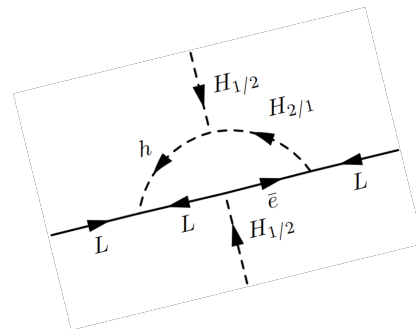
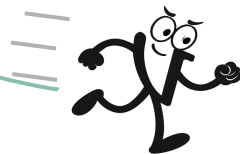




Neutrino Masses & their Temperament

PPC 2026, Sydney

James Vandeleur

School of Physics, UNSW.

M. A. Schmidt and **J Vandeleur**, *Running of neutrino mass parameters in the Zee Model*, JHEP, 6 (2026), [arXiv:2512.17235](#).



Neutrino mixing parameters

Measured neutrino **flavour oscillations**,

$$P_{\alpha\beta}(t) \equiv |\langle \nu_\beta | \nu_\alpha(t) \rangle|^2 \sim \sum_{i,j} U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j} \sin^2 \left(\frac{(m_i^2 - m_j^2)L}{4E} \right),$$

imply a basis of **massive neutrinos** that mix the neutrino flavours

$$\begin{array}{ccc} \text{flavour (interacting)} & & \text{mass (propagating)} \\ \text{basis} & \rightarrow & \text{basis} \end{array} \quad |\nu_\alpha\rangle = \sum_i U_{\alpha i}^* |\nu_i\rangle$$

$$\text{where, } U_{\text{PMNS}} = R_{23}(\theta_{23}) R_{13}(\theta_{13}, \delta_{\text{CP}}) R_{12}(\theta_{12}) \begin{pmatrix} e^{i\eta_1} & 0 & 0 \\ 0 & e^{i\eta_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

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We have measurements for the six parameters Δm_{21}^2 , $\Delta m_{3\ell}^2$, θ_{12} , θ_{13} , θ_{23} , and δ_{CP} .

A precision era

Current global analysis from NuFIT
– including JUNO's 59 days.

~ 1% level precision

$\sin^2 \theta_{12}$	$0.3088^{+0.0067}_{-0.0066}$	$\theta_{13}/^\circ$	$8.62^{+0.11}_{-0.11}$
$\theta_{12}/^\circ$	$33.76^{+0.42}_{-0.41}$	$\delta_{\text{CP}}/^\circ$	207^{+23}_{-20}
$\sin^2 \theta_{23}$	$0.470^{+0.017}_{-0.014}$	$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.537^{+0.094}_{-0.10}$
$\theta_{23}/^\circ$	$43.27^{+1.0}_{-0.82}$	$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.521^{+0.026}_{-0.018}$
$\sin^2 \theta_{13}$	$0.02249^{+0.00057}_{-0.00057}$		

NuFIT 6.1 (2025), www.nu-fit.org, global analysis.

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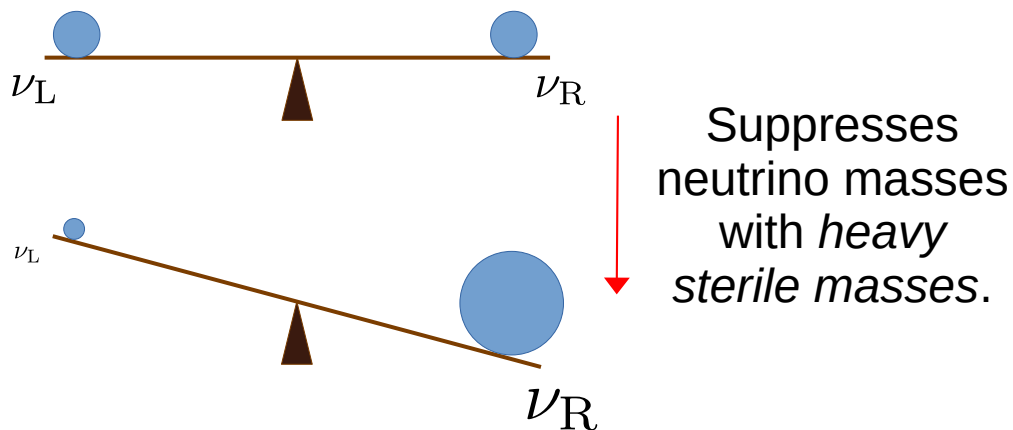
NuFIT 6.1 (2025), www.nu-fit.org, global analysis.

- JUNO predicting order **0.1%** precision for solar, reactor parameters after a few years of data taking.
- HyperK – to bring atmospheric parameters down to **~0.5%**, and CP phase 6-20°.
- DUNE, similarly, 10-15° precision in the CP phase.

Theoretical descriptions

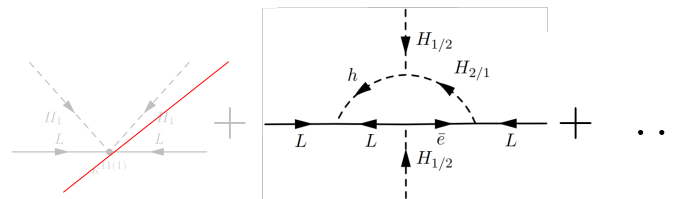
Neutrino masses need to be “unnaturally” light!

Seesaw models



Radiative models

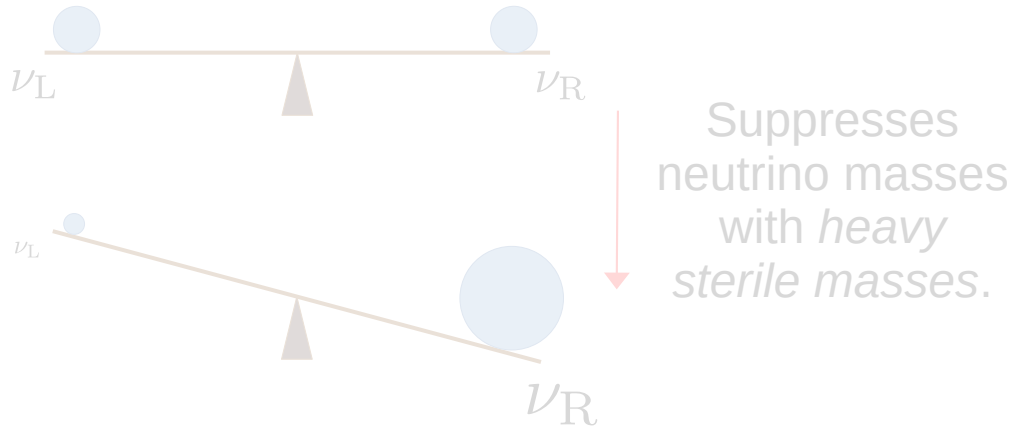
Neutrino masses are generated at *loop-order* and made small by *loop suppression*.



Theoretical descriptions

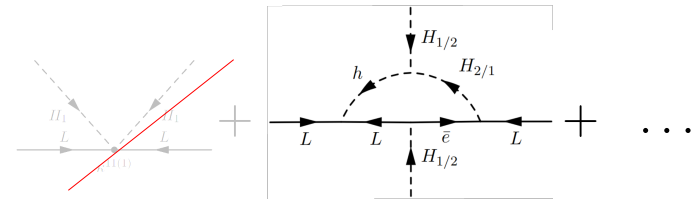
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Radiative models

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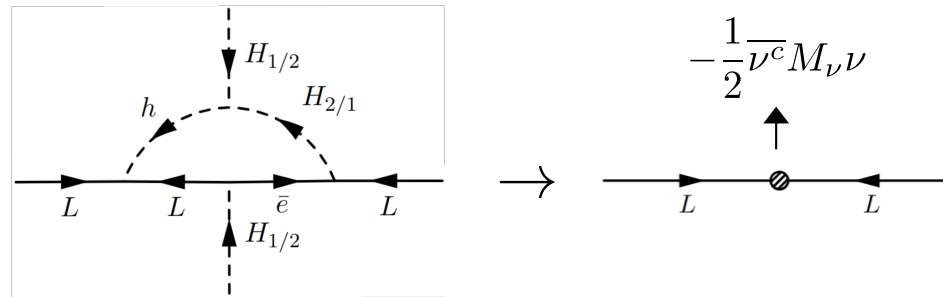
A Radiative Model

We consider the Zee model – the first and quintessential radiative model – adding,

$$H_2 \sim (1, 2, -1/2) \quad \text{and} \quad h \sim (1, 1, 1)$$

(not the excluded Zee-Wolfenstein model!)

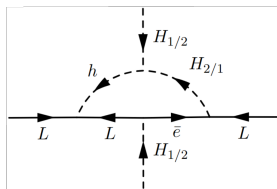
to the SM. We then generate Majorana neutrino masses at 1-loop.



NB - Most of this study is relevant to the popular 2-Higgs Doublet Model component of the Zee model.

ν masses in the Zee model

The general procedure...



$$\longrightarrow M_\nu = -\frac{v^2}{16\pi^2} \frac{\mu_{Zee}}{m_h^2 - m_{H_2}^2} \log\left(\frac{m_h^2}{m_{H_2}^2}\right) \left(f Y_e^{1\dagger} Y_e^2 + Y_e^{2T} Y_e^{1*} f^T\right)$$

Mass matrix $M_\nu \longrightarrow$ Compare to data $\longrightarrow$ Fit UV parameters

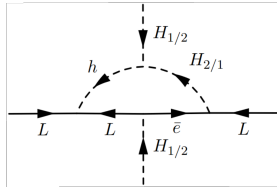
Fitted UV parameters are **compared to other observable phenomena**;

cLFV, dipole moments, *collider*, *cosmology* etc..

...at potentially different scales.

Problem of Large Logs

This log term is large for $m_h \gg m_{H_2}$ or $m_{H_2} \gg m_h$.



$$\longrightarrow M_\nu = -\frac{v^2}{16\pi^2} \frac{\mu_{Zee}}{m_h^2 - m_{H_2}^2} \log\left(\frac{m_h^2}{m_{H_2}^2}\right) \left(f Y_e^{1\dagger} Y_e^2 + Y_e^{2T} Y_e^{1*} f^T\right)$$

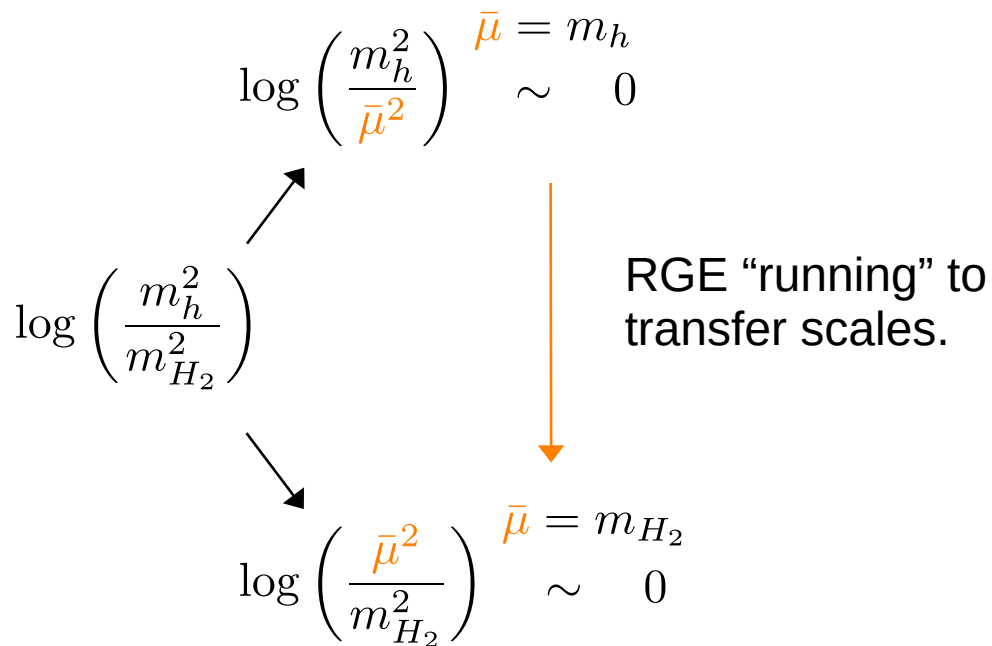


At higher orders... $\log\left(\frac{m_h^2}{m_{H_2}^2}\right)^n$

Separation of scales makes the perturbative calculations, in the **full theory**, *imprecise*.

We need EFT!

We can use the matching and running of EFT to “re-sum” these large logarithms.



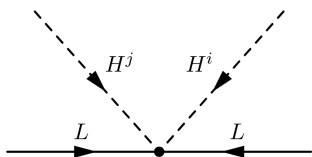
- Effective Field Theory (EFT) provides a framework to re-sum large logs.
- Requires *matching* and *running* between high and low scale theories.
- **An EFT treatment is required for state of the art precision in typical calculations of radiative mass models.**

Complete EFT Prescription

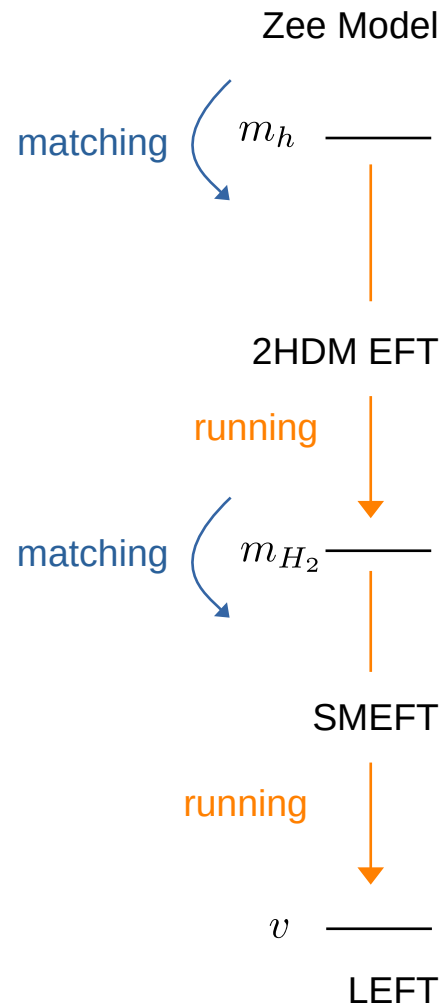
For concreteness, we assume the hierarchy $m_h \gg m_{H_2}$.
(a similar analysis could be done for the inverted choice)

This creates a ladder of **2 EFTs** between the UV scale of new physics and the Low Energy Effective Field Theory appropriate for scales where the measurement occurs.

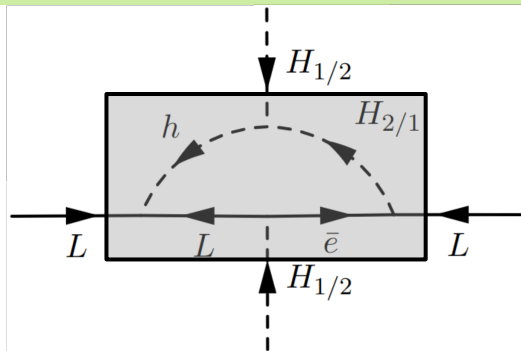
Our final goal is the dimension 5 Weinberg operator in the SMEFT which generates the neutrino mass matrix in the LEFT.



$$\mathcal{L}_{\text{SMEFT}} \supset \frac{1}{\Lambda} C_{\alpha\beta} (\bar{\tilde{L}}_{\alpha} H) (\tilde{H}^{\dagger} L_{\beta})$$



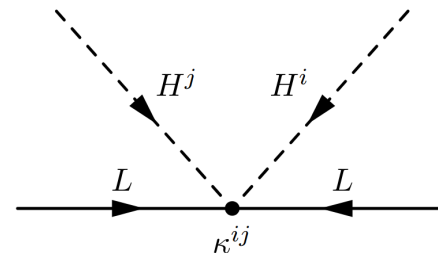
Matching (1-loop)



Expanded propagator with

$$p^2 \ll m_h^2$$

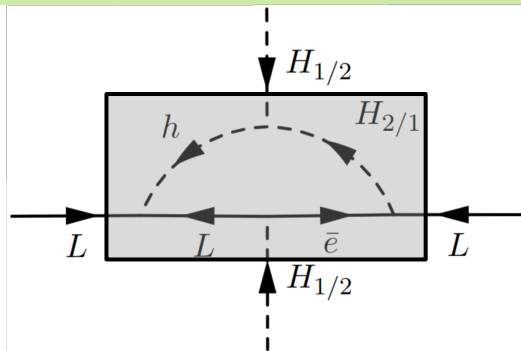
Systematic matching
with Matchete*.



Match **Zee model** to the **2HDM EFT**: $\mathcal{L}_{\text{2HDM EFT}} \supset \frac{1}{\Lambda} \kappa_{\alpha\beta}^{ij}(f, Y, Y_2, \dots) (\bar{\tilde{L}}_\alpha H^i) (\tilde{H}^{j\dagger} L_\beta)$

*Fuentes-Martín J., König M., Pagès J. et al. *A proof of concept for matchete: an automated tool for matching effective theories*. Eur. Phys. J. C 83, 662 (2023).

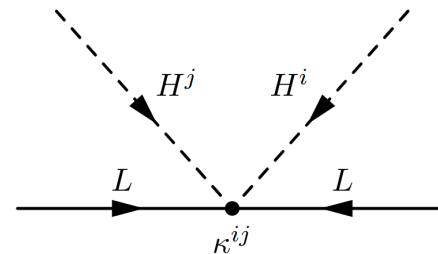
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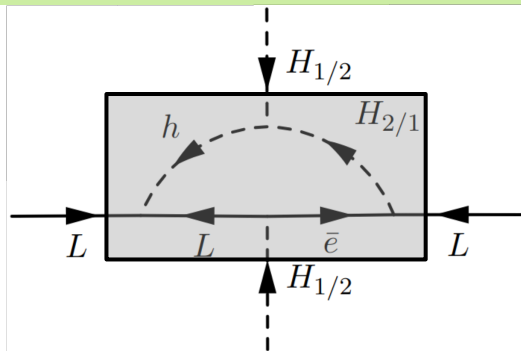
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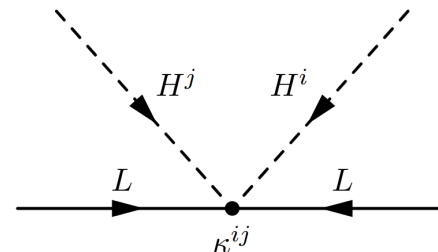
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Match **2HDM EFT** to the **SMEFT**: $\mathcal{L}_{\text{SMEFT}} \supset \frac{1}{\Lambda} C_{\alpha\beta}(\kappa, \dots) (\bar{L}_\alpha H) (\tilde{H}^\dagger L_\beta)$

If we neglect running and match these directly, we get a glimpse of the full theory result,

$$C^{(1\text{-loop})}(m_{H_2}) = \frac{1}{16\pi^2} \log\left(\frac{m_{H_2}^2}{m_h^2}\right) \frac{\mu_{\text{Zee}}}{m_h^2} (f Y_e^{1\dagger} Y_e^2 - Y_e^{2T} Y_e^{1*} f) \quad \begin{array}{l} \text{* cancellation of constant in } \left(1 + \log\left(\frac{m_{H_2}^2}{m_h^2}\right)\right) \\ \text{suggests large running contribution.} \end{array}$$

RG Running (1-loop*)

The dependence of parameters in a theory on the renormalisation scale is given by their respective beta-functions.

- **2HDM EFT**

Significant running occurs here between scales.

Facilitated by the relatively unconstrained Y_e^2 .

- **SMEFT**

Negligible running (for neutrino masses) here, although included for completeness.

* generally, for 1-loop matching, 2-loop running is required to ensure no renormalisation scheme dependence⁺. We anticipate such problems are avoided by their being only a *single* fermion bi-linear in the Weinberg operator.

Grimus W., Lavoura L. *Renormalization of the neutrino mass operators in the multi-Higgs-doublet standard model*. Eur. Phys. J. C 39, 219-227 (2005).

Thomsen, A. E. *Introducing RGBeta: a Mathematica package for the evaluation of renormalization group-functions*. Eur. Phys. J. C 81, 408 (2021).

⁺M.Bando, T. Kugo, N. Maekawa, H. Nakano, *Improving the effective potential*, Phys. Lett. B, 301 1 (1993).

Benchmarks Parameter Choices

We now have a system of **coupled diff. equations** given by the RG *running*.

We solve them with **boundary conditions** given by the *matching* (and *pheno*).

Benchmarks Parameter Choices

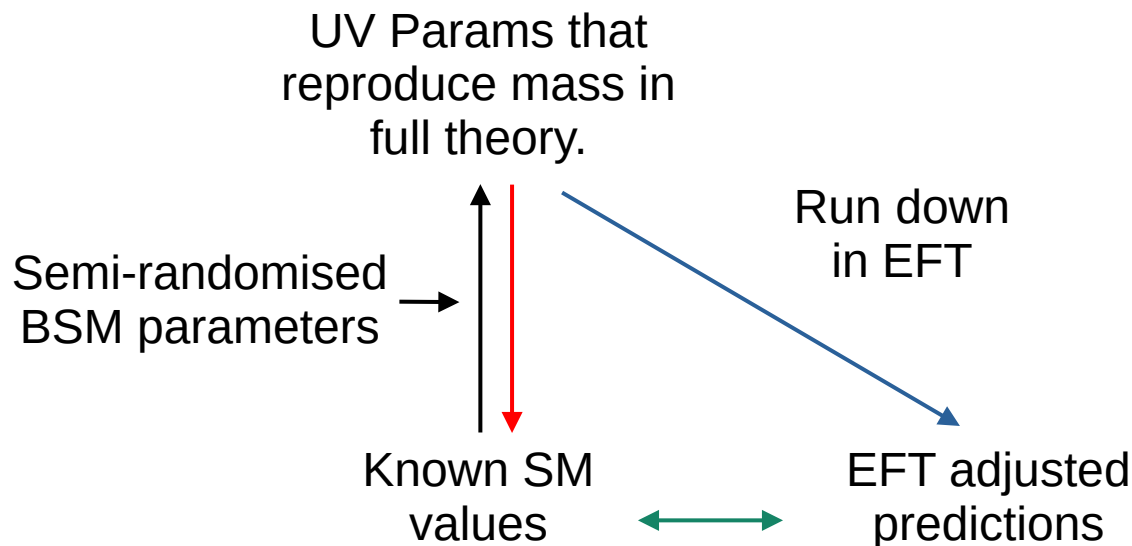
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$$m_h \sim 10^4 \text{TeV}$$

$$m_{H_2} \sim 10^2 \text{TeV}$$

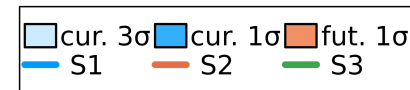
$$m_{\text{EM}} \sim 125 \text{GeV}$$



*Note on scales – RGEs and boundary conditions are scale independent, therefore running looks the same across scale choices in our analysis.

Results from EFT

$$\alpha \equiv Y_{u,d}^2 / Y_{u,d}^1$$



Rescaled parameter combinations that leave the *full theory* calculation *invariant* run independently in the EFT calculation.

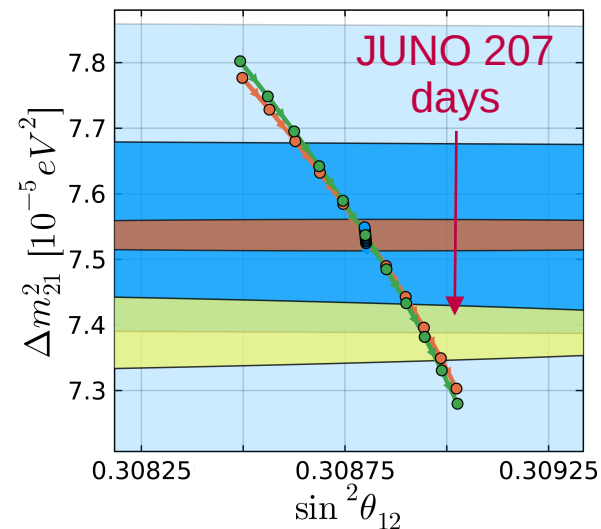
$$M_\nu^{\text{full}} \sim \left(f Y_e^{1\dagger} Y_e^2 + Y_e^{2T} Y_e^{1*} f^T \right)$$

$$\text{S1 : } f \rightarrow \gamma f, Y_e^{1,2} \rightarrow \gamma^{-1/2} Y_e^{1,2}$$

$$\text{S2 : } f \rightarrow \gamma f, Y_e^2 \rightarrow \gamma^{-1} Y_e^2$$

$$\text{S3 : } f \rightarrow \gamma^{-1} f, Y_e^1 \rightarrow \gamma Y_e^1$$

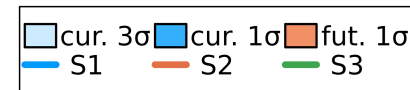
$\gamma \in [0.99, 1.01]$ along each the curve for each rescaling.



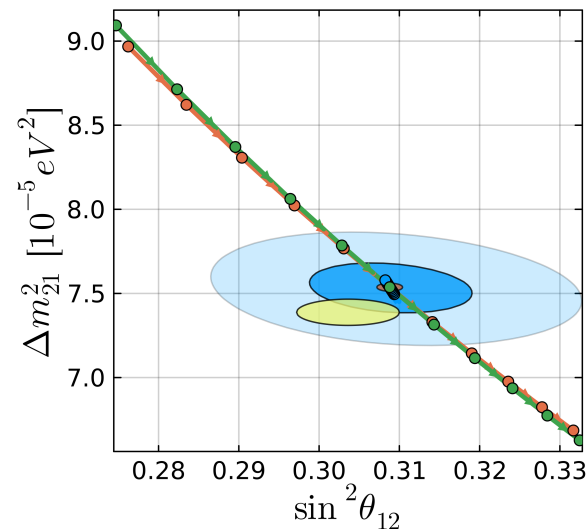
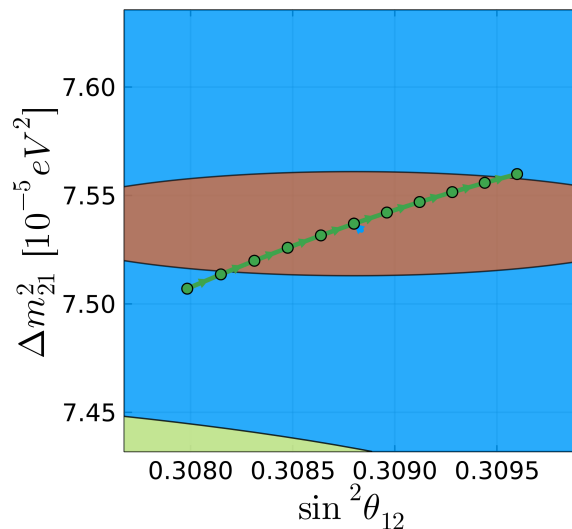
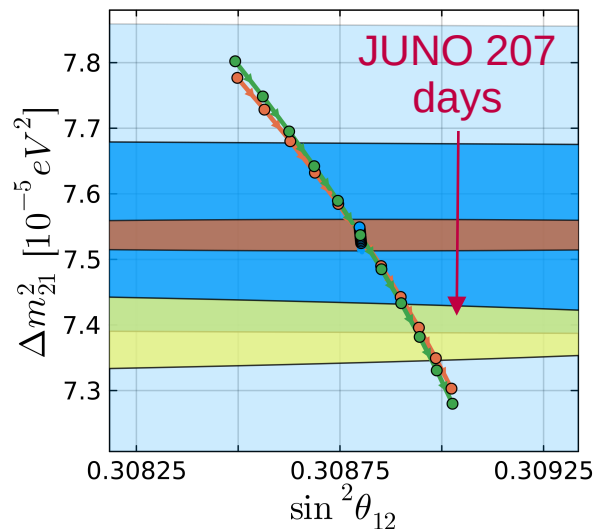
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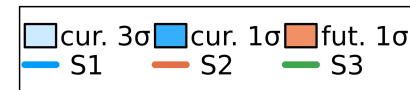
BM 1: $Y_e^2 \gg Y_e^1$ and $\alpha = 1$

BM 2: $Y_e^2 \sim Y_e^1$ and $\alpha = 1$

BM 3: $Y_e^2 \gg Y_e^1$ and $\alpha = 0$

The CP Phase

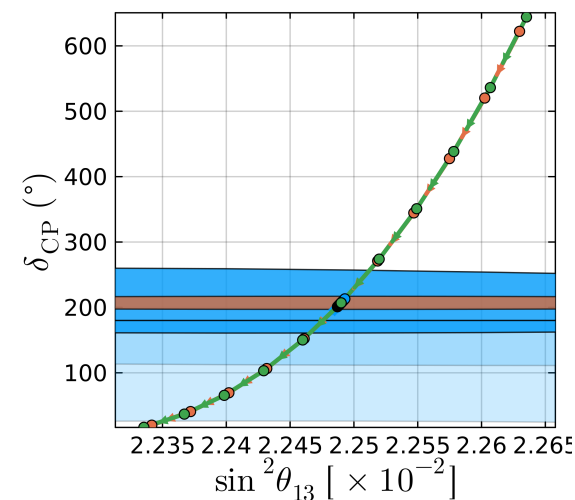
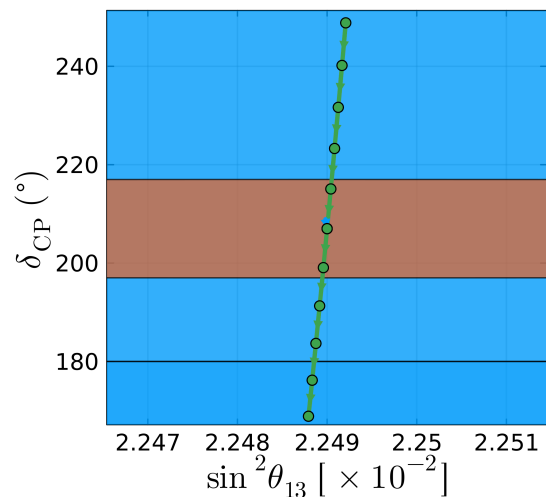
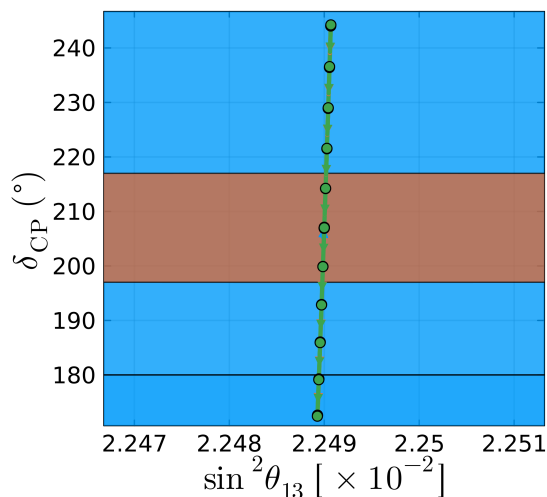
$$\alpha \equiv Y_{u,d}^2 / Y_{u,d}^1$$



CP phase is very sensitive to running. Now with only,

$$\gamma \in [0.999, 1.001]$$

HyperK, DUNE are predicting 5–15° degree precision in δ_{CP}^*

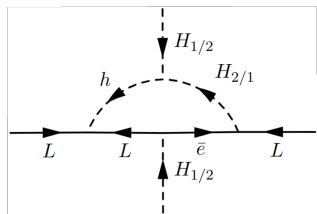


BM 1: $Y_e^2 \gg Y_e^1$ and $\alpha = 1$

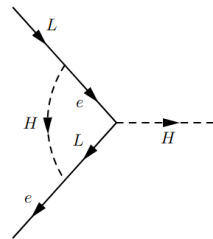
BM 2: $Y_e^2 \sim Y_e^1$ and $\alpha = 1$

BM 3: $Y_e^2 \gg Y_e^1$ and $\alpha = 0$

Generic Features



generates masses ,



generates running.

The loop-level mass leaves itself vulnerable to dependence on a large log term.

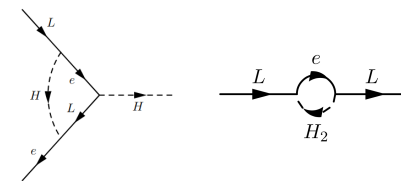
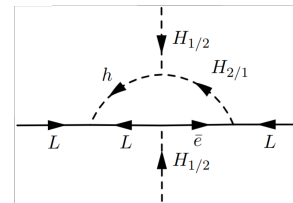
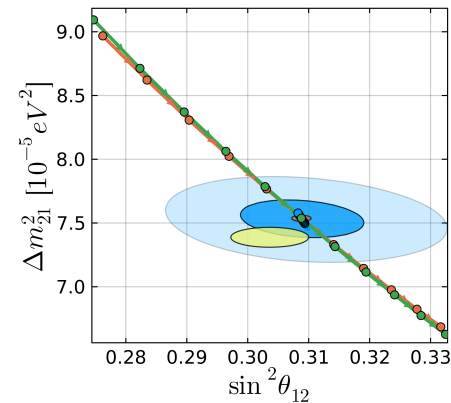
Here, this manifests after a cancellation of the constant +1 term in direct matching of the Weinberg operator.

$$C^{(1-\text{loop})}(m_{H_2}) = \frac{1}{16\pi^2} \log \left(\frac{m_{H_2}^2}{m_h^2} \right) \frac{\mu_{Zee}}{m_h^2} (f Y_e^{1\dagger} Y_e^2 - Y_e^{2T} Y_e^{1*} f)$$

RGEs will take the place of this log term, and effectively generate the low energy mass ‘themselves’.

Summary

- A full EFT framework for radiative models is likely required to match the precision of current and future neutrino mixing measurements (particularly in light of JUNO's first results).
- Current measurements of mixing parameters are sensitive to difference in full theory vs EFT for **~1% change in UV couplings**.
This becomes **~0.1% for the CP phase**.
- RG improvements can be significant in radiative mass models, where the running can act at the same order as the mass is generated (1-loop).
This effect is exaggerated when a strong cancellation in the bare matching isolates the large log term.



Backup Slides

Higgs parameters		quark masses
$m_{h_{\text{SM}}(\text{pole})} = 125.25 \text{ GeV}$		$m_u = 1.23 \text{ MeV}$
$\lambda = 0.139825$	$v = 246.604 \text{ GeV}$	$m_c = 0.621 \text{ GeV}$
lepton masses	gauge couplings	$m_t = 168.62 \text{ GeV}$
$m_e = 0.483246 \text{ MeV}$	$g_1 = 0.357266$	$m_d = 2.69 \text{ MeV}$
$m_\mu = 0.102017 \text{ GeV}$	$g_2 = 0.65096$	$m_s = 53 \text{ MeV}$
$m_\tau = 1.7329 \text{ GeV}$	$g_3 = 1.2123$	$m_b = 2.842 \text{ GeV}$

Table 1: SM running parameters in the $\overline{\text{MS}}$ scheme used as inputs for our numeric calculations. All quantities are at the scale of the top quark pole mass $m_t = 172.4 \text{ GeV}$ as calculated in [100] unless specified otherwise.

mass scales	coupling strengths
$\mu_{\text{Zee}} = m_h = 10^4 \text{ TeV}$	$\lambda_{8,9,10}^{(\text{Zee})} = 0$, $\lambda_h = 10^{-4}$
$m_{H_2} = 10^2 \text{ TeV}$	$\lambda_{1,\dots,7}$ (see table 3)

Table 2: Choice of UV scales and scalar potential couplings used in numeric calculations. The quartic coupling λ_h is chosen small but non-zero to guarantee stability of the potential for large field values of h despite having very little effect on the overall computation.

Benchmark Set 1	Benchmark Set 2
$Y_e^2 \gg Y_e^1, \alpha = 1, m_{H_2} = 100 \text{ TeV}, m_h = 10^4 \text{ TeV}$ $\lambda_{1,2} = 0.25, \lambda_{3,\dots,7} = 0$	$Y_e^2 \sim Y_e^1, \alpha = 1, m_{H_2} = 100 \text{ TeV}, m_h = 10^4 \text{ TeV}$ $\lambda_{1,2} = 0.25, \lambda_{3,\dots,7} = 0$
$f \simeq \begin{pmatrix} 0 & 7.2 \times 10^{-6} & 1.5 \times 10^{-5} \\ -7.2 \times 10^{-6} & 0 & 1.5 \times 10^{-9} \\ -1.5 \times 10^{-5} & -1.5 \times 10^{-9} & 0 \end{pmatrix}$	$f \simeq \begin{pmatrix} 0 & 0.0093 & 0.0017 \\ -0.0093 & 0 & 9.3 \times 10^{-7} \\ -0.0017 & -9.3 \times 10^{-7} & 0 \end{pmatrix}$
$Y_e^2(m_h) \simeq \begin{pmatrix} 0.36 - i0.24 & -0.11 + i0.089 & -0.080 - i0.052 \\ -0.099 - i0.11 & 0.21 + i0.42 & 0.11 - i0.026 \\ -0.11 + i0.11 & -0.0467 + i0.11 & -0.55 - i0.066 \end{pmatrix}$	$Y_e^2(m_h) \simeq \begin{pmatrix} -0.0053 + i0.0053 & 0.0053 + i0.0053 & -0.0053 + i0.0053 \\ 0.0050 + i0.0053 & 0.0054 - i0.0050 & 0.0053 - i0.00050 \\ -0.0026 - i0.0053 & 0.00020 + i0.00080 & -0.0048 - i0.0049 \end{pmatrix}$
Benchmark Set 3	
$Y_e^2 \gg Y_e^1, \alpha = 0, m_{H_2} = 100 \text{ TeV}, m_h = 10^4 \text{ TeV}$ $\lambda_{1,2} = 0.25, \lambda_{3,4,5} = 0, \lambda_6 = -\lambda_7 = 0.25$	
$f \simeq \begin{pmatrix} 0 & -1.4 \times 10^{-4} & 7.4 \times 10^{-6} \\ 1.4 \times 10^{-4} & 0 & 1.4 \times 10^{-8} \\ -7.4 \times 10^{-6} & -1.4 \times 10^{-8} & 0 \end{pmatrix}$	
$Y_e^2(m_h) \simeq \begin{pmatrix} -0.35 - i0.70 & 0.035 - i0.10 & -0.10 - i0.10 \\ 0.0096 + i0.032 & 0.72 - i0.12 & -0.027 - i0.056 \\ 0.072 - i0.10 & -0.10 + i0.10 & -0.20 - i0.19 \end{pmatrix}$	

UV parameter fitting

1. Begin with SMEFT values for $Y_{e,u,d}(m_{H_2})$ and $\lambda(m_{H_2})$.
2. Generate a random, complex matrix Y_e^2 (subject to flavour physics constraints, section 4.1).
3. Use $Y_e^2(m_{H_2})$ and $Y_e(m_{H_2})$ to determine $Y_e^1(m_{H_2})$ from the matching condition, equation (59).
4. Run Y_e^2 and $\lambda_{1,\dots,7}$ in the 2HDM EFT up to $\bar{\mu} = m_h$.
5. Generate random real parameters to construct $f(m_h)$.
6. Use the now full set of UV parameters to calculate the neutrino mass matrix in the full Zee Model, equation (20).
7. Repeat steps 2-6 in a root-finding algorithm to determine values for $Y_e^2(m_{H_2})$ and $f(m_h)$ that optimally reproduce the experimentally determined neutrino mixing parameters [88] in the full theory.

(some) Beta functions

$$\begin{aligned}
 16\pi^2 \frac{d}{dt} \kappa^{ij} = & -3g^2(2\kappa^{ij} - \kappa^{ji}) + 4 \sum_{k,l=1}^2 \kappa^{kl} \lambda_{kilj} \\
 & + 2 \sum_{k=1}^2 \left[\kappa^{kj} Y_e^{i\dagger} Y_e^k - (\kappa^{ik} + \kappa^{ki}) Y_e^{j\dagger} Y_e^k \right. \\
 & \quad \left. + Y_e^{kT} Y_e^{j*} \kappa^{ik} - Y_e^{kT} Y_e^{i*} (\kappa^{kj} + \kappa^{jk}) \right] \\
 & + \sum_{k=1}^2 [T_{ki} \kappa^{kj} + T_{kj} \kappa^{ik}] + \kappa^{ij} S + (S^T) \kappa^{ij} , \\
 \text{where, } S = & \frac{1}{2} \sum_{k=1}^2 Y_e^{k\dagger} Y_e^k \quad \text{and, } T^{ij} = \text{Tr} \left(Y_e^{i\dagger} Y_e^j + 3Y_u^{i\dagger} Y_u^j + 3Y_d^{i\dagger} Y_d^j \right) .
 \end{aligned}$$

$$16\pi^2 \frac{d}{dt} Y_e^i = \sum_{k=1}^2 \left(T_{ik} Y_e^k + Y_e^k Y_e^{k\dagger} Y_e^i + \frac{1}{2} Y_e^i Y_e^{k\dagger} Y_e^k \right) - \frac{9g_2^2 + 15g_1^2}{4} Y_e^i$$