

Emergent particle states in electroweak theory and gravity

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[2406.18402](#), [2408.07535](#), [2509.16043](#) + work in progress

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New physics from topology of vacuum states

- The global (topological) properties of the quantum vacuum in gauge theories induce novel non-perturbative interactions and influence the particle state spectrum, implying the existence of *light emergent scalar particle states*
- Within the Standard Model we predict a new emergent pseudoscalar particle state, η_w , associated with the spontaneous breaking of anomalous $U(1)_{B+L}$ symmetry
- Gravity plays important role in shaping the vacuum state => dynamical supersymmetry breaking => emergent dilaton, axion and goldstino

Topology, θ -vacua, anomalies and condensates: general considerations

- Consider SU(2) pure Yang-Mills theory (prototype for QCD and the EW sectors of the standard model or even GR)

$$\mathcal{L} = -\frac{1}{2g^2} \text{Tr} F_{\mu\nu} F^{\mu\nu} + \frac{\theta}{16\pi^2} \text{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu], \quad A_\mu = A_\mu^a \sigma^a / 2, \quad \tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

- θ -term is topological:

$$\text{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu} \equiv \partial_\mu K^\mu, \quad K^\mu = \frac{1}{6} \epsilon^{\mu\nu\rho\sigma} C_{\nu\rho\sigma}$$

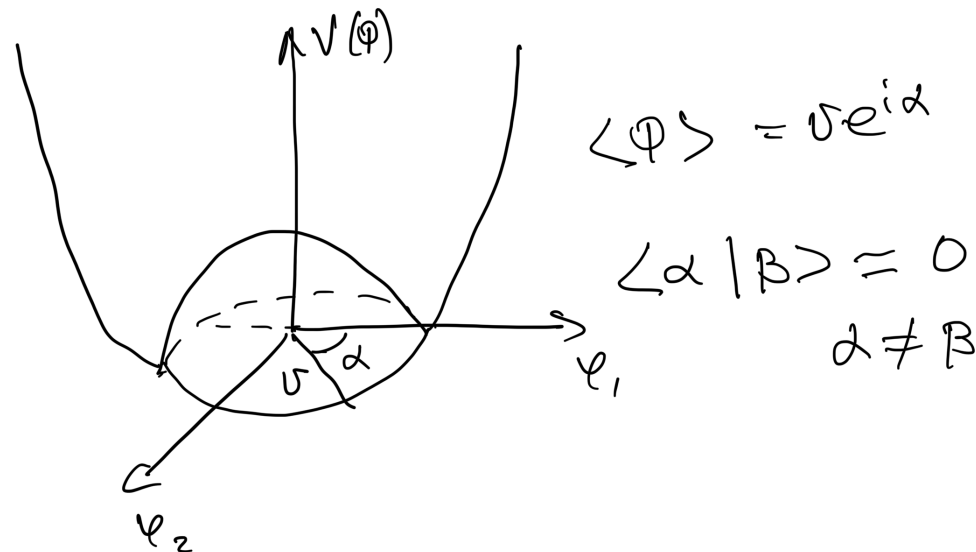
$$C_{\nu\rho\sigma} = \text{Tr} \left(A_{[\nu} \partial_\rho A_{\sigma]} - \frac{2i}{3} A_{[\nu} A_\rho A_{\sigma]} \right) - \text{Chern-Simons 3-form}$$

Topology, θ -vacua, anomalies and condensates: general considerations

- Fock space of states
disjoint union of θ sectors

$$\mathcal{F} = \coprod_{\theta \in \mathbb{R}} \mathcal{F}_\theta$$

- Similar to vacua in spontaneous symmetry breaking



Topology, θ -vacua, anomalies and condensates: general considerations

- Add (spin-1/2) fermions:

$$Z = \int \mathcal{D}A_\mu^{(\nu)} \text{Det} \left(\not{D}^{(\nu)} + M \right) \exp \left(-S[A] + \int d^4x \frac{i\theta}{16\pi^2} \text{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu} \right)$$

- If the Dirac operator admits zero-mode solutions

$$\left(\not{D}^{(\nu)} + M \right) \Psi_0^{(\nu)} = 0$$

the vacuum transitions with $\nu \neq 0$ are halted and θ becomes unobservable.

Topology, θ -vacua, anomalies and condensates: general considerations

- Fermion zero-modes are mandated by the topological index theorem

$$\partial_\mu \tilde{J}_A^\mu = \frac{2N}{16\pi^2} \text{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

Atiyah and Singer '63

$$\tilde{Q}_A(t = +\infty) - \tilde{Q}_A(t = -\infty) \equiv n_+ - n_- = 2N\nu$$

For $\nu \neq 0$, the Dirac operator exhibits spectral asymmetry in zero modes

Topology, θ -vacua, anomalies and condensates: general considerations

- Anomalous $U(1)_A$ symmetry must be spontaneously broken

$$e^{i\alpha Q_A/2N} \in U(1), \quad \left[Q_A = \int d^3\vec{x} (\tilde{J}_A^0 + 2K^0) \right]$$

- Action on the vacuum state:

$$e^{i\alpha Q_A/2N} |\theta\rangle = |\theta + \alpha\rangle$$

$$\langle \theta | \theta + \alpha \rangle = 0 \implies Q_A |\theta\rangle \neq 0$$

Anomalous symmetry is spontaneously broken

Topology, θ -vacua, anomalies and condensates: general considerations

- The order parameter for spontaneous $U(1)_A$ breaking is given by a local composite fermion operator that carries each of the fermion zero modes (the local 't Hooft operator):

$$\langle \theta | \det \left(\bar{\Psi}_i(x) \Psi_j(x) \right) | \theta \rangle = \text{const} \neq 0$$

- Spectral asymmetry of the Dirac operator in vacuum is materialised as the vacuum condensate of the fermionic composite operator
- Superselection of θ -sectors is lifted by the corresponding Goldstone

Topology, θ -vacua, anomalies and condensates: general considerations

- Naïve application of the Goldstone theorem would imply the existence of a massless particle (phase field of the order parameter)
- No massless pole in this case, but a massive (pseudo)Goldstone state:
 - θ -vacuum implies long-range (non-propagating) gauge interactions, hence spontaneous breaking of an anomalous symmetry is realised in the Higgs phase, i.e., a massive propagating dof.

Dvali '05

θ -vacua in General Relativity

- Energy/action is not positive definite
- Eigenvalues of the Dirac operator and topological properties of vacua can be affected by the topology of curved manifolds and their causal structure
- Positive energy/action theorem: The manifolds that are asymptotically Ricci flat ($R_{\mu\nu} = 0$) and admit spinors have positive energy/action and well-defined S-matrix:

$$S_{GR} \geq 0$$

Schoen and Yau '79
Witten '81

θ -vacua in General Relativity

$$S_{GR} = 0 \implies \begin{cases} \eta_{\mu\nu}, & \text{flat spacetime with trivial topology} \\ R_{\mu\nu\rho\sigma} = \pm \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} R^{\alpha\beta}{}_{\rho\sigma}, & \text{ALE gravitational instantons} \end{cases}$$

Eguchi and Hanson '78

Gibbons and Hawking '78

- Topological invariants

$$\tau = \frac{1}{24\pi^2} \int_M \text{Tr} R \tilde{R} + \text{b.terms} = -1 \text{ (EH instanton)}$$

$$\chi = \frac{1}{64\pi^2} \int_M \epsilon_{\mu\nu\rho\sigma} \epsilon_{\alpha\beta\gamma\delta} R^{\mu\nu\alpha\beta} R^{\rho\sigma\gamma\delta} + \text{b.terms} = 2 \text{ (EH instanton)}$$

- Effective GR action: $\int d^4x \frac{\sqrt{-g} M_P^2}{2} R + \theta_g \tau + c\chi$

θ -vacua in General Relativity

- Index of the Dirac operator $I_{1/2} = 0$ for all ALE instantons – no spin 1/2 zero modes!
- Spin 3/2 fermions, $I_{3/2} \neq 0$ (= 2 for EH instanton)
- Hence to nullify θ_g – vacua we must incorporate spin 3/2 fermion => promote GR to **Supergravity**(!)

QCD in the chiral symmetry limit

- In direct analogy with the Schwinger model in (1+1)d:

$$\theta - \text{term} : \quad \frac{\theta}{\pi} \tilde{F} = \frac{\theta}{2\pi} \epsilon^{\mu\nu} F_{\mu\nu} \quad \longleftrightarrow \quad \frac{\theta}{16\pi^2} \text{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu} = \frac{\theta}{16\pi^2} \frac{1}{4!} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma}$$

$$\text{Topological gauge field} : \quad A_\mu \quad \longleftrightarrow \quad C_{\mu\nu\rho}$$

$$\text{Spontaneous } U(1)_A \text{ breaking} : \quad \langle \bar{\psi}\psi \rangle \quad \longleftrightarrow \quad \text{quark condensate}$$

$$\text{Higgs effect} : \quad \text{massive } A_\mu \quad \longleftrightarrow \quad \text{massive } C_{\mu\nu\rho} \equiv \text{massive } \eta' \text{-meson}$$

- θ is unobservable in chiral QCD. “Massless quark” solution to the strong CP problem = η' as axion

Dvali '05

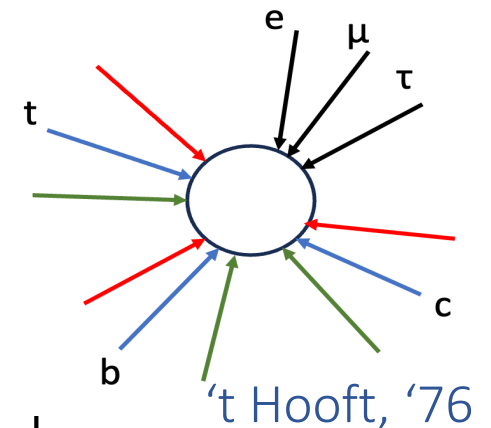
The electroweak η_w of the Standard Model

Dvali, AK, Sakhelashvili, [2408.07535](#)

- In the Standard Model $U(1)_{B+L}$ is an exact symmetry in the classical approximation and explicitly broken by quantum anomaly

$$\partial_\mu J_{B+L}^\mu = -\frac{3}{16\pi^2} W_{\mu\nu}^a \tilde{W}^{a\mu\nu} + (\text{hypercharge})$$

$$\Delta Q_{B+L} = 3\nu$$



- $U(1)_{B+L}$ must be broken also spontaneously, the order parameter being 't Hooft's local composite operator comprising of 12 fermionic (quark and lepton) operators. The phase field of the order-parameter is an emergent dof, η_w .

The modified Goldstone theorem

- Apply the Goldstone theorem with the anomaly contribution:

$$\delta(\Psi^+ \Gamma_5 \Psi) = 2i\Psi^+ \Psi$$

$$\int d^4x \left\langle \left(i\mu\Psi^+ \Gamma_5 \Psi - \frac{\alpha}{4\pi} W\tilde{W} \right) (x) , \Psi^+ \Gamma_5 \Psi(0) \right\rangle = i\langle \Psi^+ \Psi \rangle \neq 0.$$

$$\int d^4x \langle W\tilde{W}(x) , \Psi^+ \Gamma_5 \Psi \rangle_{p=0} \propto \langle \Psi^+ \Psi \rangle \implies \langle vac | W\tilde{W} | \eta \rangle = B(p) \neq 0$$

$$\langle \eta | \Psi^+ \Gamma_5 \Psi | vac \rangle = C(p) \neq 0$$

- Electroweak 3-form is Higgsed (hence θ_{ew} is unobservable):

$$FT \langle C^{(CS)}, C^{(CS)} \rangle = \frac{|B(0)|^2}{p^2 - m_\eta^2} + \dots$$

Supergravity breaking via gravitino condensate

Dvali, AK, Sakhelashvili, [2406.18402](#)

- N=1 pure Supergravity – $g_{\mu\nu}, \psi_\mu + (\text{auxiliary fields})$
- ALE gravitational instantons do not respect supersymmetry:

$$\psi_i = \psi_i^t + D_i \lambda \quad (\text{gauge } \psi_0, \gamma^\mu \psi_\mu = 0)$$
$$\gamma^i D_i \lambda = 0$$

- The necessary condition for supercharges

$$Q_\alpha = - \oint dS_i D^i \lambda_\alpha, \quad \bar{Q}^{\dot{\alpha}} = - \oint dS_i D^i \bar{\lambda}^{\dot{\alpha}}$$

is the existence of normalisable solutions to

$$D_i \lambda = D_i \bar{\lambda} = 0$$

- No normalisable spin 1/2 zero-mode solutions in ALE gravitational instantons -> supersymmetry must be broken!

Supergravity breaking via gravitino condensate

- Global U(1) R-symmetry $\psi_\mu \rightarrow e^{i\alpha} \psi_\mu$

is classically exact, broken by gravitational anomaly

$$\partial_\mu J_R^\mu \propto R\tilde{R}$$

- Scale transformations (dilatations), also anomalous:

$$T_\mu^\mu \propto \tilde{R}\tilde{R}$$

- γ -trace supercurrent anomaly:

$$\gamma^\mu S_\mu \propto R^{\mu\nu\rho\sigma} [\gamma_\rho, \gamma_\sigma] D_{[\mu} \psi_{\nu]}$$

$$X = \left(x, \gamma^\mu S_\mu, \frac{2}{3} T_\mu^\mu + i \partial_\mu J_R^\mu \right) \text{ S. Ferrara and B. Zumino, '75}$$

Supergravity breaking via gravitino condensate

- Gravitino condensate

$$\langle \psi_\mu \sigma^{\mu\nu} \psi_\nu \rangle \neq 0.$$

spontaneously breaks scale invariance, axial R-symmetry and global supersymmetry

- Emergent Goldstone supermultiplet (dilaton, R-axion, goldstino)

$$\phi, \eta_R, \lambda$$

Summary

- θ -vacuum structure and spectral asymmetry of the Dirac operator (anomaly), implies that anomalous symmetry is broken spontaneously
- The breaking is realised as the Higgs effect for a topological 3-form gauge field, rendering θ unobservable and resulting in an emergent pseudo-scalar particle
- In the case of an additional explicit violation of anomalous symmetry, the condensate persists (the spectrum of the Dirac operator is still asymmetric), with corrections from explicit breaking.
- In the electroweak sector of the Standard Model, unobservability of the θ parameter is accompanied by the emergence of a particle state, η_W .
- Dynamical supersymmetry breaking in the simplest supergravity theory => emergent dilaton, R-axion and Goldstino.

Topology, θ -vacua, anomalies and condensates: general considerations

- θ -term results from topological properties of the ground (vacuum) state that is gauge-invariant and complies with the causality (cluster decomposition) properties of the theory
- Topology of vacuum states :

$$A_0 = 0, \quad A_i = g\partial_i g^{-1}, \quad g(\vec{x}) = e^{i\alpha^a(\vec{x})\sigma^a/2} \in SU(2)$$

Jackiw and Rebbi '76

Callan, Dashen and Gross '76

$$g(\vec{x}) : S^3 \rightarrow SU(2), \quad \pi_3(SU(2)) = \mathbb{Z}$$

$$n = \frac{1}{16\pi^2} \int d^4x \text{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu} = -\frac{g^3}{96\pi^2} \int d^3\vec{x} \epsilon_{0ijk} \epsilon^{abc} A_i^a A_j^b A_k^c$$

Topology, θ -vacua, anomalies and condensates: general considerations

- The ground state

$$|n\rangle = \int \mathcal{D}\alpha^a(\vec{x}) |e^{i\alpha^a(\vec{x})\sigma^a/2} A_i^a(n)\rangle$$

- Invariant under “small” gauge transformations [$\alpha^a(\vec{x})$ is smooth on S^3]
- “Large” gauge transformations: $|n\rangle \rightarrow |n + \nu\rangle$ [ν winding #]
- Quantum transition between different $|n\rangle$ vacua are ‘mediated’ by instantons (not energy eigenstates)

Belavin, Polyakov, Schwarz and Tyupkin ‘75

Fully gauge invariant ground state:

$$|\theta\rangle = \sum_n e^{i\theta n} |n\rangle, \quad H|\theta\rangle = E_\theta |\theta\rangle$$

3-form gauge theory formulation of θ -vacua

$$\theta \text{Tr} F \tilde{F} \rightarrow \theta \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma} \quad \text{Dvali '05}$$

$$F_{\mu\nu\rho\sigma} = \partial_{[\mu} C_{\nu\rho\sigma]}, \quad C'_{\nu\rho\sigma} = C_{\mu\nu\rho} + \partial_\nu \omega_{\rho\sigma} + \partial_\sigma \omega_{\nu\rho} + \partial_\rho \omega_{\sigma\nu}$$

- Gauge redundancy $\rightarrow$ no propagating dof
- Nevertheless, one observes a long-range Coulomb-type correlation:

$$\text{F.T.} \langle \theta | C_{\mu\nu\rho}(x), C_{\alpha\beta\gamma}(0) | \theta \rangle_{p \rightarrow 0} \propto \frac{\rho(0)}{p^2} + \dots, \quad \text{Luscher '78, Veneziano '79}$$

θ is observable (Coulomb phase, Luscher pole)

- Spontaneous breaking of anomalous symmetry – Higgsing 3-form

$$\mathcal{L} = -\frac{1}{2 \cdot 4!} F_{\mu\nu\rho\sigma} F^{\mu\nu\rho\sigma} + \frac{1}{2} (\partial_\mu \eta)^2 + \frac{m}{3!} \epsilon^{\mu\nu\rho\sigma} \partial_\mu \eta C_{\nu\rho\sigma}$$

$$\Downarrow$$

$$\mathcal{L}_{\text{equiv.}} = -\frac{1}{2 \cdot 4!} F_{\mu\nu\rho\sigma} F^{\mu\nu\rho\sigma} + \frac{m^2}{2 \cdot 3!} \left(C_{\nu\rho\sigma} - \frac{1}{m} \partial_{[\nu} B_{\nu\rho]} \right)^2.$$

$$\Downarrow$$

$$\mathcal{L}'_{\text{equiv.}} = \frac{1}{2} (\partial_\mu \eta)^2 - \frac{m^2}{2} \eta^2$$

Dvali '05
Dvali, Jackiw and Pi, '06

3-form gauge theory formulation of θ -vacua

$$\text{F.T.} \langle \theta | C_{\mu\nu\rho}(x), C_{\alpha\beta\gamma}(0) | \theta \rangle_{p \rightarrow 0} \propto \begin{cases} \frac{\rho(0)}{p^2} + \dots, & \theta \text{ observable (Coulomb phase, Luscher pole)} \\ \frac{\rho(0)}{p^2 - m_\eta^2} + \dots, & \theta \text{ unobservable (Higgs phase, propagating dof!)} \end{cases}$$

- Topological susceptibility of vacuum

$$\chi = \text{F.T.} \langle \theta | \text{Tr} F \tilde{F}(x), \text{Tr} F \tilde{F}(0) | \theta \rangle_{p \rightarrow 0} = \begin{cases} \rho(0) = \text{const.}, & \theta \text{ observable} \\ 0, & \theta \text{ unobservable} \\ & (\text{massive pseudo-scalar dof!}) \end{cases}$$

These are very generic considerations, applicable also to massive gauge-invariant theories in the Higgs phase

Calculations in full Standard Model

- 12-fermion composite order parameter:

$$\hat{O} \propto \det [(qqq)_i l_j], \quad i = 1, \dots, N_f = 3$$

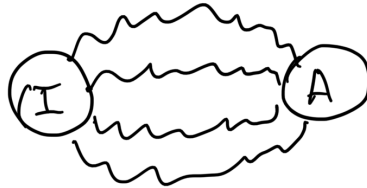
$$\begin{aligned} \langle \theta | \hat{O}(x) | \theta \rangle &\sim \left(\frac{2\pi}{\alpha_2(v_{ew})} \right)^4 e^{-\frac{2\pi}{\alpha_2(v_{ew})} v_{ew}^{6N_f}} \\ &\times \int \frac{d(\rho v_{ew})}{\rho v_{ew}} e^{-2\pi^2 (\rho v_{ew})^2} (\rho v_{ew})^{(43-44N_f)/6} \end{aligned}$$

- Naïve ‘non-perturbative’ running implies UV scale:

$$\Lambda_{\text{naive}} = v_{ew}^{\frac{12\pi}{\alpha_2(44N_f - 43)}} \approx 5 \cdot 10^4 \text{TeV}$$

Calculations in full Standard Model

- However, the non-perturbative vacuum gets rearranged at much lower scales:



$$\Lambda \lesssim \frac{2m_W}{\pi\alpha_2} \approx 14\text{TeV (sphaleron scale)}$$

- Vacuum is dominated by topologically trivial ($v=0$) instanton – ant-instanton correlated configurations and the vacuum condensate ‘evaporates’
- The emergent state cease to exist as 1-particle state:

$$\rho(p \rightarrow f_\eta) \rightarrow 0, \quad f_\eta \sim \Lambda$$

Preliminary, work in progress

Properties of η_W

- Mass:

- Witten-Veneziano relation (can be reproduced using EFT and anomalous Ward identities):

$$m_\eta^2 f_\eta^2 \propto \Lambda^4 e^{-\frac{2\pi}{\alpha_2}}, \quad m_\eta \sim 10^{-20} - 10^{-17} \text{ eV}$$

- Couples via anomaly with electroweak gauge bosons
 - No coupling to 2-photons (no mixed B+L-electromagnetic anomaly) – avoid astrophysical bounds
 - Direct couplings to fermions are exponentially suppressed; only through gauge boson loops (2-loop radiative couplings at best)

Properties of η_w (preliminary)

$$\mathcal{L}_{\eta_w VV} = \frac{c_w}{f_\eta} \eta_w W_\mu^+ \tilde{W}^{-\mu} + \frac{c_z}{f_\eta} \eta_w Z_\mu \tilde{Z}^\mu + \frac{c_{z\gamma}}{f_\eta} \eta_w Z_\mu \tilde{\gamma}^\mu ;$$

$$c_w = \frac{3\alpha}{8\pi \sin^2 \theta_W}, \quad c_z = \frac{3\alpha \cos 2\theta_w}{2\pi \sin^2 2\theta_w}, \quad c_{z\gamma} = \frac{3\alpha}{2\pi} \frac{1}{\sin 2\theta_w}$$

- Processes with missing energy signature, e.g., rare meson decay:

$$B \rightarrow K(\pi) + \text{inv}, \quad K \rightarrow \pi + \text{inv}, \quad \text{etc.}$$

