

WARM INGREDIENTS IN A COLD SOUP

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PPC | 2026



AGENDA

- Freeze-in production of dark matter
- Non-cold relic in cosmological perturbation theory
- Cosmological constraints

DARK MATTER PRODUCTION

Aim Saturate dark matter relic

$$\Omega_\Phi h^2 \sim 0.12$$

Method Gravitational interaction / Other weak Interactions

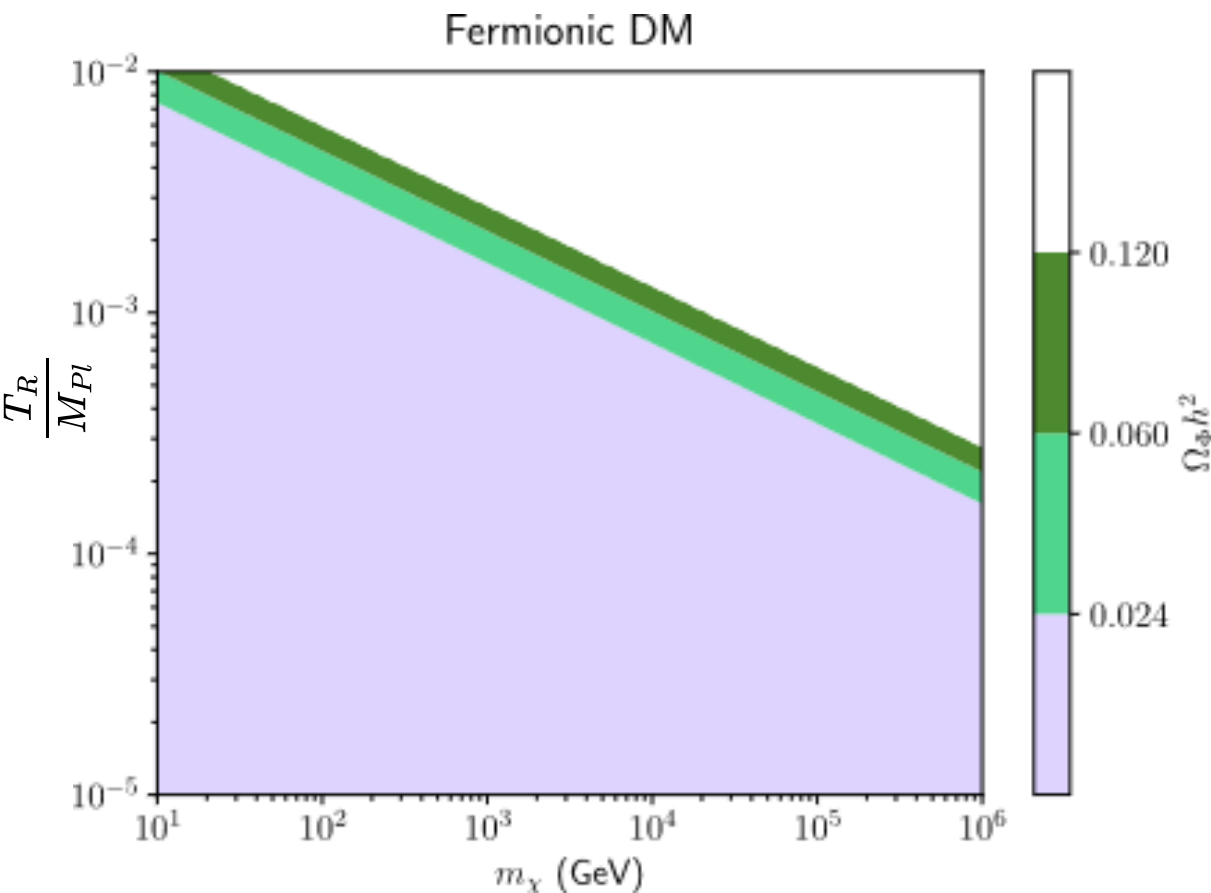
$$S \ni \frac{1}{M_{Pl}} \int_V d^4x \hat{h}_{\mu\nu} \left(T^{\mu\nu} + \tilde{T}^{\mu\nu} \right)$$

Problem Gravity too weak to saturate relic; requires very high reheating temperature

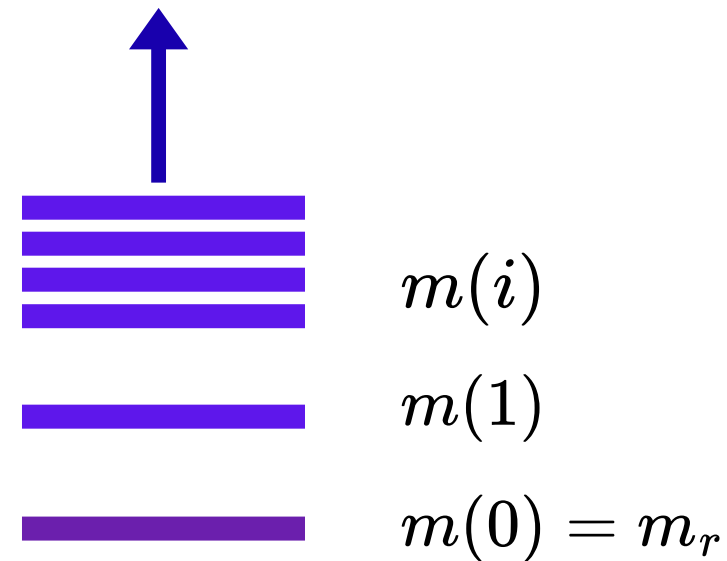
$$\Omega_\Phi h^2 \sim \left(\frac{T_R}{M_{Pl}} \right)^3$$

Solution Extra-dimensions to bring down the scale of gravity from M_{Pl} to Λ_π

$$S \ni -\frac{1}{\sqrt{6}\Lambda_\pi} \int_V d^4x \hat{r} \left(T^\mu_\mu + \tilde{T}^\mu_\mu \right)$$



Tower of Scalars



Model parameters

$$T_R \quad \Lambda_\pi \quad m_r \quad m_\Phi$$

Reheating temperature greater than radion mass

- All KK modes are produced
- Cumbersome to solve coupled differential equation for all of them

Reheating temperature smaller than radion mass

- Only radion is produced which decays into dark matter
- Simplification: Only dark matter freezes in

Coupling

Dark matter is a feebly interacting massive particle (FIMP) with coupling strength of the order of $\frac{1}{\Lambda_\pi}$

Initial Conditions

Negligible amount of dark matter present and the SM bath and DM bath are out of equilibrium

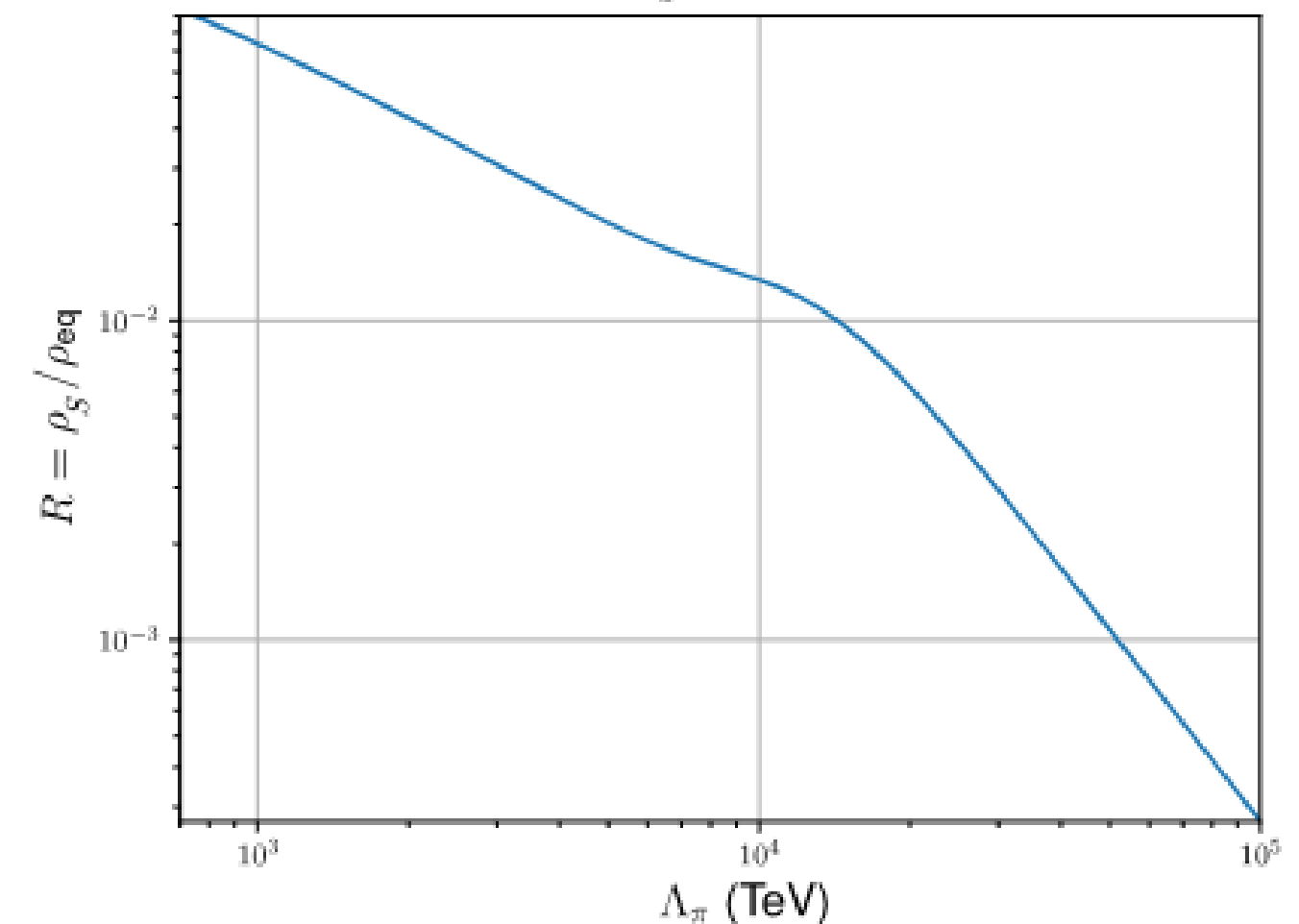
Phase space distribution

PSD is not known a priori and its shape depends on the details of the production process

What motivates freeze-in for our model ?

1. Freeze-out case of RS model has already been explored in previous paper ([arXiv:2507.21218](#))
2. We are free to choose Λ_π (10000 TeV) to any large value (smaller than the Planck scale) to induce feeble coupling
3. PSD inherits a "non-cold" component of the DM species leading to small scale matter power suppression

Scalar DM; $m_r = 100$ GeV; $m_g = 10$ KeV; $T_R = 5$ GeV; $T_{SM} = T_R/3$



DARK MATTER PRODUCTION

Master Equation: Boltzmann Equation

$$\frac{df_{\Phi}}{dt} = \mathcal{C}[f_{\Phi}]$$

Collision functional:

$$\mathcal{C}[g] = \vec{\mathcal{C}}[g] + \overleftarrow{\mathcal{C}}[g]$$

Forward $\rightarrow$ $\vec{\mathcal{C}}[g] = \frac{T e^{-E/T}}{512\pi^3 E p n!} \int_{4m_{\chi}^2}^{\infty} \frac{ds}{p^*(s)\sqrt{s}} \left(e^{-E^{\min}(s)/T} - e^{-E^{\max}(s)/T} \right) \times \int_{t^{\min}}^{t^{\max}} dt |\mathcal{M}_{\Phi\Phi \rightarrow \chi\chi}|^2$

Backward $\leftarrow$ $\overleftarrow{\mathcal{C}}[g] = -\frac{g(p, T)}{512\pi^3 E p^3 n!} \int_{4m_{\chi}^2}^{\infty} \frac{ds}{p^*(s)\sqrt{s}} \left(F(E^{\max}(s)) - F(E^{\min}(s)) \right) \times \int_{t^{\min}}^{t^{\max}} dt |\mathcal{M}_{\chi\chi \rightarrow \Phi\Phi}|^2$

$$\mathcal{C}[f_{\Phi}] = \frac{1}{512\pi^2 g_{\Phi}^* E p n!} \int_{4m_{\Phi}^2}^{\infty} \frac{ds}{p^*(s)\sqrt{s}} \int_{t^{\min}(s)}^{t^{\max}(s)} dt |\mathcal{M}_{\zeta\zeta \rightarrow \Phi\Phi}|^2 \times$$

$$\left[T e^{-E/T} \left(e^{-E^{\min}(s)/T} - e^{-E^{\max}(s)/T} \right) - f_{\Phi} \left(F(E^{\max}(s)) - F(E^{\min}(s)) \right) \right]$$

DARK MATTER PRODUCTION

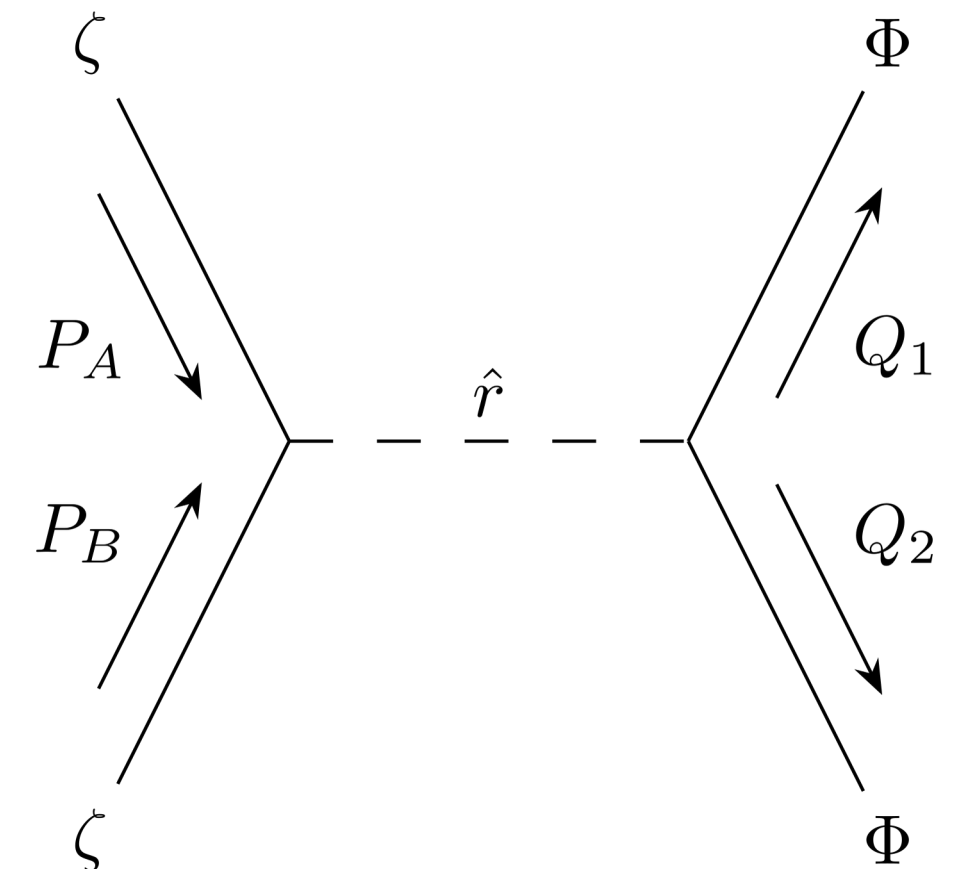
Interaction and Amplitudes:

$$\mathcal{M}_{HH \rightarrow \Phi\Phi}^2 = \frac{16\pi^2}{3\Lambda_\pi^2} \left[\sqrt{1 - \frac{4m_H^2}{m_r^2}} \right]^{-1} v_\Phi(m_r, m_\Phi) \beta(r \rightarrow HH) \delta(s - m_r^2)$$

Spin dependent factor
Branching Ratio

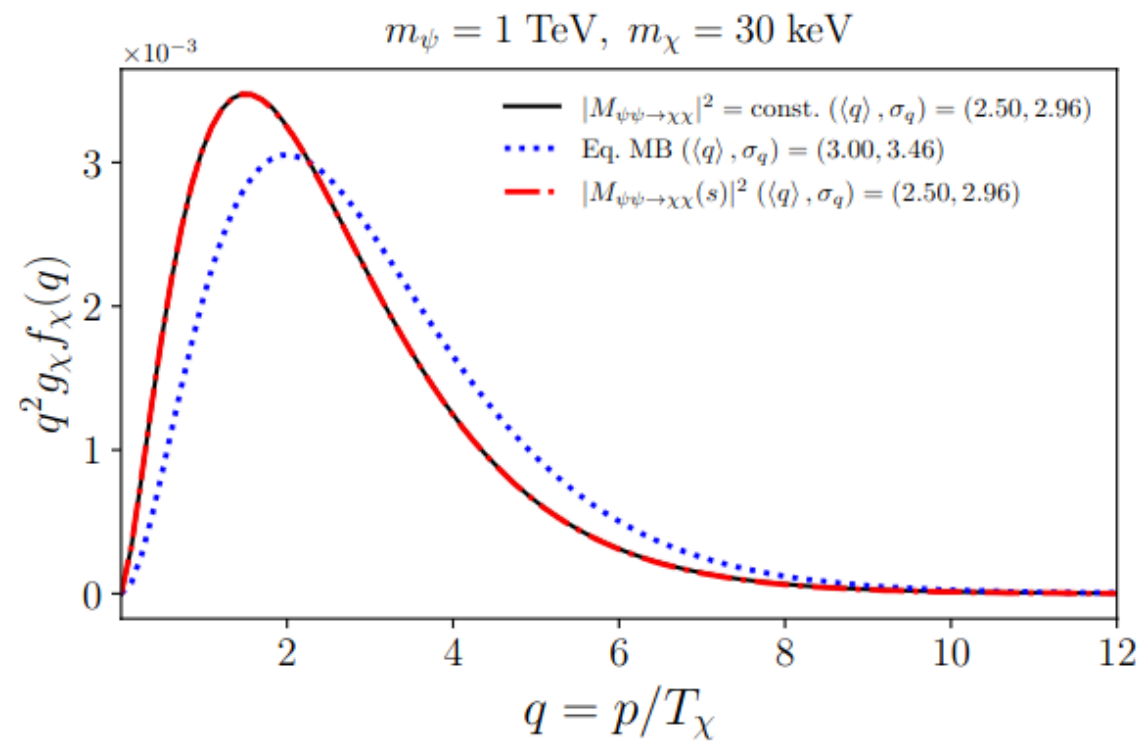
Fermion	$v_\chi(m_r, m_\chi) = 2m_\chi^2 (m_r^2 - 4m_\chi^2)$
Scalar	$v_S(m_r, m_S) = (m_r^2 + 2m_S^2)^2$
Vector	$v_V(m_r, m_V) = m_r^4 - 4m_r^2 m_V^2 + 12m_V^4$

- Matrix elements for forward and backward processes are identical
- SM bath is always in equilibrium: Maxwell-Boltzmann distribution
- Spin statistics are ignored

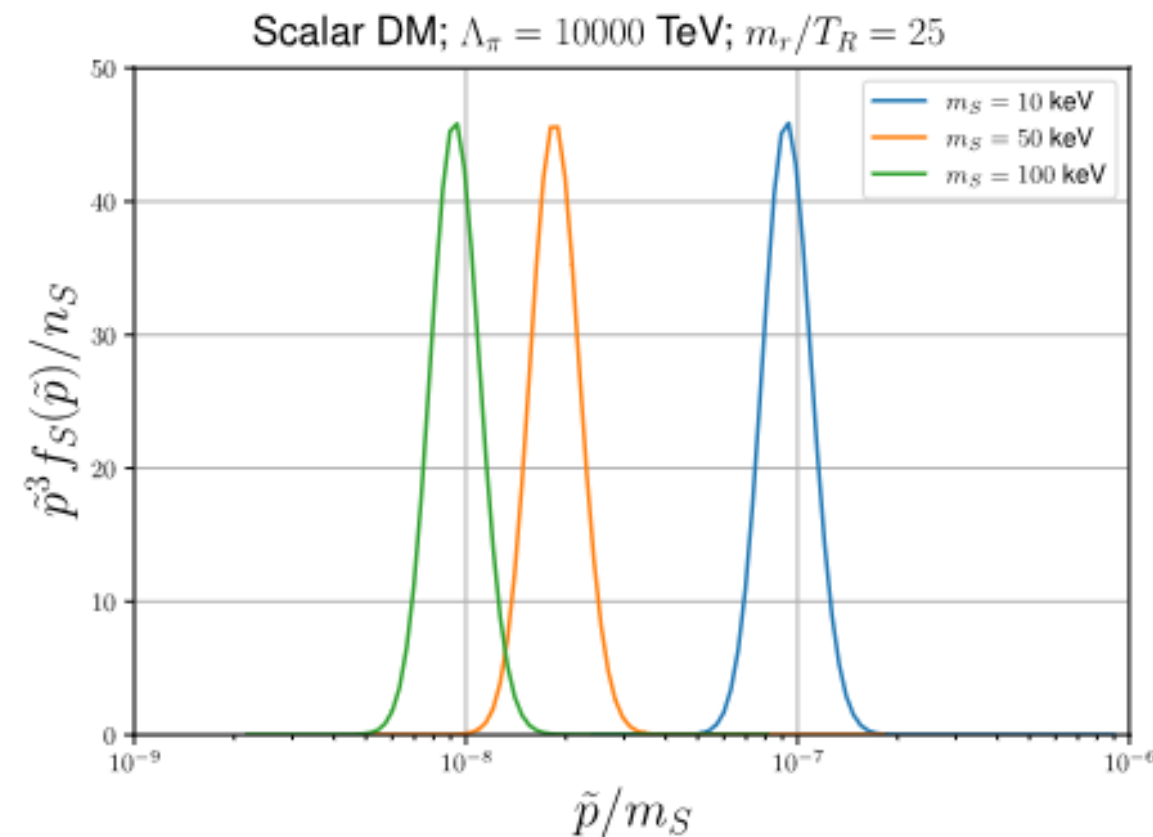


NON-THERMAL PHASE SPACE DISTRIBUTION

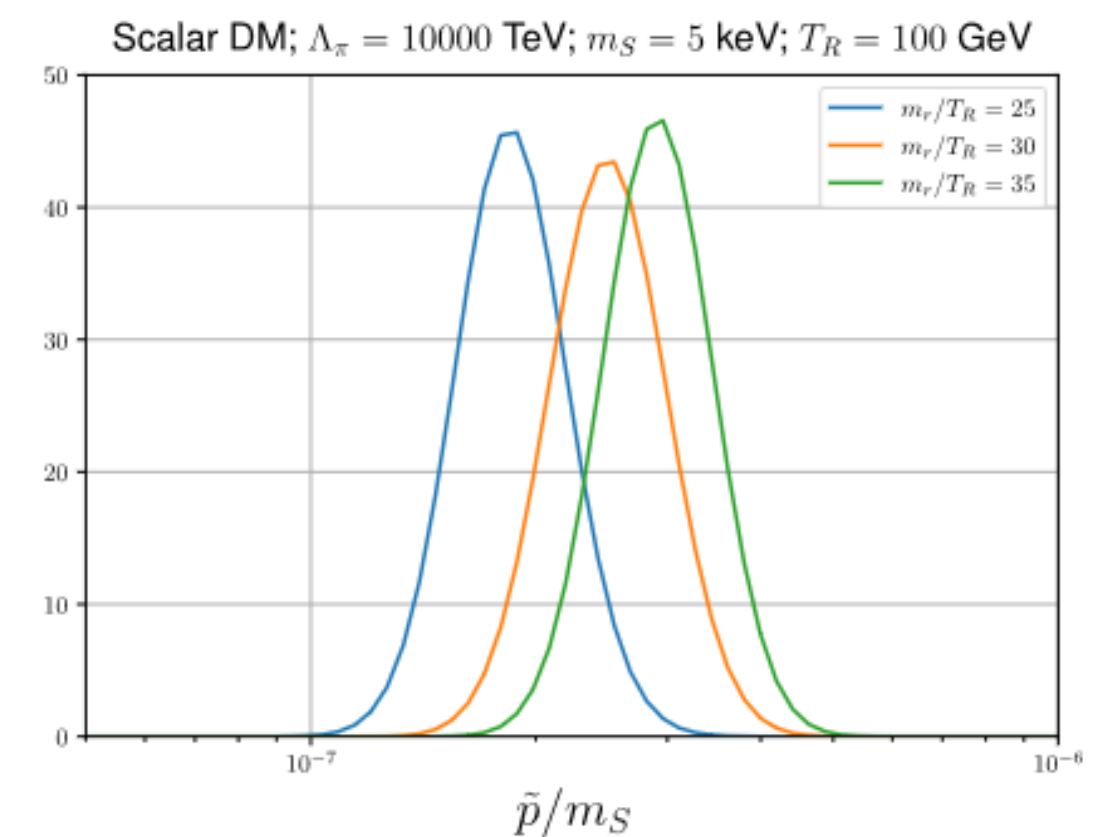
The shape of the PSD is determined by the form of the collision function

$$C[f_\Phi = 0] \approx \frac{T}{96\pi g_\Phi^*} \frac{\bar{v}_\Phi(m_r, m_\Phi)}{p^2 \Lambda_\pi^2} e^{-\left(\frac{p}{T} + \frac{m_r^2}{4pT}\right)} \quad \text{with a peak at} \quad p = \frac{m_r}{2} \left(\sqrt{1 + \frac{4T^2}{m_r^2}} - \frac{2T}{m_r} \right)$$


F. D'Eramo and A. Lenoci, JCAP 10, 045 (2021), arXiv:2012.01446 [hep-ph]



Changing mass of the DM species

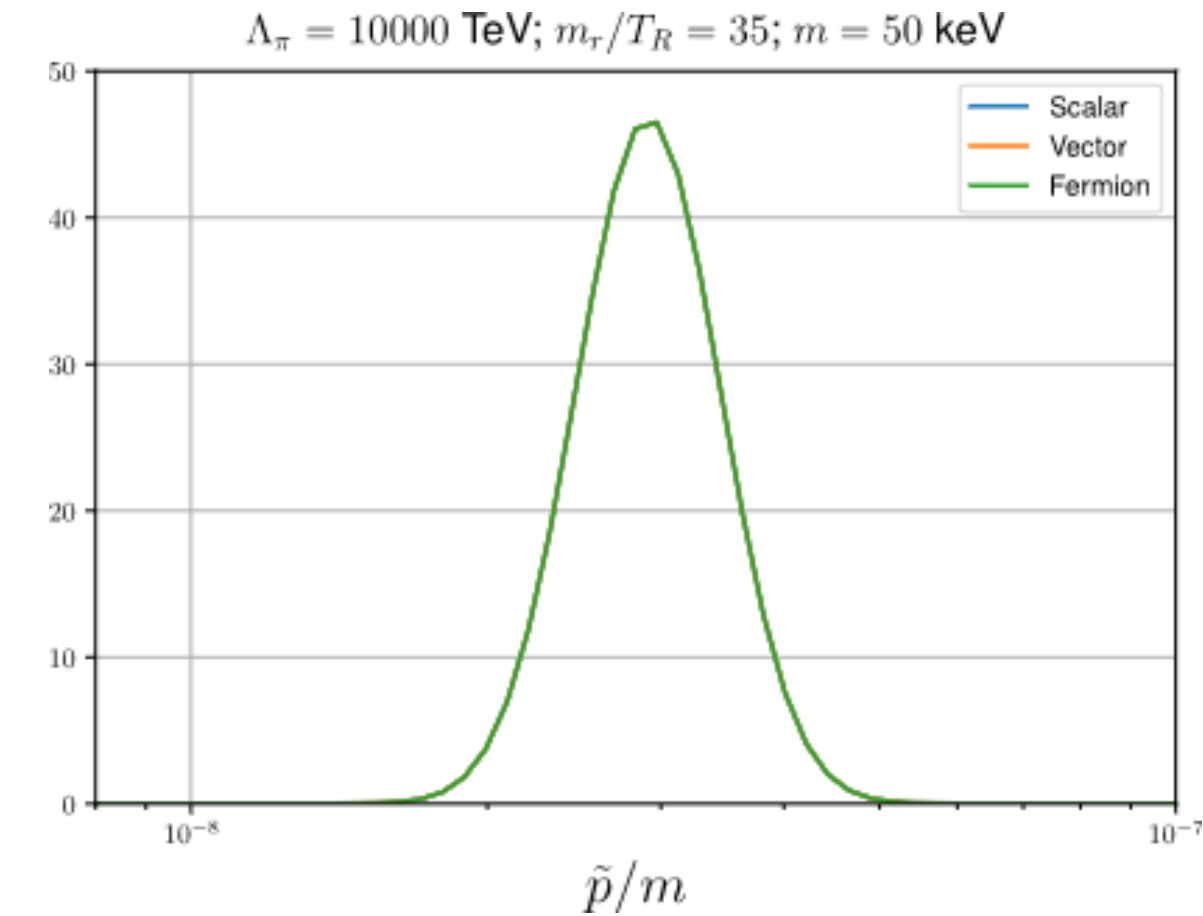
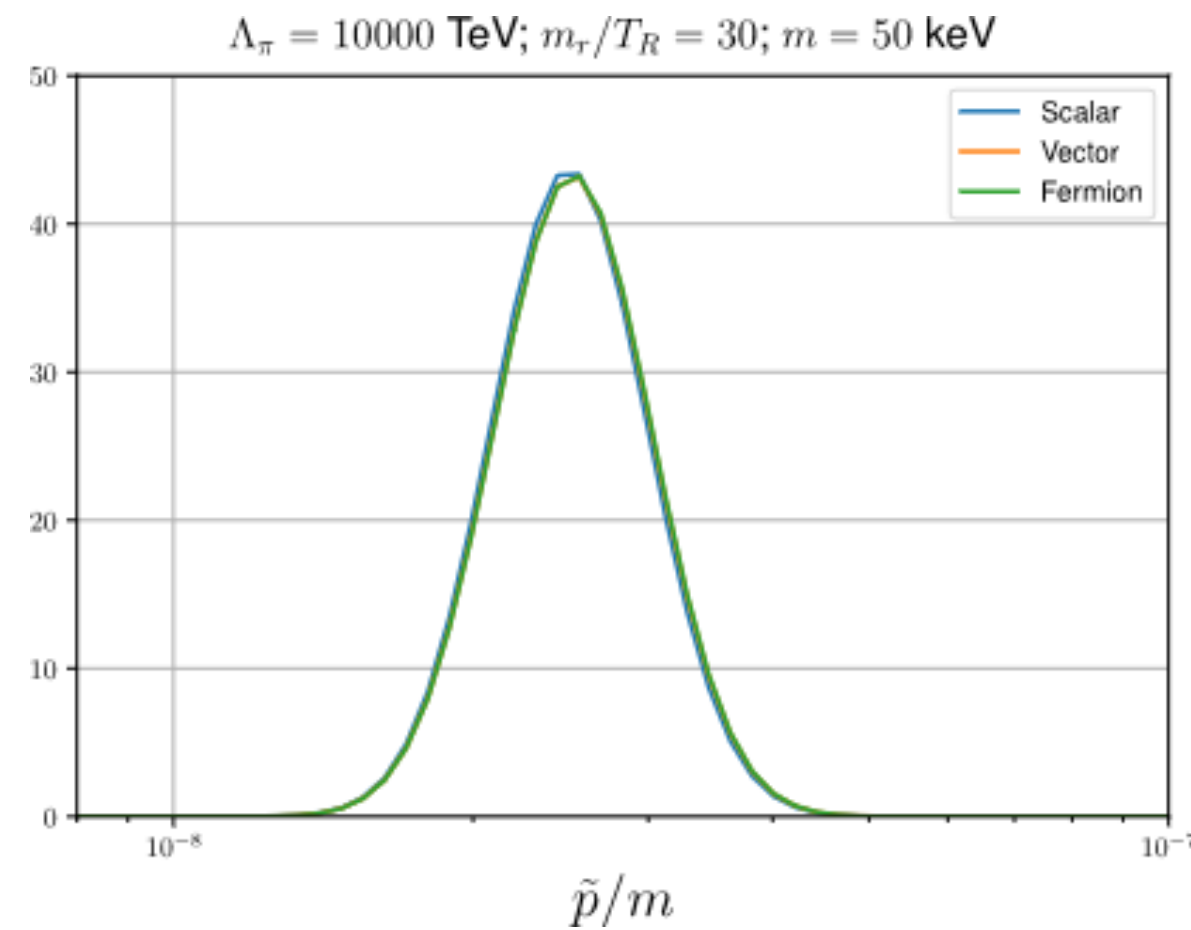
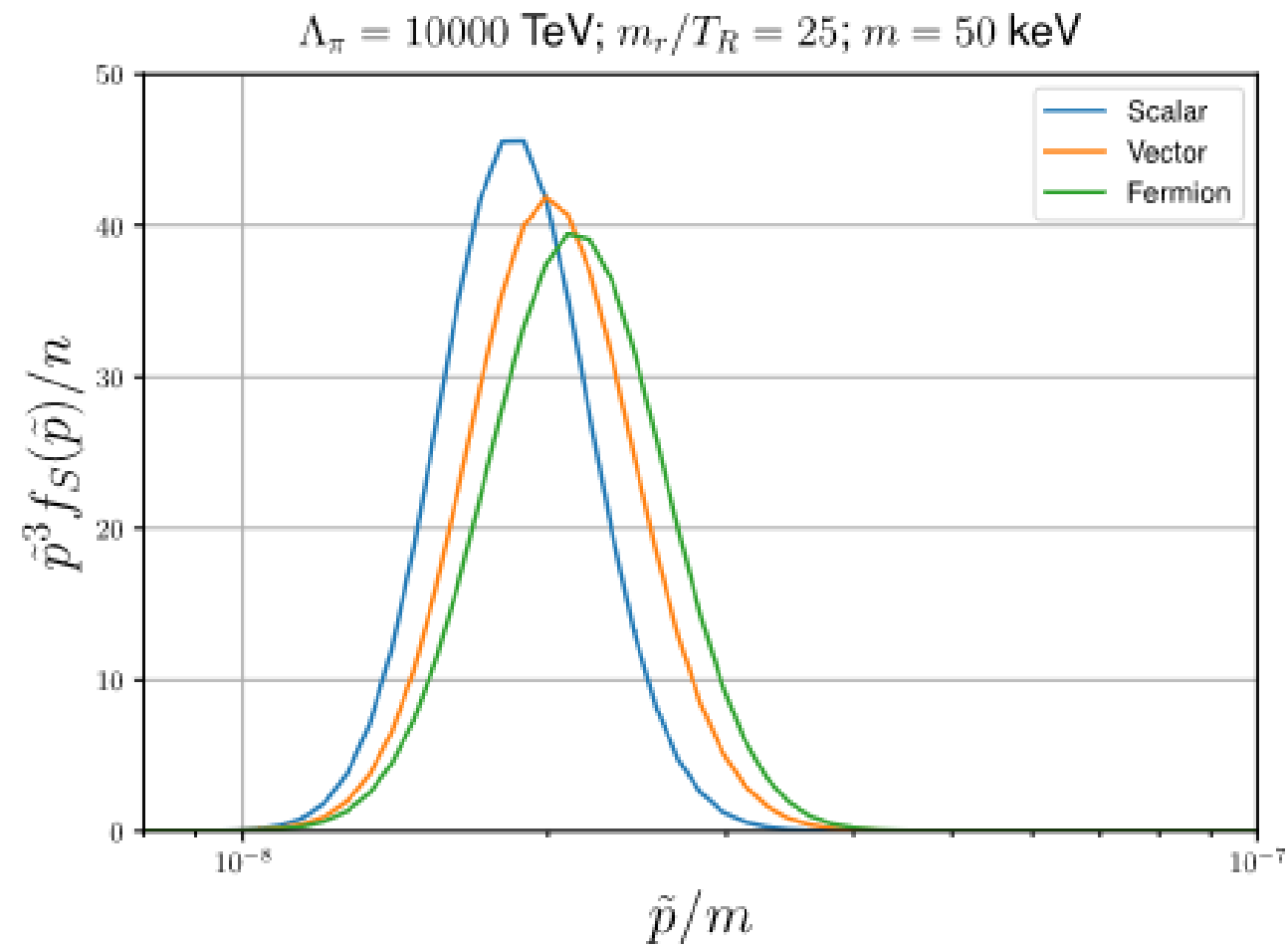


Changing mass of the radion



Warmer Relic

SPIN DEPENDENCE OF THE PSD



Interaction dependent factors in the limit of large radion mass

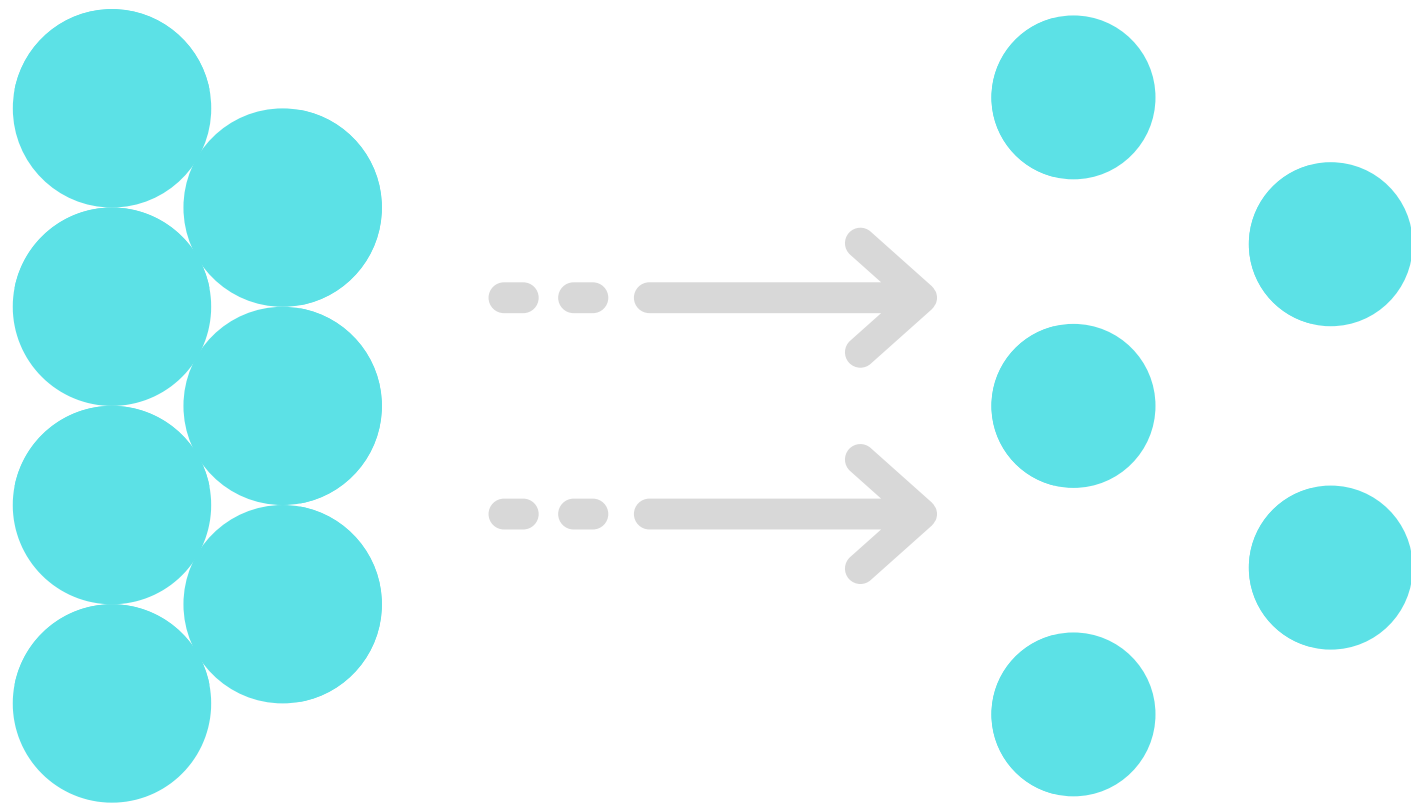
$$v_S(m_r, m_S) = m_r^4$$

$$v_V(m_r, m_V) = m_r^4$$

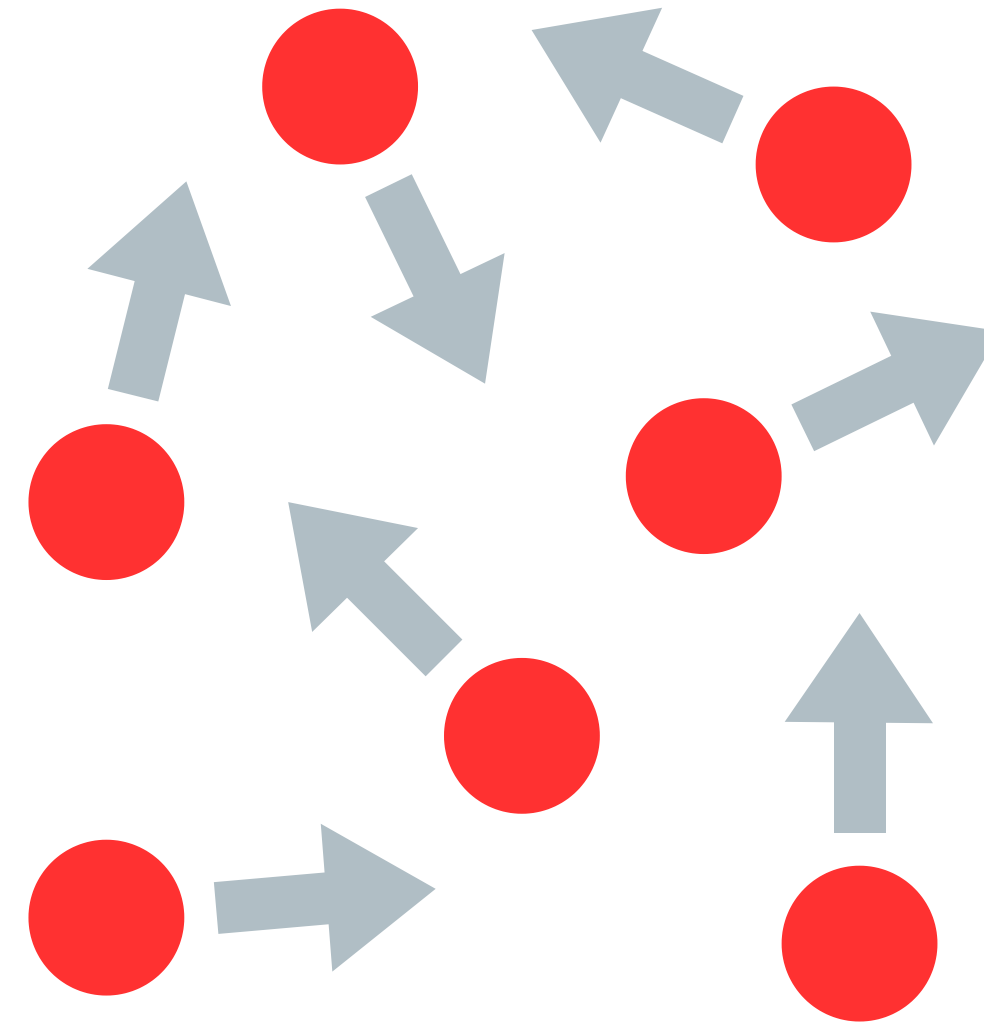
$$v_\chi(m_r, m_\chi) = 2m_\chi^2 m_r^2$$

FROM NON-THERMAL PSD TO A NON-COLD RELIC

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Cold dark matter has negligible velocity-distribution at production and it forms gravitational potential wells, seeds of structure formation.



Non-cold relic has significant velocity spread at production. Particles escape gravitational wells, washing out structure below free-streaming length.

J. Lesgourgues and T. Tram, JCAP 09, 032 (2011), arXiv:1104.2935 [astro-ph.CO].

1 Seeds of anisotropies: Perturbations to the metric and PSD

$$G_{\mu\nu} = \bar{G}_{\mu\nu} + \delta G_{\mu\nu}$$

$$f(q) = f_0(q)[1 + \Psi(x^i, q, n_j, \tau)]$$

$$\Psi(\vec{k}, \hat{n}, q, \tau) = \sum_{l=0}^{\infty} (-i)^l (2l+1) \Psi_l(\vec{k}, q, \tau) P_l(\hat{k} \cdot \hat{n})$$

$$\delta \ll 1$$

2 Components of energy-momentum tensor

$$T_0^0 = -a^4 \int q^2 dq d\Omega \sqrt{q^2 + m^2 a^2} f_0(q) (1 + \Psi)$$

$$T_i^0 = a^4 \int q^2 dq d\Omega q n_i f_0(q) \Psi$$

$$T_j^i = a^4 \int q^2 dq d\Omega \frac{q^2 n_i n_j}{\sqrt{q^2 + m^2 a^2}} f_0(q) (1 + \Psi)$$

3 Master Equations: Einstein's Equations and Boltzmann Equation

$$\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu}$$

$$\dot{\Psi}_0 = -\frac{qk}{\epsilon} \Psi_1 + \frac{1}{6} \dot{h} \frac{d \ln f_0}{d \ln q}$$

$$\dot{\Psi}_1 = \frac{qk}{3\epsilon} (\Psi_0 - 2\Psi_2)$$

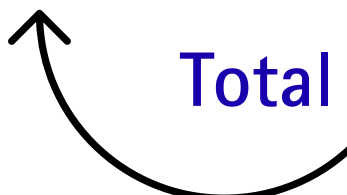
$$\dot{\Psi}_2 = \frac{qk}{5\epsilon} (2\Psi_1 - 3\Psi_3) - \left(\frac{1}{15} \dot{h} + \frac{2}{5} \dot{\eta} \right) \frac{d \ln f_0}{d \ln q}$$

$$\dot{\Psi}_l = \frac{qk}{(2l+1)\epsilon} [l\Psi_{l-1} - (l+1)\Psi_{l+1}], \quad l \geq 3$$

4 Observable: Matter Power Spectrum

$$P_m(k, z) = |\delta_m(k, z)|^2 P_R(k)$$

Total matter contrast



LYMAN-ALPHA CONSTRAINTS

- We access the deviation of the obtained matter power spectrum from the reference model through the δA criterion.

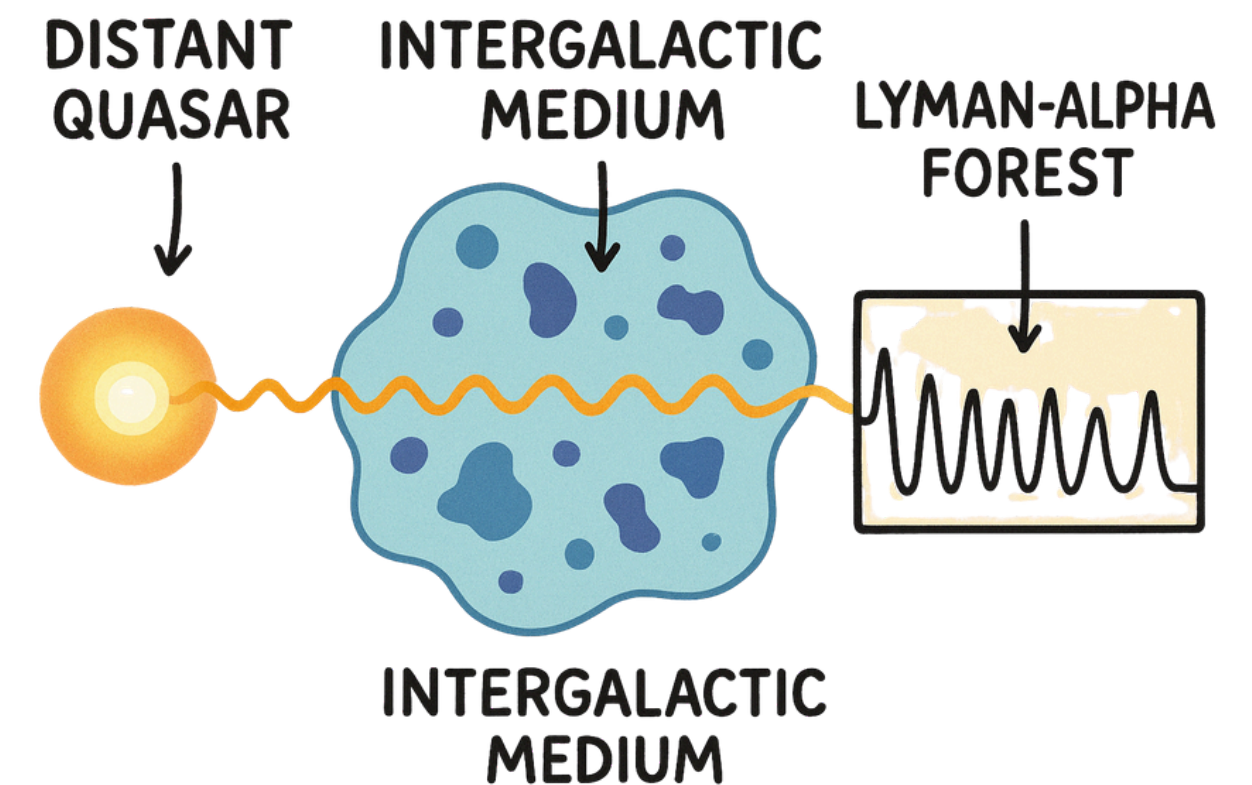
$$P_{1D}(k) = \int_k^{k_{\max}} \frac{dk'}{2\pi} k' P(k'), \quad A = \int_{k_{\min}}^{k_{\max}} dk \frac{P_{1D}(k)}{P_{1D}^{\Lambda\text{CDM}}(k)}, \quad \delta A = \frac{A_{\Lambda\text{CDM}} - A}{A_{\Lambda\text{CDM}}}$$

- We calculate the δA value for a reference warm dark matter model whose power spectrum is parameterized by

$$P(k) = \left[1 + (\alpha k)^{2\nu} \right]^{-10/\nu} P_{\Lambda\text{CDM}}(k)$$

- The reference serves as an estimate of the minimal power spectrum suppression for models that are excluded at 95% confidence level by the Lyman- α measurements.

arXiv:1704.07838v2

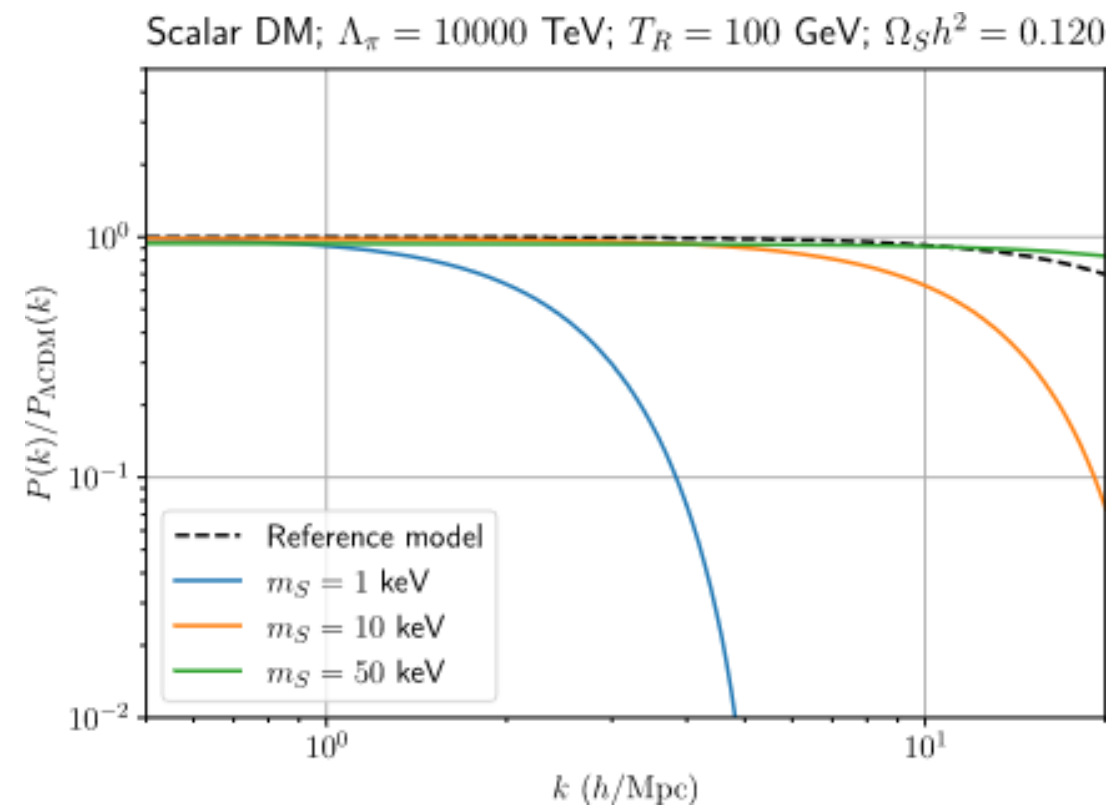


$$\delta A > \delta A_{REF}$$

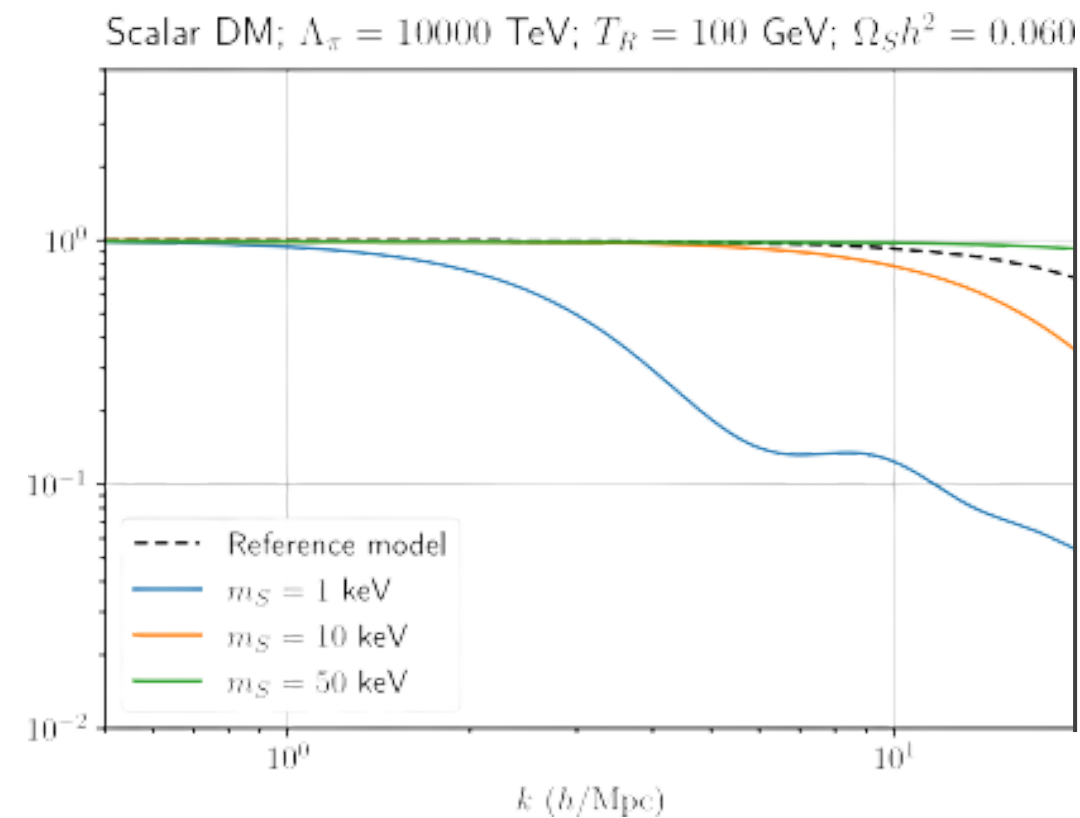
More power suppression
than allowed by
Lyman- α

TRANSFER-FUNCTION

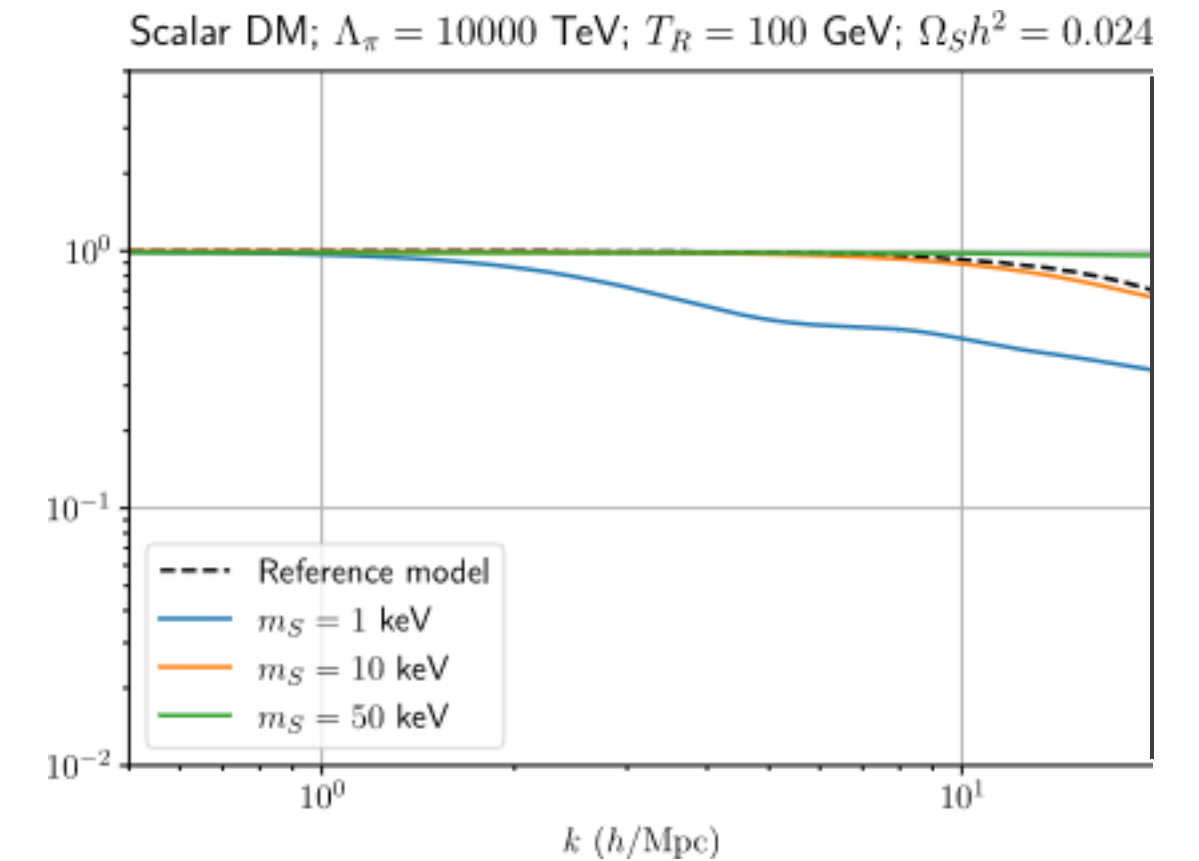
100 % NCDM



50 % NCDM



20 % NCDM

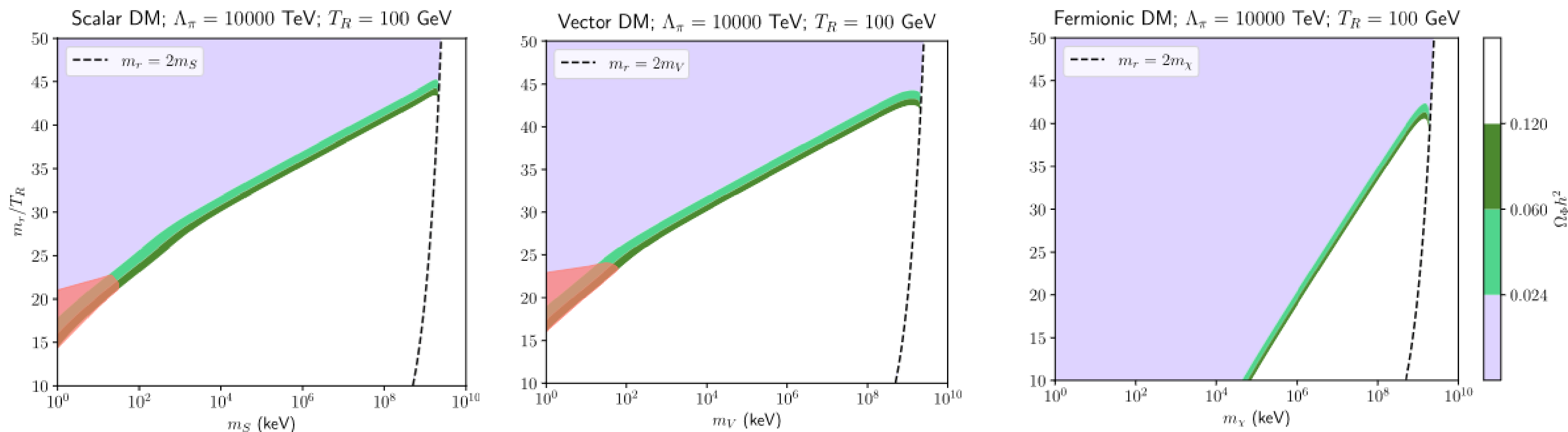


Sharp drop-off in power

Lower dark matter mass

Higher NCDM fraction

MODEL PARAMETER SPACE



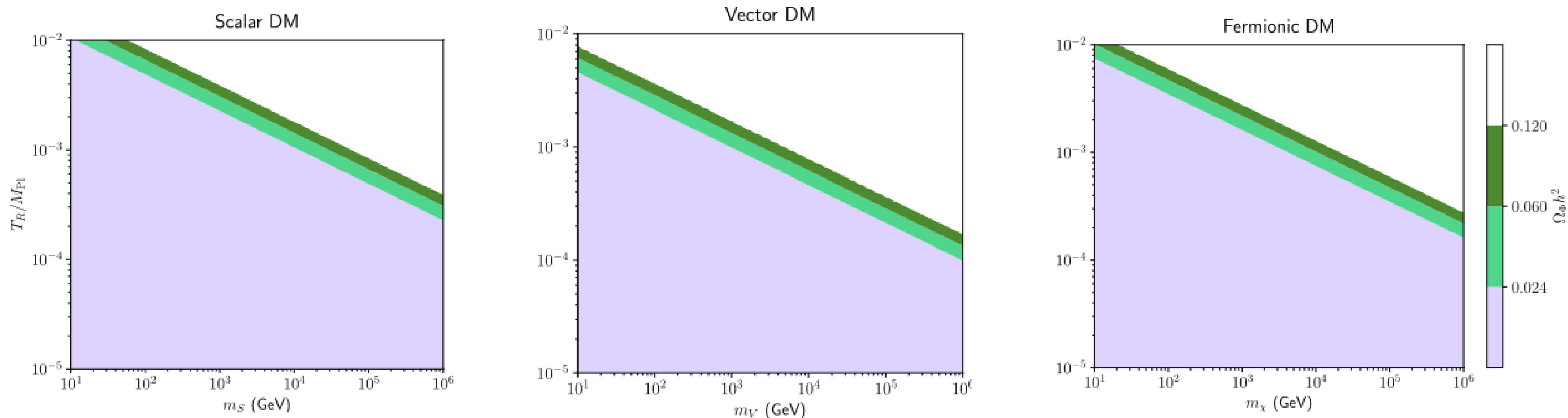
Fermion DM parameter space is
unconstrained

- Extra-dimensional models offer a solution to produce enough dark matter through freeze-in without invoking high reheating temperatures
- Backward contribution to collision function need not always be negligible
- Non-thermal phase space distribution gives a non-cold component to the dark matter relic
- Non-cold component leads to suppression in power in the standard cold dark matter power spectrum
- For our model the parameter space is unconstrained by Lyman- α for a fermionic dark matter particle



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Parameter Space for graviton mediated DM production



Can we actually allow for higher reheating temperatures ?

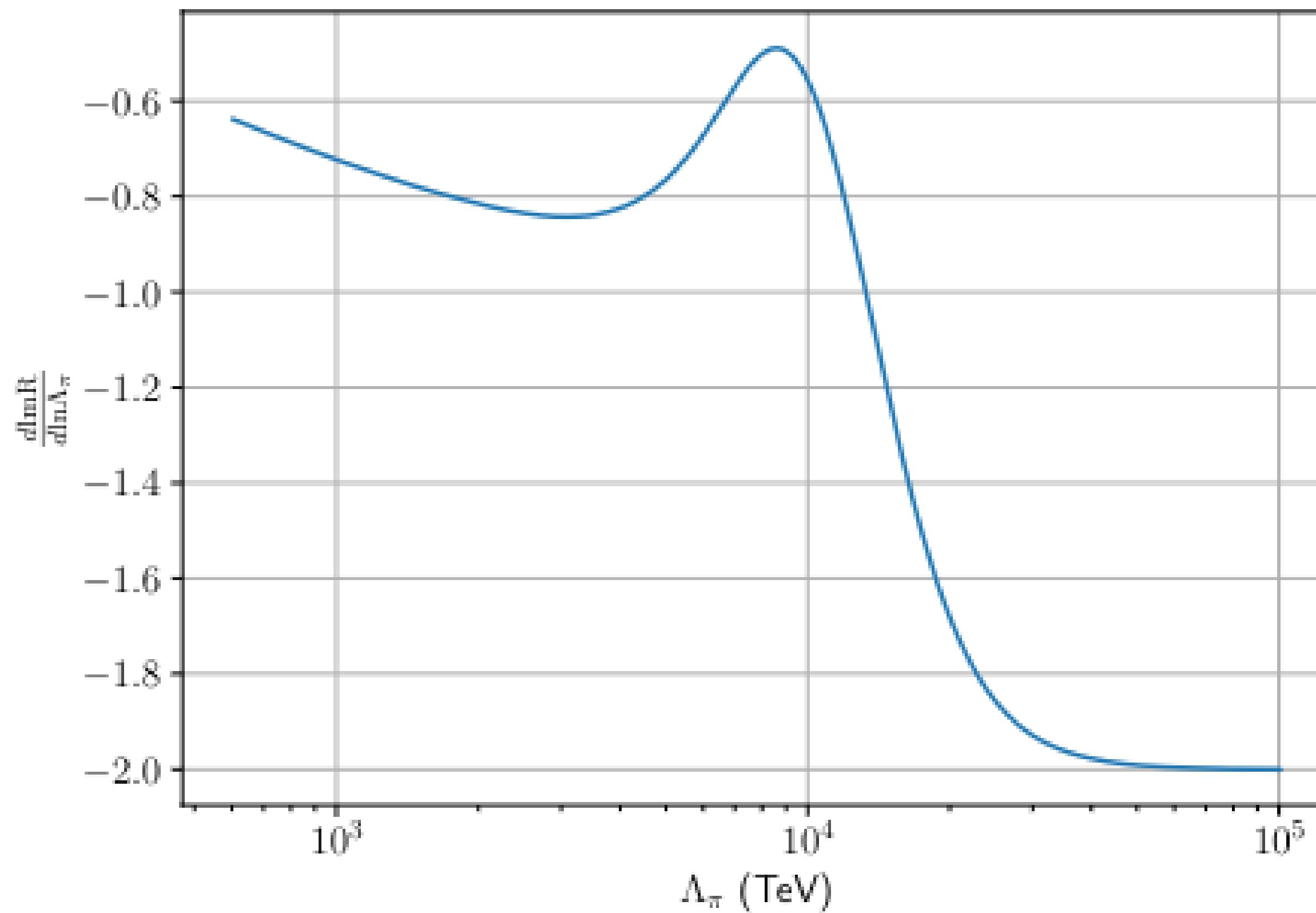
- Specific to inflationary models- some models do predict high reheating temperatures like 10^{13} GeV etc.
- But DM mass needed 10^{10} GeV very tiny relic. Model building is difficult.

Massive graviton as a solution to using simple gravity ?

- Naive expectation for 2-2 scattering rates to scale as $\frac{s^3}{m_g^4 M_{Pl}^2}$
- But the actual effective theory cut-off scale is much lower than that

Back Reaction Collision Integral

$$m_r = 100 \text{ GeV}; m_\chi = 10 \text{ KeV}; T_R = 5.0 \text{ GeV}; T_{\text{SM}} = 3.0 \text{ GeV}$$



Lyman-alpha constraints

- lower and upper limits of k , 0.5–20 h/Mpc
- $\delta A > \delta A_{REF}$ (0.16 for 3.5 keV WDM)
- Lyman-alpha forest is flux power spectrum but the bias factor is not too different between CDM and NCDM
- Primordial power spectrum $\frac{k^3}{2\pi^2} \mathcal{P}_R(k) = A \left(\frac{k}{\mathcal{K}} \right)^{n-1}$