

Nonlinear-Supersymmetric General Relativity(NLSGR) of Nature

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1. Motivation:

@ The success of **GR** and **SM(GWS) model**,
still **many unsolved fundamental problems are left**, e.g.,

- Unification of two SMs,
- Space-time dimension **four**,
- Three-generations structure of quarks and leptons,
- Tiny Neutrino mass M_ν , proton stability in GUT
- Dark Matter, Dark Energy; $\rho_{D.E.} \sim (M_\nu)^4 \Leftrightarrow \Lambda CDM$, Inflation

$\Rightarrow$ **SUSY, SUGRA**,

- Origin of SUSY breaking ?, $\dots$ etc.

@ As for the **three-generations structure**
among **SO(N) LSUSY sP**, $N = 10$ gives **SM with just 3 generations**,

@ How to construct **N=10 SUSY with gravity**
despite the **No-Go** theorem for $N > 8$ by the **S-matrix** argument.

- To circumvent the No-Go theorem **the degeneracy of space-time** is essential.

- **Dirac particle survivors** of $SO(10)$ sP after *tentative* superHiggs-like mechanism

$SU(3)$	Q_e	$SU(2) \otimes U(1)$
<u>1</u>	0 -1 -2	$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} (N)$ (E)
<u>3</u>	5/3 2/3 -1/3 -4/3	$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \begin{pmatrix} h \\ o \end{pmatrix} \begin{pmatrix} a \\ f \end{pmatrix} \begin{pmatrix} g \\ m \end{pmatrix} \begin{pmatrix} r \\ i \\ n \end{pmatrix}$
<u>6</u>	4/3 1/3 -2/3	$\begin{pmatrix} P \\ Q \\ R \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$
<u>8</u>	0 -1	$\begin{pmatrix} N_1 \\ E_1 \end{pmatrix} \begin{pmatrix} N_2 \\ E_2 \end{pmatrix}$

- Prediction: New **color-singlet** states, N^0 (sterile ν ?) and $E^{\pm 2}$

A quick review of NLSUSY:

- Take (flat) space-time specified by x^a for $SO(1,3)$ and ψ_α for $SL(2,C)$.
- Consider one form $\omega^a = dx^a + \frac{\kappa^2}{2i}(\bar{\psi}\gamma^a d\psi - d\bar{\psi}\gamma^a\psi)$,
 κ is an **arbitrary** constant with the dimension l^{+2} .

- $\delta\omega^a = 0$ under $\delta x^a = \frac{i\kappa^2}{2}(\bar{\zeta}\gamma^a\psi - \bar{\psi}\gamma^a\zeta)$ and $\delta\psi = \zeta$ with a **global** spinor parameter ζ .

- An invariant action($\sim$ **invariant volume**) is obtained:

$$S = -\frac{1}{2\kappa^2} \int \omega^0 \wedge \omega^1 \wedge \omega^2 \wedge \omega^3 = \int d^4x L_{VA},$$

L_{VA} is **N=1 Volkov-Akulov model of NLSUSY** given by

$$L_{VA} = -\frac{1}{2\kappa^2}|w_{VA}| = -\frac{1}{2\kappa^2} \left[1 - t^a_a + \frac{1}{2}(t^a_a t^b_b - t^a_b t^b_a) + \dots \right],$$

$$|w_{VA}| = \det w^a_b = \det(\delta^a_b + t^a_b), \quad t^a_b = -i\kappa^2(\bar{\psi}\gamma^a\partial_b\psi - \partial_b\bar{\psi}\gamma^a\psi),$$

which is invariant under **N=1 NLSUSY** transformation:

$$\delta_\zeta\psi = \frac{1}{\kappa}\zeta - i\kappa(\bar{\zeta}\gamma^a\psi - \bar{\psi}\gamma^a\zeta)\partial_a\psi, \quad [\delta_1, \delta_2] = \delta_P.$$

- ψ is **Nambu-Goldstone(NG) fermion** for $\frac{\text{supertranslation}}{\text{translation}}$.
- ψ is quantized **canonically** in compatible with **SUSY** algebra.
- Conserved supercurrent, supercharge:

$$J^0 \longrightarrow Q \sim c\psi + (\dots\partial_a\psi\dots)$$

2. Nonlinear-Supersymmetric General Relativity(NLSGR)

@ Considering as an ultimate shape of nature,
new (unstable) *Riemann* space-time inspired by nonlinear(NL) SUSY:

The tangent space is specified by the Minkowski coordinate x^a for SO(1,3),
and Grassmann coordinate ψ_α for SL(2,C)

@ The geometrical arguments of Einstein general relativity(GR)
can be extended to new (unstable) space-time.

• Unified vierbein $\tilde{e}^a_\mu(x)$ of new space-time:

$$\tilde{e}^a_\mu(x) = e^a_\mu + t^a_\mu(\psi) \equiv w^a_b e^b_\mu, \quad w^a_b = \delta^a_b + t^a_b,$$

$$t^a_b(\psi) = \frac{\kappa^2}{2i}(\bar{\psi}^I \gamma^a \partial_b \psi^I - \partial_b \bar{\psi}^I \gamma^a \psi^I), \quad (I = 1, 2, \dots, N)$$

$$\tilde{e}^\mu_a(x) = e^\mu_a - t^\mu_a + t^\mu_\rho t^\rho_a - t^\mu_\sigma t^\sigma_\rho t^\rho_a + t^\mu_\kappa t^\kappa_\sigma t^\sigma_\rho t^\rho_a + \dots + O(t)^{4N} \equiv e^\mu_b w^b_a,$$

$$\tilde{e}^a_\mu(x) \tilde{e}^\mu_b(x) = \delta^a_b, \quad \tilde{e}^a_\mu(x) \tilde{e}^\nu_a(x) = \delta^\nu_\mu,$$

$$t^a_\mu(\psi) = \frac{\kappa^2}{2i}(\bar{\psi}^I \gamma^a \partial_\mu \psi^I - \partial_\mu \bar{\psi}^I \gamma^a \psi^I), \quad (I = 1, 2, \dots, N)$$

(Note that Grassmann *odd* d.o.f. induces the imaginary part of $\tilde{e}^a_\mu(x)$.)

- N -extended NLSGR action of EH-type for new space-time:

$$L_{\text{NLSGR}}(\tilde{e}) = -\frac{c^4}{16\pi G} |\tilde{e}| \{R(\tilde{e}) + \Lambda\},$$

$$|\tilde{e}| = \det \tilde{e}^a_\mu = \det(e^a_\mu + t^a_\mu(\psi)),$$

- $\tilde{e}^a_\mu(x) (= e^a_\mu + t^a_\mu(\psi))$: **the unified vierbein of new space-time**
- $e^a_\mu(x)$: **the ordinary vierbein for the local SO(1,3) d.o.f. of GR,**
- $t^a_\mu(\psi(x))$: **the mimic vierbein for the local SL(2,C) d.o.f. composed of the stress-energy-momentum of NG fermion $\psi(x)^I$ (called *superons*),**
- $R(\tilde{e})$: **Ricci scalar curvature of new space-time computed in terms of $\tilde{e}^a_\mu$,**
- $\tilde{g}_{\mu\nu} \equiv \tilde{e}^a_\mu \eta_{ab} \tilde{e}^b_\nu$, $\tilde{g}^{\mu\nu}(x) \equiv \tilde{e}^\mu_a(x) \eta^{ab} \tilde{e}^\nu_b(x)$: **metric tensors of new space-time.**
- G : **the Newton gravitational constant.**
- $\Lambda > 0$: **cosmological constant**
- **The arbitrary constant κ^2 of NLSUSY is fixed at $\kappa^2 = (\frac{c^4 \Lambda}{8\pi G})^{-1}$,**
- $\Lambda > 0$ in the action L_{NLSGR} allows **negative dark energy density interpretation of $O(\frac{\Lambda}{G})$**
- **No-go theorem for $N > 8$ SUSY has been circumvented by the global NLSUSY structure, i.e. by the degeneracy of the vacuum.**
- **Note that $SO(1, D-1) \cong SL(d, C)$, i.e. $\frac{D(D-1)}{2} = 2(d^2 - 1)$ holds for only $D = 4, d = 2$. NLSGR scenario predicts 4 dimensional space-time.**

- **Space-time symmetries** ($\sim sP$):

$$[\text{new NLSUSY}] \otimes [\text{local GL}(4, \mathbb{R})] \otimes [\text{local Lorentz}]$$

- **Internal symmetries** for N-extended NLSUSY GR (N-superons ψ^I ($I = 1, 2, \dots, N$)):

$$[\text{global SO}(N)] \otimes [\text{local U}(1)^N] \otimes [\text{chiral}]. \quad (1)$$

For example:

@ Invariance under the new NLSUSY transformation;

$$\delta_\zeta \psi^I = \frac{1}{\kappa} \zeta^I - i\kappa \bar{\zeta}^J \gamma^\rho \psi^J \partial_\rho \psi^I, \quad \delta_\zeta e^a_\mu = i\kappa \bar{\zeta}^J \gamma^\rho \psi^J \partial_{[\mu} e^a_{\rho]}. \quad (2)$$

induces **GL(4,R) transformations** on $\tilde{e}_\mu^a$ and the unified metric $\tilde{g}_{\mu\nu}$

$$\delta_\zeta \tilde{e}_\mu^a = \xi^\nu \partial_\nu \tilde{e}_\mu^a + \partial_\mu \xi^\nu \tilde{e}_\nu^a, \quad \delta_\zeta \tilde{g}_{\mu\nu} = \xi^\kappa \partial_\kappa \tilde{g}_{\mu\nu} + \partial_\mu \xi^\kappa \tilde{g}_{\kappa\nu} + \partial_\nu \xi^\kappa \tilde{g}_{\mu\kappa},$$

where ζ is a constant spinor parameter, $\partial_{[\rho} e^a_{\mu]} = \partial_\rho e^a_\mu - \partial_\mu e^a_\rho$ and $\xi^\rho = -i\kappa \bar{\zeta}^I \gamma^\rho \psi^I$.

- **Commutators of two new NLSUSY transformation on ψ^I and e^a_μ close on GL(4,R),**

$$[\delta_{\zeta_1}, \delta_{\zeta_2}] \psi^I = \Xi^\mu \partial_\mu \psi^I, \quad [\delta_{\zeta_1}, \delta_{\zeta_2}] e^a_\mu = \Xi^\rho \partial_\rho e^a_\mu + e^a_\rho \partial_\mu \Xi^\rho, \quad (3)$$

where $\Xi^\mu = 2i\bar{\zeta}_1^I \gamma^\mu \zeta_2^I - \xi_1^\rho \xi_2^\sigma e_a{}^\mu \partial_{[\rho} e^a_{\sigma]}$.

- New NLSUSY is the square-root of $GL(4,R)$;

$$[\delta_1, \delta_2] = \delta_{GL(4,R)}, \quad i.e. \quad \delta \sim \sqrt{\delta_{GL(4,R)}}.$$

- @ The ordinary local $GL(4,R)$ invariance is manifest by the construction.
- @ Invariance under new local Lorentz transformation;

$$\delta_L \psi^I = -\frac{i}{2} \epsilon_{ab} \sigma^{ab} \psi^I, \quad \delta_L e^a{}_\mu = \epsilon^a{}_b e^b{}_\mu + \frac{\kappa^4}{4} \epsilon^{abcd} \bar{\psi}^I \gamma_5 \gamma_d \psi^I$$

induce the familiar form of the local Lorentz transformation on $\tilde{e}^a{}_\mu$:

$$\delta_L \tilde{e}^a{}_\mu = \epsilon^a{}_b \tilde{e}^b{}_\mu$$

with the local parameter $\epsilon_{ab} = (1/2) \epsilon_{[ab]}(x)$
and close the algebra, e. g., **the new form** on $e^a{}_\mu(x)$

$$[\delta_{L_1}, \delta_{L_2}] e^a{}_\mu = \beta^a{}_b e^b{}_\mu + \frac{\kappa^4}{4} \epsilon^{abcd} \bar{\psi}^j \gamma_5 \gamma_d \psi^j (\partial_\mu \beta_{bc}),$$

where $\beta_{ab} = -\beta_{ba}$ is given by $\beta_{ab} = \epsilon_{2ac} \epsilon_1^c{}_b - \epsilon_{2bc} \epsilon_1^c{}_a$.

3. Big Collapse of NLSGR [Cosmology]

@ Due to the NLSUSY structure of space-time, L_{NLSGR} would collapse (called **Big Collapse(BC)**) spontaneously toward the vacuum configuration with NG particle.
i. e., to ordinary EH action of graviton e^a_μ , NLSUSY action of Nambu-Goldstone(NG) fermion (called **superon**) and their gravitational interaction $\bar{\psi}\gamma^a\partial_b\psi e^b_\mu$ (called **superon-graviton model(SGM)**).

$$L_{SGM}(e, \psi) = -\frac{c^4}{16\pi G}|e|\{R(e) + |w(\psi^I)_{VA}|\Lambda + \tilde{T}(e, \psi^I)\}.$$

• The variation with respect to e^a_μ producee Einstein equation for SGM equipting with **negative dark energy density** :

$$R_{\mu\nu}(e) - \frac{1}{2}g_{\mu\nu}R(e) = \frac{8\pi G}{c^4}\{\tilde{T}_{\mu\nu}(e, \psi) - g_{\mu\nu}\frac{c^4\Lambda}{8\pi G}\}. \quad (4)$$

• **Big collapse(BC)** induces **3 dimensional (space-like region) rapid expansion** of space-time by **Pauli exclsionm principle(Quantum Inflation)**:

$$ds^2 = \tilde{g}_{\mu\nu}(x)dx^\mu dx^\nu = \{g_{\mu\nu} + \underline{\Phi_{\mu\nu}(e, \psi)}\}dx^\mu dx^\nu.$$

$$\{\psi(x)_a, \bar{\psi}(y)_b\}_{x_0=y_0} = 0 \quad \Rightarrow \quad \{\psi(x)_a, \bar{\psi}(y)_b\}_{x_0=y_0} = \delta^{(3)}(\mathbf{x} - \mathbf{y})\delta_{ab}$$

$$(\{ \psi(x)_a, \bar{\psi}(y)_b \} = 0 \quad \Rightarrow \quad \{ \psi(x)_a, \bar{\psi}(y)_b \} = iS(x - y)\delta_{ab})$$

- The conserved supercurrent $S^{I\mu} = i\sqrt{\frac{c^4\Lambda}{8\pi G}}\gamma^\mu\psi^I + (\cdots\partial\psi, \psi\cdots)$.
couples the superon(NG fermion) to the vacuum
 $\langle \psi_\beta^J | S_\alpha^{I\mu} | 0 \rangle = i\sqrt{\frac{c^4\Lambda}{8\pi G}}\delta^{IJ}(\gamma^\mu)_{\alpha\beta}$ with the strength $g_{sv} \equiv \sqrt{\frac{c^4\Lambda}{8\pi G}} = \kappa^{-1}$,
the fundamental constant characterizing Big Collapse of Nature.

4. BC and Beginning of Big Bang(BB) Universe:: [Particle Physics]

@ Simultaneously at BC the universal attractive force graviton would dictate the evolution (vacuum) of $SGM(L_{SGM})$ by producing all possible gravitational-composites of superons which span the (massless) irreducible representation of the helicity states of the $LSUSY$ $SO(N)$ super-Poincaré(sP) symmetry of nature.

- This can be regarded as **the Beginning of the Big Bang** of the universe.
- Considering that the (low energy) leading term of the supercharges of both L- and NL-representation contains the fundamental fermion.
- The linearization of NLSUSY may give the related tractable (equivalent) LSUSY theory, which elucidates the vacuum structure of NLSUSY theory.
- The vacuum structure of SGM is studied by:

linearizing NLSUSY in SGM and finding the equivalent LSUSY theory

(called **NL/L SUSY relation**).

- The global (NL and L) SUSY algebra plays an essential role.
- It is shown in the toy model that the ordinary LSUSY model for the low energy particle physics can emerge in the (flat space) vacuum of $L_{NLSGR/SGM}$, where all particles are the superon-composite eigenstates of LSUSY algebra.
- **NL/L SUSY relation for N=2 SUSY in $d = 2$ means:**

$$L_{\mathbf{N=2LSUSYQED}}(v_a, \lambda^i, A, \phi^i, D, \chi, F^i) = f(\xi, \xi^i) L_{\mathbf{N=2NLSUSY}}(\psi^j) + [\text{surface terms}],$$

where

$$v^a = -\frac{i}{2} \xi \kappa \epsilon^{ij} \bar{\psi}^i \gamma^a \psi^j |w|, \quad \lambda^i = \xi \psi^i |w|, \quad A = \frac{1}{2} \xi \kappa \bar{\psi}^i \psi^i |w|, \quad D = \frac{\xi}{\kappa} |w| \quad \cdots \quad \text{etc.}$$

SUSY compositeness,

therefore the vacuum of N -NLSUSY is given by that of N -LSUSY..

Broken LSUSY gauge theory is encoded in the true vacuum of NLSUSY theory as composite of NG fermion.

- NL/L SUSY relation bridges (high energy) Extreme Universe and (low energy) particle physics and elucidate relations between them. e. g., $e = \tilde{f}(\xi, \xi^i)$, etc.

- For linearizing directly NLSGR/SGM

we must find the adequate (SUGRA-analogue) **broken global LSUSY** theory expressed in terms of the composites of superon $\psi(x)$ and e^a_μ , and achieves

NL/L SUSY relation (*equivalence*):

$$L_{\text{NLSGR}}(w) = L_{\text{SGM}}(e, \psi) = L_{\text{LSUSY}}(e^a_\mu, \psi_\nu(e, \psi), M(e, \psi), N(e, \psi), \dots).$$

which elucidates the vacuum structure of SGM.

5. Prospect

@ NLSGR((SGM) paradigm can bridge naturally the cosmology and the low energy particle physics, which provides new insights into unsolved problems of cosmology, particle physics and mysterious relations between them, e.g. the space-time dimension *four*, the origin of mass and SUSY breaking, the dark energy and the dark matter, the dark energy density $\approx (\text{neutrino mass})^4$ the tiny neutrino mass, inflation of space-time, the three-generations structure of quarks and leptons, the rapid expansion of space-time, the magnitude of the bare gauge coupling constant, matter dominance of nature , **the shape and the role of black hole** etc.

- Many important problems are yet to be studied, e.g., global SUSY for higher spins (5/2 3), for SUGRA and the direct linearization of (N=10) SGM, etc.

6. Summary

NLSGR(SGM) for unity of nature:

- Ultimate entity; **Unstable** $d = 4$ **space-time**: $[x^a, \psi_\alpha^N; x^\mu]$ described by $[L_{\text{NLSGR}}(w^a_\mu)]$: **Global SUSY NLSGR on New space-time** with $\Lambda > 0$

$\Rightarrow$ **Big Collapse (BC)**

- The creation of **ordinary Riemann space-time** [graviton e^a_μ] and **massless NG fermionic matter** [superon ψ_α^N], **rapid expansion by Pauli principle** $[L_{\text{SGM}} = L_{\text{EH}}(e) - \Lambda + T(\psi.e)]$: **Einstein GR with $\Lambda > 0$ and N superon**

- The universal attractive force graviton dictates the evolution of $[L_{\text{SGM}}$ by forming **gravitational composite global LSUSY supermultiplet** corresponding to (massless) eigenstates of space-time symmetry **SO(10) sP.**

$\Rightarrow$ **Ignition of Big Bang of the universe toward Λ CDM scenario**

- NL/L SUSY relation elucidates the vacuum structure of NLSUSY SGM.

For some details:

1. K. Shima, Invited talk at International Conference, *100 Years Werner Heisenberg - Works and Impact*-, September 26-30, 2001., Bamberg, Germany. Fortschr. Phys 50(2002) 5-7. 517-523, eds. D. Papenfuss, D. Luest and W. P. Schuleich.
2. K. Shima, Preliminary temporal report: arXiv:2012.01646[hep-th](2020)