

Barrier Tunnelling of a Particle in QFT

Aditya S. Tejas^{1,2}

Cedric Simenel^{1,2} and Anthony G. Williams^{2,3,4}

¹Department of Fundamental and Theoretical Physics, The Australian National University

²ARC Centre of Excellence for Dark Matter Particle Physics (CDMPP)

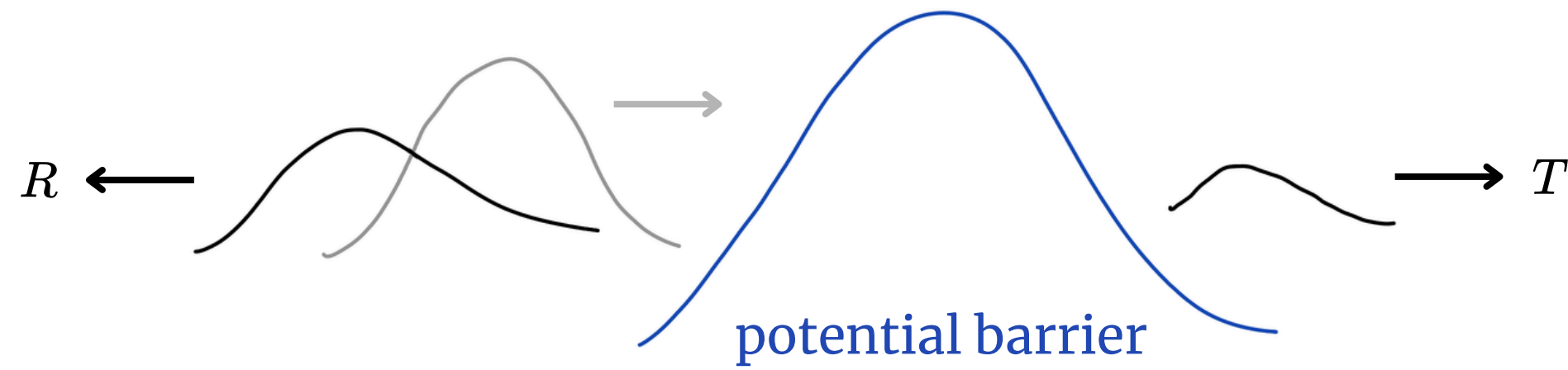
³Department of Physics, Adelaide University

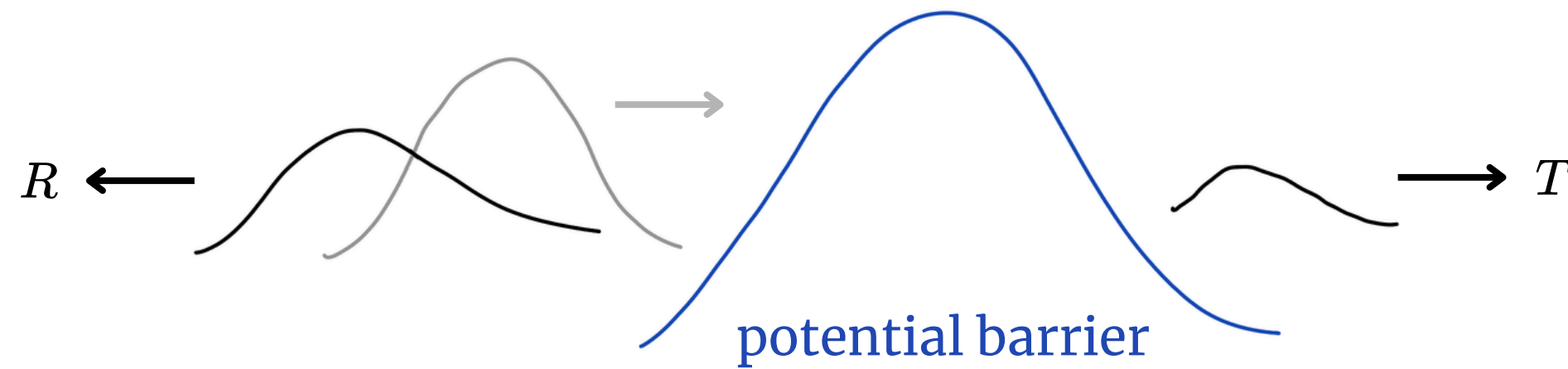
⁴Centre of the Subatomic Structure of Matter (CSSM)



Australian
National
University







- Astrophysical processes: Stellar fusion, nucleosynthesis, alpha-decay, etc.
- Semiconductors
- Medical sensing & natural biological processes
- Scanning tunnelling microscopy
- Fundamental physics :)

Quantum tunnelling is Non-Perturbative

Quantum tunnelling is Non-Perturbative

1D scattering $\langle p|S|p\rangle = \sum_{n=1}^{\infty} \langle p|S|p\rangle^{(n)}$

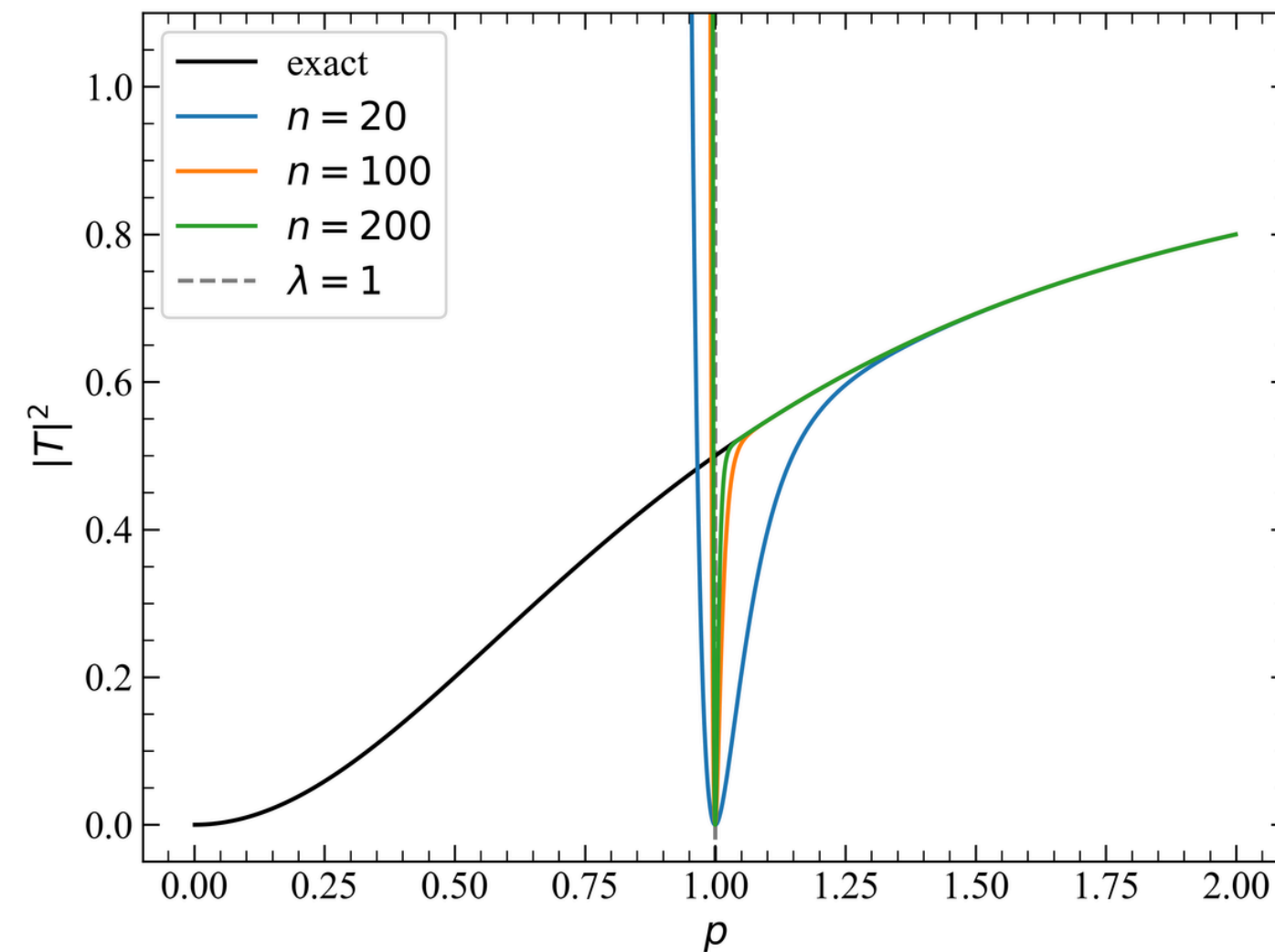
1D scattering $\langle p|S|p\rangle = \sum_{n=1}^{\infty} \langle p|S|p\rangle^{(n)}$

- $|T| \sim e^{-1/g}$ in the tunnelling regime

1D scattering $\langle p|S|p\rangle = \sum_{n=1}^{\infty} \langle p|S|p\rangle^{(n)}$

- $|T| \sim e^{-1/g}$ in the tunnelling regime

-



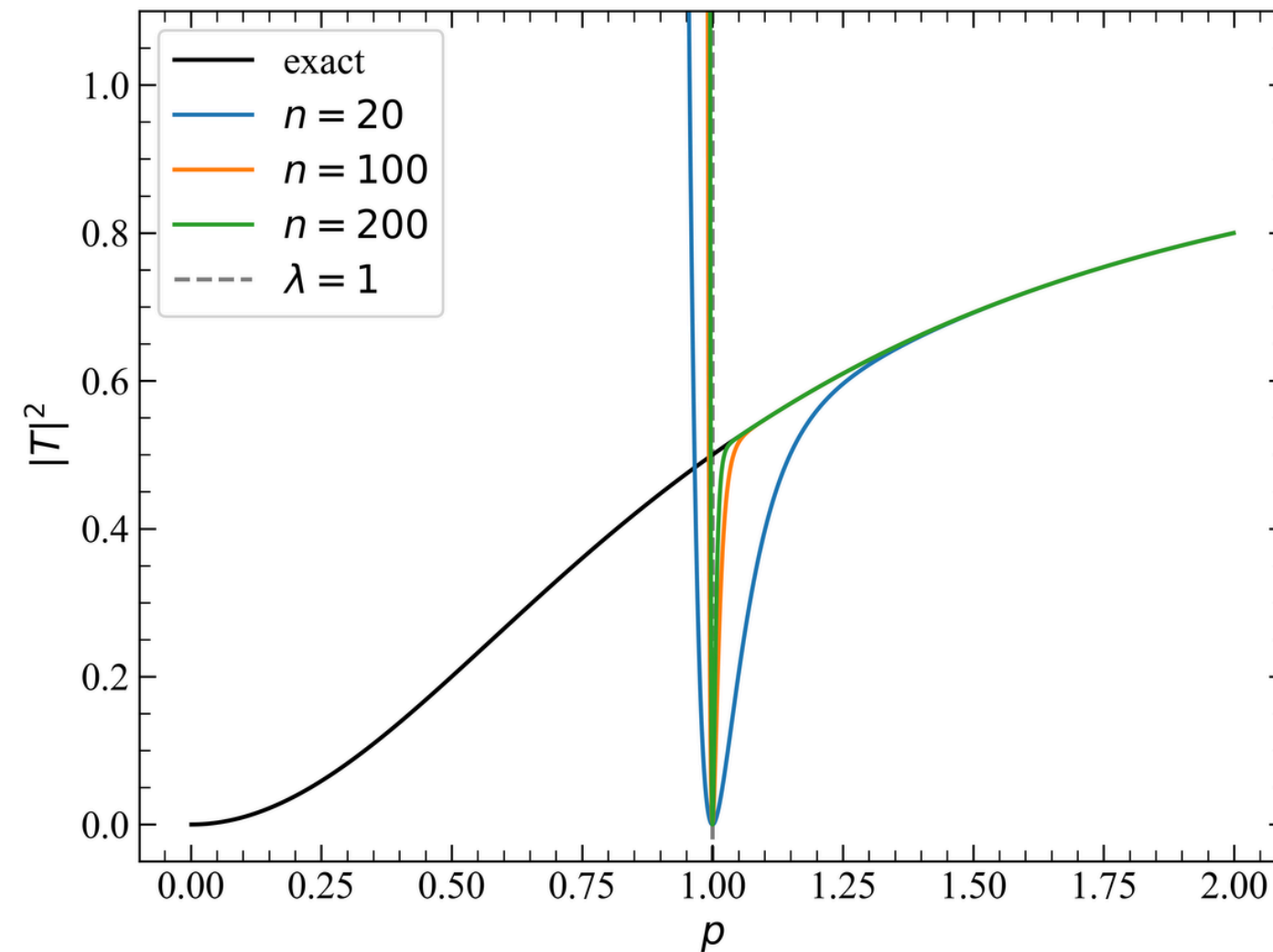
scalar particle through Dirac-delta barrier

transmission $T(p) = \frac{1}{1 + i\lambda/p}$

1D scattering $\langle p|S|p\rangle = \sum_{n=1}^{\infty} \langle p|S|p\rangle^{(n)}$

- $|T| \sim e^{-1/g}$ in the tunnelling regime

-



scalar particle through Dirac-delta barrier

transmission $T(p) = \frac{1}{1 + i\lambda/p}$

Need a way to deal with infinite sums.

How Accurate is Quantum Mechanics?

- Quantum mechanics is incomplete.
- Quantum field theory allows processes like virtual particle exchange and pair production.

How Accurate is Quantum Mechanics?

- Quantum mechanics is incomplete.
- Quantum field theory allows processes like virtual particle exchange and pair production.

Common use of QFT

$$\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}}$$

perturbation theory

How Accurate is Quantum Mechanics?

- Quantum mechanics is incomplete.
- Quantum field theory allows processes like virtual particle exchange and pair production.

Common use of QFT

$$\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}}$$

perturbation theory

Addition of a classical background

$$\mathcal{L} = \mathcal{L}_{\text{free}}(\phi, \varphi) + \mathcal{L}_{\text{int}}(\phi, \varphi) + V(x)f(\phi)$$

interacting
particle

potential

tunnelling
particle

infinite resummation

How Accurate is Quantum Mechanics?

- Quantum mechanics is incomplete.
- Quantum field theory allows processes like virtual particle exchange and pair production.

Common use of QFT

$$\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}}$$

perturbation theory

Addition of a classical background

$$\mathcal{L} = \mathcal{L}_{\text{free}}(\phi, \cancel{\varphi}) + \cancel{\mathcal{L}_{\text{int}}(\phi, \varphi)} + V(x)f(\phi) \quad \sim \text{RQM}$$

interacting
particle

potential

tunnelling
particle

infinite resummation

- Quantum mechanics is incomplete.
- Quantum field theory allows processes like virtual particle exchange and pair production.

Common use of QFT

$$\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}}$$

perturbation theory

Addition of a classical background

$$\mathcal{L} = \mathcal{L}_{\text{free}}(\phi, \varphi) + \cancel{\mathcal{L}_{\text{int}}(\phi, \varphi)} + V(x)f(\phi) \quad \sim \text{RQM}$$

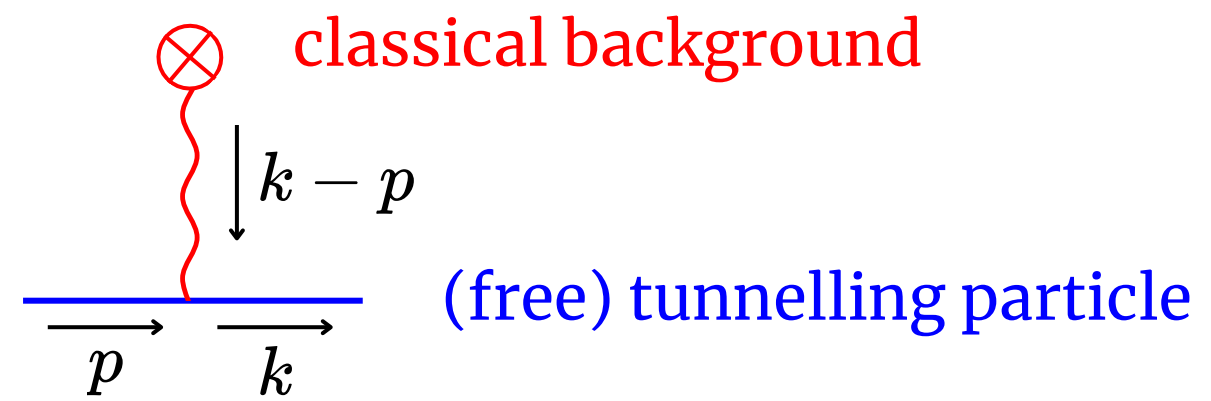
interacting
particle

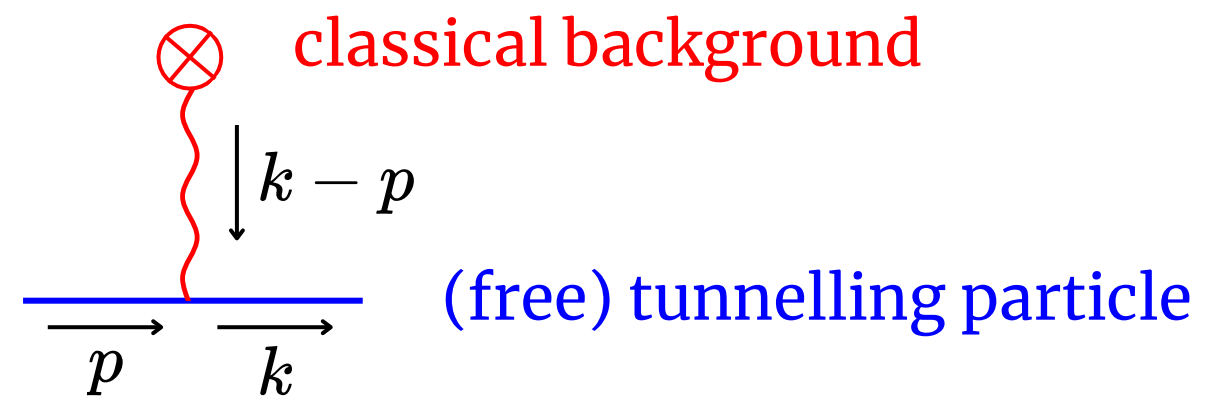
potential

tunnelling
particle

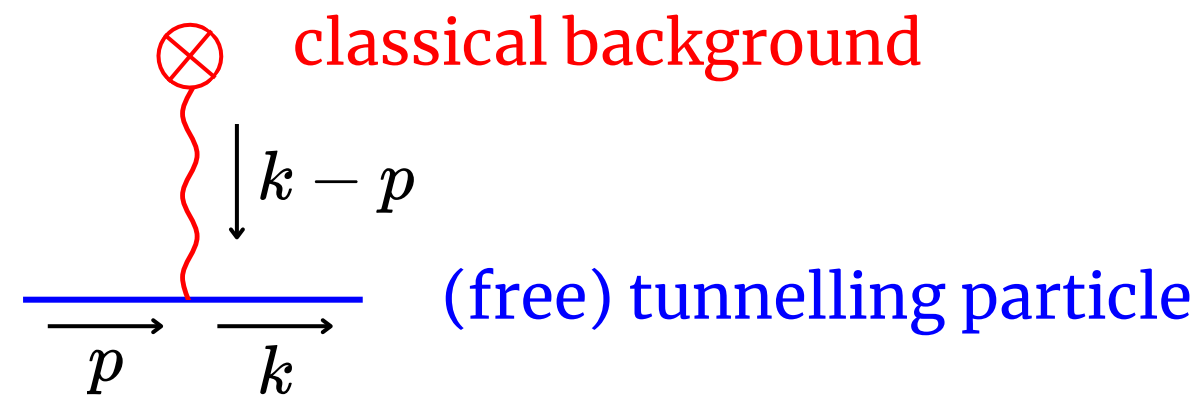
infinite resummation

Aim: Quantify beyond RQM corrections to particle tunnelling observables.



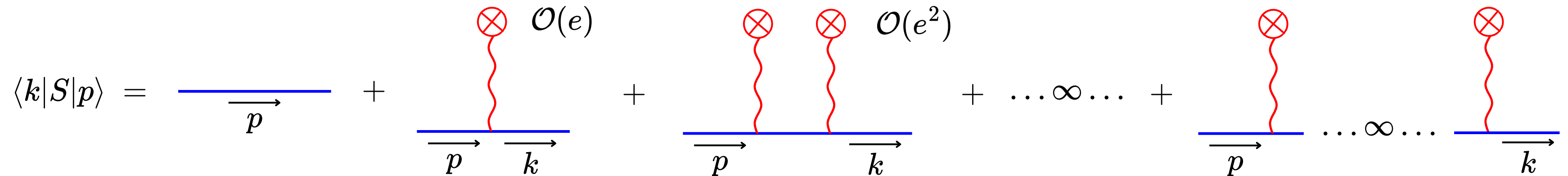


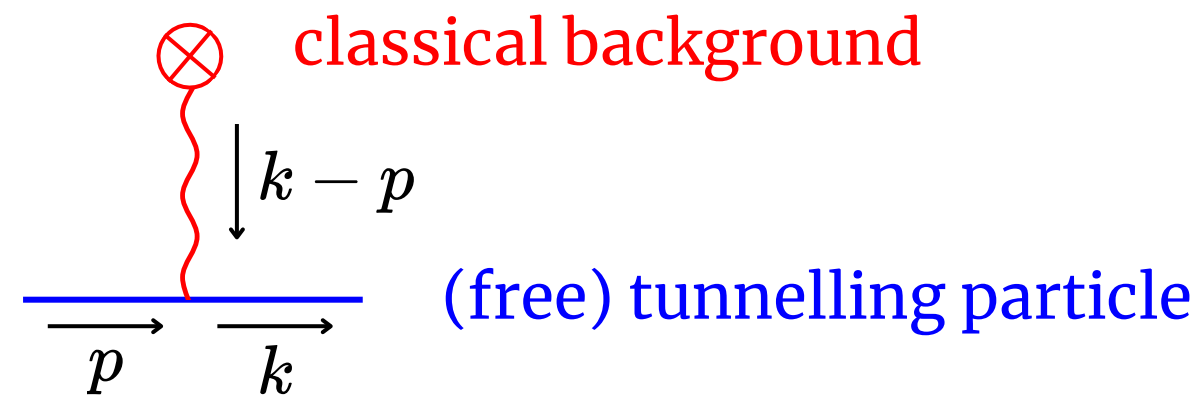
e.g. $\mathcal{L}_{\text{KG}}(\phi) + eV(x)\phi^2$
 $\mathcal{L}_{\text{D}}(\psi) + eV_{\mu}(x)\bar{\psi}\gamma^{\mu}\psi$



e.g. $\mathcal{L}_{\text{KG}}(\phi) + eV(x)\phi^2$
 $\mathcal{L}_{\text{D}}(\psi) + eV_\mu(x)\bar{\psi}\gamma^\mu\psi$

Scattering matrix elements:

$$\langle k|S|p\rangle = \text{---}\overrightarrow{p}\text{---} + \text{---}\overrightarrow{p}\text{---}\overrightarrow{k}\text{---} \text{---}\mathcal{O}(e) + \text{---}\overrightarrow{p}\text{---}\text{---}\overrightarrow{k}\text{---} \text{---}\mathcal{O}(e^2) + \dots \infty \dots + \text{---}\overrightarrow{p}\text{---} \dots \infty \dots \text{---}\overrightarrow{k}\text{---}$$




e.g. $\mathcal{L}_{\text{KG}}(\phi) + eV(x)\phi^2$
 $\mathcal{L}_{\text{D}}(\psi) + eV_\mu(x)\bar{\psi}\gamma^\mu\psi$

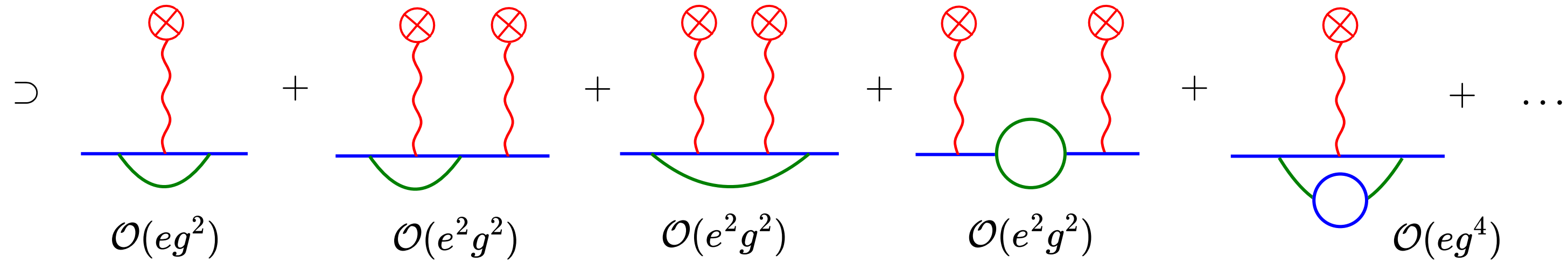
Scattering matrix elements:

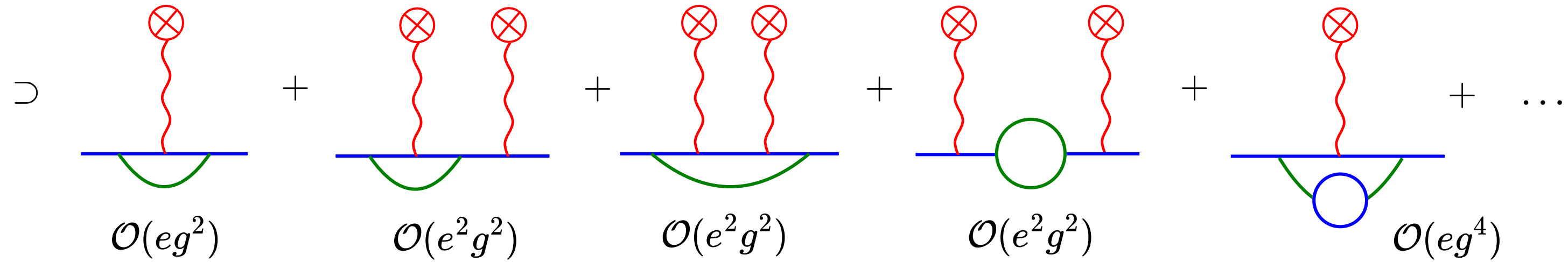
$$\langle k|S|p\rangle = \text{---}\overrightarrow{p}\text{---} + \text{---}\overrightarrow{p}\text{---}\overrightarrow{k}\text{---} \text{---}\mathcal{O}(e) + \text{---}\overrightarrow{p}\text{---}\text{---}\overrightarrow{k}\text{---} \text{---}\mathcal{O}(e^2) + \dots \infty \dots + \text{---}\overrightarrow{p}\text{---} \dots \infty \dots \text{---}\overrightarrow{k}\text{---}$$

RQM scattering amplitudes using QFT framework:

- Zielinski, R., et. al. *Eur. Phys. J. C* 84, 992 (2024) – scalar + scalar (Dirac-delta barrier)
- Fleming, M., et. al. *arXiv.2608.05511* (2026) – spinor + vector $V_0(z)$

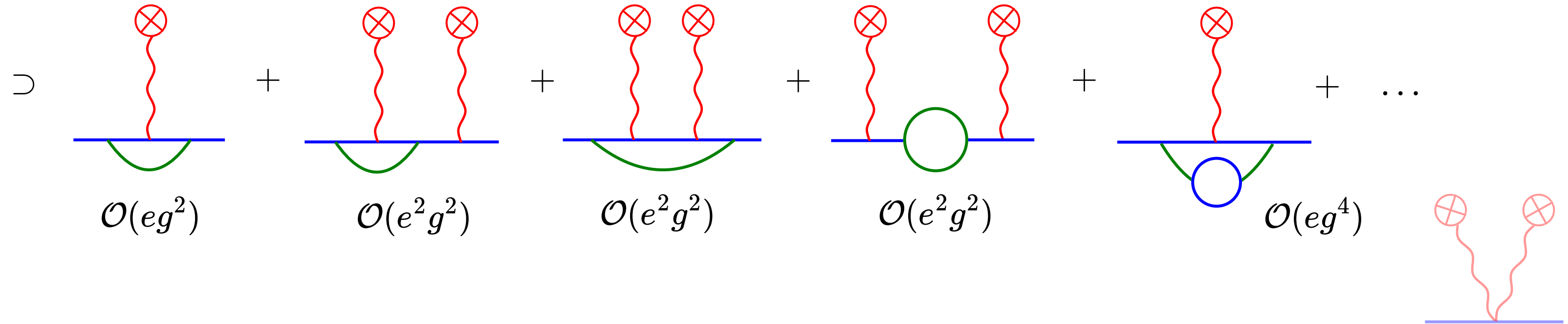
Introducing Interactions



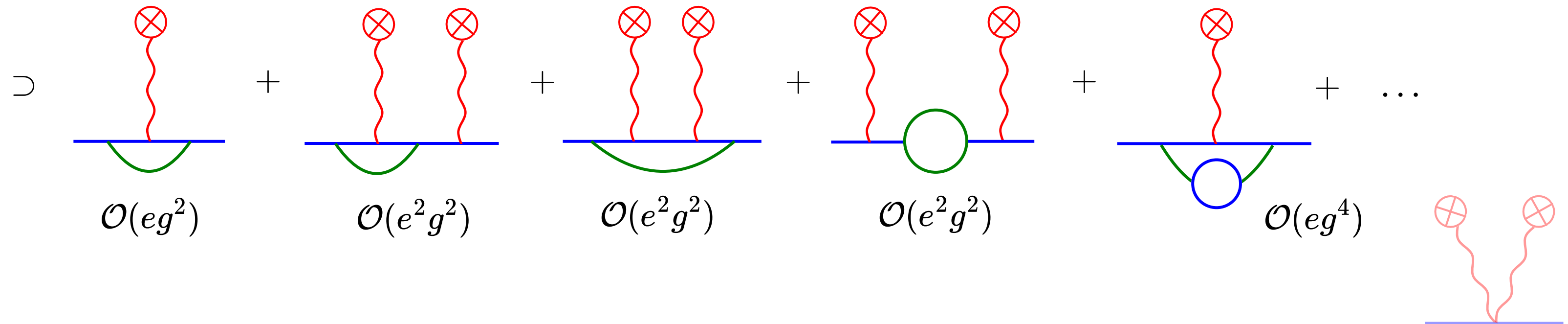


e.g. $\mathcal{L}_{\text{KG}}(\phi) + eV(x)\phi^2 + gA\phi^2, \quad \mathcal{L}_{\text{D}}(\psi) + eV_\mu(x)\bar{\psi}\gamma^\mu\psi + gA_\mu\bar{\psi}\gamma^\mu\psi$

Introducing Interactions



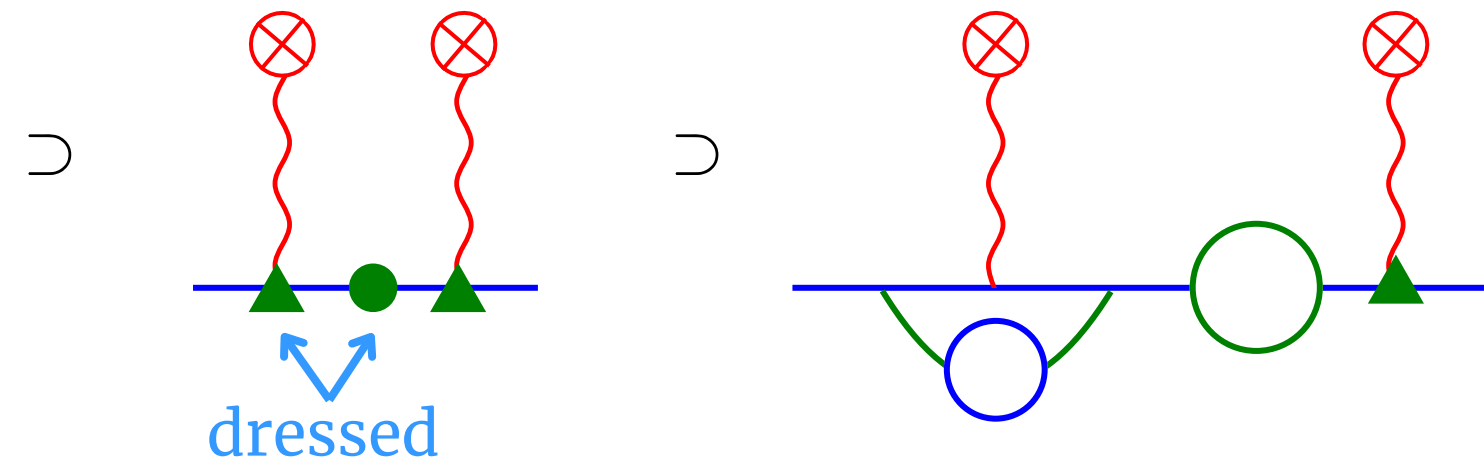
e.g. $\mathcal{L}_{\text{KG}}(\phi) + e\textcolor{red}{V}(\textcolor{red}{x})\phi^{\textcolor{blue}{2}} + g\textcolor{green}{A}\phi^2, \quad \mathcal{L}_{\text{D}}(\psi) + e\textcolor{red}{V}_{\mu}(\textcolor{red}{x})\bar{\psi}\gamma^{\mu}\psi + g\textcolor{green}{A}_{\mu}\bar{\psi}\gamma^{\mu}\psi$



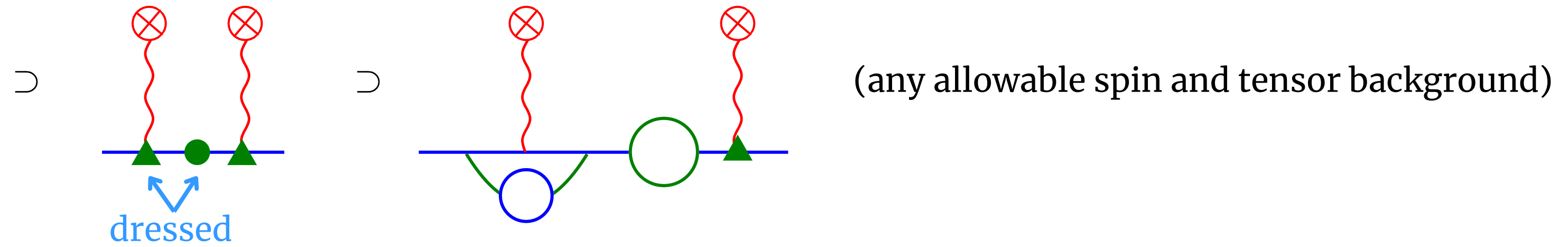
e.g. $\mathcal{L}_{\text{KG}}(\phi) + e\mathbf{V}(\mathbf{x})\phi^2 + g\mathbf{A}\phi^2, \quad \mathcal{L}_{\text{D}}(\psi) + e\mathbf{V}_\mu(\mathbf{x})\bar{\psi}\gamma^\mu\psi + g\mathbf{A}_\mu\bar{\psi}\gamma^\mu\psi$

QFT corrections:

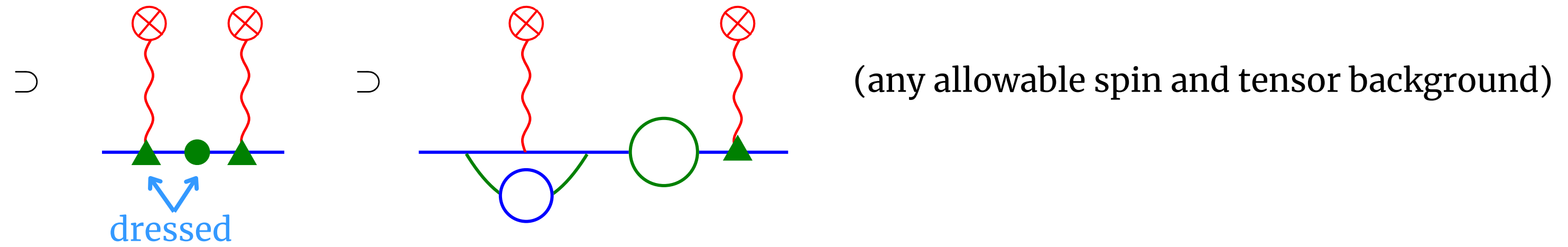
- Zielinski, R., et. al. *Eur. Phys. J. C* 84, 967 (2024) – scalar + scalar + scalar $\xrightarrow{*}$ G_{RQM} based equation
 - Numerical solver – Desheng Wang (Masters thesis, unpublished)
- Fleming, M. Honours thesis (ANU, 2025) – spinor + vector $\varphi_0(z)$ + photon $\xrightarrow{*}$ position space ODE for T/R.
 - Preliminary numerical results.

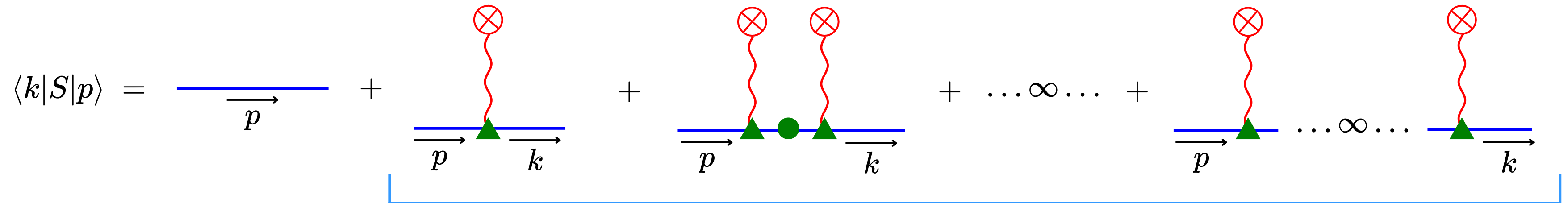


(any allowable spin and tensor background)

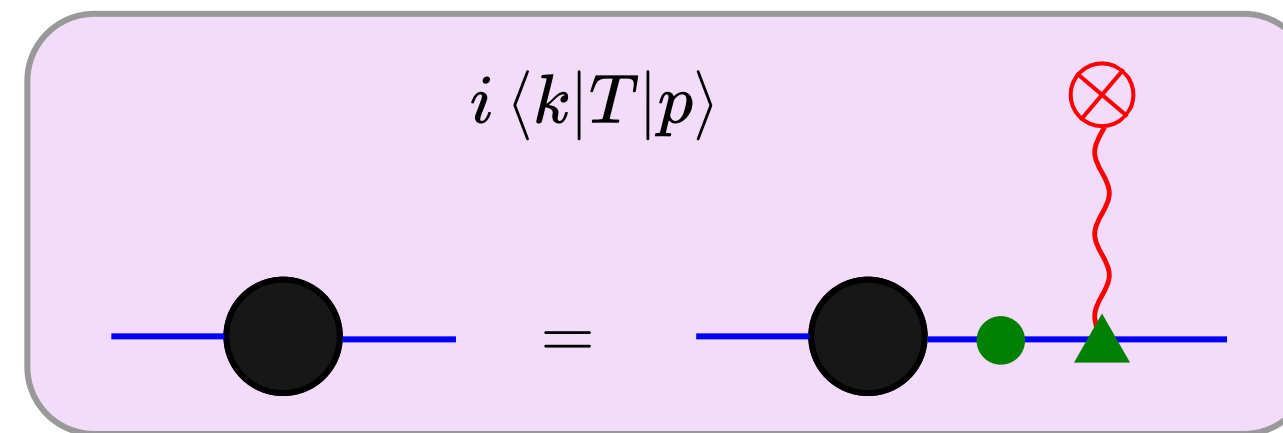


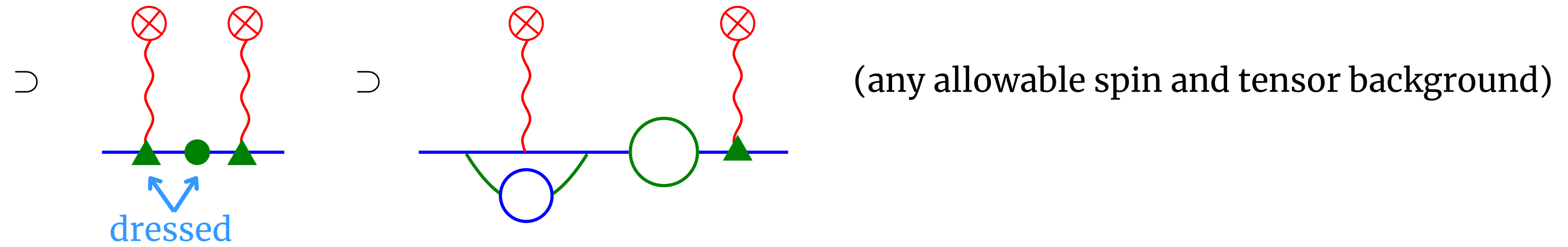
$$\langle k|S|p\rangle = \begin{array}{c} \text{---} \xrightarrow{p} \text{---} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \xrightarrow{p} \text{---} \xrightarrow{k} \text{---} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \xrightarrow{p} \text{---} \text{---} \xrightarrow{k} \text{---} \\ \text{---} \end{array} + \dots \infty \dots + \begin{array}{c} \text{---} \xrightarrow{p} \text{---} \dots \infty \dots \text{---} \xrightarrow{k} \text{---} \\ \text{---} \end{array}$$



$$\langle k|S|p\rangle = \begin{array}{c} \xrightarrow{p} \end{array} + \begin{array}{c} \xrightarrow{p} \quad \xrightarrow{k} \end{array} + \begin{array}{c} \xrightarrow{p} \quad \xrightarrow{k} \end{array} + \dots \infty \dots + \begin{array}{c} \xrightarrow{p} \quad \dots \infty \dots \quad \xrightarrow{k} \end{array}$$


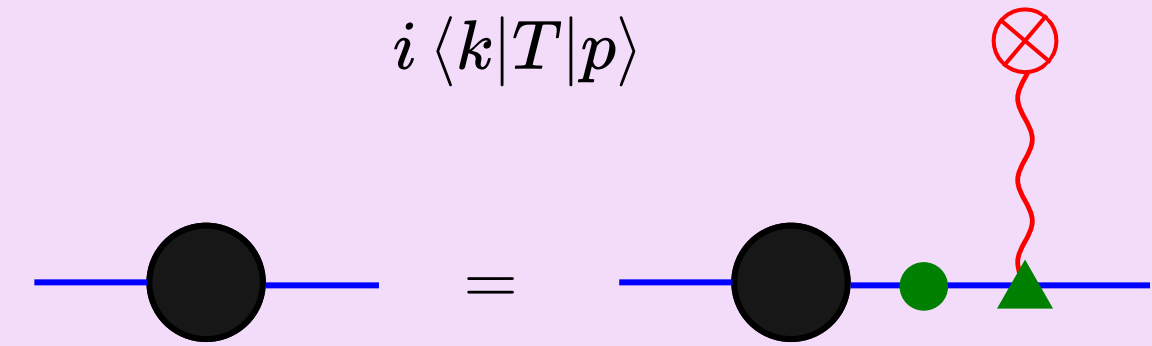
$i \langle k|T|p\rangle$





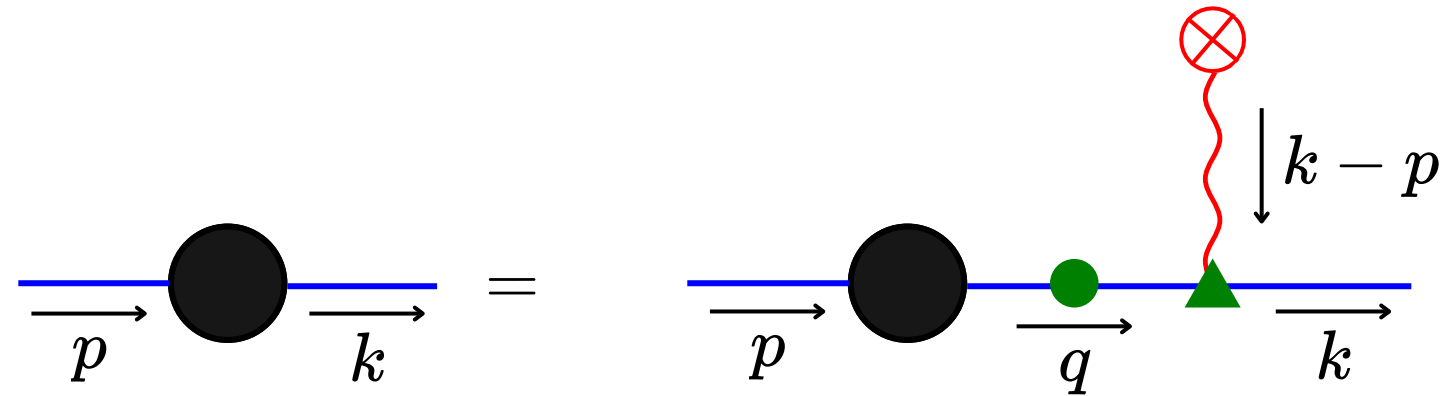
$$\langle k|S|p\rangle = \begin{array}{c} \xrightarrow{p} \end{array} + \begin{array}{c} \xrightarrow{p} \quad \xrightarrow{k} \end{array} + \begin{array}{c} \xrightarrow{p} \quad \xrightarrow{k} \end{array} + \dots \infty \dots + \begin{array}{c} \xrightarrow{p} \quad \dots \infty \dots \quad \xrightarrow{k} \end{array}$$

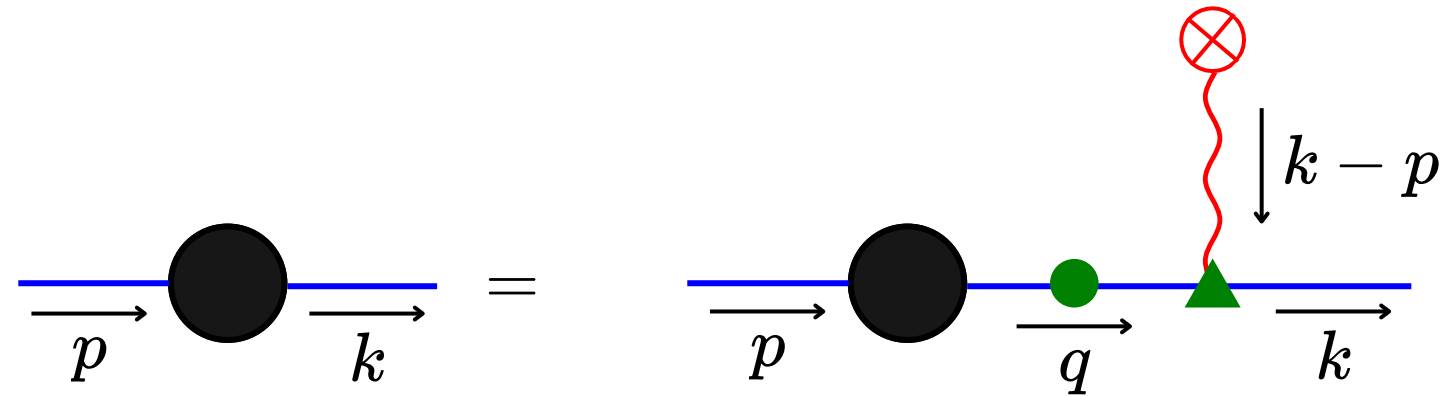
$i \langle k|T|p\rangle$



~ Field theoretic
Lippmann-Schwinger equation

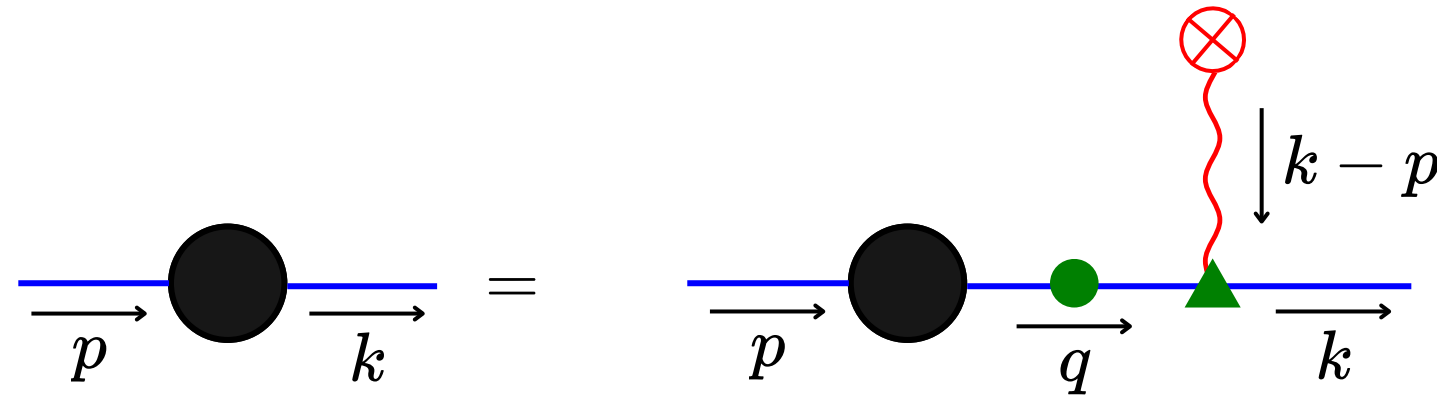
QFT Lippmann-Schwinger Equations (LSE)





cf. QM LSE: $T = V + VG_0T$

$$\tilde{T}(k, p) = -\tilde{\mathcal{V}}(k, p) - i \int \frac{d^4q}{(2\pi)^4} \tilde{\mathcal{V}}(k, q) \tilde{G}_F(q) \tilde{T}(q, p)$$



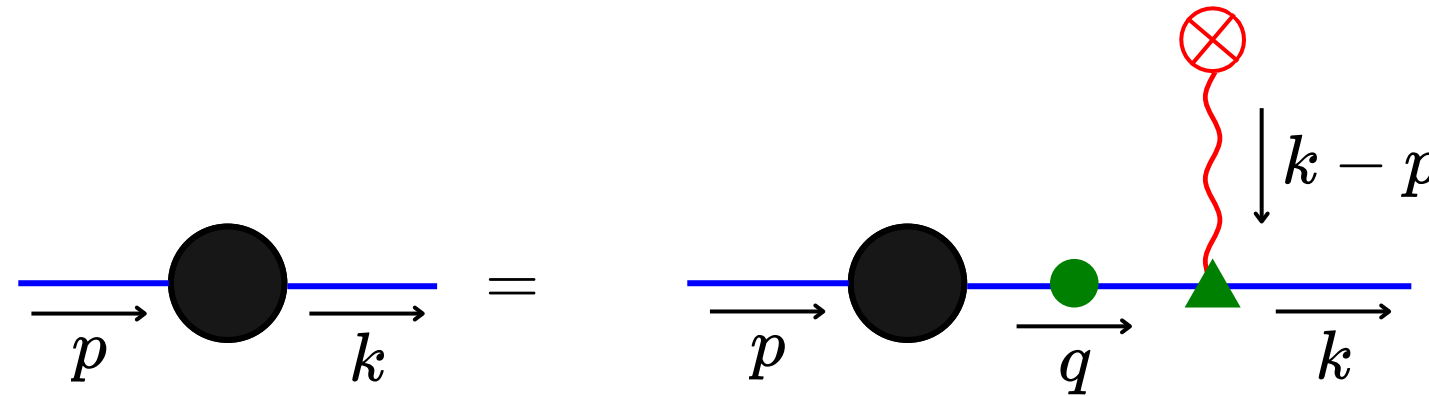
cf. QM LSE: $T = V + VG_0T$

$$\tilde{T}(k, p) = -\tilde{\mathcal{V}}(k, p) - i \int \frac{d^4q}{(2\pi)^4} \tilde{\mathcal{V}}(k, q) \tilde{G}_F(q) \tilde{T}(q, p)$$

↑ dressed potential
 ← Feynman propagator

$$e\Gamma_3^{\nu_1, \dots, \nu_n}(k, p) \tilde{V}_{\nu_1, \dots, \nu_n}(k - p)$$

← three-point function ▲



cf. QM LSE: $T = V + VG_0T$

$$\tilde{T}(k, p) = -\tilde{\mathcal{V}}(k, p) - i \int \frac{d^4q}{(2\pi)^4} \tilde{\mathcal{V}}(k, q) \tilde{G}_F(q) \tilde{T}(q, p)$$

↑ dressed potential
 ← Feynman propagator

$$e\Gamma_3^{\nu_1, \dots, \nu_n}(k, p) \tilde{V}_{\nu_1, \dots, \nu_n}(k - p)$$

← three-point function ▲

RQM $\longrightarrow$ ● = $\cdot$ and ▲ = $\cdot$ (bare)

- Lundberg, L. *Comm. Math. Phys.* 31(3): 243–257 (1973).
- Sakhnovich, L. *arXiv:1801.05370* (2018)

$$\tilde{V}_{\nu_1, \dots, \nu_i}(q) = (2\pi)^3 \delta(q^0) \delta^2(\vec{q}_\perp) \tilde{V}_{\nu_1, \dots, \nu_i}(q^3)$$

$$\tilde{V}_{\nu_1, \dots, \nu_i}(q^3) \equiv \int_{-\infty}^{\infty} dz V_{\nu_1, \dots, \nu_i}(z) e^{-iq^3 z}$$

$$\tilde{T}(k^3, p^3) = -\tilde{\mathcal{V}}(k^3, p^3) - i \int_{-\infty}^{\infty} \frac{dq^3}{2\pi} \tilde{\mathcal{V}}(k^3, q^3) \tilde{G}(q^3) \tilde{T}(q^3, p^3)$$

$(p^0, \mathbf{p}_\perp)$ conserved

set $\mathbf{p}_\perp = \mathbf{0}$

transmission $t(p^3) = 1 + \frac{i}{2p^3} \bar{u}(p^3) \tilde{T}(p^3, p^3) u(p^3),$

reflection $r(p^3) = \frac{i}{2p^3} \bar{u}(-p^3) \tilde{T}(-p^3, p^3) u(p^3).$

Momentum space

$$\tilde{T}(k^3, p^3) = -\tilde{\mathcal{V}}(k^3, p^3) - i \int_{-\infty}^{\infty} \frac{dq^3}{2\pi} \tilde{\mathcal{V}}(k^3, q^3) \tilde{G}_F(q^3) \tilde{T}(q^3, p^3)$$

Position space

$$\mathcal{T}_{p^3}(z) \equiv \int \frac{dq^3}{2\pi} \tilde{G}_F(q^3) T(q^3, p^3) e^{iq^3 z}$$

$$\mathcal{T}_{p^3}(z'') = - \int_{-\infty}^{\infty} dz V(z) \left(F_{p^3}(z'' - z) e^{ip^3 z} + i \int_{-\infty}^{\infty} dz' \tilde{F}(z - z') \mathcal{T}_{p^3}(z) \right)$$

$$F_{q^3}(w) \equiv \int \frac{dk^3}{2\pi} \tilde{G}_F(k^3) \Gamma_3(k^3, q^3) e^{ik^3 w}$$

$$\tilde{F}(w) \equiv \int \frac{dq^3}{2\pi} F_{q^3}(z'' - z) e^{iq^3 w}$$

- With the same setup and approximation $\longrightarrow$ Fleming, M. Honours thesis (ANU, 2025).
- For QM parallel see Marinov, M. S. & Segev, B. *J. Phys. A: Math. Gen.* 29 2839 (1996).

Neutral scalar ϕ and A .

$$\mathcal{L} = \mathcal{L}_{\text{KG}}(\phi, m) + \mathcal{L}_{\text{KG}}(A, \mu) - \frac{1}{2}gA\phi^2 - \frac{1}{2}eV(z)\phi^2$$

$$\text{take } V(z) = V_0\delta(z)$$

Make an analytic approximation: **

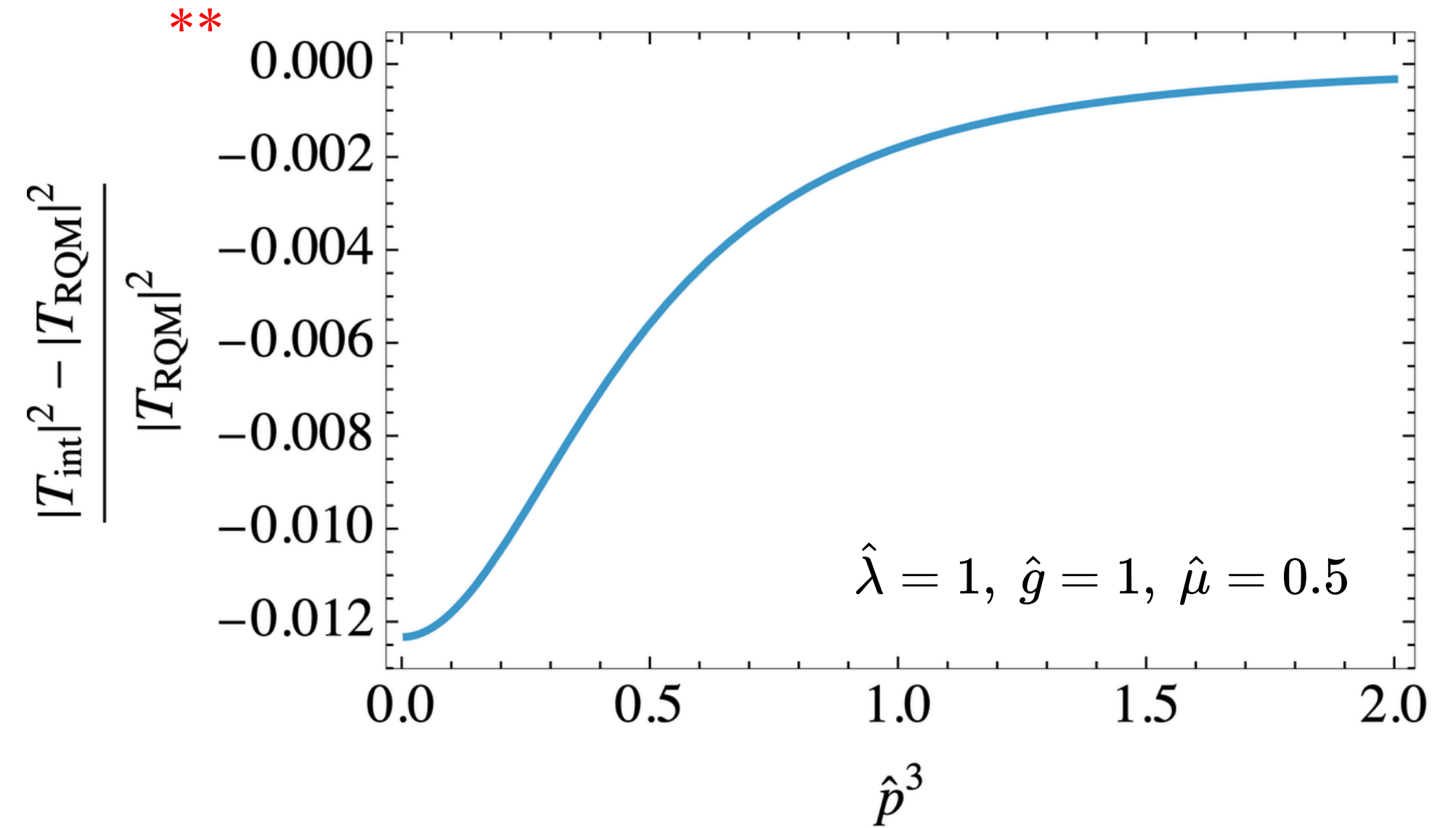
- essentially $G_F(q^3) = G_F^0(q^3)$
- consider one-loop $\Gamma_3(k^3, q^3)$ analytic in q^3

$$F_{q^3}(w) = \frac{1}{2p^3} (\Theta(w)\Gamma_3(p^3, p^3)e^{ip^3w} + \Theta(-w)\Gamma_3(-p^3, p^3)e^{-ip^3w}) + \mathcal{O}(Ce^{-\alpha|z|})$$

Problem simplifies to a system of linear equations.

Dimensionless parameters:

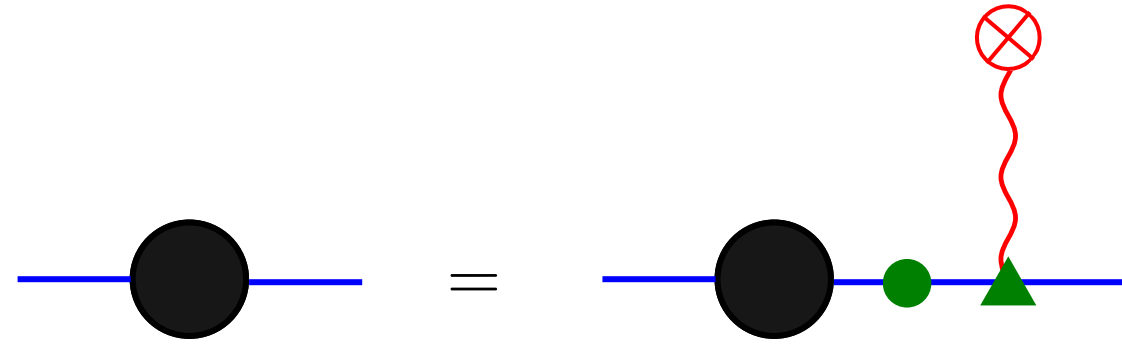
- Barrier strength - $\hat{\lambda} = eV_0/m$
- Momentum - $\hat{p}^3 = p^3/m$
- Mediator mass - $\hat{\mu} = \mu/m$
- Interaction strength - $\hat{g} = g/m$



QFT corrections effectively raised the barrier in this case.

$$T_{\text{int}}(p^3) = T_{\text{RQM}}(p^3) \left[1 + f(p^3) \delta\Gamma_3^{\parallel}(p^3) + h(p^3) \delta\Gamma_3^{\times}(p^3) \right]$$

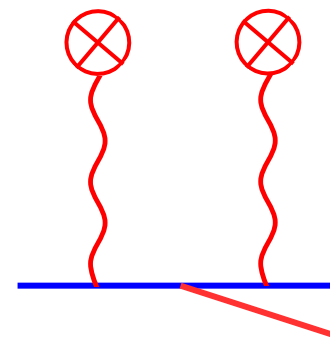
- Framework to quantify QFT corrections to quantum tunnelling.



- any spin and tensor background
- restricted topology

Next steps:

- Direct numerical solution in momentum space (e.g. Nyström method).
- Quantify/improve analytic approximation.
- Emission whilst tunnelling? Bremsstrahlung (scalar QED).



- Other applications, e.g. very low-energy fusion, strong-field ionisation, etc.