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Electromagnetic instantons and asymmetric Hawking radiation of black holes

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[Archil Kobakhidze, EL, 2604.02841]

Non-trivial topology of the Schwarzschild manifold supports a physical θ -term for electromagnetism, and Euclidean dyon solutions which lead to an asymmetric flux of left- and right-polarisations in the Hawking radiation.

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Euclidean Schwarzschild manifold

Path integrals on the Black Hole background

We study pure electromagnetism on the background of a black hole

$$Z = \int \mathrm{D}A|_{\text{g.f.}} e^{iS[A, g_{\text{Sch.}}]}$$

In the limit of a large black hole, $G \rightarrow 0$, $M \rightarrow \infty$, $GM = \text{const.}$
(‘Hawking limit’ $\Rightarrow$ finite temperature + no backreaction).

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(‘Hawking limit’ $\Rightarrow$ finite temperature + no backreaction).

In this limit, we can Wick rotate

$$\longrightarrow \int \mathrm{D}A|_{\text{g.f.}} e^{-S_{\text{E}}[A, g_{\text{Sch.}}^{\text{E}}]}$$

$$\mathrm{d}s^2 = +\left(1 - \frac{r_s}{r}\right)\mathrm{d}\tau^2 + \left(1 - \frac{r_s}{r}\right)^{-1}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)$$

Euclidean Schwarzschild manifold

[Gibbons, Hawking, 1977]

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Regularity $\implies \tau \sim \tau + \beta$, $\beta = 8\pi GM = \frac{1}{T_H} \implies$ Thermality!

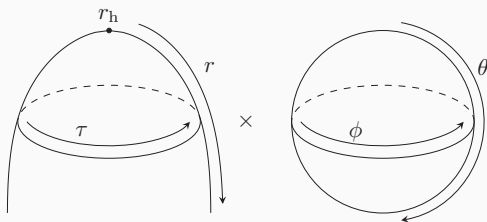
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This is the 'cigar' geometry



Path integrals on the Black Hole background

This path integral is still extremely hard to handle.

Leading term gives Gibbons–Hawking result,

$$Z \sim e^{-S_{\text{GHY}}}, \quad S \sim \frac{A_{\text{h}}^2}{4G}$$

Can we do more? (non-perturbatively?)

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Can we do more? (non-perturbatively?) **Yes**

We can compute the complete *electromagnetic* saddle-point approximation

$$Z \simeq \sum_{\text{solns.}} e^{-S}$$

Topology

Classical solutions?

$$Z \simeq \sum_{\text{sols?}} e^{-S_{\text{EM}}}, \quad S_{\text{EM}} = \frac{1}{2e^2} \int F \wedge \star F$$

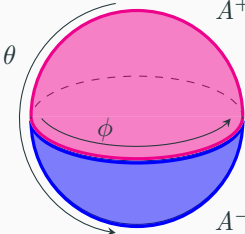
The equations of motion tell us that F is *harmonic*

$$d \star F = 0, \quad (dF = 0)$$

Hodge theory $\implies$ exactly one harmonic 2-form per cohomology class $H^2(M)$.

Wu–Yang monopole

Famously, the Wu–Yang monopole [Wu and Yang, 1975] of charge m ,

$$F = \frac{m}{2} \sin \theta \, d\theta \wedge d\phi$$

$$A^+ = \frac{m}{2}(+1 - \cos \theta)d\phi$$
$$A^+ - A^- = d\alpha, \quad \alpha = m\phi$$
$$A^- = \frac{m}{2}(-1 - \cos \theta)d\phi$$

Forcing $e^{i\alpha(\phi)} = e^{i\alpha(\phi+2\pi)}$ to be single-valued recovers the Dirac quantisation condition $m \in H^2(S^2) \cong \mathbb{Z}$.

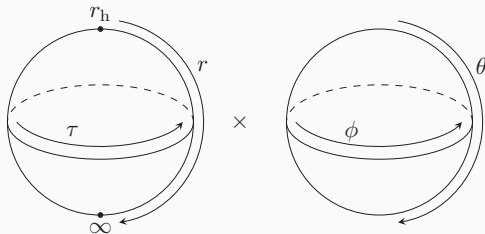
Compactified Euclidean Schwarzschild manifold

We need *finite* action, $\int F \wedge \star F < \infty$. On asymptotically locally flat spaces, these “ L^2 ” harmonic forms can be classified by compactifying $M \rightarrow M \cup \{\infty\}$. [Hausel et al, 2003, math/0207169v2]

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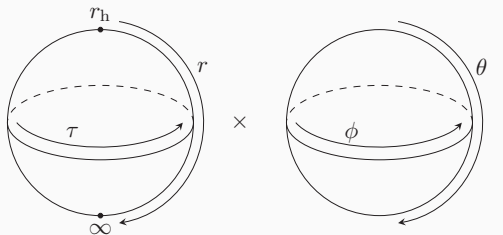
This changes the topology to $S^2 \times S^2$.



Compactified Euclidean Schwarzschild manifold

The harmonic forms then correspond to

$$H^2(S^2 \times S^2) \cong H^2(S^2) \times H^2(S^2) \cong \mathbb{Z} \oplus \mathbb{Z}$$



electric charge n

magnetic charge m

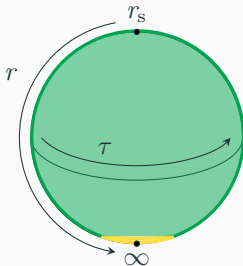
Dual Dirac quantisation

Electric charges have an analogous Wu–Yang patching

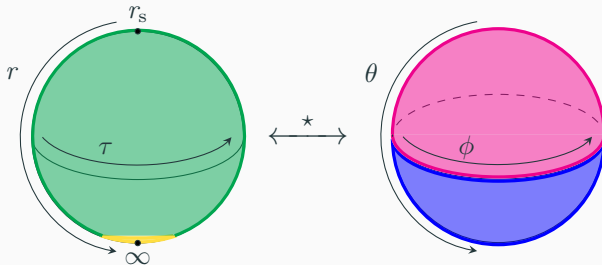
$$A_{\tau}^{r < \infty} = -\frac{n}{2r_s} \left(1 - \frac{r_s}{r}\right), \quad A_{\tau}^{\infty} = 0$$

Gauge transformation $\alpha = \frac{n}{2r_s}\tau$ at infinity.

Matching $\alpha(\tau) \sim \alpha(\tau + \beta) \implies n \in \mathbb{Z}$ (dual Dirac quantisation).



The electromagnetic (Hodge) duality swaps the two spheres, swapping the charges.



$$\star\left(\frac{1}{2r^2}d\tau \wedge dr\right) = \frac{1}{2}\sin\theta d\theta \wedge d\phi$$

All solutions to the Maxwell equations (harmonic 2-forms) are integer charge dyons, $(n, m) \in \mathbb{Z} \oplus \mathbb{Z}$

$$F = \frac{n}{2r^2} d\tau \wedge dr + \frac{m}{2} \sin \theta d\theta \wedge d\phi$$

see also Etesi and Hausel, 2001 [hep-th/0003239]

Semiclassical thermodynamics

Return to the saddle-point approximation

$$Z = \int \mathrm{D}A|_{\text{g.f.}} e^{-S_{\text{EM}}} \simeq \sum_{n,m} e^{-S_{\text{EM}}}, \quad S_{\text{EM}} = \frac{2\pi^2}{e^2}(n^2 + m^2)$$

(factor out gravity).

Thermal correlators

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The energy-momentum tensor and physical electric and magnetic charges are undisturbed

$$\langle T_{\mu\nu} \rangle \simeq \frac{1}{Z} \sum_{n,m} T_{\mu\nu} e^{-S_{\text{EM}}} \propto \frac{1}{Z} \sum_{n,m} (m^2 - n^2) \exp\left[-\frac{2\pi^2}{e^2}(n^2 + m^2)\right] = 0$$

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¹(Anti-)self-dual solutions $n = \pm m$ have $T_{\mu\nu} = 0$, so solve gravitational EoM too and are full Euclidean solutions.

Is anything different?

The Pontryagin ('instanton') number

$$P = \frac{1}{8\pi^2} \int F \wedge F = \frac{1}{32\pi^2} \int d^4x F_{\mu\nu} \tilde{F}^{\mu\nu}$$

Is always zero on flat space. This is CP odd, and analogous to the instanton number in QCD.

However, on the Euclidean Schwarzschild manifold

$$P = \frac{1}{8\pi^2} \int F \wedge F = nm$$

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CP odd!

The *chiral photon current* $J_h = \star(A \wedge F)$ counts the flow of right-minus left-handed polarised photons. It's divergence is

$$d \star J_h = F \wedge F$$

In the thermal ensemble this has a net flux

$$\langle \Phi_h \rangle = \int_{r \rightarrow \infty} \langle J_h^r \rangle r^2 d\Omega = -32\pi^2 i T_H e^{-4\pi^2/e^2} \sin \theta_{EM}$$

$\implies$ Chiral asymmetry in the Hawking photon flux!

Further directions: realistic
models and phenomenology

N.B. Unlike in QCD, these are *not* a winding of the vector potential A , superposing in the vacuum, but rather windings of F itself.

$$\hat{G}_n^{\text{large}}|0\rangle = |n\rangle, \quad |\theta\rangle = \sum_n e^{-in\theta} |n\rangle$$

Our $F \neq 0$ are not degenerate classical vacuum solutions!

No vacuum interpretation for θ_{EM} seems available?

Non-abelian embeddings

QCD? $EM \subset EW \subset GUT$?

- We can embed these dyons into any non-abelian theory, in one generator
- θ_{EM} descends from $\theta_{\text{non-abelian}}$.
- + New essentially non-abelian modes with continuous moduli; not purely topological.²

²For an unbroken $SU(2)$ theory see [Charap and Duff, 1977] (These include the abelian modes—see Etesi and Hausel, 2001 [hep-th/0003239])

Fermions

By the Atiyah–Singer index theorems [Atiyah and Singer, 1975], there are fermionic zero modes on the dyon background. These lead to

1. The fermionic chiral anomaly $\partial_\mu J_A^\mu \propto F_{\mu\nu} \tilde{F}^{\mu\nu}$

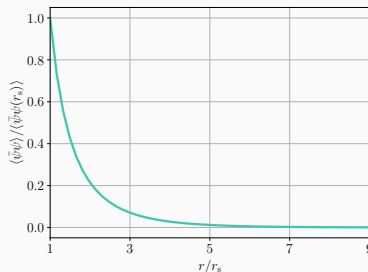
(c.f. monopoles
[Callan, 1982])

2. A chiral condensate

$$\langle \bar{\psi}\psi \rangle \propto e^{-4\pi^2/e^2} \frac{e^{-r/2r_s}}{(r/r_s)^{3/2}} \cos \theta_{\text{EM}}$$

localised around the black hole (*massless limit*).

The phase of the condensate acts as an axion, screening θ_{EM} at scales $\lesssim 1/m$.



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...but is lifted by the running of e around microscopic black holes

$\Rightarrow$ Large effects with a population of $\sim$ GUT scale primordial black holes (PBHs)?

$\Rightarrow$ Evaporate to leave an excess chiral charge ($\Rightarrow$ Baryogenesis?)

Quantum gravity?

Also the possibility of **virtual** black holes? [Hawking, 1996]

Here semi-classicality is a bad assumption.

Similar results have been recently proposed for virtual wormholes [Garcia Garcia and Maderazo, 2512.13833]

Black Holes and Abelian Instantons

Isabel Garcia Garcia and Elliot Maderazo

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ABSTRACT: We argue that the electromagnetic θ -term is a physical parameter of the Standard Model coupled to gravity. Specifically, in the context of 4-dimensional Einstein-Maxwell

We show that **quantum gravity is unnecessary.**

Axions couple to the local density

$$\langle F_{\mu\nu} \tilde{F}^{\mu\nu} \rangle \propto \frac{1}{r^4} e^{-4\pi^2/e^2} \sin \theta_{\text{EM}}$$

This lifts $\langle a \rangle$ away from $\bar{\theta}_{\text{QCD}}$, leading to CP violations in QCD near the black hole.

Without QCD effects, $\langle a \rangle \propto \frac{1}{r} \sin \theta_{\text{EM}}$.

$\Rightarrow$ Modification of early universe axion cosmology in the presence of PBHs?

$\Rightarrow$...or even just virtual black holes

Semiclassical black hole+quantum electromagnetism supports

- Euclidean thermal spectrum of dyons with non-zero $\int F \wedge F \propto nm$,
- a physical θ -term for electromagnetism activated by gravitational background,
- an asymmetry flux of left- and right- polarisations in the Hawking radiation.

$$\langle \Phi_h \rangle = \int_{r \rightarrow \infty} \langle J_h^r \rangle r^2 d\Omega = -32\pi^2 i T_H e^{-4\pi^2/e^2} \sin \theta_{\text{EM}}$$

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Thank you!

Backup slides

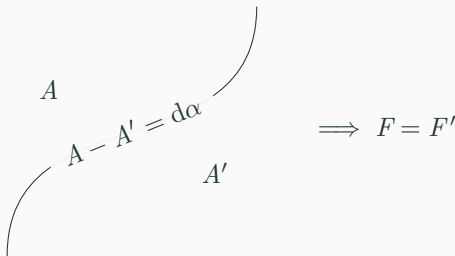
Electromagnetism and forms

The Electromagnetic field strength

$$F = \frac{1}{2} F_{\mu\nu} dx^\mu \wedge dx^\nu$$

is a **closed** 2-form, $dF = 0$.

Locally, it can be written as the exterior derivative of a 1-form potential, $F = dA$, but A may have some non-trivial patching by local gauge transformations $A \rightarrow A + d\alpha$.

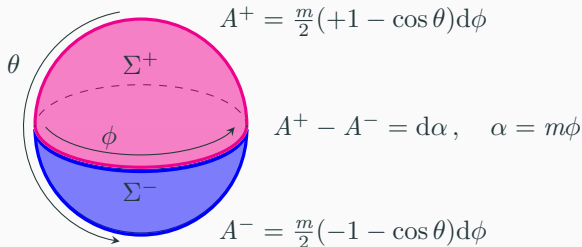

$$\begin{array}{c} A \\ \text{---} A - A' = d\alpha \text{---} \\ A' \end{array} \implies F = F'$$

Wu-Yang monopole

In fact, the magnetic charge exactly measures the winding of this local gauge transformation

$$\frac{1}{2\pi} \int_{\Sigma} F = \frac{1}{2\pi} \int_{\Sigma^+} dA^+ + \frac{1}{2\pi} \int_{\Sigma^-} dA^- = \frac{1}{2\pi} \int_{\partial\Sigma^+} (A^+ - A^-) = m$$

For some Gaussian 2-surface Σ , with north Σ^+ and south Σ^- parts.



Cohomology classes

The failure for F to be exact (*A global*) is measured by the *cohomology class* of F

$$H^2(M) \equiv \frac{\text{closed 2-forms}}{\text{exact 2-forms}}$$

For the monopole

$$H^2(S^2) \cong \mathbb{Z}$$

labelled by magnetic charge.

Labels of cohomology groups

The charges are, dually

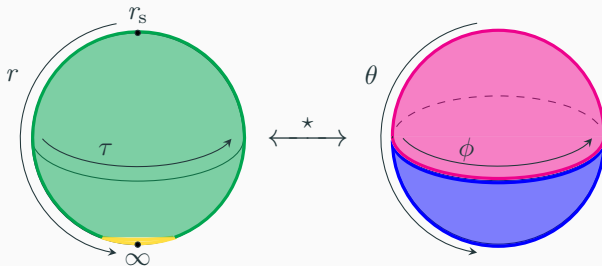
$$n = \frac{1}{2\pi} \int_{\Sigma} \star F,$$

$$n = \frac{1}{2\pi} \int_T F,$$

$$m = \frac{1}{2\pi} \int_{\Sigma} F,$$

$$m = \frac{1}{2\pi} \int_T \star F,$$

Taking the dual in either cohomology (swapping $F \leftrightarrow \star F$) or homology ($\Sigma \leftrightarrow T$) where Σ wraps around $S^2_{(\theta,\phi)}$ and T around $S^2_{(\tau,r)}$



QFT at finite temperature is equivalent to the corresponding Euclidean theory, in periodic imaginary time of period $\beta = \frac{1}{T}$

$$Z = \int_{\Phi(0,\mathbf{x})=\Phi(\beta,\mathbf{x})} \mathrm{D}\Phi \, e^{-S_{\mathrm{E}}[\Phi]}$$

Pontryagin number factorisation

Because F splits into orthogonal electric and magnetic parts

$$F = F_e(r, \tau) + F_m(\theta, \phi), \quad F \wedge F = 2F_e \wedge F_m$$

and the Pontryagin charge P factorises into the product of quantised electric and magnetic charges,

$$\begin{aligned} P &\equiv \frac{1}{8\pi^2} \int_{\mathcal{M}} F \wedge F = \frac{1}{4\pi^2} \int_{\mathcal{M}} F_e \wedge F_m \\ &= \left(\frac{1}{2\pi} \int_{S^2_{(\tau, r)}} F_e \right) \left(\frac{1}{2\pi} \int_{S^2_{(\theta, \phi)}} F_m \right) = nm. \end{aligned}$$

Time evolution?

It may seem like these solutions lack time evolution, so don't contribute to any 'tunnelling'-like effects? Two considerations

1. They are not vacuum-vacuum transitions anyway
2. The same appears in thermal instantons in flat space!
There are gauges where they can be made completely smooth and *periodic*.

$A_0 = 0$ gauge can be taken locally to recover an tunnelling interpretation [Dunne and Tekin, 2001, hep-th/0011169v3], but this misses essential topological modes (namely, non-trivial holonomy $\int d\tau A_0$).

See [Kobakhidze, EL, 2606.XXXXX]

Energy-momentum tensor components

The dyons have energy-momentum tensor $T_{\mu\nu} = (m^2 - n^2)t_{\mu\nu}$, where

$$\begin{aligned}t_{\tau\tau} &= \frac{1}{2r^4} \left(1 - \frac{r_s}{r}\right), & t_{rr} &= \frac{1}{2r^4} \left(1 - \frac{r_s}{r}\right)^{-1}, \\t_{\theta\theta} &= -\frac{1}{2r^2}, & t_{\phi\phi} &= -\frac{\sin^2 \theta}{2r^2},\end{aligned}$$

In particular, the (anti-)self-dual dyons $n = \pm m$ have $T_{\mu\nu} = 0$.

No continuous moduli

Discrete cohomology group $\implies$ no continuous moduli.

c.f. e.g. abelian instantons around monopoles [Csáki et al, 2406.13738]

Abelian Instantons and Monopole Scattering

Csaba Csáki,^a Rotem Ovadia,^b Ofri Telem,^b John Terning^c and Shimon Yankielowicz^d

- Unlike non-Abelian instantons, the vortex part of Abelian instantons is not a vacuum solution of the Euclidean EOMs. Rather, it is the saddle-point of the gauge path integral *in the presence of external sources*. The latter appears in correlators over electrically charged fermions in a monopole background. Moreover, Abelian instantons have no moduli space and **no collective coordinates**, as they are “pinned” to the sources that generate them.