



Nuclear Rates for BBN from an EFT Built out of Amplitudes

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Based on:

TMPT, arXiv:2608.23699 [nucl-th]

TMPT, arXiv:2607.05514 [nucl-th]

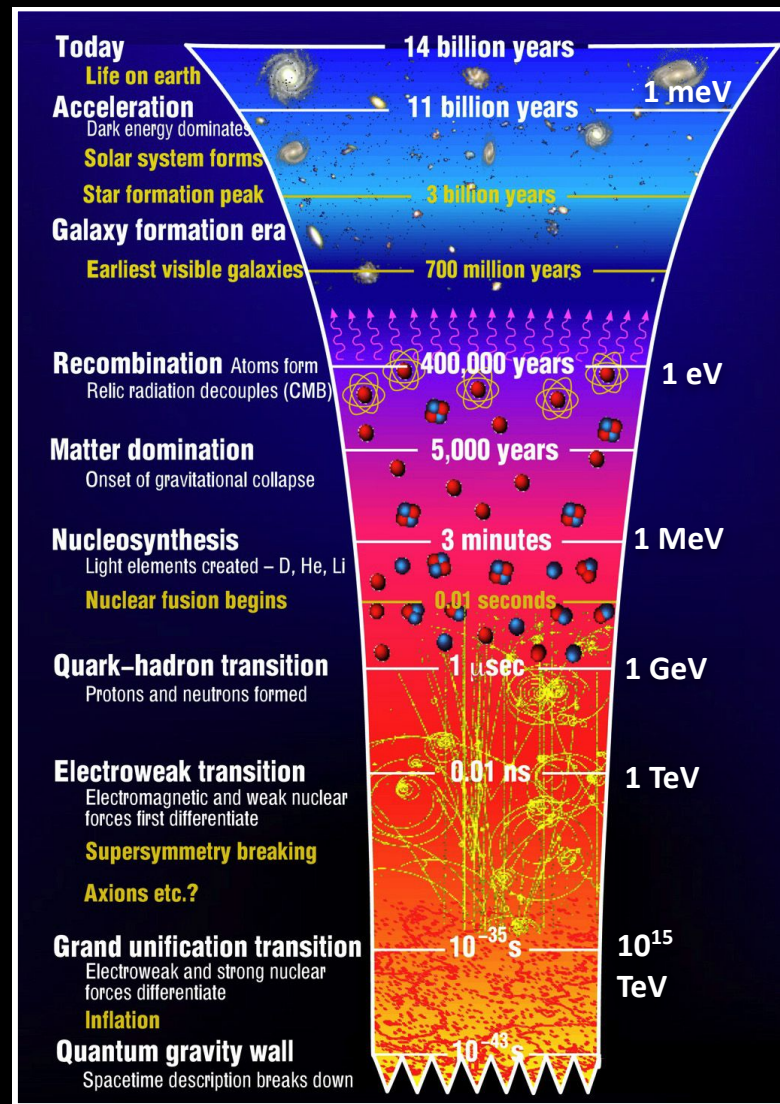
PRyMordial:

Burns, TMPT, Valli, Eur. Phys. J.C. 84 (2024) 1, 86
arXiv: 2307.07061 [hep-ph]

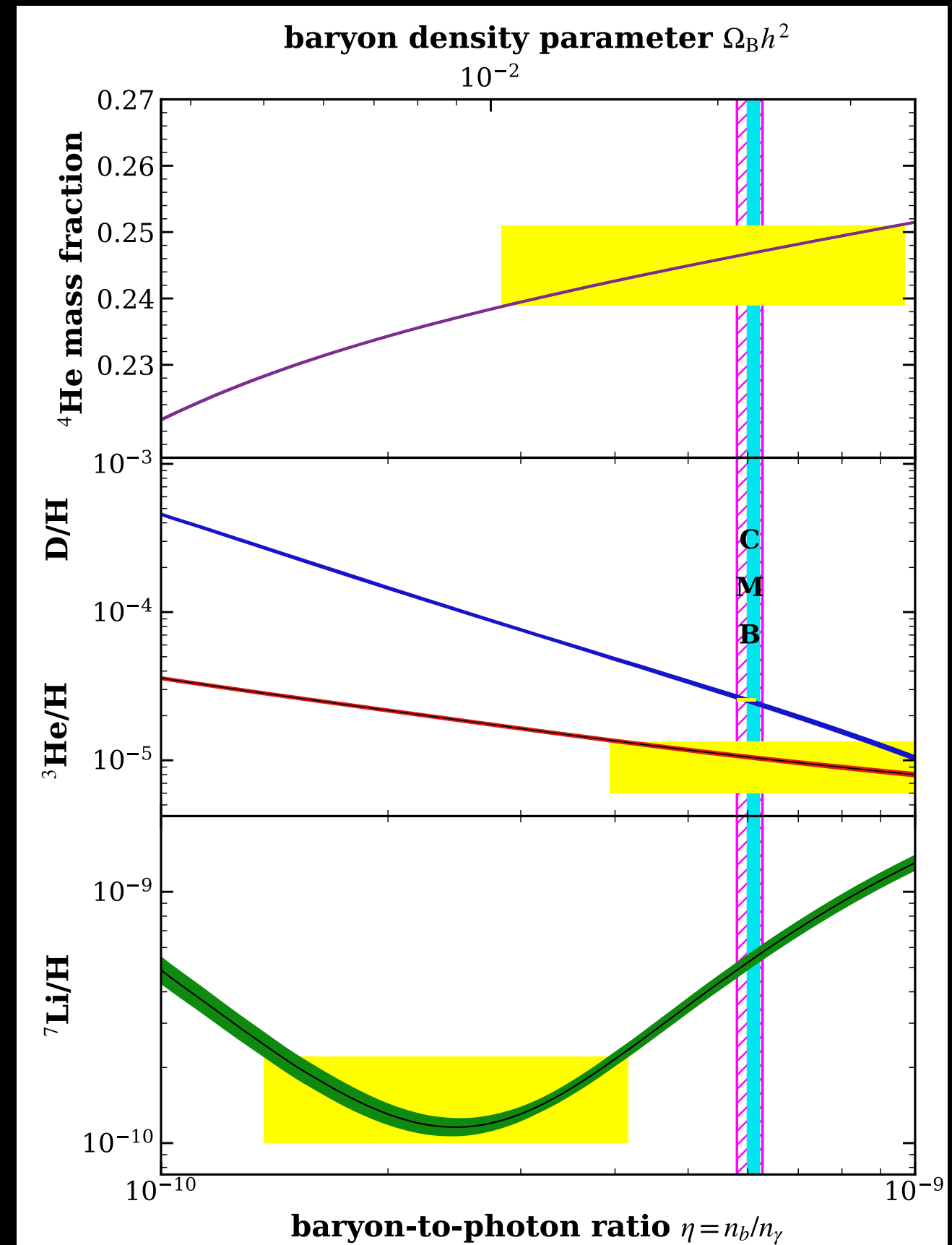


PPC 2026
September 1, 2026

Big Bang Nucleosynthesis

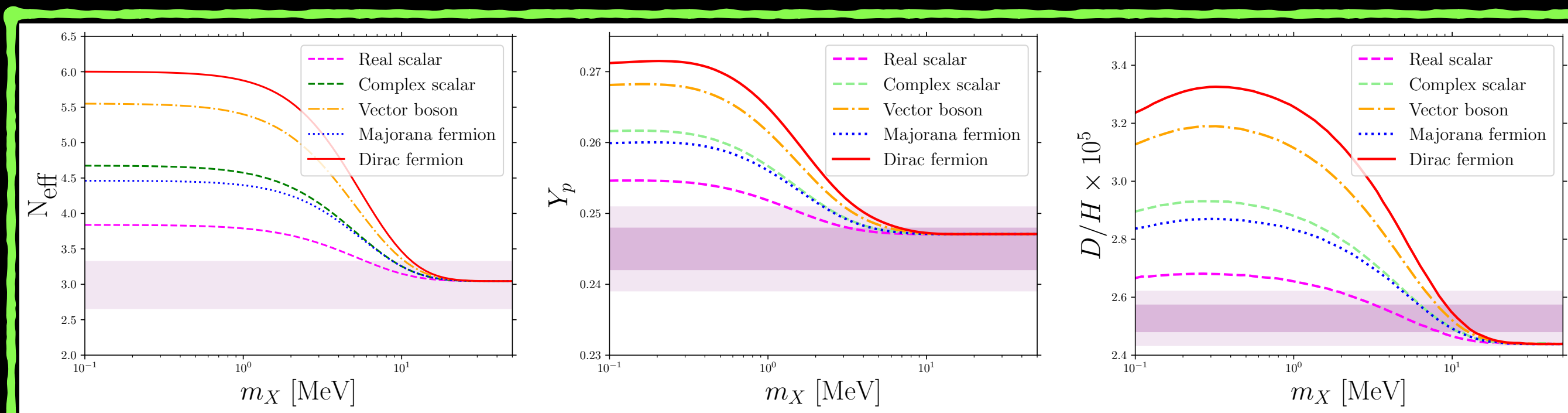
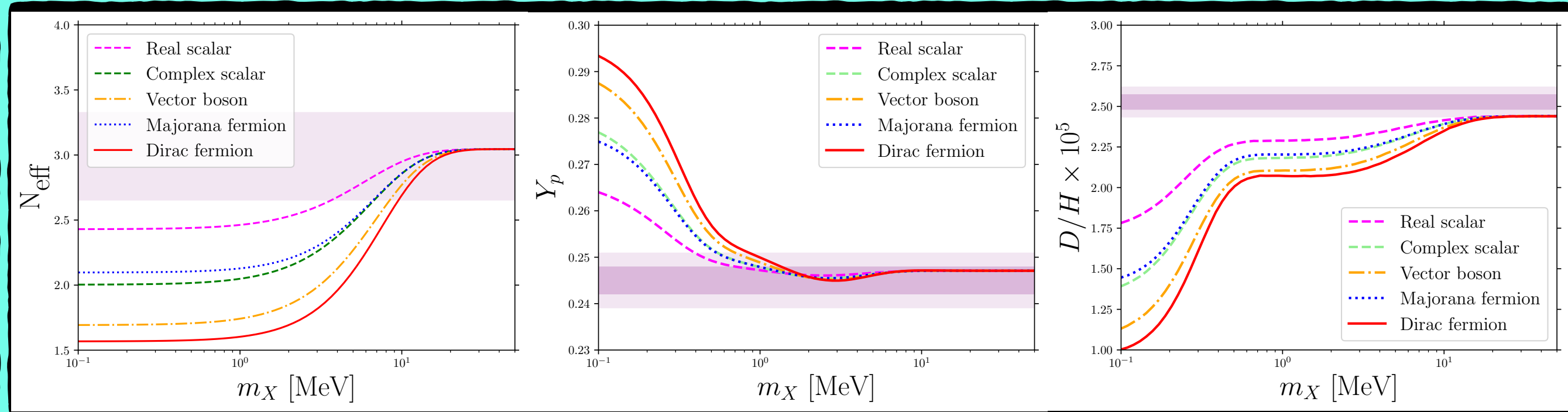


BBN is remarkably successful, connecting particle & nuclear physics to cosmology, and making a convincing case that we understand the Universe pretty well back to temperatures of O(MeV) — and it places important constraints on new physics.



For Example...

Electrophillic-thermalized X particle



Neutrino-phillic-thermalized X particle

BBN Observations

To compare with theory, we need quality measurements of the primordial abundances of the light elements: D , ${}^3\text{He}$, ${}^4\text{He}$, ${}^7\text{Li}$



Helium-4 observed in extragalactic HII regions

Emission spectra of gas clouds (detailed line modeling required)



Deuterium observed in Quasar absorption systems

Damped Lyman- α spectra from intervening gas along l.o.s.



Helium-3 observed in the Solar neighborhood

Solar winds, meteorites, ISM ... stellar nucleosynthesis uncertainties

Cooke et al, arXiv:2605.00122 [astro-ph.CO]



Lithium-7 in the atmosphere of dwarf halo stars

Physics of convection, depletion indicators ... needs support from data

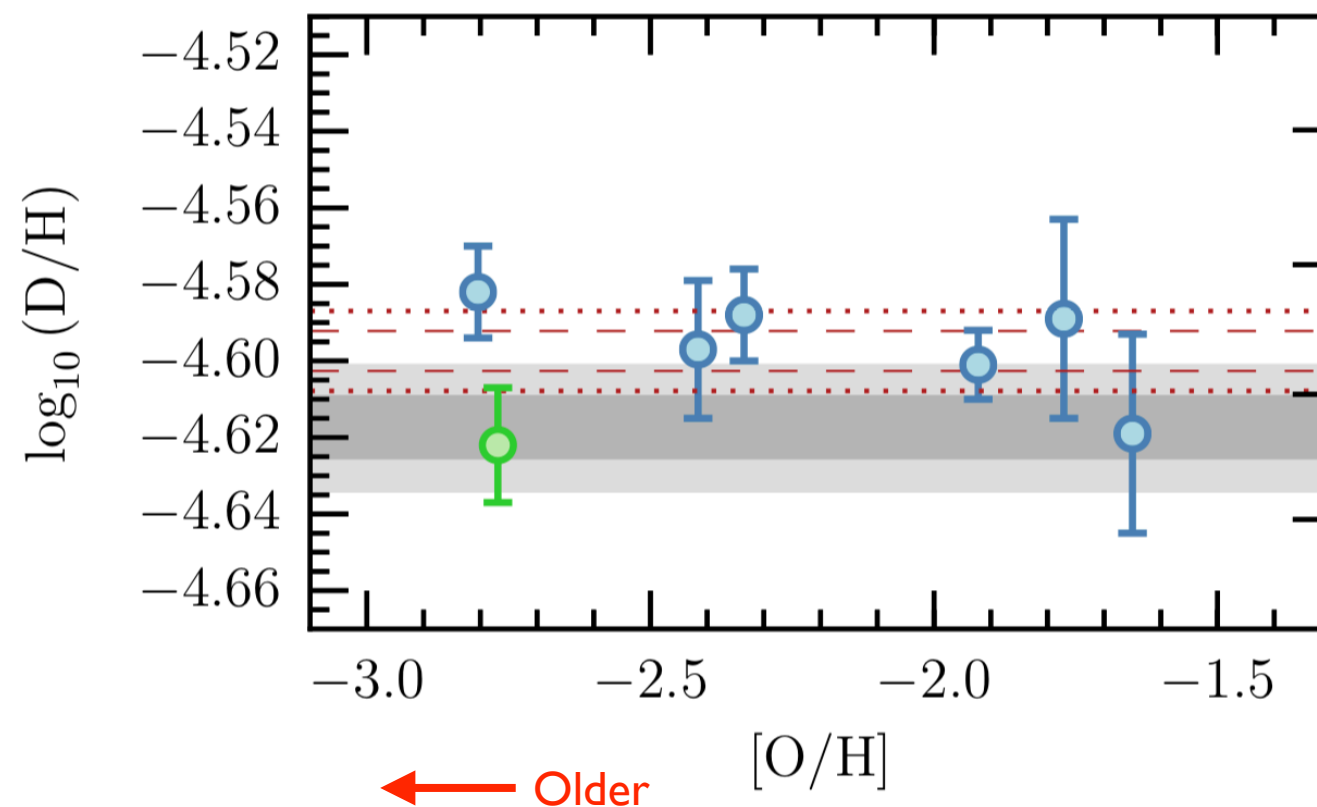
E.g. Fields, Olive arXiv:2204.03167

Deuterium

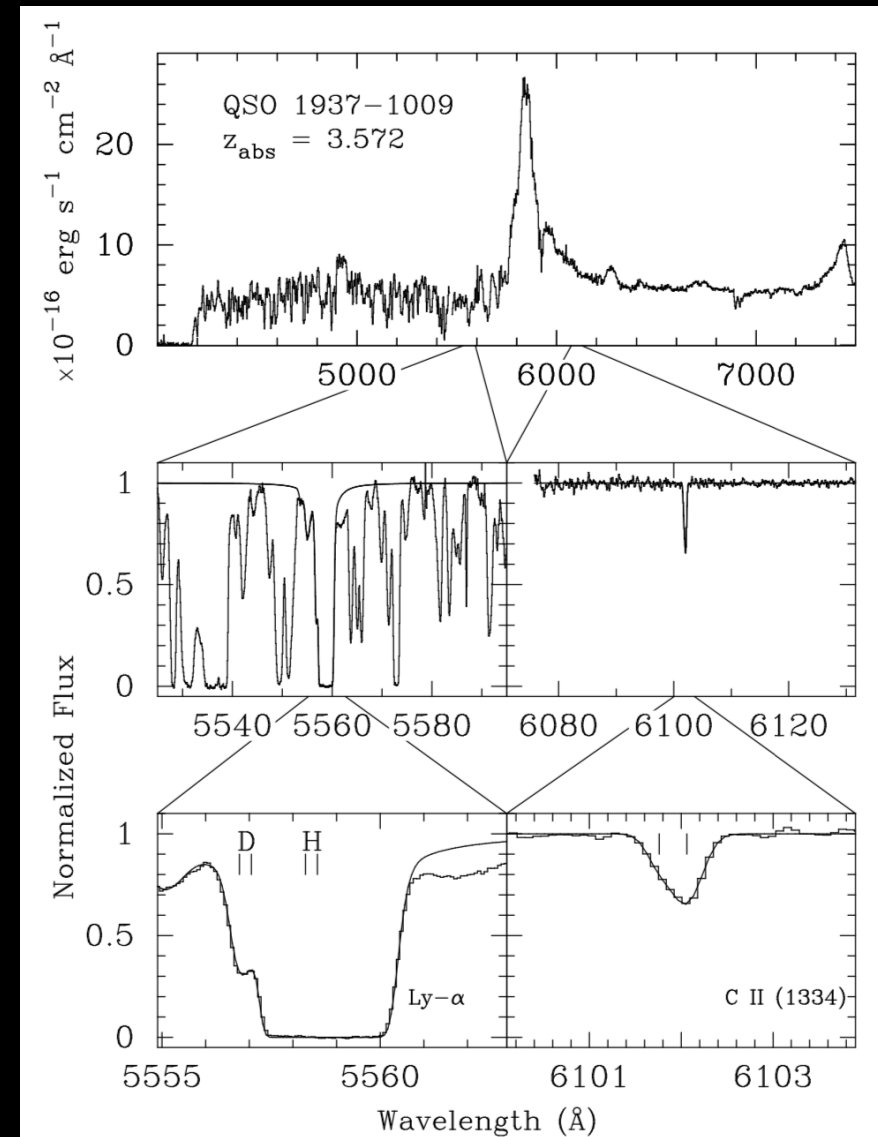
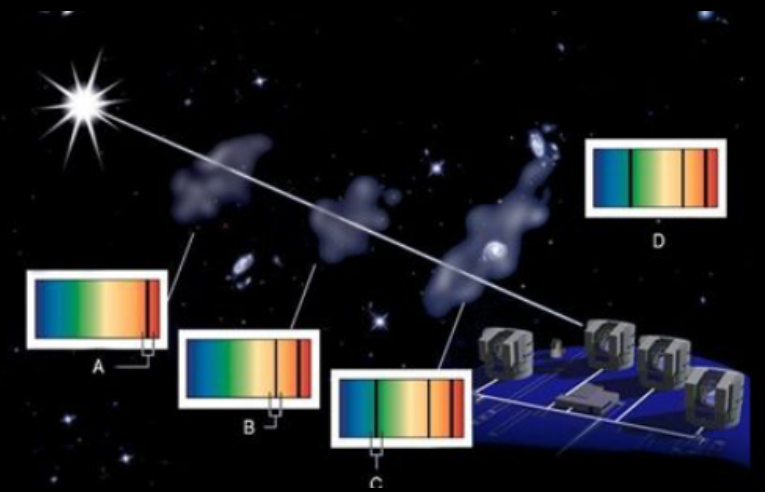
PDG 2021:

$$(D/H) \times 10^5 = 2.547 \pm 0.025$$

% level
measurement



arXiv: 1710.11129



astro-ph/9803071

PRyMordial

Burns, TMPT, Valli,
arXiv: 2307.07061

- **PRyMordial** is a new tool to investigate BBN both within the SM and beyond.
- It's written in python, and runs very quickly using DiffEq.jl from the Sci Machine Learning kit implemented in Julia.
- It fully models neutrino decoupling and computes N_{eff} , allowing for precise constraints on dark sectors.
- PRyMordial allows for precision treatment of electroweak rates, including mass effects and $O(\alpha)$ corrections.
- It implements network of $O(10/50)$ nuclear reactions in a modular format, making it easy to compare different nuclear treatments.

Escudero, arXiv:1812.05605
& arXiv: 2001.04466

Based on PRIMAT's Mathematica
implementation, but in python!

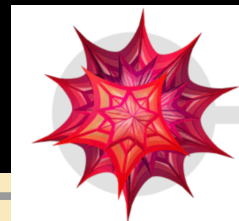
NACRE & PRIMAT
approaches



AlterBBN

NEW AlterBBN v2.2

arXiv:1106.1363,
arXiv:1806.11095



PRIMAT

PRImordial MATter

arXiv:1806.11095,
arXiv: 2011.11320



PRyMordial

Fortran

ParthENoPE

Public Algorithm Evaluating the Nucleosynthesis of Primordial Elements

arXiv:0705.0290, arXiv:1712.04378, arXiv:2103.05027

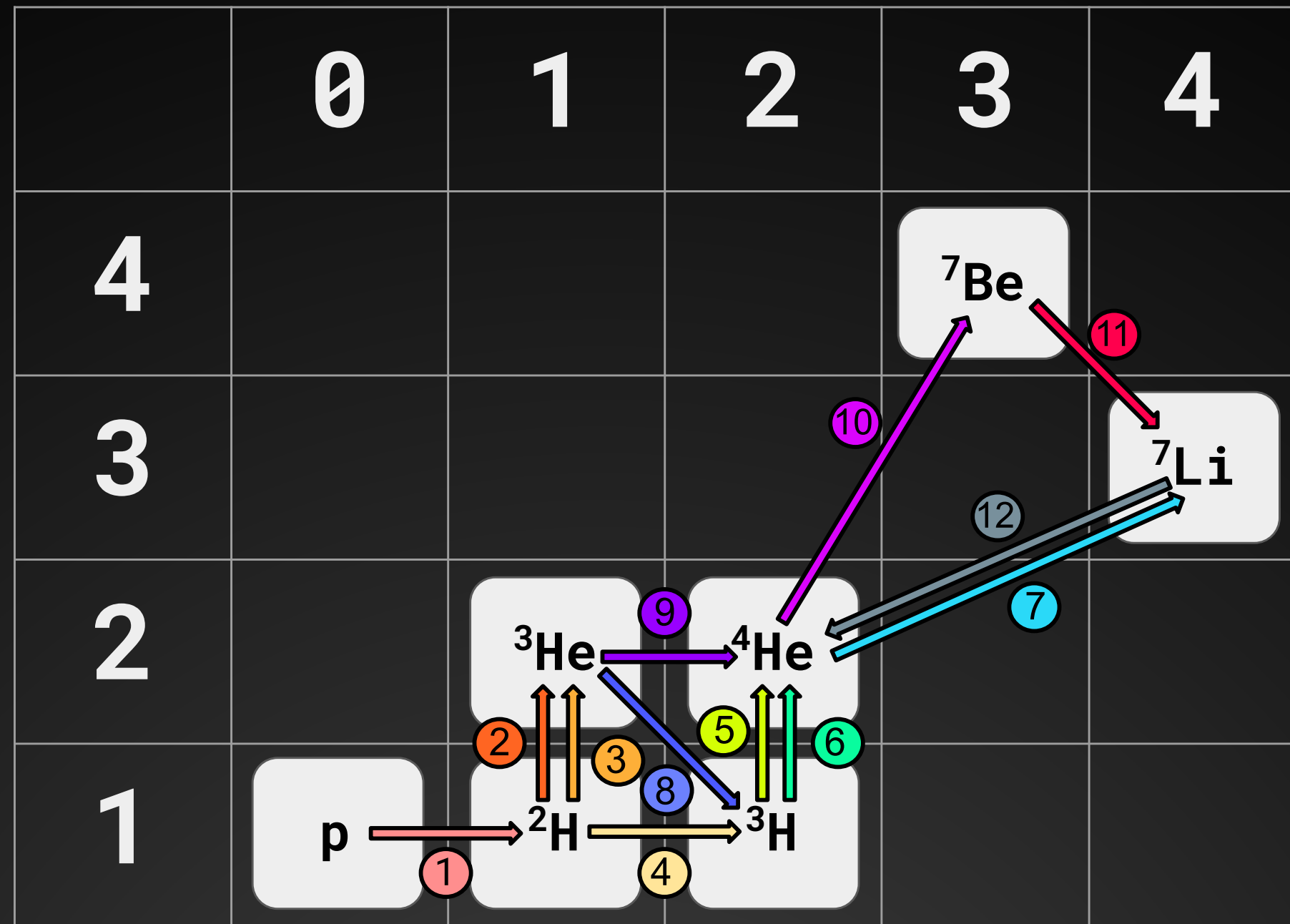
Nuclear Rates

Essential Nuclear Reactions

1. $n + p \rightarrow {}^2\text{H} + \gamma$
2. ${}^2\text{H} + p \rightarrow {}^3\text{He} + \gamma$
3. ${}^2\text{H} + {}^2\text{H} \rightarrow {}^3\text{He} + n$
4. ${}^2\text{H} + {}^2\text{H} \rightarrow {}^3\text{H} + p$
5. ${}^3\text{H} + p \rightarrow {}^4\text{He} + \gamma$
6. ${}^3\text{H} + {}^2\text{H} \rightarrow {}^4\text{He} + n$
7. ${}^3\text{H} + {}^4\text{He} \rightarrow {}^7\text{Li} + \gamma$
8. ${}^3\text{He} + n \rightarrow {}^3\text{H} + p$
9. ${}^3\text{He} + {}^2\text{H} \rightarrow {}^4\text{He} + p$
10. ${}^3\text{He} + {}^4\text{He} \rightarrow {}^7\text{Be} + \gamma$
11. ${}^7\text{Be} + n \rightarrow {}^7\text{Li} + p$
12. ${}^7\text{Li} + p \rightarrow {}^4\text{He} + {}^4\text{He}$

Atomic Number

Neutron Number

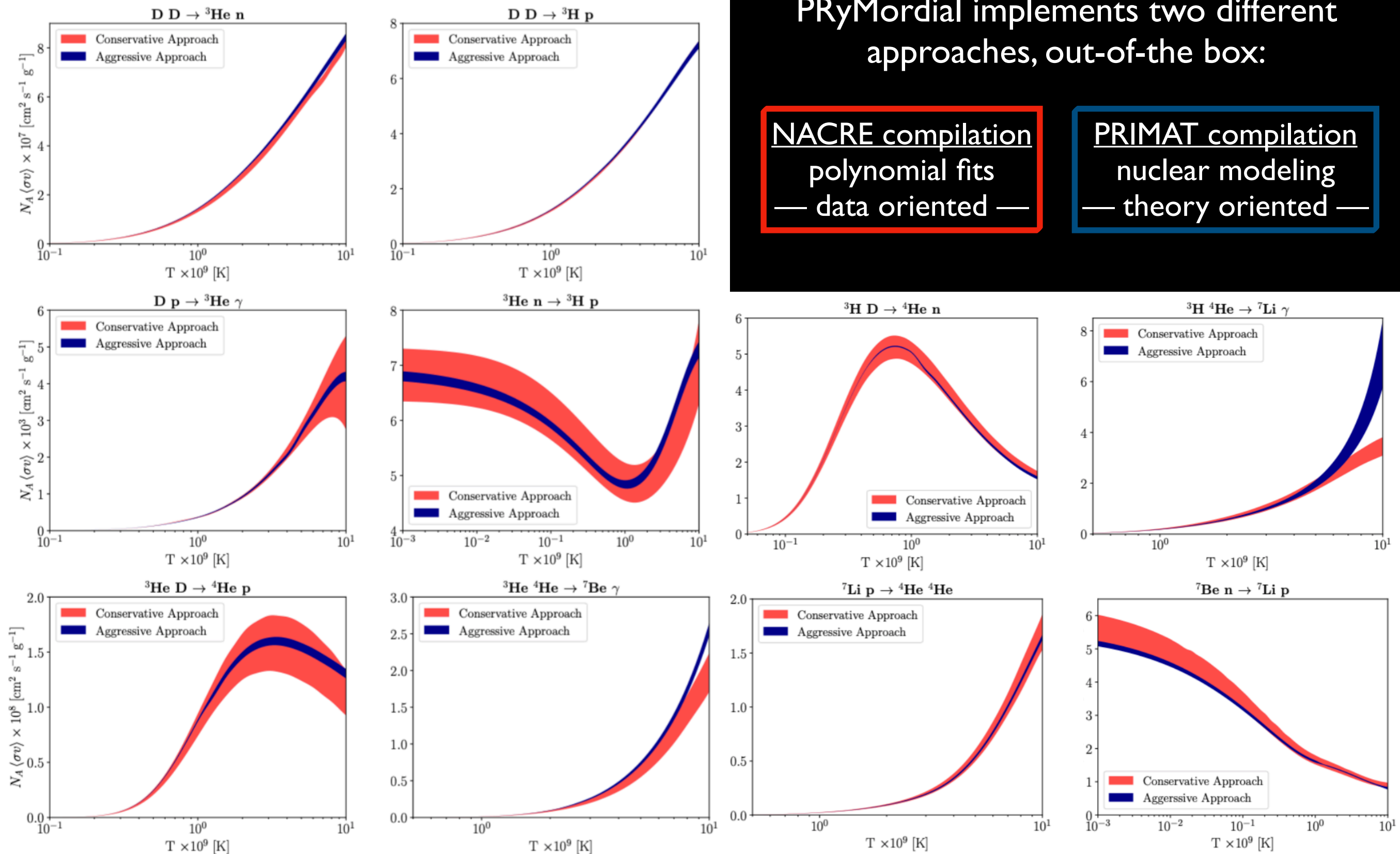


Nuclear Rates

PRyMordial implements two different approaches, out-of-the box:

NACRE compilation
polynomial fits
— data oriented —

PRIMAT compilation
nuclear modeling
— theory oriented —



Machine Learning

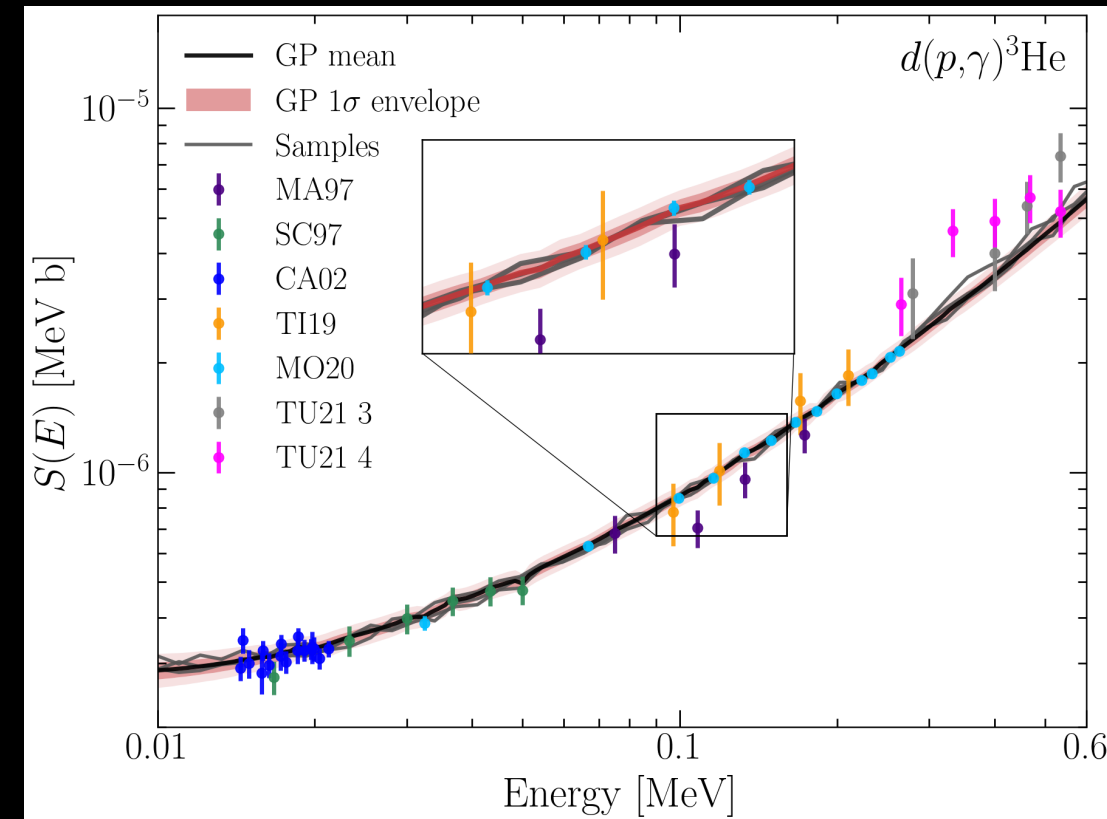
- Recent work applies Gaussian-process ML regression of the raw S-factor data for the deuterium burning processes— using no nuclear theory at all.

Launders, Giovanetti, Liu,
[arXiv:2604.16600]

- Based on this treatment, they find:

$$10^5 \text{ D/H} = 2.442 \pm 0.040$$

- $\sim 1.7\sigma$ below the Cooke et al. measurement (2.0σ with Planck+ACT+SPT).
- They identify the relevant window of energies for deuterium production/burning to be **14 keV – 0.6 MeV**.
- The burning processes are the most important to further pin down.
- I'm pursuing a different approach— using EFT to try to control theory uncertainties.



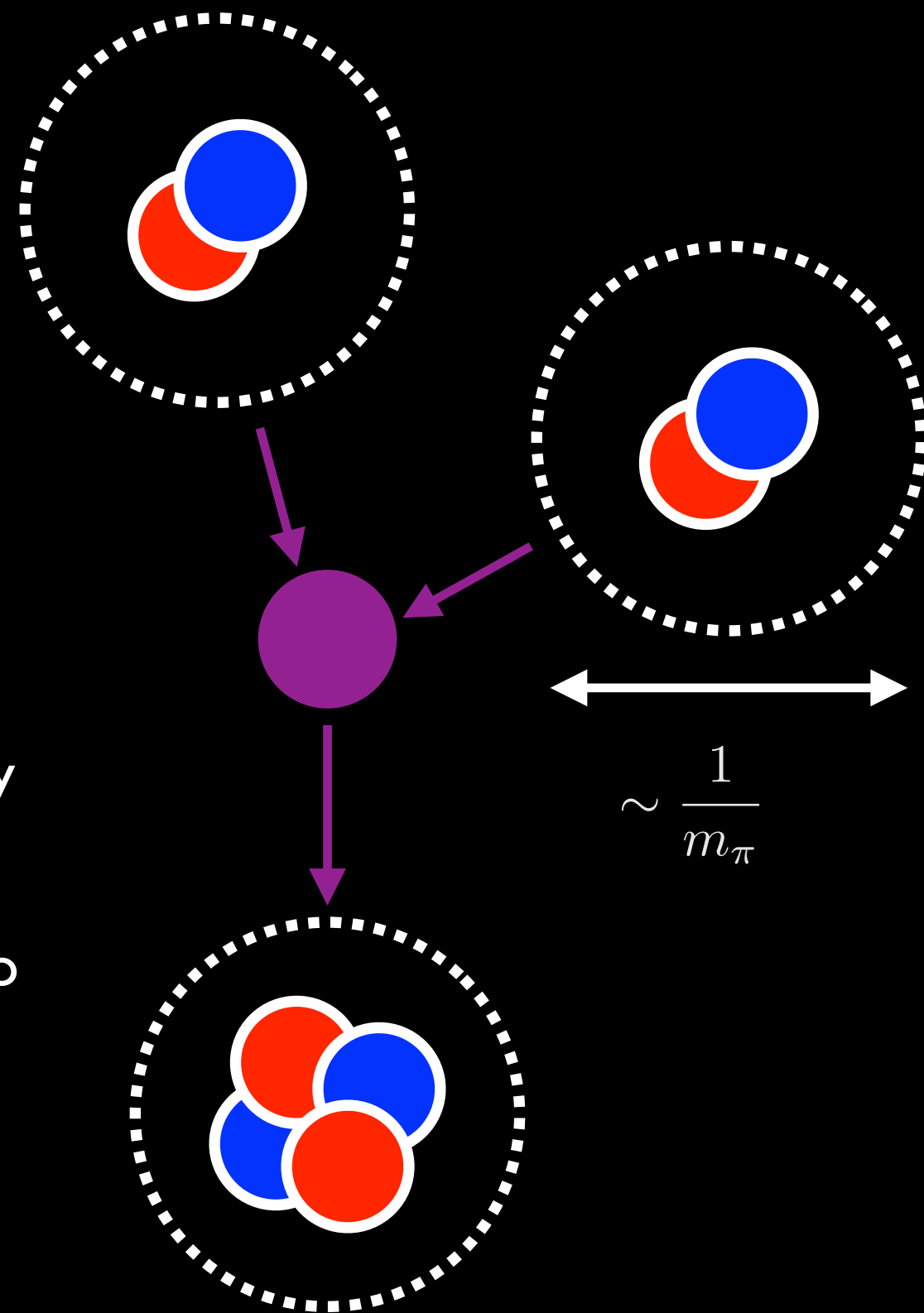
Reaction	% Change			
	D/H	Y_P	${}^3\text{He}/\text{H}$	${}^7\text{Li}/\text{H}$
$p(n, \gamma)d$	-0.0884	0.0005	0.0371	-0.0739
$d(p, \gamma){}^3\text{He}$	-0.7700	-0.0004	0.8370	-0.2358
$d(d, n){}^3\text{He}$	-0.5952	0.0051	0.2222	-0.0737
$d(d, p)t$	-0.5000	0.0042	-0.2791	-0.0517

Percent change in prediction corresponding to a 1σ variation of the rate from its central value.

Giovanetti et al, PRD 112 (2025) 6, 063531, [2408.14538]

An EFT for Nuclei

- I work with an effective field theory (EFT) that keeps the nuclear states as degrees of freedom.
- At typical BBN temperatures, the momenta are typically much smaller than m_π , so nuclear interactions look point-like. There are some similarities to the nuclear ‘pionless EFT’
- Every state of a nucleus looks like a separate ‘particle’, and excitation/de-excitation processes look like decays, etc.
- Deviations from the point-like limit are encoded by higher dimensional local operators in the EFT.
- Nuclei have higher spin states for which there is no relativistic QFT description.
 - One could use heavy particle effective theory (like HQET); instead, I will use the on-shell amplitude program which bypasses a Lagrangian entirely.



On-Shell Amplitudes

One-particle states are labeled by the **little group** — the rotations that survive once the momentum is fixed: helicity for a massless leg, spin for a massive one.

Momenta are parameterized by spinors — the **only** building blocks:

$$k_{\alpha\dot{\alpha}} = \lambda_{\alpha} \tilde{\lambda}_{\dot{\alpha}} \quad (m = 0), \quad p_{\alpha\dot{\alpha}} = \lambda_{\alpha}^I \tilde{\lambda}_{\dot{\alpha} I} \quad (p^2 = m^2), \quad \langle ij \rangle, \quad [ij]$$

Amplitudes are bracket polynomials of fixed little-group weight:

$$\lambda_i \rightarrow t_i \lambda_i, \quad \tilde{\lambda}_i \rightarrow t_i^{-1} \tilde{\lambda}_i \quad \implies \quad M \rightarrow \left(\prod_i t_i^{-2h_i} \right) M$$

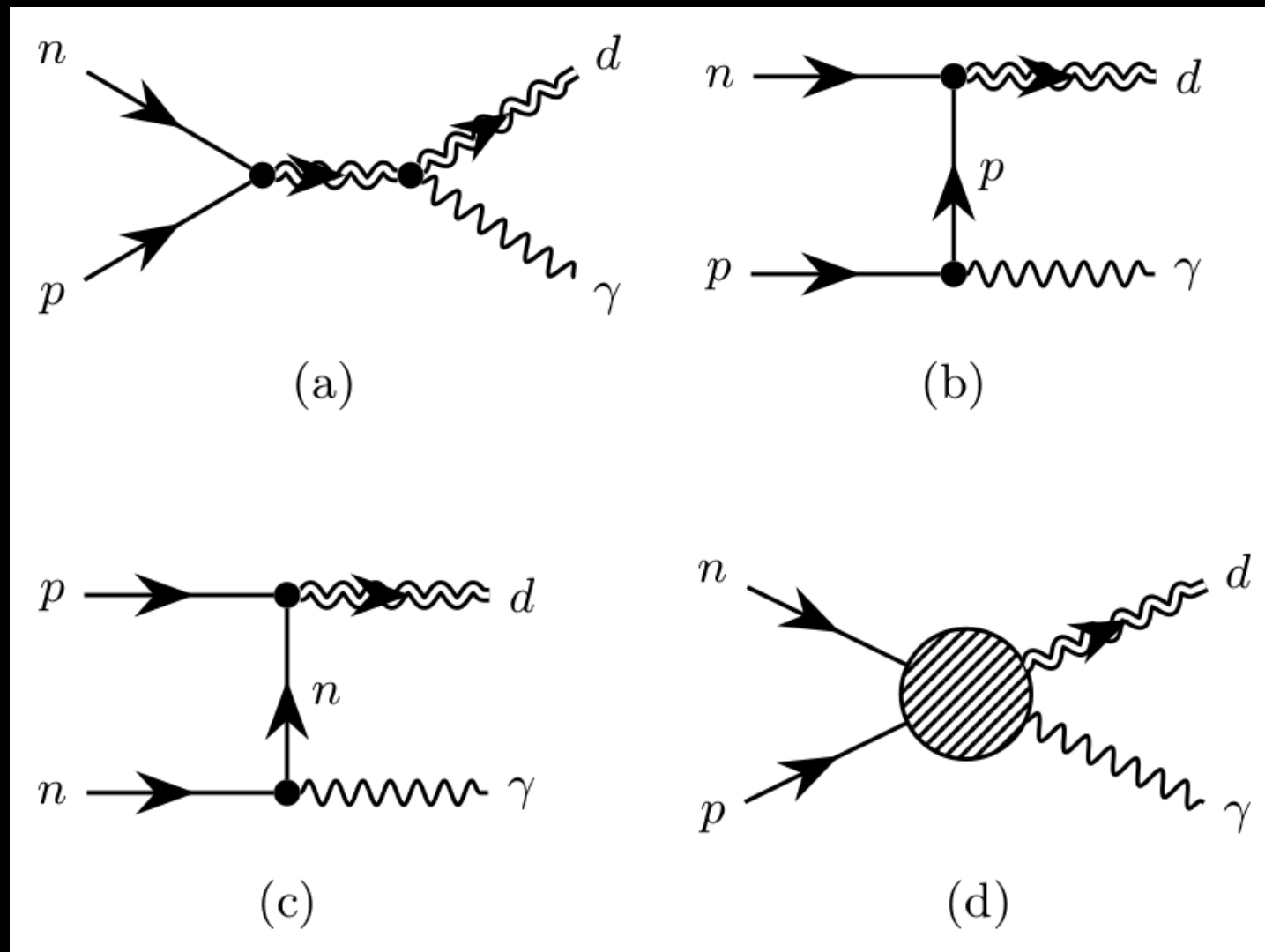
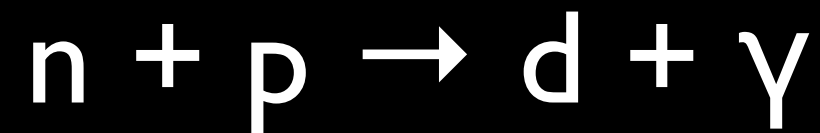
This is so restrictive that every **three-point amplitude is fixed by symmetry alone**, up to a finite number of over-all constants — the couplings.

Higher point amplitudes are assembled: 4-point amplitudes glue the 3-point vertices across every factorization pole, plus contact (“boundary”) terms:

$$M_4 = \sum_{\text{channels}} \frac{\sum_{\text{spins}} M_3 M_3 \big|_{P^2 \rightarrow m_I^2}}{P^2 - m_I^2} + M_4^{\text{boundary}}$$

For massive particles:
Arkani-Hamed, Huang, Huang,
JHEP 11 (2021) 070 [arXiv:1709.04891]

Deuterium Production



$n (p, d) \gamma$

How to speak 'nuclear':

(: +
) : +
 , : →

In the amplitude language, deuterium Production is built out of three factorized poles + local boundary (contact) terms.

Strong Vertex: $d - n - p$

Massive spinor-helicity: bold spinors $|i\rangle, |i]$ carry the little group of each massive leg. Symmetry fixes each 3-point amplitude up to constants.

Spins (1, 1/2, 1/2): twelve naive structures collapse to **two** conserving parity:

$$\mathcal{E}^{(1)} = \langle d n \rangle \langle d p \rangle + [d n][d p], \quad \mathcal{E}^{(2)} = \langle d n \rangle [d p] + [d n] \langle d p \rangle$$

$$M_3 = (\tilde{C}_1 \mathcal{E}^{(1)} + \tilde{C}_2 \mathcal{E}^{(2)}) / m_d$$

Diagonalizing the $n+p$ partial waves gives the deuteron's own S and D states:

$$\mathcal{E}^{(S)} = \mathcal{E}^{(1)} - \mathcal{E}^{(2)} = (\langle d n \rangle - [d n])(\langle d p \rangle - [d p]) \quad {}^3S_1$$

$$\mathcal{E}^{(D)} = \mathcal{E}^{(1)} + \mathcal{E}^{(2)} = (\langle d n \rangle + [d n])(\langle d p \rangle + [d p]) \quad {}^3D_1$$

$$\text{with couplings } \tilde{C}_S = 1/2(\tilde{C}_1 - \tilde{C}_2), \quad \tilde{C}_D = 1/2(\tilde{C}_1 + \tilde{C}_2)$$

The magnitude of the coupling can be related to the deuteron's 'asymptotic normalization constant' (ANC):

$$|M_3(d \leftrightarrow np)|_{\text{phys}}^2 = 8\pi \frac{m_d m_n m_p}{\mu^2} A_S^2, \quad \tilde{C}_S^{\text{ANC}} = \frac{A_S \sqrt{2\pi m_d}}{m_N} = 1.4367$$

Electromagnetic Amplitudes

A spin-1/2 nucleon radiates through just two structures per helicity:

$$M_{X, IJ}^{+1} = Z_X e x \langle \mathbf{a}^I \mathbf{b}^J \rangle + \frac{e \kappa_X}{2m_X} [\mathbf{a}^I \boldsymbol{\gamma}] [\mathbf{b}^J \boldsymbol{\gamma}],$$

$$M_{X, IJ}^{-1} = Z_X e \tilde{x} [\mathbf{a}^I \mathbf{b}^J] + \frac{e \kappa_X}{2m_X} \langle \mathbf{a}^I \boldsymbol{\gamma} \rangle \langle \mathbf{b}^J \boldsymbol{\gamma} \rangle$$

Z is the electric charge and κ is the anomalous moment.

The deuteron is a charged spin-1 field with three form factors,
fixed by $Z=1, \mu_d, Q_d$:

$$M_d^{+1} = \frac{Z_d e}{m_d} x \langle \mathbf{a} \mathbf{b} \rangle^2 + \frac{e g_1^d}{m_d^2} x^2 \langle \boldsymbol{\gamma} \mathbf{a} \rangle \langle \boldsymbol{\gamma} \mathbf{b} \rangle \langle \mathbf{a} \mathbf{b} \rangle + \frac{e g_2^d}{m_d^3} x^3 \langle \boldsymbol{\gamma} \mathbf{a} \rangle^2 \langle \boldsymbol{\gamma} \mathbf{b} \rangle^2$$

All of these are measured much more precisely than the nuclear inputs, and far far below the precision necessary for BBN.

Boundary Terms

Boundary terms encode the higher order EFT corrections. At a given dimension they can be enumerated algorithmically.

De Angelis, JHEP 08, 299,
arXiv:2202.02681.

At leading order (dimension 6), there are **five parity-even structures**:

$$\mathcal{B}_1 = \langle \mathbf{14} \rangle^2 \langle \mathbf{23} \rangle,$$

$$\mathcal{B}_2 = \langle \mathbf{12} \rangle \langle \mathbf{14} \rangle \langle \mathbf{34} \rangle,$$

$$\mathcal{B}_3 = \langle \mathbf{14} \rangle^2 [\mathbf{23}],$$

$$\mathcal{B}_4 = \langle \mathbf{14} \rangle \langle \mathbf{24} \rangle [\mathbf{13}],$$

$$\mathcal{B}_5 = \langle \mathbf{14} \rangle \langle \mathbf{34} \rangle [\mathbf{12}]$$

$$\frac{e c_a}{\Lambda^3} \mathcal{B}_a$$

legs $(l, 2, 3, 4) = (\bar{d}, n, p, \gamma)$; the c_a are dimensionless and $O(l)$ by NDA

Projecting onto two-nucleon channels maps the five c_a onto multipoles:

$$c_{M1}^V \propto 2c_1 - c_2 + 2c_3 + c_4 - c_5 \quad ({}^1S_0, T=1)$$

$$c_{E2} \propto c_2 + c_4 + c_5 \quad ({}^3S_1, T=0)$$

At dimension 7, an electric dipole contributes to the **3P_0 channel**:

$$c_{E1}^{(1)} = c_{E1}^{(2)} = 0, \quad c_{E1} = \sum_J \frac{2J+1}{9} c_{E1}^{(J)} = \frac{1}{9} c_{E1}^{(0)}$$

Cross Section

Putting this all together into the unpolarized cross section:

$$\sigma(np \rightarrow d\gamma) = \frac{\alpha(\gamma^2 + p_{\text{rel}}^2)}{4p_{\text{rel}}} \left[2 |\mathcal{A}_{M1}|^2 + \sum_{J=0}^2 |\mathcal{A}_{E1}^{(J)}|^2 \right]$$

There are two multipoles adding incoherently at tree level:

$$\mathcal{A}_{M1} = \sqrt{2m_d} E_\gamma U_{M1}, \quad \mathcal{A}_{E1}^{(J)} = \sqrt{2m_d} \frac{w_J p_{\text{rel}}}{\gamma^2 + p_{\text{rel}}^2} U_{E1}$$

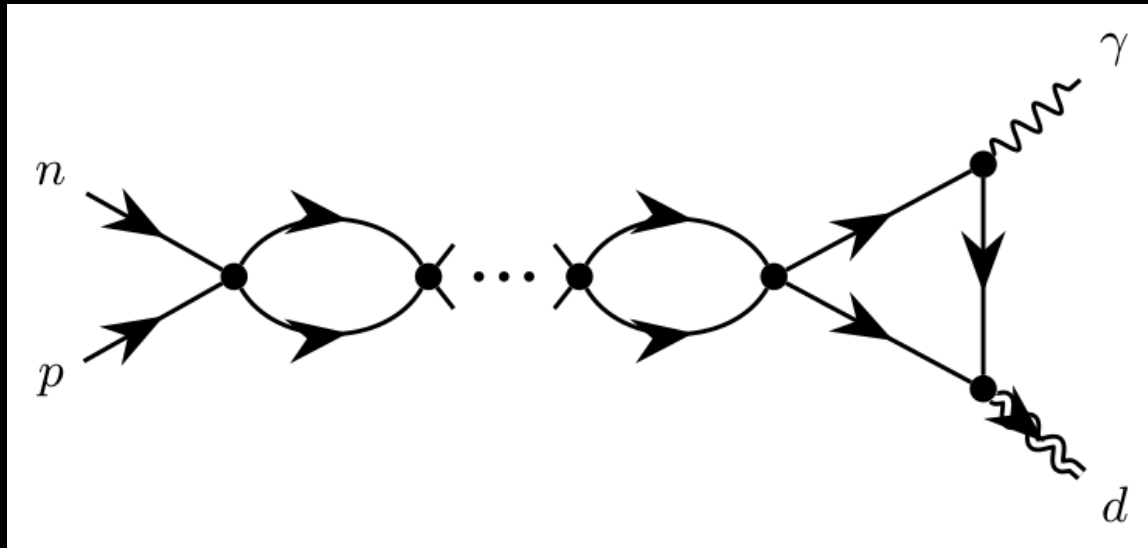
$$U_{M1} = \frac{\tilde{C}_S}{m_d} (1 + \kappa_V) P_N + \frac{c_{M1}}{\Lambda^3}, \quad U_{E1} = \frac{e_{\text{eff}}}{e} \frac{\tilde{C}_S}{m_d} + \frac{E_\gamma^2 c_{E1}}{\Lambda^3}$$

$$(1 + \kappa_V) \equiv \mu_p - \mu_n \quad P_N \equiv \frac{1}{2m_N E_\gamma} \quad e_{\text{eff}} \equiv e \frac{Z_n m_p - Z_p m_n}{m_n + m_p} \simeq -\frac{1}{2}e$$

The M1 (isovector, $\mu_p - \mu_n$) contribution dominates at threshold.

E1 (proton convection) takes over above **E $\simeq$ 0.2 MeV**.

Rescattering



$$T \equiv VR,$$

$$\begin{aligned} R(s_{np}) &= 1 + 4m_N^2 V\Pi + (4m_N^2 V\Pi)^2 + \dots \\ &= \frac{1}{1 - 4m_N^2 V(s_{np}) \Pi(s_{np})}, \end{aligned}$$

$$\begin{aligned} \Pi(s_{np}) &= \Pi(0) + \frac{s_{np}}{\pi} \int_{4m_N^2}^{\infty} \frac{\text{Im } \Pi(s')}{s' (s' - s_{np})} ds' \\ &= \frac{B_0(s_{np}; m_N, m_N)}{16\pi^2}, \end{aligned}$$

The 1S_0 pair rescatters $O(1)$ times: a virtual state sits **66 keV below threshold**. The ladder must be resummed — we do this dispersively, from the on-shell amplitudes themselves.

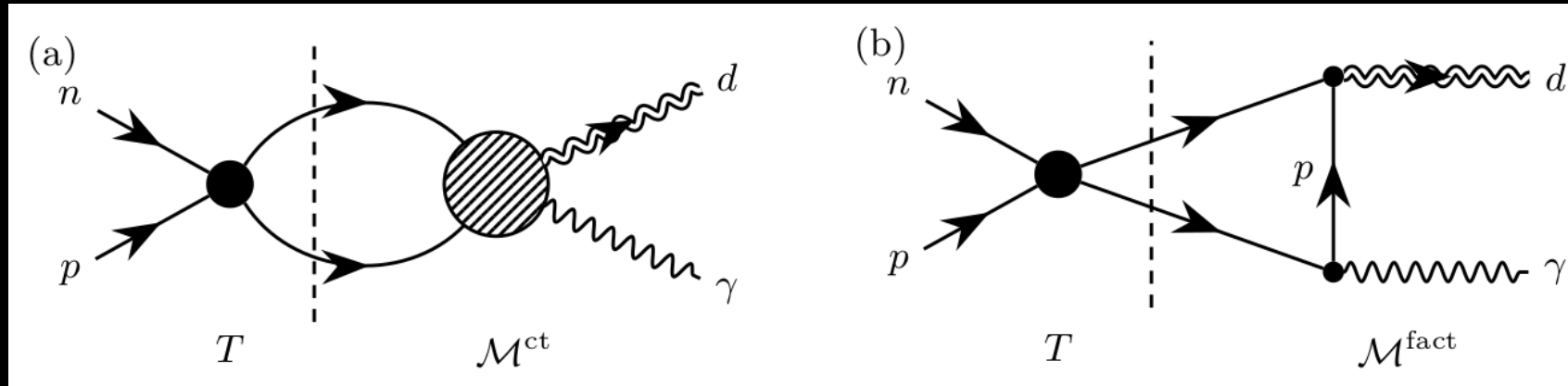
The way to think about this in the EFT language is as a contribution from four-nucleon scattering amplitudes, at loop level. One loop is a good approximation to the rescattering, but it is large enough that it must be resummed in σ .

$$V(s_{np}) = \begin{cases} C_{^1S_0}(s_{np}) & (^1S_0) \\ \frac{\tilde{C}_S^2}{s_{np} - m_d^2} + C_{^3S_1}(s_{np}) & (^3S_1) \end{cases}$$

$$T_{^1S_0} = \frac{4\pi/m_N}{-1/a_0 + \frac{1}{2}r_0 p_{\text{rel}}^2 - ip_{\text{rel}}}$$

The ‘effective range expansion’ (ERE)

Dressed Amplitude



The final re-scattering dresses the tree level amplitudes into loops:

$$\mathcal{A}_{M1} = \sqrt{\frac{2}{m_d}} \frac{1}{8\pi} T_1 S_0 \left[(1 + \kappa_V) \tilde{C}_S W(p_{\text{rel}}^2) + (\gamma^2 + p_{\text{rel}}^2) \ell_1 \right]$$

$$W(p_{\text{rel}}^2) = \gamma - \frac{1}{a_0} - \frac{(r_0 + \rho_d) \gamma^2}{4} + \frac{(r_0 - \rho_d) p_{\text{rel}}^2}{4}$$

(ρ_d is just a rewriting of the strong coupling & ℓ_1 is the dibaryon-literature form of c_{M1}).

The electric channel is also dressed, but the scattering is weak enough not to resum:

$$\mathcal{A}_{E1}^{(J)} = \sqrt{\frac{2}{m_d}} \frac{w_J p_{\text{rel}} m_d}{\gamma^2 + p_{\text{rel}}^2} \left[\frac{e_{\text{eff}}}{e} \frac{\tilde{C}_S}{m_d} (1 + \delta_{E1}^{(J)}) + \delta_{J0} \frac{9 E_\gamma^2 c_{E1}}{\Lambda^3} e^{i\delta_{3P_0}} \cos \delta_{3P_0} \right],$$

$$\delta_{E1}^{(J)} = \sin \delta_{3P_J} e^{i\delta_{3P_J}} \left(\frac{\gamma(\gamma^2 + 3p_{\text{rel}}^2)}{2p_{\text{rel}}^3} + i \right)$$

Joint Bayesian Fit

We construct a joint Bayesian fit that allows the EFT inputs to float. The fit includes measurements at threshold, measurements of the crossed-channel photo-disintegration process at the Shanghai Laser Electron Gamma Source (SLEGS), and measured ERE parameters.

Y. Chen et al., PRL136, 052701 (2026), [arXiv:2509.11743].

$$\begin{aligned}
 -2 \ln \mathcal{P}(\theta) = & \sum_{k=1}^2 \frac{[\sigma(E_{\text{th}}; \theta) - \sigma_{\text{th}}^{(k)}]^2}{(\delta\sigma_{\text{th}}^{(k)})^2} + \sum_{i=1}^{22} \frac{[\sigma_i^{\text{fold}}(\theta) - \sigma_i^{\text{f}}]^2}{(\delta\sigma_i^{\text{f}})^2} \\
 & + \left(\frac{\lambda_S - 1}{\delta_S} \right)^2 + \left(\frac{\tilde{C}_S - \tilde{C}_S^{\text{ANC}}}{\sigma_{\tilde{C}_S}} \right)^2 + \left(\frac{a_0 - \bar{a}_0}{\sigma_{a_0}} \right)^2 + \left(\frac{r_0 - \bar{r}_0}{\sigma_{r_0}} \right)^2
 \end{aligned}$$

The SLEGS data, published in 2026, publishes cross section measurements folded over their (measured) beam energy spectrum $D(E)$:

$$\sigma_i^{\text{fold}}(\theta) = \lambda_S \int_{S_n}^{E_{\text{max}}^{(i)}} D_i(E_\gamma) f(E_\gamma) \sigma(E_{\text{cm}}(E_\gamma); \theta) dE_\gamma, \quad f = \frac{2}{3} \frac{p_{\text{rel}}^2}{k^2}$$

λ_S is the over-all scale, assigned the 3.6% systematic reported by SLEGS and treated as a nuisance parameter.

Fit Results

The nuclear inputs come out very consistent with the input measurements.

The magnetic contact term comes out with posterior:

$$c_{M1} = +0.162 \pm 0.015$$

This represents the well-known 10% impulse deficit usually ascribed to meson-exchange currents.

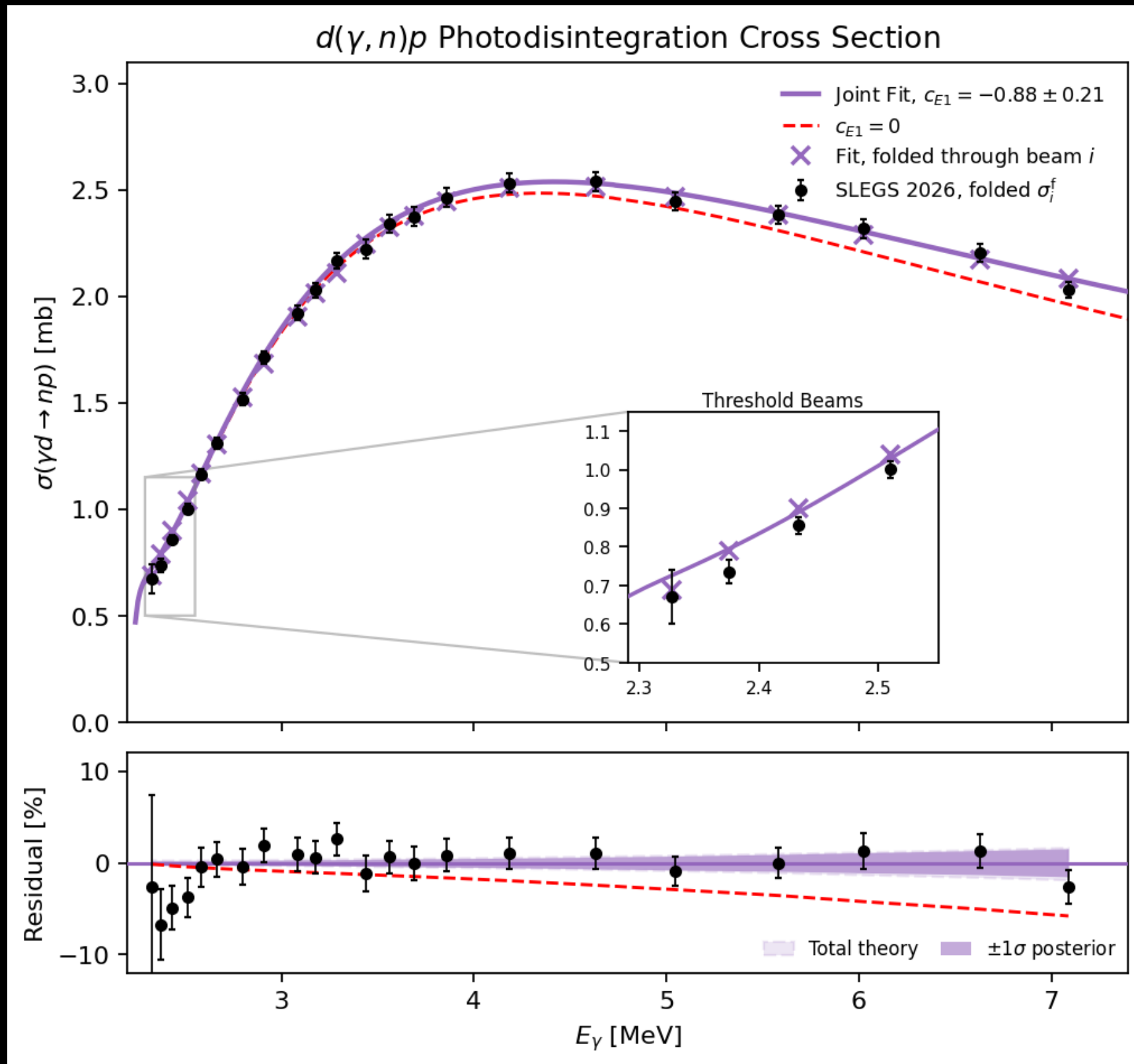
Parameter	Prior	Posterior
$\tilde{C}_S$	1.4367 ± 0.0015 [48]	1.4369 ± 0.0015
a_0 [fm]	-23.715 ± 0.020	-23.717 ± 0.020
r_0 [fm]	$+2.706 \pm 0.060$	$+2.712 \pm 0.060$
c_{M1}	Flat	$+0.162 \pm 0.015$
c_{E1}	Flat	$-0.88 \pm 0.21^{+0.16}_{-0.08} \pm 0.03$
λ_S	1.000 ± 0.036 [13]	0.981 ± 0.006
ρ_d [fm]	Derived	$+1.763 \pm 0.005$
ℓ_1 [fm]	Derived	$+1.19 \pm 0.11$
L_{E1} [fm ³]	Derived	$+14.9 \pm 3.5^{+1.3}_{-2.6} \pm 0.5$
g_A [10 ⁴]	Derived	$+3.074 \pm 0.067$
g_B [10 ⁴]	Derived	-3.070 ± 0.067
$g_A + g_B$	Derived	$+42.22 \pm 0.04$

The constraint on the E1 contact term is driven by the modern SLEGS data, and shifts by 3.5σ from the pre-2000 extraction.

$$c_{E1} = -0.88 \pm 0.21^{+0.16}_{-0.08} \pm 0.03, \quad L_{E1} = +14.9 \pm 3.5^{+1.3}_{-2.6} \pm 0.5 \text{ fm}^3$$

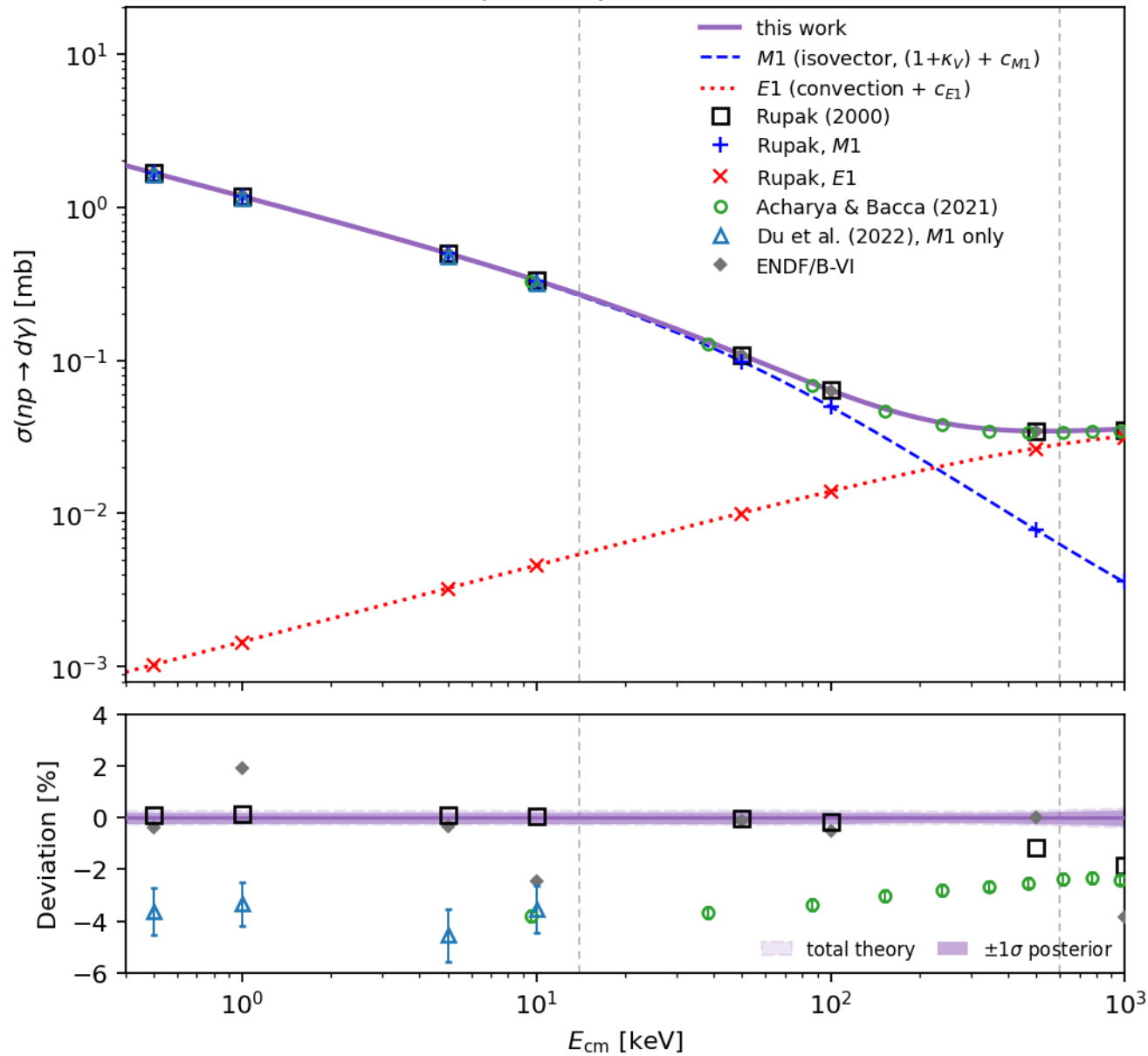
Both contact terms are fit to be $O(1)$, consistent with the expectations from naive dimensional analysis power-counting.

Confronting SLEGS

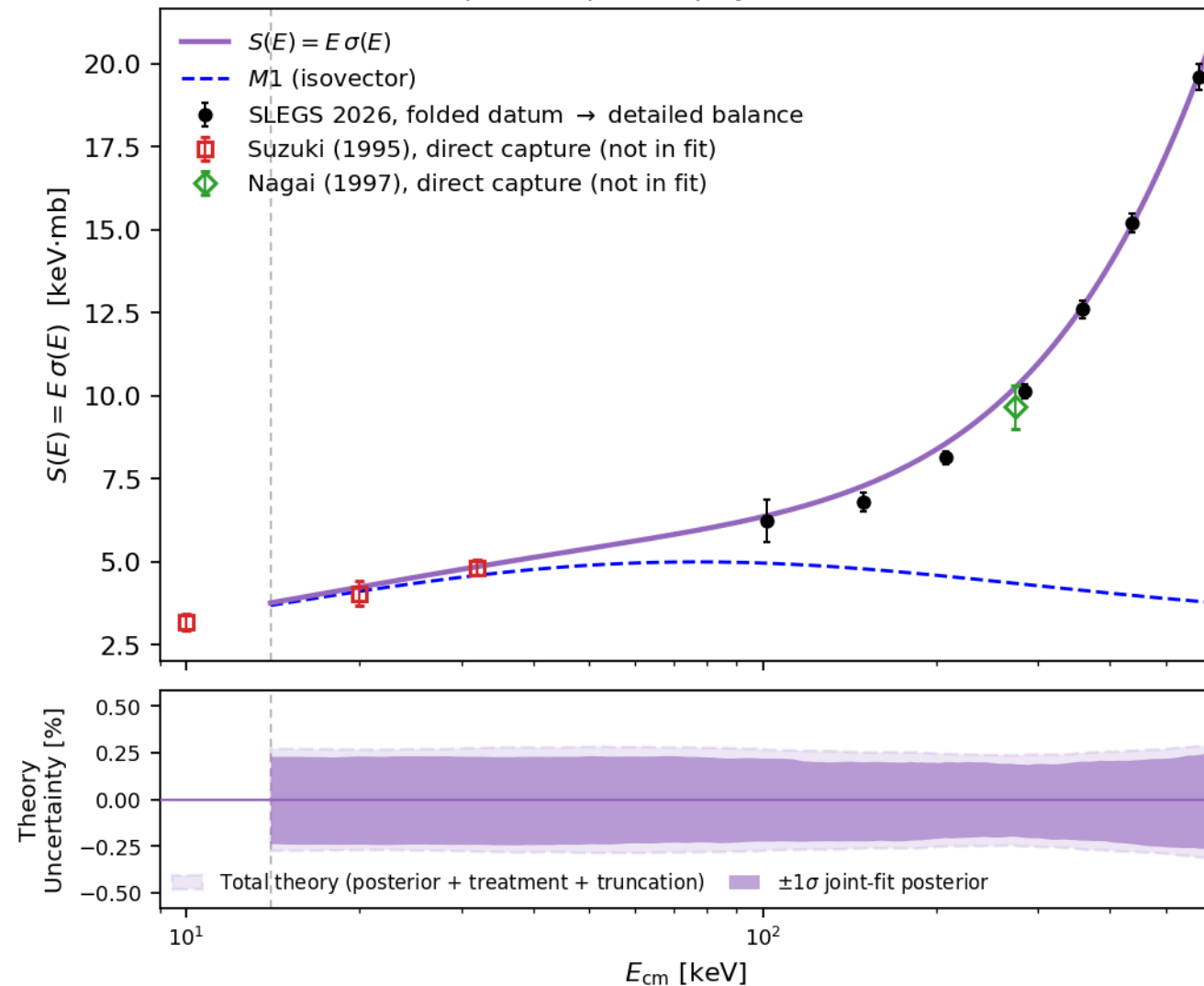


$\sigma(E)$ & S-Factor

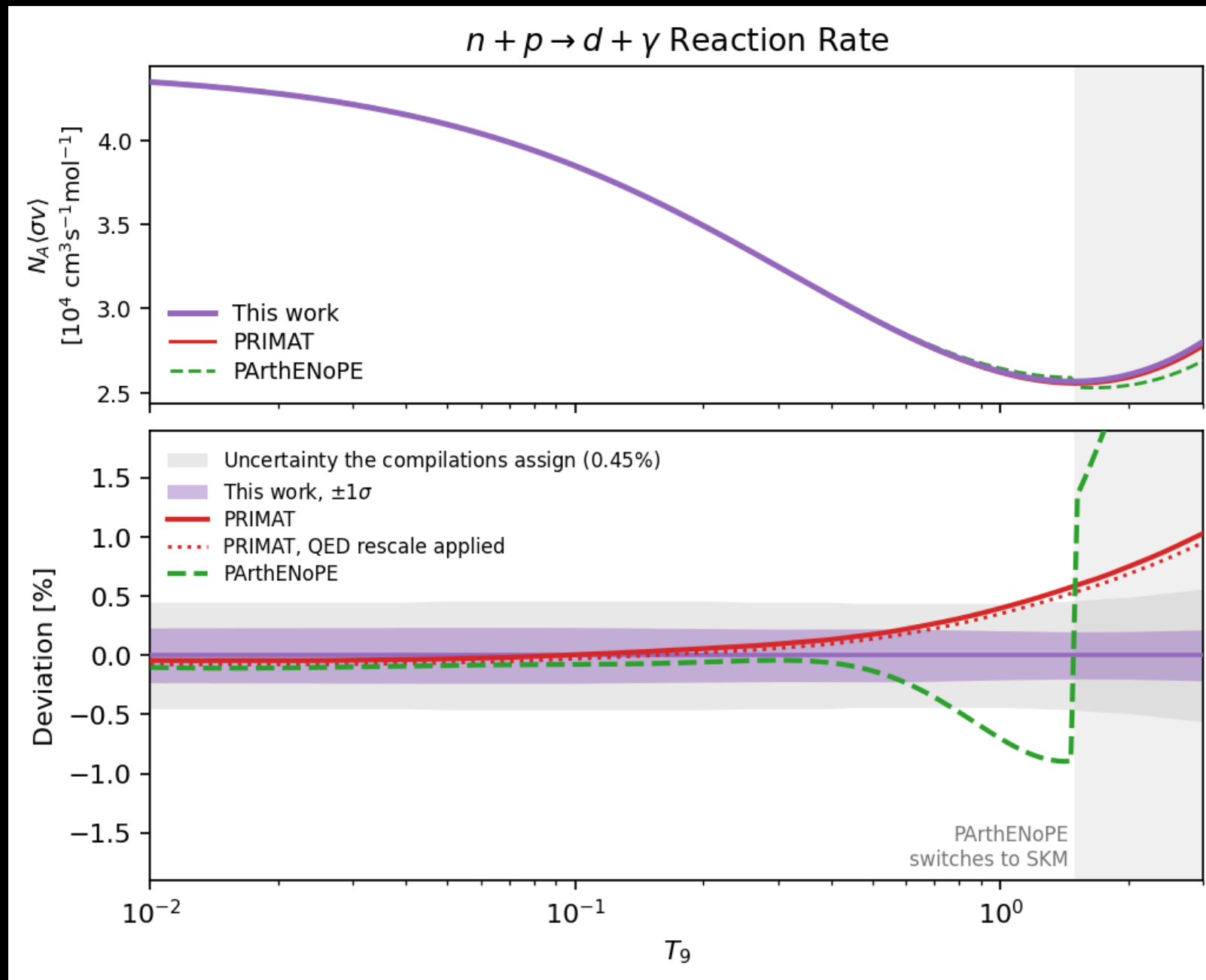
$n + p \rightarrow d + \gamma$ Cross Section



$n + p \rightarrow d + \gamma$ Astrophysical S-Factor



Thermonuclear Rate



Total theory uncertainty:

0.25–0.27%

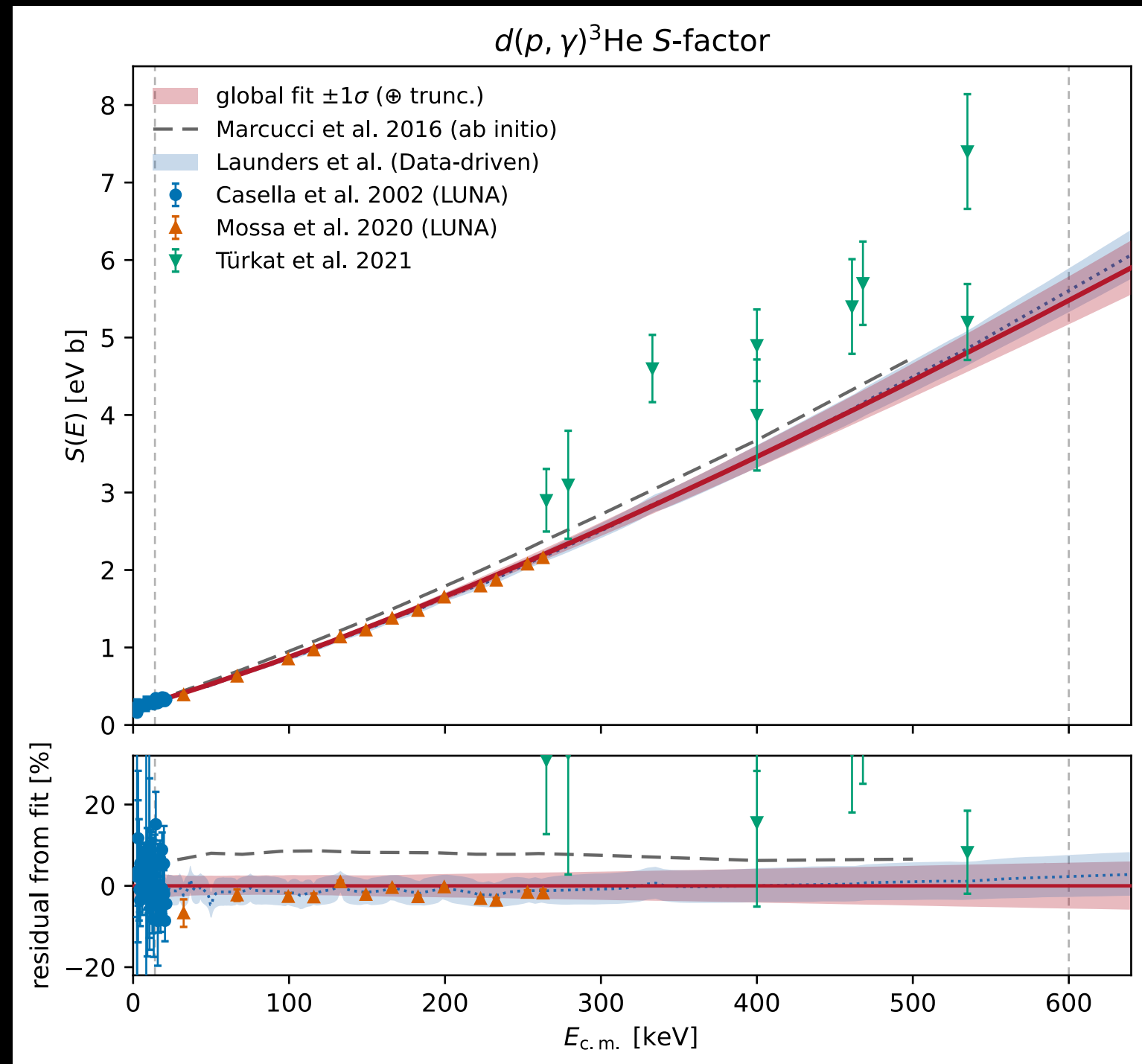
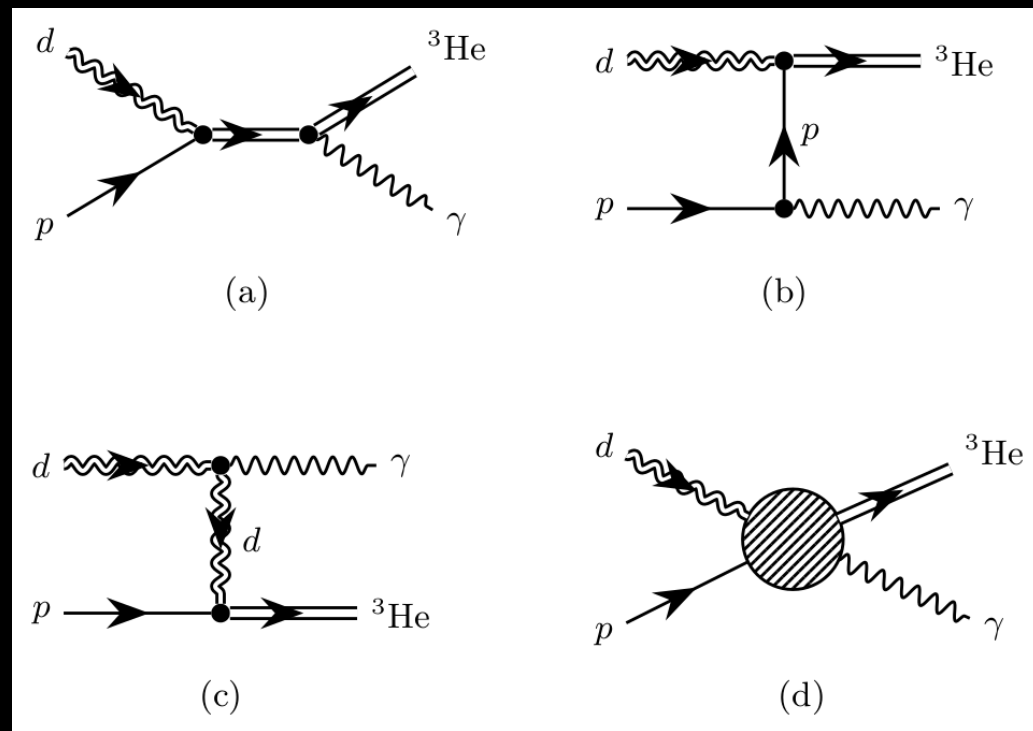
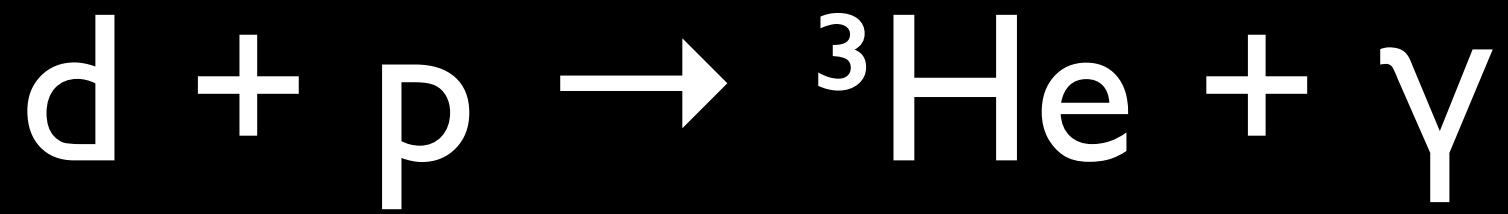
across $T_9 = 0.01$ – 3 , with no high-temperature switch.

Propagated through P_RyMordial with the full uncertainty:

$$\Delta(\text{D}/\text{H}) = -0.06\%$$

$$\delta(\text{D}/\text{H}): 0.089\% \rightarrow 0.050\%$$

The production rate is essentially **retired** from the primordial D/H error budget.

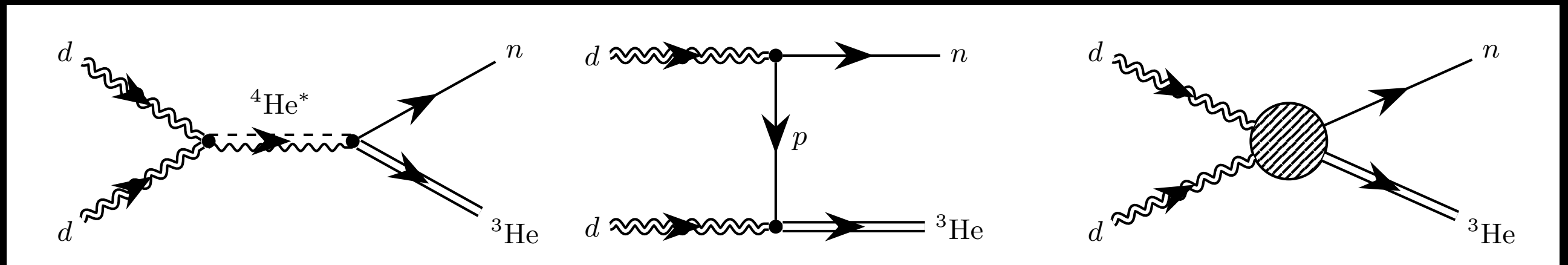


$$S(0) = 0.209 \pm 0.008 \text{ keV b}, \quad t_{E1} = -0.146 \pm 0.021, \quad \chi^2/\text{dof} = 0.83$$

Fit to the LUNA + World Data

Outlook

I am currently working on the two d+d burning reactions:
 $d+d \rightarrow n+{}^3\text{He}$ and $d+d \rightarrow p+t$



$d+d \rightarrow n+{}^3\text{He}$ and $d+d \rightarrow p+t$ reuse the **same d–n–p vertex** from both capture processes — correlations across all four reactions are built in from the start.

They also include exchanges of the tower of He-4 states.

A simultaneous amplitude-level fit of the full deuterium network is the goal.

Outlook

- An EFT built from on-shell amplitudes of the nuclear states now covers both radiative captures necessary to describe the production of primordial deuterium during BBN, at few-per-mille precision.
- $n+p \rightarrow d\gamma$: rate to 0.25–0.27%, D/H error contribution halved — production retired from the uncertainty budget.
- We also make predictions for some polarization observables, which are not important for BBN, but allow us to further test our amplitude/EFT machinery.

Coming up:

$d+d$ burning — plus a global fit of the whole deuterium network!

Thank you!

The BBN Era

At the qualitative level, BBN is fairly easy to understand.

For $T \gtrsim 10$ MeV, the SM contains relativistic species ($\gamma, e^\pm, \nu$):

$$H^2 \simeq \frac{1}{3M_{Pl}^2} \frac{\pi^2}{30} \left(2 + 2 \times 2 \times \frac{7}{8} + 2 \times \frac{7}{8} N_\nu \right) T_\gamma^4$$

(plus non-relativistic: nucleons with abundance set by the baryon asymmetry, and dark matter)

From the 2nd law of thermodynamics and energy-momentum conservation:

$$d(sa^3)/dt = 0 \quad , \quad s = (\rho + P)/T_\gamma$$

One can derive the temperature of the photons as a function of time/scale factor:

$$T_\gamma(t) \quad , \quad T_\gamma(a)$$

The BBN Era

As the Universe expands and cools, eventually the weak interactions freeze out:

$$H \sim \Gamma_{weak} \sim n \langle \sigma v \rangle \sim T^3 \times G_F^2 T^2 \Rightarrow T_{f.o.}^{(weak)} \sim \text{MeV}$$

Based on entropy conservation, for $T_\gamma \leq m_e$:

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \Leftrightarrow N_{eff} \equiv \frac{8}{7} \left(\frac{11}{4}\right)^{4/3} \left(\frac{\rho_{tot.} - \rho_\gamma}{\rho_\gamma}\right) = 3.044$$

SM prediction

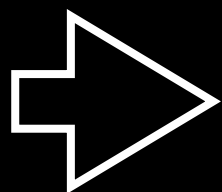
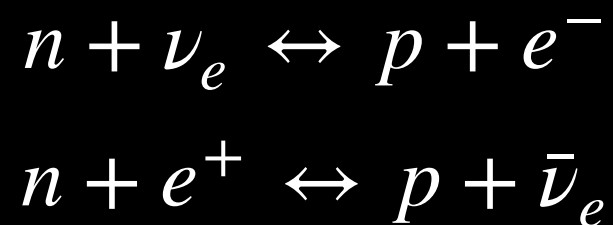
e.g., arXiv:2210.10307

Thus, one can relate the temperature of the neutrinos to the temperature of the photons:

$$T_\gamma(t) , T_\gamma(a) , \boxed{T_\nu(T_\gamma)}$$

The BBN Era

$T_\nu(T_\gamma)$ is relevant for the β -equilibrium, $n \leftrightarrow p$:



$$(n_n/n_p)|_{T \gtrsim \text{MeV}} \simeq \exp(-Q/T)$$

$$Q \equiv m_n - m_p \simeq 1.3 \text{ MeV}$$

$$(n_n/n_p)|_{T \simeq \text{MeV}} \simeq 1/6$$

Nucleosynthesis naively starts at $T_{nucl.} \sim B_D \simeq 2.2 \text{ MeV} \dots$ BUT:

$$\Gamma(n + p \rightarrow D + \gamma) \sim n_B \langle \sigma v \rangle_{D\gamma}$$

$$\Gamma(n + p \leftarrow D + \gamma) \sim n_\gamma \exp(-B_D/T_\gamma) \langle \sigma v \rangle_{D\gamma}$$

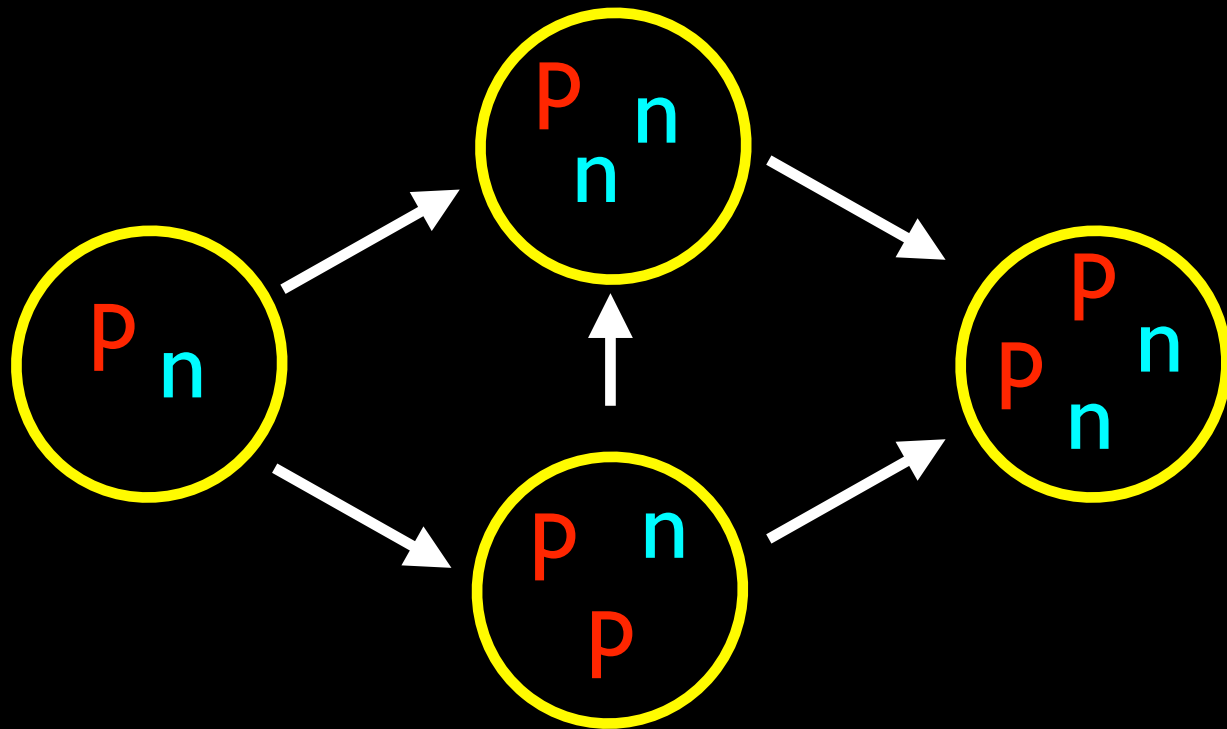
It really starts at $T_{nucl.}$ such that: $\eta_B \simeq \exp(-B_D/T_{nucl.})$

The BBN Era

The “deuterium bottleneck” implies $T_{nucl} \simeq 0.1$ MeV. After that :

Essentially all of the neutrons are bound into helium-4...

$$(n_n/n_p)|_{T \simeq 0.1 \text{ MeV}} \simeq 1/7$$



$$Y_P \equiv \frac{m_{4He}}{m_B} \simeq \frac{4(n_n/2)}{n_n + n_p} \simeq 0.25$$

Baryon mass fraction in helium-4

$\mathcal{O}(10^{-5})$ residual amount of deuterium and helium-3 relative to p .

Lithium-7 “survives” in a much smaller relative abundance, $\mathcal{O}(10^{-10})$.

Precision BBN

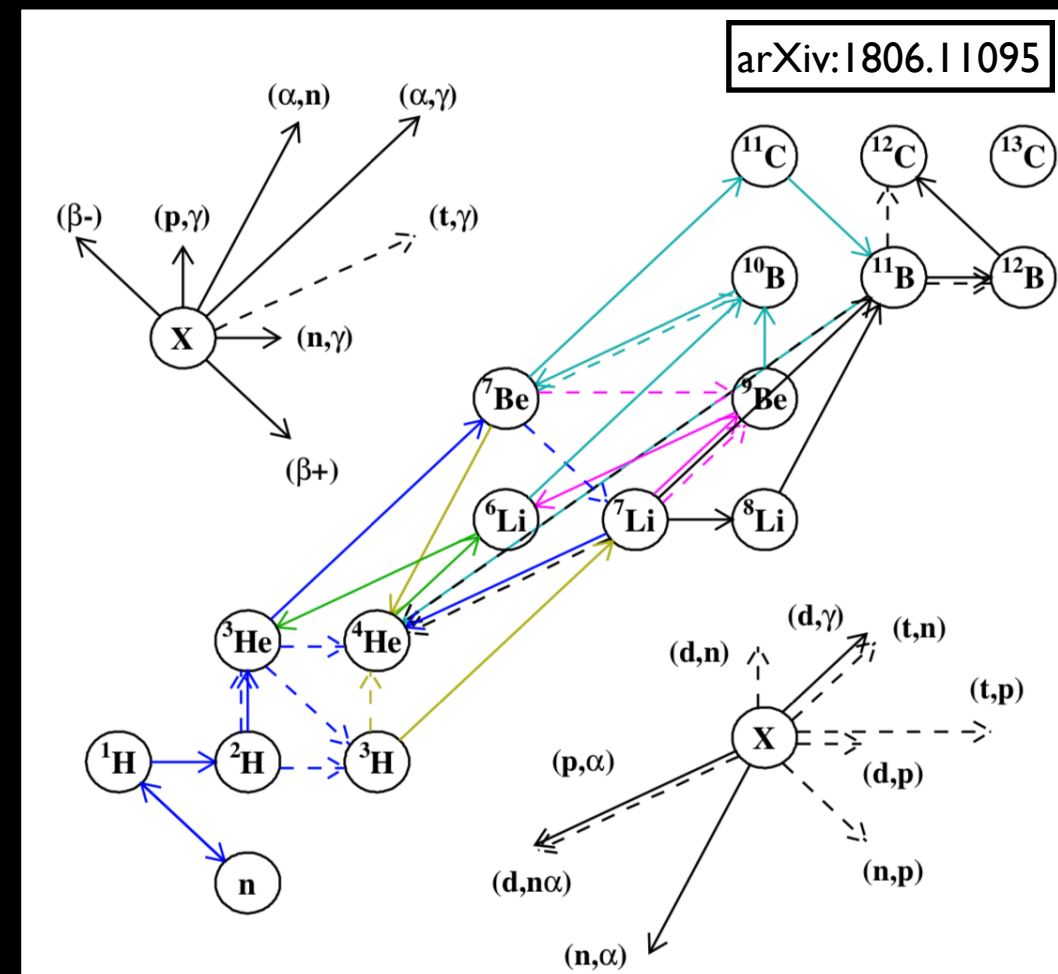
- A significant shift in the primordial abundance of ^4He is both interesting and alarming.
- The new result is about 2.5σ from the old one, which resulted in a very consistent picture with theoretical predictions from BBN in the SM.
- How significant are the differences, really? What role do theory uncertainties play?
- What role do other measurements such as deuterium play? Are there tensions within BBN?
- Is this a hint for BSM physics? Are there surprises about cosmology to be gleaned from understanding BBN at the % level?
- The answers to these questions require us to go beyond the qualitative picture of BBN to more precise modeling of the web of nuclear reactions, with a framework that allows us to more robustly explore uncertainties...

PDG 2021:

$$Y_P = 0.245 \pm 0.003$$

Subaru 2022:

$$Y_P = 0.2370^{+0.0034}_{-0.0033}$$

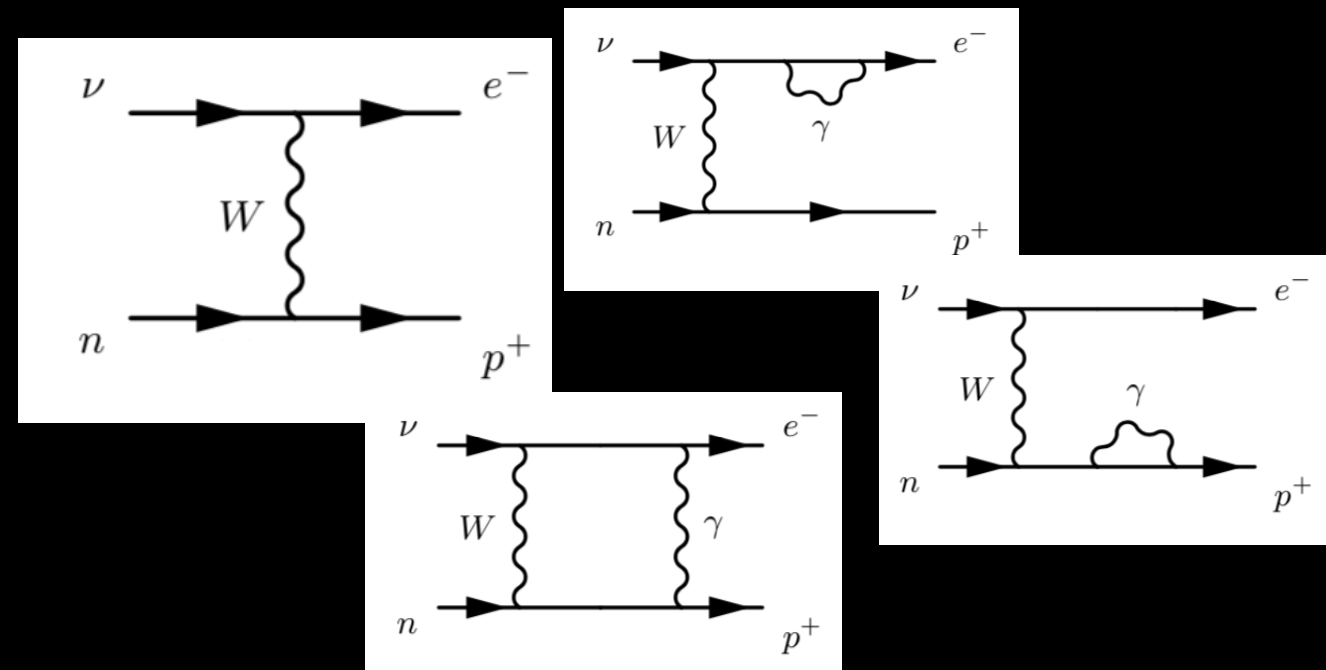


$$\dot{Y}_i \sim n_B \left(\langle \sigma v \rangle_{kl \rightarrow ij} Y_k Y_l - \langle \sigma v \rangle_{ij \rightarrow kl} Y_i Y_j \right)$$

PRyMordial

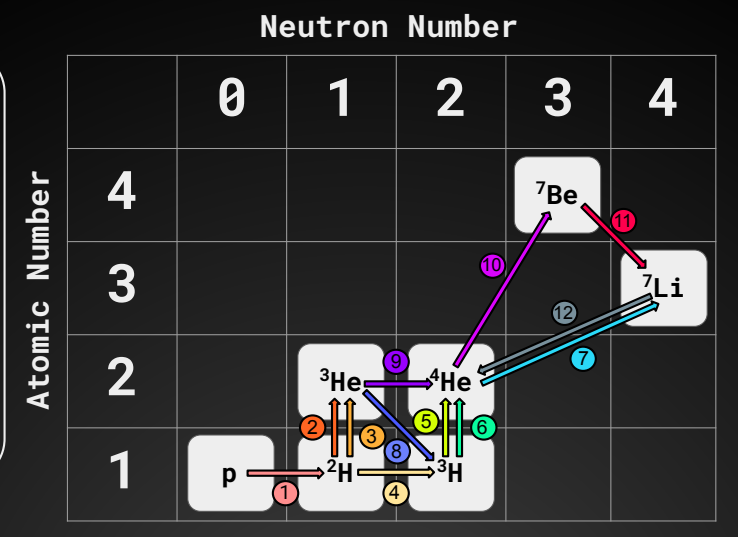
Burns, TMPT, Valli,
arXiv: 2307.07061

- To answer these questions, we have developed a new tool (**PRyMordial**) to investigate BBN both within the SM and beyond.
- PRyMordial is written in python, and runs very quickly using DiffEq.jl from the Sci Machine Learning kit implemented in Julia.
- PRyMordial fully models neutrino decoupling and computes $N_{\text{eff.}}$, allowing for precise constraints on dark sectors.
- PRyMordial allows for precision treatment of electroweak rates, including mass effects and $O(\alpha)$ corrections.
- PRyMordial implements network of $O(10/50)$ nuclear reactions in a modular format, making it easy to compare different treatments of T-dependence.



Essential Nuclear Reactions

1. $n+p \rightarrow {}^2\text{H}+\gamma$
2. ${}^2\text{H}+p \rightarrow {}^3\text{He}+\gamma$
3. ${}^2\text{H}+{}^2\text{H} \rightarrow {}^3\text{He}+n$
4. ${}^2\text{H}+{}^2\text{H} \rightarrow {}^3\text{H}+p$
5. ${}^3\text{H}+p \rightarrow {}^4\text{He}+\gamma$
6. ${}^3\text{H}+{}^2\text{H} \rightarrow {}^4\text{He}+n$
7. ${}^3\text{H}+{}^4\text{He} \rightarrow {}^7\text{Li}+\gamma$
8. ${}^3\text{He}+n \rightarrow {}^3\text{H}+p$
9. ${}^3\text{He}+{}^2\text{H} \rightarrow {}^4\text{He}+p$
10. ${}^3\text{He}+{}^4\text{He} \rightarrow {}^7\text{Be}+\gamma$
11. ${}^7\text{Be}+n \rightarrow {}^7\text{Li}+p$
12. ${}^7\text{Li}+p \rightarrow {}^4\text{He}+{}^4\text{He}$



PRyMordial



wikiHow: PRyMordial

PRyM_init.py

PRyM_thermo.py

PRyM_nTOp.py

PRyM_eval_nTOp.py

PRyM_nuclear_net.py

PRyM_main.py

#7

#0

#1

#2 : bckg + N_{eff}

#3 : weak rates

#5 : interpolation

#6 : ODE systems

#4

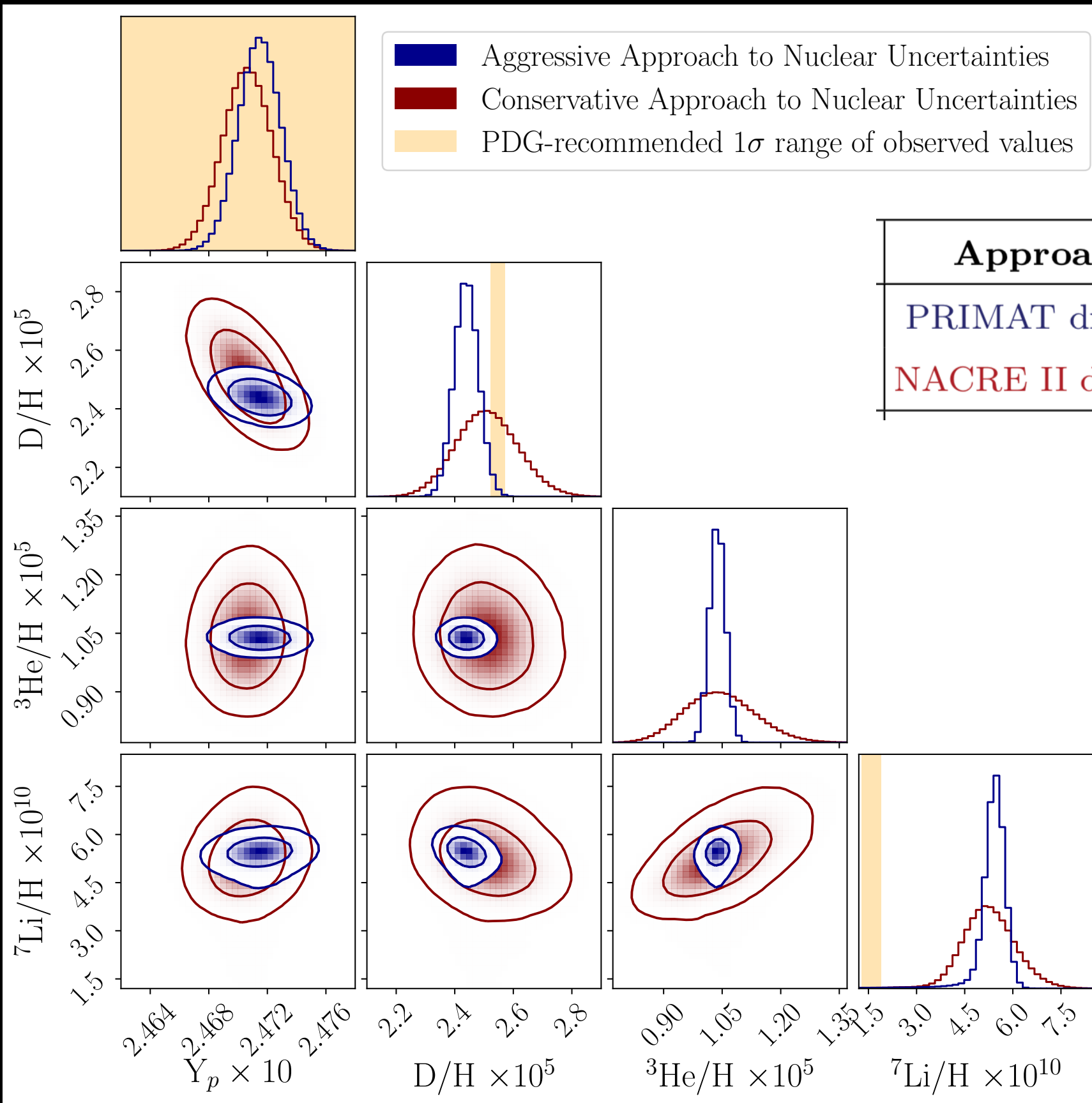
runPRyM_XX_example.py

```
#####  
PRyMordial results  
#####  
Neff --> 3.0452714140682438  
Yp --> 0.24709402881702736  
D/H x 10^5 --> 2.438952842054179  
(He3+T)/H x 10^5 --> 1.03826086327731  
(Li7+Be7)/H x 10^10 --> 5.545267496680167
```



```
--- running time: 16.100466012954712 seconds ---  
mbp-di-mauro:PRyMordial_vBeta mauro_87$
```

PRyMordial: SM BBN



Approach	$Y_P \times 10$	$D/H \times 10^5$
PRIMAT driven	2.4715(14)	2.439(36)
NACRE II driven	2.4706(16)	2.51(10)

0.5% 1.5% vs 4%

Theoretical
Uncertainty Estimates

Launders et al.: three rates dominate D/H

Reaction	D/H	Reaction	D/H
$p(n, \gamma)d$	-0.0884	$t(\alpha, \gamma)^7\text{Li}$	-0.0007
$d(p, \gamma)^3\text{He}$	-0.7700	$^3\text{He}(n, p)t$	0.0367
$d(d, n)^3\text{He}$	-0.5952	$^3\text{He}(d, p)^4\text{He}$	-0.0389
$d(d, p)t$	-0.5000	$^3\text{He}(\alpha, \gamma)^7\text{Be}$	0.0002
$t(p, \gamma)^4\text{He}$	-0.0129	$^7\text{Be}(n, p)^7\text{Li}$	0.0003
$t(d, n)^4\text{He}$	-0.0005	$^7\text{Li}(p, \alpha)^4\text{He}$	-0.0158

+1 σ rate shift

change in D/H

d(p, γ) -0.77%

d(d,n) -0.60%

d(d,p)t -0.50%

BBN energies expose a clean EFT hierarchy

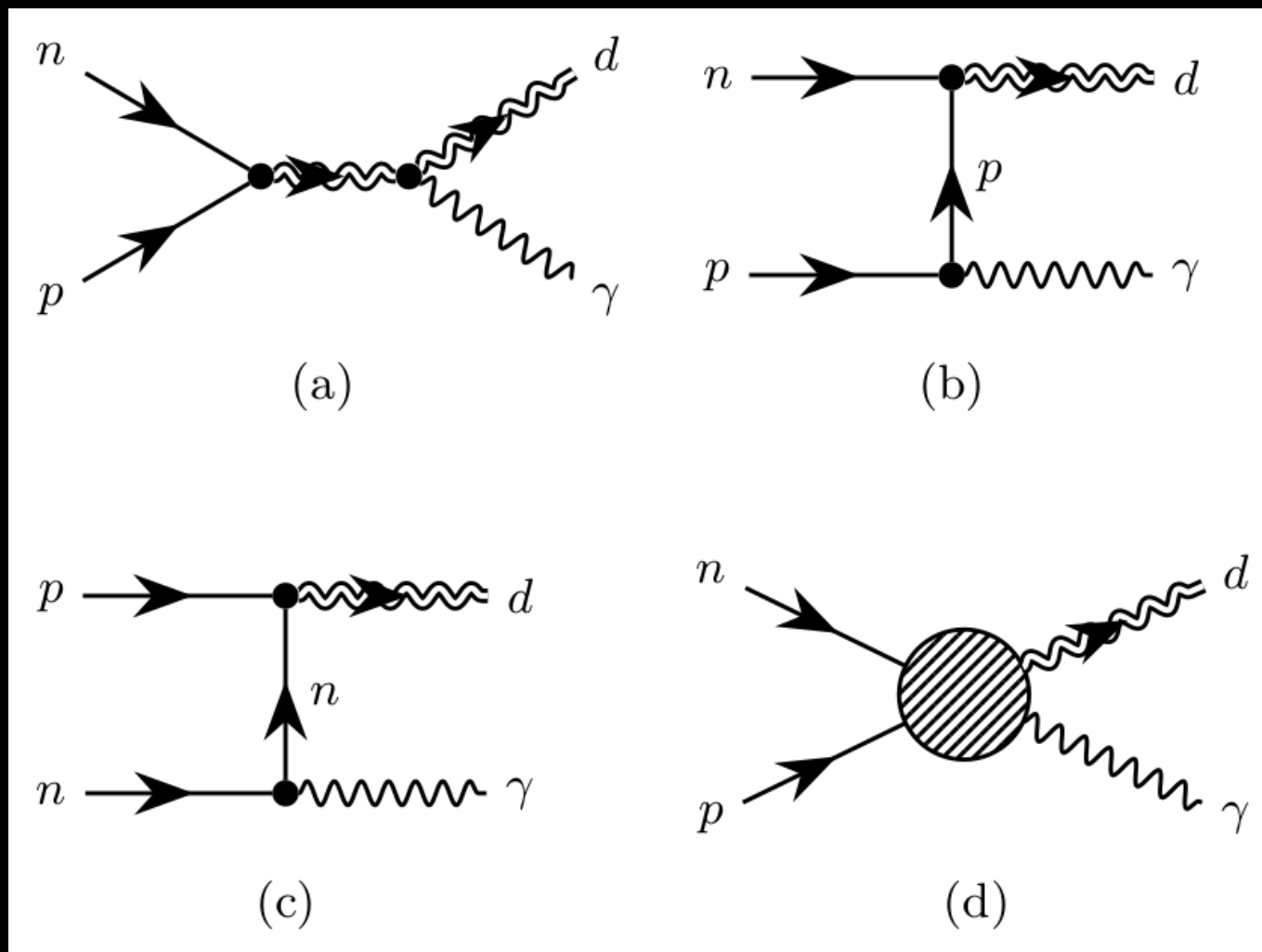


Degrees of freedom

n p d γ

**Pions are unresolved; finite nuclear size
appears as an expansion in local contact terms.**

Gluing: the Factorized Amplitude



One pole per radiating leg; each residue is a product of on-shell 3-point amplitudes.

η is an arbitrary reference spinor — the QED **gauge freedom**, made visible.

(d) collects the local boundary terms.

$$x_p = \frac{\langle \eta | p | \gamma \rangle}{m_p \langle \eta \gamma \rangle}, \quad x_d = \frac{\langle \eta | \bar{d} | \gamma \rangle}{m_d \langle \eta \gamma \rangle}, \quad \langle \eta | i | \gamma \rangle \equiv \langle \eta i \rangle [i \gamma]$$

Selection Rules Pick Two Constants

Projecting onto two-nucleon channels maps the five c_a one-to-one onto multipoles:

$$\begin{aligned} c_{M1}^V &\propto 2c_1 - c_2 + 2c_3 + c_4 - c_5 & ({}^1S_0, T=1) \\ c_{E2} &\propto c_2 + c_4 + c_5 & ({}^3S_1, T=0) \end{aligned}$$

and a Jacob–Wick projection shows the $\Delta W=7$ electric dipole reaches **one entrance channel only**:

$$c_{E1}^{(1)} = c_{E1}^{(2)} = 0, \quad c_{E1} = \sum_J \frac{2J+1}{9} c_{E1}^{(J)} = \frac{1}{9} c_{E1}^{(0)}$$

The unpolarized fit measures the 3P_0 constant itself — a prediction polarization can test.

Gauge Invariance is a Derivation

Shifting η along the photon momentum probes the Ward identity:

$$k_\mu M_{\text{fact}}^\mu = e \sum_{i=\{p,\bar{d}\}} Q_i \left[V_{\text{strong}}^{(i)} - V_{\text{strong}}^{(0)} \right] \xrightarrow{+ \mathcal{B}_W} k_\mu \mathcal{M}^\mu = 0$$

Charge conservation kills the unshifted piece; the momentum-dependence of the strong vertex does not cancel — it **derives** a unique seagull B_{W} with no free parameter.

Verified numerically component-by-component: η -independence at the 10^{-11} level, all twelve massive-leg spin states, both photon helicities.

Gauge invariance fixes the local completion

Gauge variation of the pole terms

$$k_\mu M_{\text{fact}}^\mu = e \sum_{i=\{p,\bar{d}\}} Q_i \left[V_{\text{strong}}^{(i)} - V_{\text{strong}}^{(0)} \right], \quad (11)$$

Ward-completing boundary term

$$\mathcal{B}_W^{+1} = -\frac{eZ_d\tilde{C}_1}{2m_d^2\langle\eta\gamma\rangle} \text{sym}_{(I_1I_2)} \left\{ [d^{I_2}\gamma] (\langle nd^{I_1}\rangle\langle\eta p\rangle + \langle n\eta\rangle\langle d^{I_1}p\rangle) - \langle d^{I_1}\eta\rangle ([nd^{I_2}][\gamma p] + [n\gamma][d^{I_2}p]) \right\}, \quad (12)$$

The boundary term removes the reference-spinor dependence and enforces the Ward identity.

The Ward identity fixes the local completion.

Fit Inputs

Quantity	Source	Value
Data		
σ_{th} (2200 m/s)	Thermal capture [59]	334.2 ± 0.5 mb
σ_{th} (2200 m/s)	Thermal capture [60]	332.6 ± 0.7 mb
$\sigma^{\text{f}}(\gamma d \rightarrow np)$	SLEGS 22 points [13]	2.33–7.09 MeV
λ_S	SLEGS normalization [13]	1.000 ± 0.036
Theory		
$\tilde{C}_S$	ANC A_S [48]	1.4367 ± 0.0015
a_0 [fm]	np 1S_0 [54]	-23.715 ± 0.020
r_0 [fm]	PWA93 [54]	$+2.706 \pm 0.060$
Fixed		
$1 + \kappa_V$	[61]	4.706
e_{eff}/e	[61]	−0.500
Λ	Eq. (14)	162.0 MeV
$a_1^{(J)}$ [fm ³]	NijmII [55]	−2.468, 1.529, −0.2844
Fit		
c_{M1}	Flat prior	Isovector $M1$
c_{E1}	Flat prior	$E1$