

Quantum (non)equivalence of dual massive p -form gauge theories

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Electromagnetism is a gauge theory

Electromagnetism is described by a U(1) Abelian gauge symmetry.

Vector gauge field: A_μ **Gauge transformation:** $A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \theta$

Bulk action: $S = -\frac{1}{4} \int d^4x \sqrt{-g} F_{\mu\nu} F^{\mu\nu}$ **Field strength tensor:** $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$

Equations of motion: $\nabla_\mu F^{\mu\nu} = 0$ **Bianchi identity:** $\nabla_\mu F_{\nu\rho} + \nabla_\nu F_{\rho\mu} + \nabla_\rho F_{\mu\nu} = 0$

$$\begin{aligned}\vec{\nabla} \cdot \vec{E} &= 0 \\ \vec{\nabla} \times \vec{B} &= 0\end{aligned}$$

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Dual theory of electromagnetism

The classical theory in vacuum is unchanged under the exchange of electric and magnetic fields.

Duality transformation: $F_{\mu\nu} \rightarrow \tilde{F}_{\mu\nu} = -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$

Equations of motion: $\nabla_\mu F^{\mu\nu} = 0 \rightarrow \nabla_\mu \tilde{F}_{\nu\rho} + \nabla_\nu \tilde{F}_{\rho\mu} + \nabla_\rho \tilde{F}_{\mu\nu} = 0$

$$\begin{aligned}\vec{\nabla} \cdot \vec{B} &= 0 \\ \vec{\nabla} \times \vec{E} &= 0\end{aligned}$$

Bianchi identity: $\nabla_\mu F_{\nu\rho} + \nabla_\nu F_{\rho\mu} + \nabla_\rho F_{\mu\nu} = 0 \rightarrow \nabla_\mu \tilde{F}^{\mu\nu} = 0$

$$\begin{aligned}\vec{\nabla} \times \vec{B} &= 0 \\ \vec{\nabla} \cdot \vec{E} &= 0\end{aligned}$$

Under a duality transformation, Equations of motion $\leftrightarrow$ Bianchi identity

Gauge theory of massless p -forms

Vector gauge theory is an example of a 1-form gauge theory.

We generalise to arbitrary p -form gauge theories in d dimensions.

	Differential form	Tensor notation
p -form	A_p	$A_{\mu_1 \dots \mu_p}$
Gauge transformation	$A_p \rightarrow A'_p = A_p + d\theta_{p-1}$	$A_{\mu_1 \dots \mu_p} \rightarrow A'_{\mu_1 \dots \mu_p} = A_{\mu_1 \dots \mu_p} + \partial_{[\mu_1} \theta_{\mu_2 \dots \mu_p]}$
$(p + 1)$ -form field strength	$F \equiv dA_p$	$F_{\mu_0 \mu_1 \dots \mu_p} = p \partial_{[\mu_0} A_{\mu_1 \dots \mu_p]}$
Bulk action	$S = -\frac{1}{2} \int F \wedge \star F$	$-\frac{1}{2 \cdot (p + 1)!} \int d^d x \sqrt{-g} F_{\mu_0 \mu_1 \dots \mu_p} F^{\mu_0 \mu_1 \dots \mu_p}$
Equations of motion	$d \star F = 0$	$\nabla_{\mu_0} F^{\mu_0 \mu_1 \dots \mu_p} = 0$
Bianchi identity	$dF = 0$	$\partial_{[\alpha} F_{\mu_0 \mu_1 \dots \mu_p]} = 0$

Classical duality of massless p -forms

Equations of motion of A_p imply F is closed. It can be written as a sum of

- An exact form dC_r where $r = d - p - 2$.
- Harmonic form F_h satisfying $\square F_h = 0$.

Harmonic k -forms are classified by the de Rham cohomology group of degree k over the manifold $\mathcal{M}$.

$$H^k(\mathcal{M}) = \frac{\text{Closed } k\text{-forms}}{\text{Exact } k\text{-forms}}$$

Classically, harmonic forms do not contribute to local dynamics. Bianchi identity of p -form gives us the equations of motion of C_r

$$dF = 0 \implies d \star dC_r = 0$$

With the proper prescription of gauge redundancy, C_r describes a classically equivalent gauge theory.

Massless p -forms are classically dual to massless $(d - p - 2)$ -forms

Massless 2-form are classically dual to a massless pseudoscalar field

Consider a 2-form gauge field $B_{\mu\nu}$ with field strength tensor $H_{\mu\nu\rho}$.

Duality transformation: $H_{\mu\nu\rho} = -\frac{1}{3!}\epsilon_{\mu\nu\rho\sigma}\partial^\sigma\phi$

Equations of motion: $\nabla_\mu H^{\mu\nu\rho} = 0 \rightarrow (\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu)\phi = 0$

Bianchi identity: $\nabla_{[\mu} H_{\nu\rho\sigma]} = 0 \rightarrow \nabla_\mu \nabla^\mu \phi = 0$

Sanity check: $B_{\mu\nu}$ carries only a single propagating degree of freedom, just like ϕ .

- (Gauss constraint) B^{0i} are static fields
- (Gauge redundancy) $B_{\mu\nu} \rightarrow B_{\mu\nu} + \partial_{[\mu}\theta_{\nu]}$
 - Gauge parameter itself has a redundancy $\theta_\nu \rightarrow \theta'_\nu = \theta_\nu + d\xi$

Quantum non-equivalence of massless p -forms

Duff and Nieuwenhuizen¹ have shown that the duality is broken by quantum corrections.

Assume we are working with a compact, smooth manifold.

Consider the *Euclidean* generating functional for the gauge-fixed massless p -form

$$Z_p^0 = \int \mathcal{D}A \Big|_{\text{g.f.}} \exp(-S_p^0[A])$$

The result is a tower of determinants of the Hodge-de Rham operator $\square$.

$$Z_p^0 = (\det \square_p)^{-1/2} (\det \square_{p-1})^1 (\det \square_{p-2})^{-3/2} \dots (\det \square_0)^{(-1)^{p+1}(p+1)/2}$$

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Gauge field

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Gauge field

Ghost fields

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Gauge field

Ghost-for-ghost fields

Ghost fields

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Quantum non-equivalence of massless p -forms

$$Z_p^0 = (\det \square_p)^{-1/2} (\det \square_{p-1})^1 (\det \square_{p-2})^{-3/2} \dots (\det \square_0)^{(-1)^{p+1}(p+1)/2}$$

Zero modes of the Hodge-de Rham operator have zero action – must be removed by appropriate counterterms.

$$\Delta S_p^0 = \frac{1}{\varepsilon} (B_p - 2B_{p-1} + \dots + (-1)^{p+1}(p+1)B_0)$$

B_k is the k -th Betti number – counts number of normalisable harmonic k -forms in the manifold.

- Depends on de Rham cohomology group $H^k(\mathcal{M})$.

The difference between counterterms of a p -form and its classically dual partner is the **Gauss-Bonnet invariant**, defined by the *Euler characteristic* χ .

$$\Delta S_p^0 - \Delta S_{d-p-2}^0 = \frac{(-1)^p}{\varepsilon} \chi$$

Massive p -form gauge theories

The action for a massive p -form gauge theory is

$$S_{\text{mass.}} = -\frac{1}{2} \int F \wedge \star F - \frac{m^2}{2} \int A \wedge \star A$$

Equations of motion: $d \star F + (-1)^p m^2 \star A = 0$

Mass term explicitly breaks gauge invariance.

We would like to write an action for a massive p -form in a manifestly gauge invariant way.

The BF theory

To describe a massive p -form, we construct the following parent action

$$S_{BF} = -\frac{1}{2} \int F \wedge \star F - \frac{1}{2} \int H \wedge \star H + m \int B \wedge F$$

where B is a massless q -form with $q \equiv (d - p - 1)$ and field strength $H \equiv dB$.

Gauge invariant under $A \rightarrow A' = A + d\theta$ and $B \rightarrow B' = B + d\omega$

Integrating out B ,

$$S_{\text{mass.}} = -\frac{1}{2} \int F \wedge \star F - \frac{m^2}{2} \int (A - d\Omega) \wedge \star (A - d\Omega)$$

Ω is a Stueckelberg field with gauge redundancy $\Omega \rightarrow \Omega' = \Omega + \theta$.

Classical equivalence of massive p -forms

$$S_{BF} = -\frac{1}{2} \int F \wedge \star F - \frac{1}{2} \int H \wedge \star H + m \int B \wedge F$$

In the BF theory, we could have chosen to integrate out the p -form instead to obtain a massive q -form theory instead.

Massive p -forms are classically dual to massive $(d - p - 1)$ -forms

In four dimensions, the duality of massive p -forms is between

1-form $\leftrightarrow$ 2-form

0-form $\leftrightarrow$ 3-form

Quantum non-equivalence of massive p -forms

We show that the quantum duality is, again, broken by quantum corrections.

- Assume compact, smooth manifold.

The generating functional of the Euclidean BF theory is

$$Z = \int \mathcal{D}A \mathcal{D}B \Big|_{\text{g.f.}} \exp(-S_{BF}[A, B])$$

We first integrate out B

$$\int \mathcal{D}B \Big|_{\text{g.f.}} \exp\left(-\frac{1}{2} \int H \wedge \star H - im \int B \wedge F\right) = Z_q^0 \exp\left(\frac{m^2}{2} \int F \wedge \star \square^{-1} F\right)$$

Quantum non-equivalence of massive p -forms

$$Z_q^0 \exp\left(\frac{m^2}{2} \int F \wedge \star \square^{-1} F\right)$$

Exponential term is the mass term for the transverse components of the p -form.

$$\frac{m^2}{2} \int F \wedge \star \square^{-1} F = -\frac{m^2}{2} \int A \wedge \star \Pi_T A$$

Generating functional with B integrated out is

$$Z = Z_q^0 \int \mathcal{D}A \Big|_{\text{g.f.}} \exp\left(-\frac{1}{2} \int F \wedge \star F - \frac{m^2}{2} \int A \wedge \star \Pi_T A\right)$$

Quantum non-equivalence of massive p -forms

Need to restore longitudinal modes of massive p -form theory.

Introduce massless $(p - 1)$ -form π with gauge redundancy $\pi \rightarrow \pi' = \pi + \theta$.

$$Z = \frac{Z_q^0}{Z_{p-1}^0} \int \mathcal{D}A \mathcal{D}\pi \Big|_{\text{g.f.}} \exp \left(-\frac{1}{2} \int F \wedge \star F - \frac{m^2}{2} \int A \wedge \star \Pi_T A - \frac{1}{2} \int d\pi \wedge \star d\pi \right)$$

We absorb the longitudinal modes π into the p -form so that the path integral is the generating functional of the massive p -form

$$Z = \frac{Z_q^0}{Z_{p-1}^0} Z_p^m$$

Quantum non-equivalence of massive p -forms

$$Z = \frac{Z_q^0}{Z_{p-1}^0} Z_p^m$$

We could also reverse the order of integration to instead get

$$Z = \frac{Z_p^0}{Z_{q-1}^0} Z_q^m$$

The ratio between a massive p -form and its dual partner is

$$\frac{Z_p^m}{Z_q^m} = \frac{Z_p^0}{Z_q^0} \frac{Z_{p-1}^0}{Z_{q-1}^0}$$

Quantum non-equivalence of massive p -forms

$$\frac{Z_p^m}{Z_q^m} = \frac{Z_p^0}{Z_q^0} \frac{Z_{p-1}^0}{Z_{q-1}^0}$$

Equivalence is broken due to the failure of the dualisation of the massless longitudinal modes.

Once again, difference is due to different counterterms to cancel out normalisable zero modes.

- Depends on de Rham cohomology group $H^k(\mathcal{M})$.

$$\Delta S_p^m - \Delta S_{d-p-1}^m = \frac{(-1)^p}{\varepsilon} \chi$$

Euclidean Schwarzschild spacetime

$$ds^2 = \left(1 - \frac{r_h}{r}\right) d\tau^2 + \left(1 - \frac{r_h}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

There are two distinct harmonic 2-forms which are normalisable zero modes.

$$\Delta S_0^m - \Delta S_3^m = \frac{2}{\varepsilon} \quad \Delta S_1^m - \Delta S_2^m = -\frac{2}{\varepsilon}$$

Euclidean de Sitter spacetime

$$ds^2 = \left(1 - \frac{r^2}{a^2}\right) d\tau^2 + \left(1 - \frac{r^2}{a^2}\right)^{-1} dr^2 + r^2 d\Omega^2$$

The only harmonic forms are a 0-form and a d -form.

$$\Delta S_0^m - \Delta S_3^m = \frac{2}{\varepsilon} \quad \Delta S_1^m - \Delta S_2^m = -\frac{2}{\varepsilon}$$

Eguchi-Hanson instantons

$$ds^2 = \left(1 - \frac{a^4}{r^4}\right)^{-1} dr^2 + r^2(\sigma_x^2 + \sigma_y^2) + r^2 \left(1 - \frac{a^4}{r^4}\right) \sigma_z^2$$

σ_i are SU(2) Cartan forms satisfying the algebra $d\sigma_i = \epsilon_{ijk} d\sigma_j \wedge d\sigma_k$.

The only harmonic form is a single 2-form.

$$\Delta S_0^m - \Delta S_3^m = \frac{1}{\varepsilon} \quad \Delta S_1^m - \Delta S_2^m = -\frac{1}{\varepsilon}$$

Conclusion

- Massless p -forms are classically dual to massless $(d - p - 2)$ -forms.
- It is known that the equivalence is broken by quantum corrections in topologically non-trivial manifolds which have normalisable zero modes.

$$H^k(\mathcal{M}) = \frac{\text{Closed } k\text{-forms}}{\text{Exact } k\text{-forms}}$$

- Massive p -forms are classically dual to massive $(d - p - 1)$ -forms.
- We have shown that the equivalence is similarly broken, namely due to the failure of the dualisation of the massless longitudinal modes.

$$\frac{Z_p^m}{Z_q^m} = \frac{Z_p^0}{Z_q^0} \frac{Z_{p-1}^0}{Z_{q-1}^0}$$

$$\Delta S_p^m - \Delta S_{d-p-1}^m = \frac{(-1)^p}{\varepsilon} \chi$$

- The non-equivalence has been explicitly shown for spacetimes with non-trivial topologies.