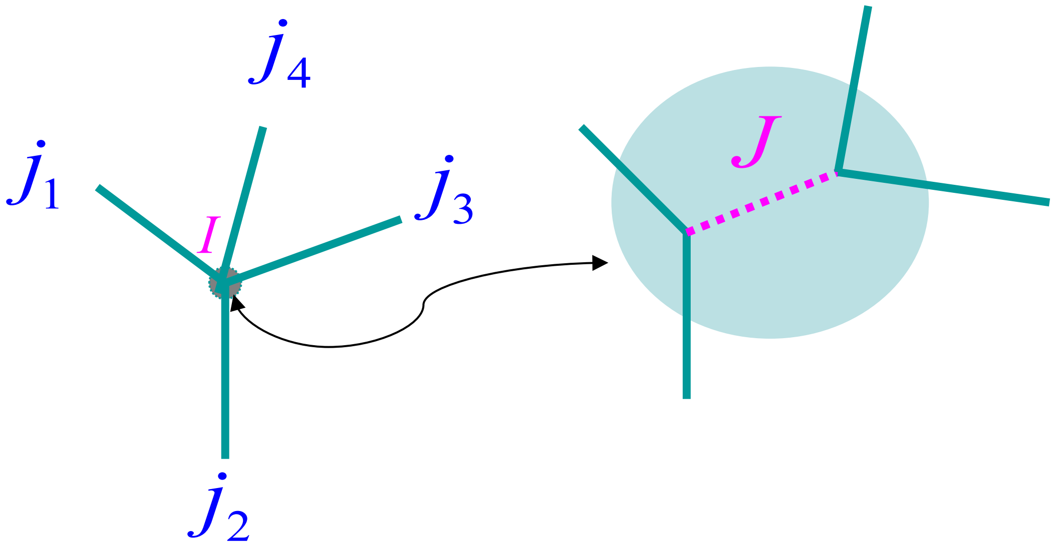
Quantum entanglement and quantum geometry

- Intertwiner **(编织结)**



$$\dim(I) = 1$$

$$V(I) = 0$$



$$j_1 = j_2 = j_3 = j_4 = \frac{1}{2}$$

$$J = 0,1$$

$$\dim(I) = 2$$

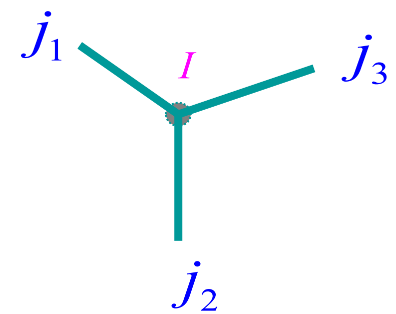
Quantum entanglement and quantum geometry

- Volume operator and eigenvalues

$$\hat{V} = l_p^3 \sqrt{|\widehat{W}|}$$

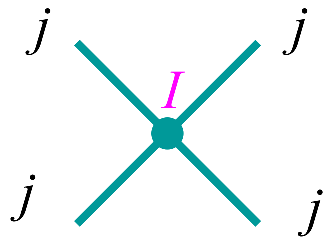
$$W_{\nu\lambda} = \langle I_\nu | \widehat{W} | I_\lambda \rangle$$

$$\widehat{W}|I_3\rangle = 0$$



$$\widehat{W} = \frac{1}{8}\varepsilon^{ijk}(J_i^{(1)}J_j^{(2)}J_k^{(3)} - J_i^{(1)}J_j^{(2)}J_k^{(4)} + J_i^{(1)}J_j^{(3)}J_k^{(4)} - J_i^{(2)}J_j^{(3)}J_k^{(4)})$$

$$\widehat{W}|I_4\rangle \neq 0$$



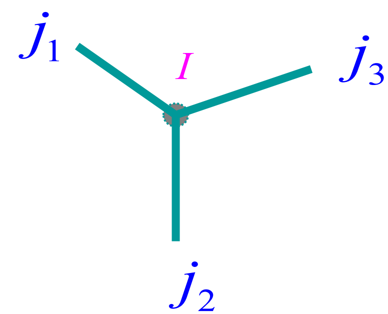
$$\hat{J}_i^{(p)} = \frac{1}{2}\int_{S_2^p} \varepsilon_{abc}\hat{E}_i^a \, dx^b dx^c$$

Quantum entanglement and quantum geometry

Y. Li, M. Han, M. Grassl and B. Zeng, (2017) , ArXiv:1612.04504.

- Tri-valent SU(2) invariant perfect tensors

$$|\psi_3\rangle = \sum_{m_1 m_2 m_3} \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} |m_1 m_2 m_3\rangle$$

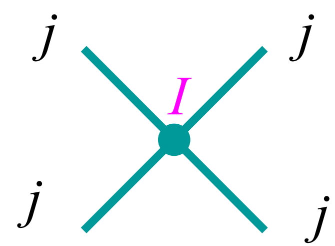


Perfect tensors

$$S_{tot} = S_1 + S_2 + S_3 = \ln(2j_1 + 1) + \ln(2j_1 + 1) + \ln(2j_3 + 1)$$

- No Four-valent SU(2) invariant perfect tensors

Perfect tensors $S_{12} = S_{13} = S_{14} = \ln 4 \qquad (j = \frac{1}{2})$



$$S_{tot} = S_{12} + S_{13} + S_{14} = 3 \ln 4$$

$$|\psi_4\rangle = \sum_{J=0,1} \alpha(J) |I(J)\rangle$$
$$= \sum_{J=0,1} \frac{\alpha(J)}{\sqrt{2J+1}} C^{j\,j\,J}_{m_1 m_2 M} C^{j\,j\,J}_{m_3 m_4 N} (-1)^{J-M} \delta^{M,-N} |m_1 m_2 m_3 m_4\rangle$$

Quantum entanglement and quantum geometry

- Conjecture

> The emergence of the space with non-zero volume in three dimensions is the reflection of the non-perfectness of SU(2)-invariant tensors.

- The measure of the non-perfectness

Maximally entangled SU(2)-invariant states $S_{tot}^{\max}$

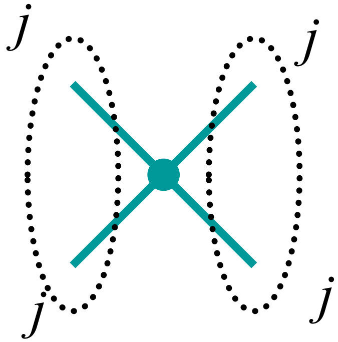
$$\Delta S = S_{tot}^{pt} - S_{tot}^{\max}$$

$$\boxed{\delta = \Delta S / S_{tot}^{pt}}$$

Quantum entanglement and quantum geometry

- Entanglement of boundary states

$$|\psi_4\rangle = \sum_{J=0,\dots 2j} \alpha(J)|I(J)\rangle$$
$$= \sum_{J=0,\dots,2j} \frac{\alpha(J)}{\sqrt{2J+1}} C^{j\,j\,J}_{m_1 m_2 M} C^{j\,j\,J}_{m_3 m_4 N} (-1)^{J-M} \delta^{M,-N} |m_1 m_2 m_3 m_4\rangle$$



$$\rho_{ik} = Tr_{ik} \frac{|\psi_4\rangle\langle\psi_4|}{\langle\psi_4|\psi_4\rangle} \qquad S_{ik} = -Tr(\rho_{ik} \ln \rho_{ik})$$

Y. Ling, Y.Xiao, M.Wu. **arXiv:1907.01215**

- SU(2)-invariant maximally entangled states

Determine 2j+1 complex numbers in intertwiner：

$$\alpha(J), \quad J = 0, \dots, 2j$$

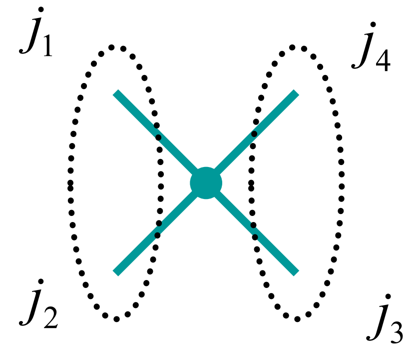
$$j_i = \frac{1}{2}, i = 1 \dots 4 \qquad \alpha(0) = \frac{1}{\sqrt{2}}, \alpha(1) = \pm \frac{i}{\sqrt{2}}$$

$$j_i = 1, i = 1 \dots 4 \qquad \alpha(0) = \frac{\sqrt{2}}{3}, \alpha(1) = \pm \frac{i}{\sqrt{2}}, \alpha(2) = -\frac{1}{3}\sqrt{\frac{5}{2}}$$

Quantum entanglement and quantum geometry

- E.g.:

$$j_i = \frac{1}{2}, i = 1\ldots4$$



Y. Ling, Y.Xiao, M.Wu. **arXiv:1907.01215**

Perfect tensor

$$S_{12} = S_{13} = S_{14} = \ln 4$$

$$S_{tot}^{pt} = S_{12} + S_{13} + S_{14} = 3\ln 4$$

Maximally entangled states $S_{tot}^{\max} = S_{12} + S_{13} + S_{14} = 3\ln(2\sqrt{3})$

$$\Delta S \cong 0.43 \qquad \delta \cong 0.104$$

$$j_i = 1, i = 1\ldots4$$

$$S_{tot}^{\max} = S_{12} + S_{13} + S_{14} = \frac{5}{3}\ln 2 + \frac{9}{2}\ln 3$$

Perfect tensor

$$S_{12} = S_{13} = S_{14} = 2\ln 3$$

$$S_{tot}^{pt} = S_{12} + S_{13} + S_{14} = 6\ln 3$$

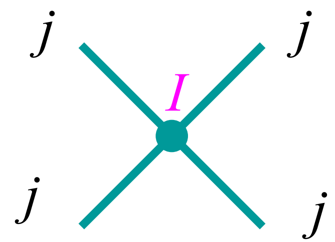
Maximally entangled states $S_{tot}^{\max} = S_{12} + S_{13} + S_{14} = \frac{5}{3}\ln 2 + \frac{9}{2}\ln 3$

$$\Delta S \cong 0.49 \qquad \delta \cong 0.075$$

Quantum entanglement and quantum geometry

- **Eigenstates and Eigenvalues of volume operator**

$$\hat{V} = l_p^3\sqrt{|\widehat{W}|} \qquad W_{\nu\lambda} = \langle I_\nu|\widehat{W}|I_\lambda\rangle$$



$$j_1 = j_2 = j_3 = j_4 = 1/2 \;\rightarrow\; W = \begin{pmatrix} 0 & -\frac{\sqrt{3}}{8}i \\ \frac{\sqrt{3}}{8}i & 0 \end{pmatrix} \;\rightarrow\; W_\pm = \pm\frac{\sqrt{3}}{8} \qquad V = l_p^3\sqrt{\frac{\sqrt{3}}{8}}$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}|J=0\rangle \pm \frac{i}{\sqrt{2}}|J=1\rangle$$

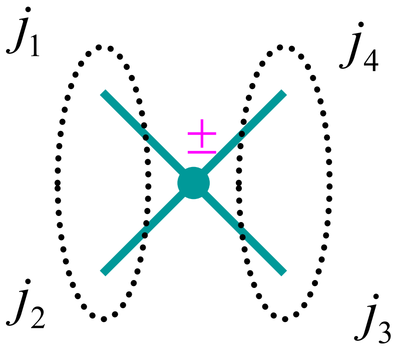
Maximally entangled states have definite orientation!

$$j_1 = j_2 = j_3 = j_4 = 1 \;\rightarrow\; W = \begin{pmatrix} 0 & -\frac{i}{\sqrt{3}} & 0 \\ \frac{i}{\sqrt{3}} & 0 & -\frac{i}{2}\sqrt{\frac{5}{3}} \\ 0 & 0 & \frac{i}{2}\sqrt{\frac{5}{3}} \end{pmatrix} \;\rightarrow\; W_\pm = \pm\frac{\sqrt{3}}{2}, W_3 = 0 \qquad V = l_p^3\sqrt{\frac{\sqrt{3}}{2}}, 0$$

$$|\pm\rangle = \frac{\sqrt{2}}{3}|J=0\rangle \pm \frac{i}{\sqrt{2}}|J=1\rangle - \frac{1}{3}\sqrt{\frac{5}{2}}|J=2\rangle$$

Quantum entanglement and quantum geometry

- Numerical Check



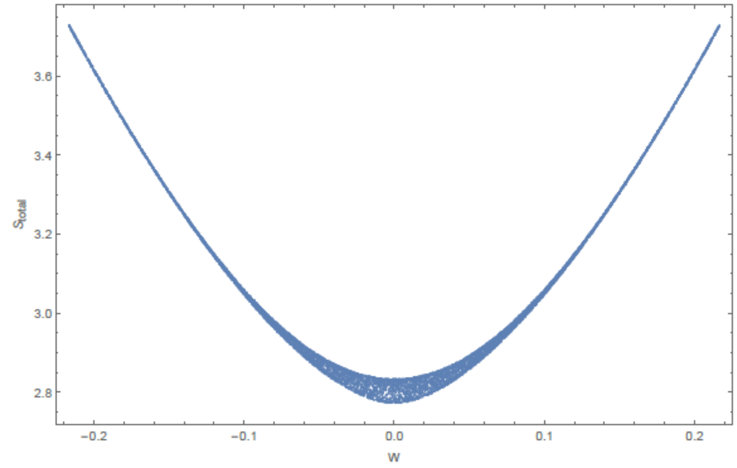
Y. Ling, Y.Xiao, M.Wu. **arXiv:1907.01215**

$$j_i = \frac{1}{2}, i = 1 \ldots 4$$

$$|\psi_4\rangle = \sum_{J=0,1} \alpha(J)\, C^{j\,j\,J}_{m_1 m_2 M}\, C^{j\,j\,J}_{m_3 m_4 N}\, (-1)^{J-M} \delta^{M,-N} |m_1 m_2 m_3 m_4\rangle$$
$$\equiv C_+|+\rangle + C_-|-\rangle$$

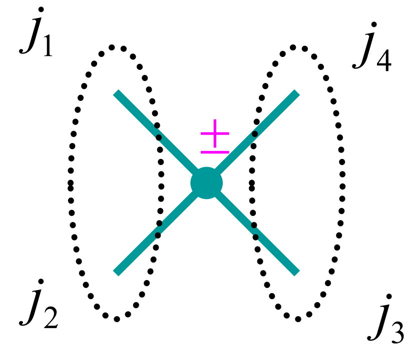
$$\langle \widehat{W} \rangle = \frac{\langle \psi_4 | \widehat{W} | \psi_4 \rangle}{\langle \psi_4 | \psi_4 \rangle} = \frac{\sqrt{3}}{8} \frac{|C_+|^2 - |C_-|^2}{|C_+|^2 + |C_-|^2}$$

$$\langle \hat{V} \rangle = \sqrt{\frac{\sqrt{3}}{8}}$$



Quantum entanglement and quantum geometry

- **Numerical Check**

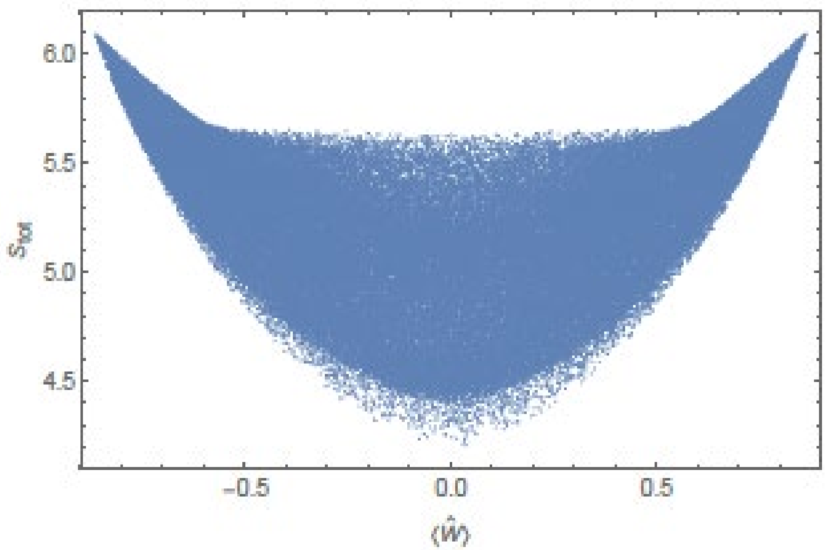


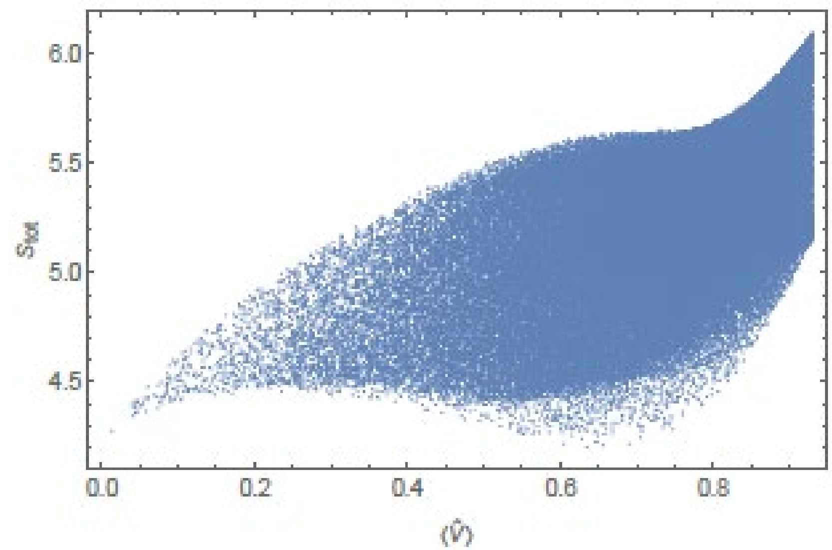
$j_i = 1, i = 1 \ldots 4$

$$|\psi_4\rangle = \alpha_+|\lambda_+\rangle + \alpha_0|\lambda_0\rangle + \alpha_-|\lambda_-\rangle$$

$$\langle \widehat{W} \rangle = \frac{\sqrt{3}}{2} \frac{|\alpha_+|^2 - |\alpha_-|^2}{|\alpha_+|^2 + |\alpha_0|^2 + |\alpha_-|^2}$$

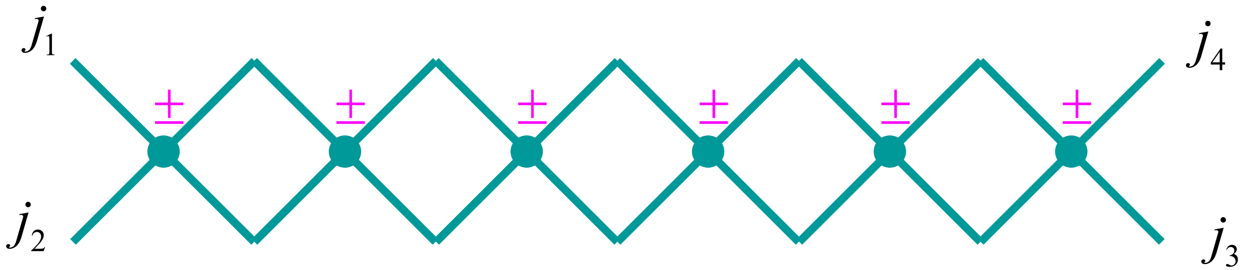
$$\langle \hat{V} \rangle = \sqrt{\frac{\sqrt{3}}{2} \frac{|\alpha_+|^2 + |\alpha_-|^2}{|\alpha_+|^2 + |\alpha_0|^2 + |\alpha_-|^2}}$$

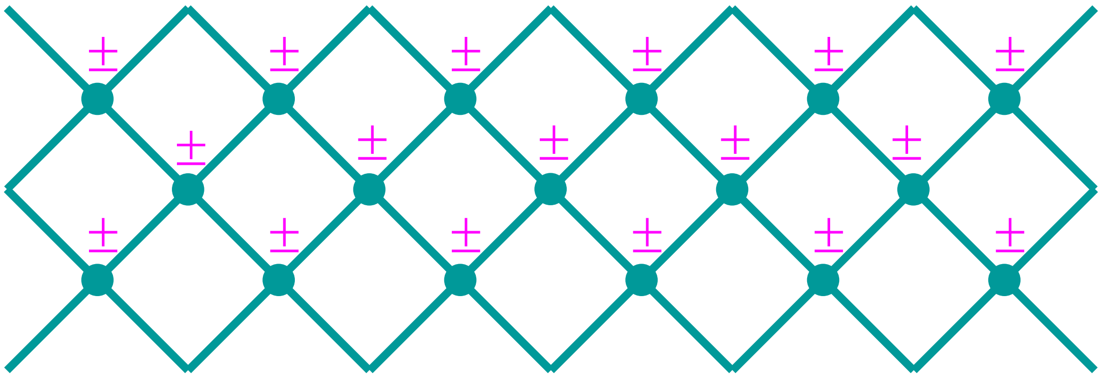




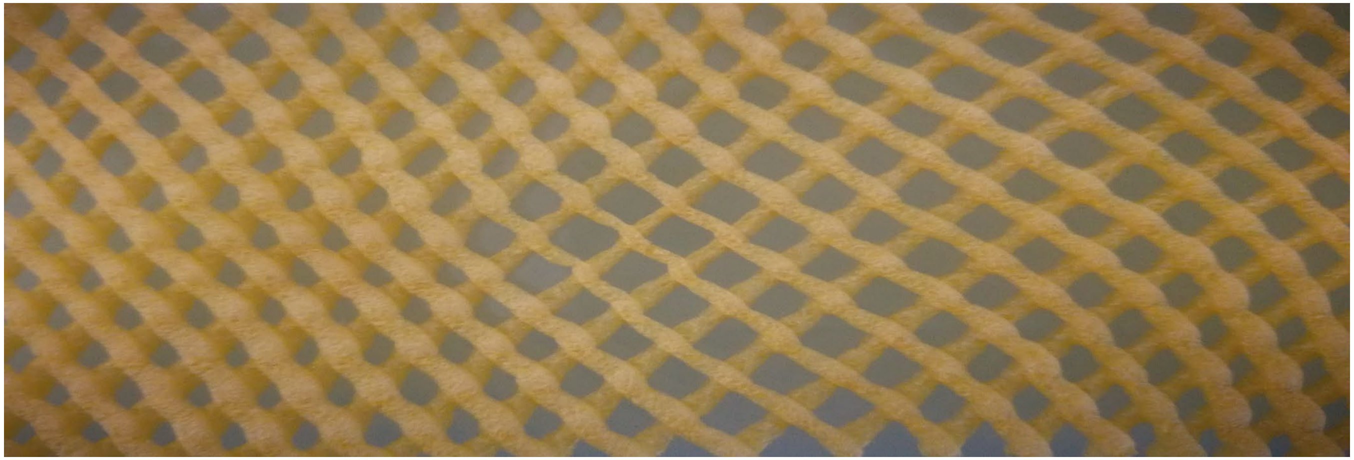
Quantum entanglement and quantum geometry

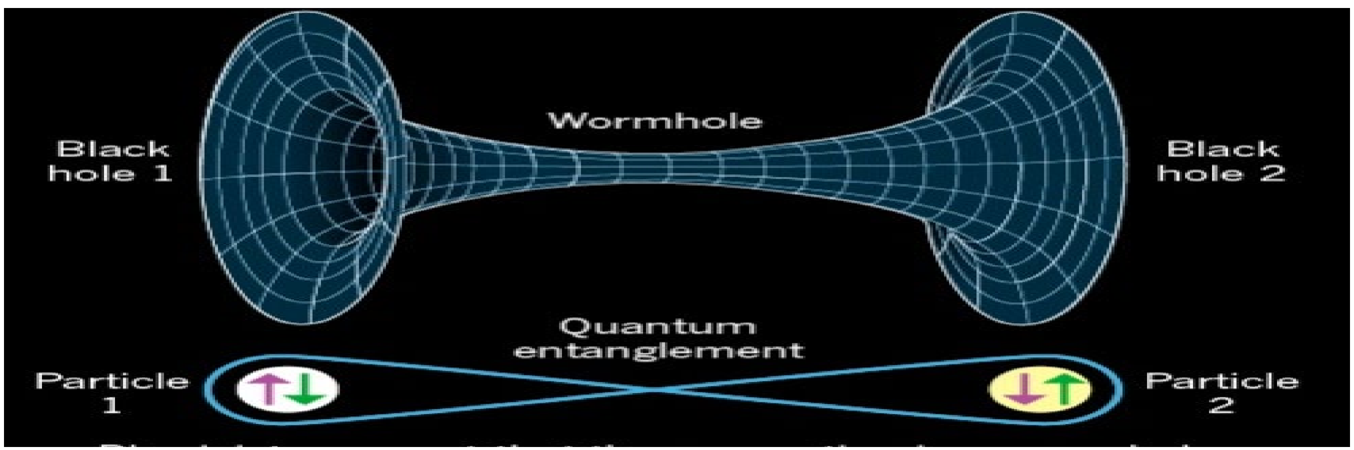
$j_i = \frac{1}{2}, i = 1 \ldots 4$





Quantum entanglement and quantum geometry





Quantum entanglement and quantum geometry

- **AI for Quantum Gravity is on the road!**

1. Suppose you knew nothing about LQG.

2. Feed DS4 with the ArXiv paper, for instance, arXiv:1907.01215.

3. Input commands and ask the DS4 to design a Mathematica codes to compute the eigenvalues of the volume operator when acting on 4-valent vertex with J= ½, 1 with detailed requirement.

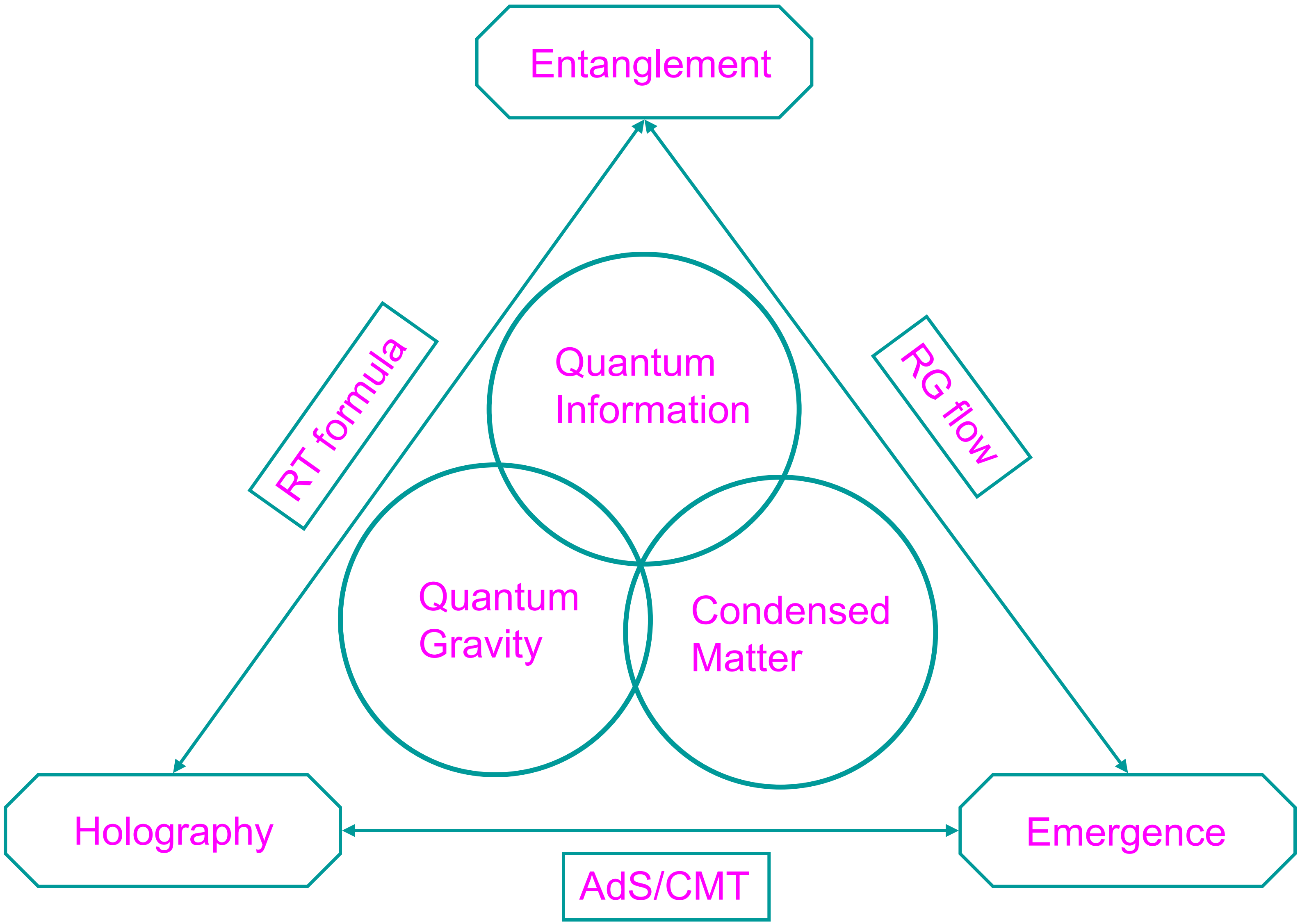
4. Using this Mathematica codes to compute the eigenvalues of the volume operator with higher spins which do not appear in arXiv:1907.01215.

5. Check the answers in gr-qc/9602023!

Quantum entanglement and quantum geometry

```
(* ============================================================ *)
(*  Mathematica program to compute volume eigenvalues       *)
(*  for a 4-valent vertex with edges j (configurable)       *)
(*                                                           *)
(*  Based on: Y. Ling, Y. Xiao, M. Wu, arXiv:1907.01215     *)
(*  "Note on quantum entanglement and quantum geometry"     *)
(*                                                           *)
(*  Strategy:                                                *)
(*  1. Build SU(2) generators as exact symbolic matrices    *)
(*  2. Build tensor product operators via KroneckerProduct   *)
(*  3. Compute W operator exactly (symbolic)                 *)
(*  4. Project onto intertwiner basis                        *)
(*  5. Diagonalize                                           *)
(* ============================================================ *)

ClearAll["Global`*"];

Print["=========================================================================="];
Print["  Volume eigenvalues for 4-valent vertex (exact symbolic)"];
Print["  Based on Y. Ling, Y. Xiao, M. Wu, arXiv:1907.01215"];
Print["=========================================================================="];

(* ================================================= *)
(*  USER CONFIGURABLE PARAMETER                     *)
(* ================================================= *)

spin = 2;  (* <---- CHANGE THIS VALUE AS NEEDED *)

Print["\nConfigured spin: j = ", spin];

(* ================================================= *)
(* SU(2) generators as EXACT symbolic matrices   *)
(* Ordering: |j,j> = index 1, ..., |j,-j> = d   *)
(* ================================================= *)
```

Summary:


Entanglement
RT formula
RG flow
Quantum
Information
Quantum
Gravity
Condensed
Matter
Holography
Emergence
AdS/CMT

Thanks!