

# Cosmic expansion amplifies gravitational waves from first-order phase transitions

**Xiao Wang**

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*Based on arXiv:2606.31571*

In collaboration with Chi Tian and Csaba Balazs



**MONASH**  
University

# Outline

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- **Introduction**
- **Effects of CE on FOPTs**
- **Impacts on GWs**
- **Summary**

# Introduction

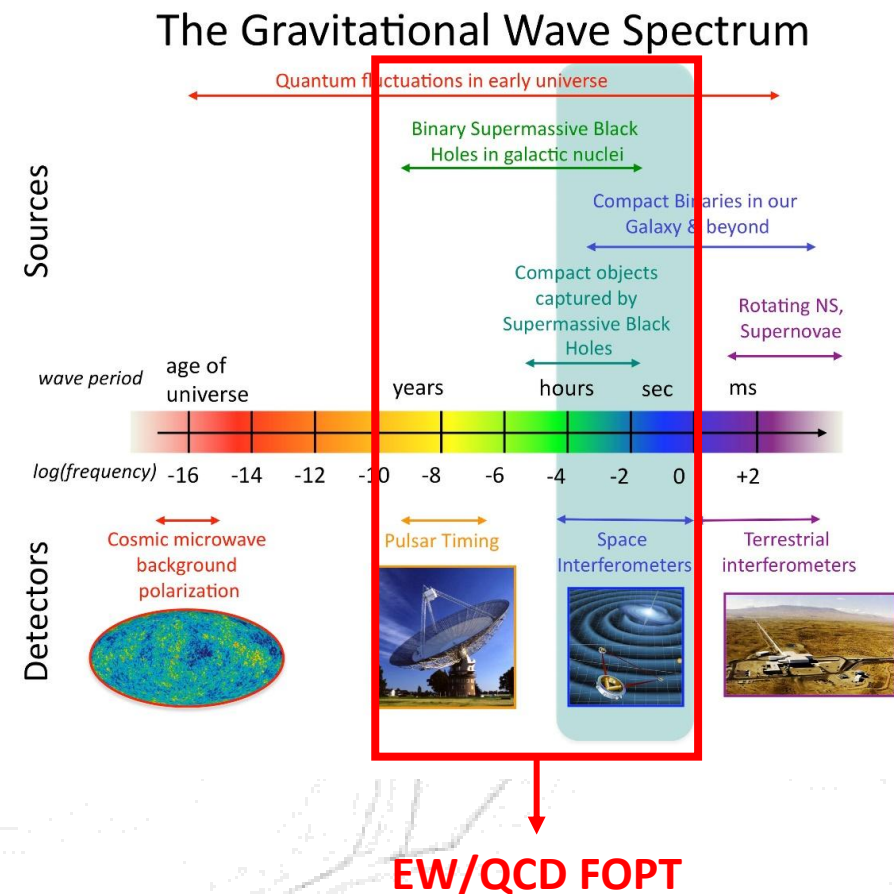


The detection of GWs by LIGO and Virgo opens a new era for *astrophysics* and *cosmology*.

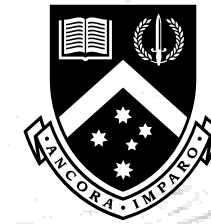
It also offers a novel probe for the physics beyond the Standard Model of particle physics.

## Sources of GWs:

- *Astrophysical origin* :  
black hole, neutron star, etc.
- *Cosmological origin* :  
inflation, domain wall, cosmic string,  
**and FOPT**



# Introduction



## Typical GW sources during FOPTs:

- Bubble collisions
- Sound wave
- Turbulence

## ➤ Other sources of GWs during phase transition:

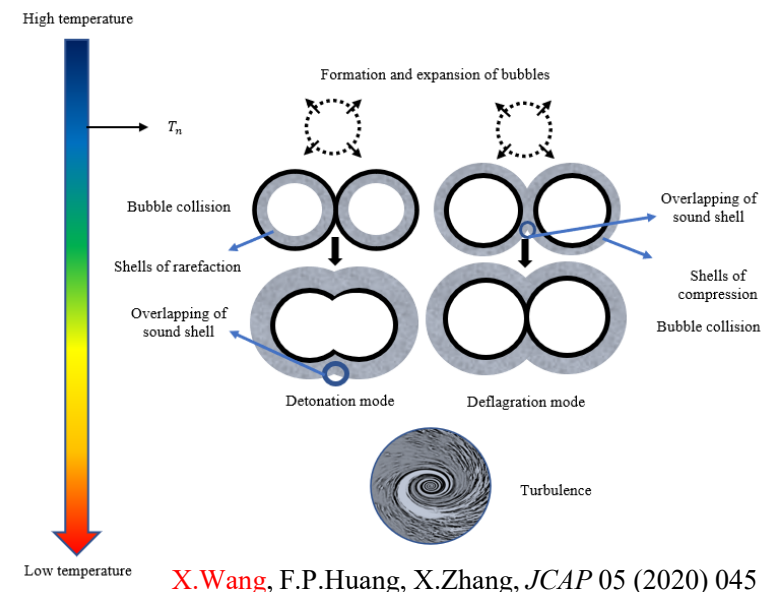
- ✓ Particles produced by bubble collisions

(Inomata et al. arXiv: 2412.17912)

- ✓ Particle splitting/losing momentum process across the bubble wall

(Wenyuan Ai. arXiv:2508.02794, Qiu et al, arXiv:2508.04314)

- ✓ ...



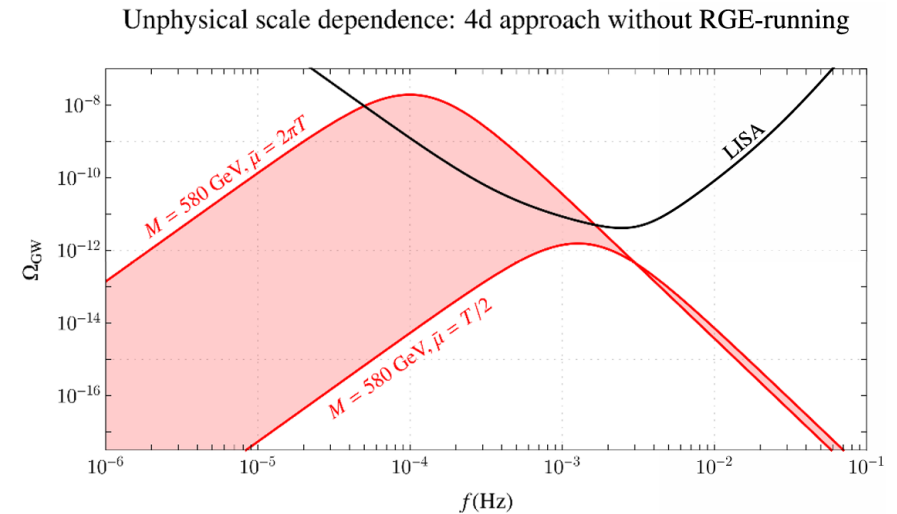
# Introduction



## Challenge:

***Theoretical uncertainties in predicting GWs from FOPTs could be Large!!!***

- Microscopic calculation
  - *Effective potential, nucleation rate, etc.*
- Macroscopic calculation
  - *GW production modeling (hydrodynamics, etc.)*



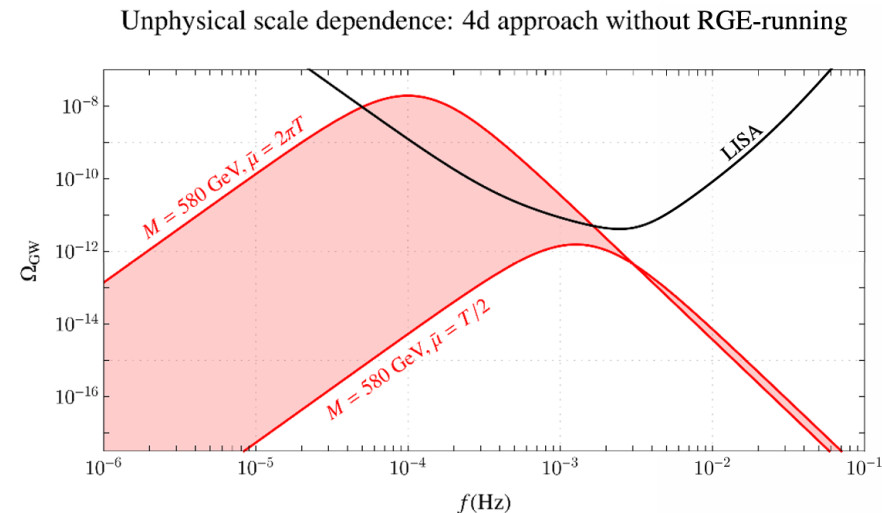
Croon, et. al. JHEP 04 (2021) 055

# Introduction



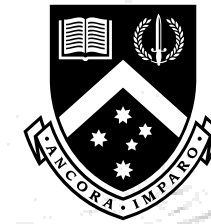
Theoretical uncertainties of GWs prediction from FOPTs could be Large!!!

- Microscopic calculation
  - *Effective potential, nucleation rate, etc.*
- Macroscopic calculation
  - *GW production modeling (hydrodynamics, etc.)*  
**(Cosmic expansion could play an important factor.)**



Croon, et. al. JHEP 04 (2021) 055

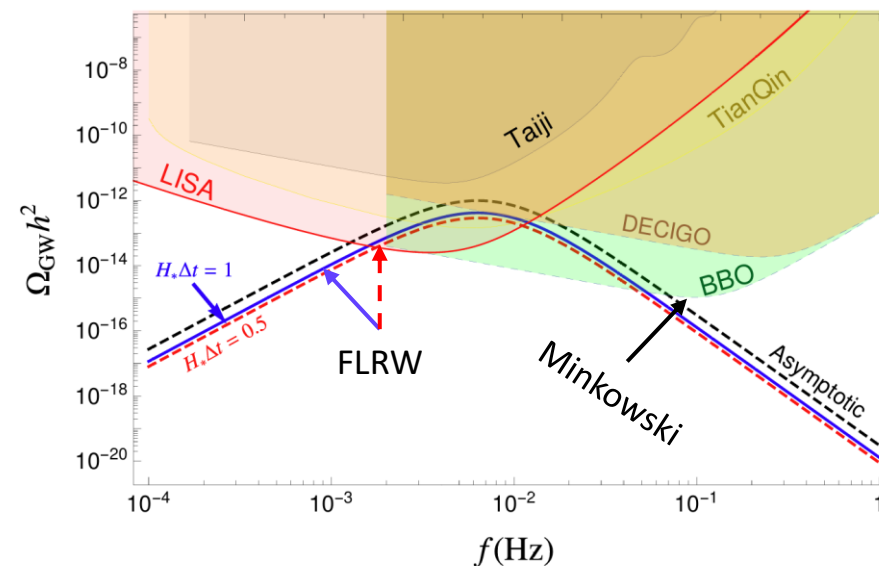
# Introduction



## Previous studies on the effect of cosmic expansion on GWs from sound waves

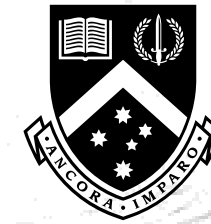
*(based on semi-analytical method --- sound shell model)*

- A **suppressed** gravitational wave signal
- A **shifted** peak frequency



Guo, et. al. JCAP 01 (2021) 001

# Introduction

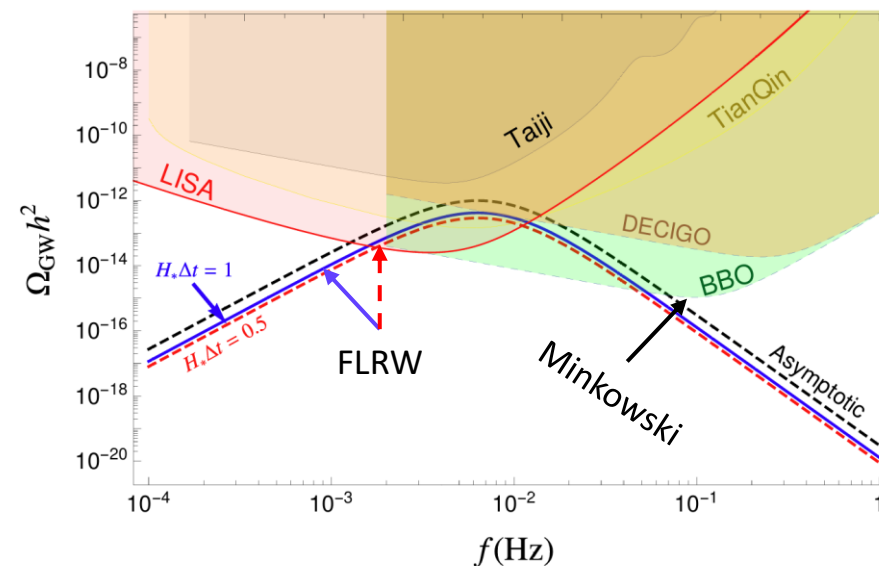


## Previous studies on the effect of cosmic expansion on GWs from sound waves

(based on semi-analytical method --- sound shell model)

- A **suppressed** gravitational wave signal
- A **shifted** peak frequency

**But something missing!!**



Guo, et. al. JCAP 01 (2021) 001

# Introduction



What cosmic expansion does:

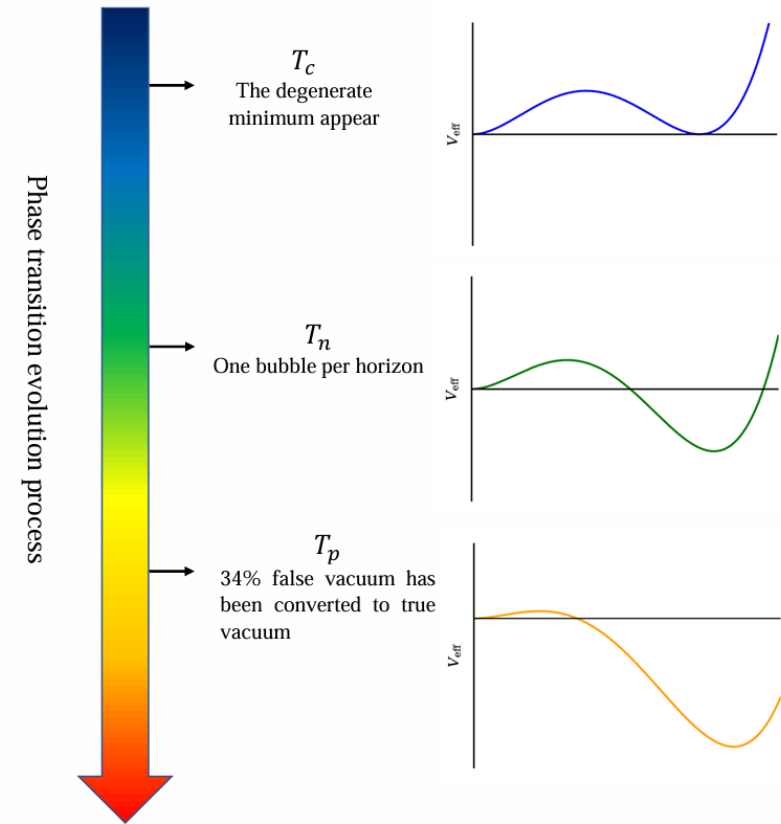
- ✓ Increasing physical volume
- ✓ Diluting energy density



Decreasing temperature

Thermal phase transitions:

- Controlled by the temperature-dependent effective potential
- Varying potential can affect a lot of things



# Introduction

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**What cosmic expansion does:**

- ✓ Increasing physical volume
- ✓ Diluting energy density

**Effects on the phase transition:**

- **FOPT Strength:** Time-dependent variations (overlooked)
- **Bubble wall velocity:** Time-dependent variations (overlooked)
- **Hydrodynamics:** Modified fluid evolution (overlooked)
- **Bubble separation:** Changed mean distances

# Introduction

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## Question:

- How do we incorporate these effects self-consistently into the calculation?

## Solution:

- Numerical simulations
  - ✓ We use **Higgsless simulations**, which can self-consistently include these effects.

*Jinno, Konstandin, et al. JCAP 02 (2023) 011*

A Higgsless simulation models the plasma and bubble-wall dynamics without explicitly simulating the microscopic Higgs or scalar field. Instead, it relies on solving the macroscopic hydrodynamic equations alongside a chosen equation of state. This approach is highly efficient and allows us to self-consistently calculate the resulting gravitational-wave spectrum from the phase transition

# Introduction



## Basic setups:

- Neglecting the contribution of the scalar field to the energy-momentum tensor of the phase transitions system

$$T^{\mu\nu} = T_{\phi}^{\mu\nu} + T_{pl}^{\mu\nu} \approx T_{pl}^{\mu\nu} = (\rho + p)U^{\mu}U^{\nu} + g^{\mu\nu}p$$

- Modeling FOPTs with the bag model equation of state

$$\rho = a_* T^4 + \epsilon, \quad p = \frac{1}{3} a_* T^4 - \epsilon$$

the universe consists of radiation and vacuum energy.

- Factorizing the thermodynamic quantities

$$X(t, x) = \tilde{X}(t, x) X_b(t)$$

where  $X_b(t)$  accounts for the effect of cosmic expansion.

- Simulating Bubble nucleation history with constant wall velocity and exponential nucleation rate

$$v_w = \text{const}, \quad \Gamma = H_n^4 \exp \beta t$$

# Effects of CE on FOPTs



- **Growing** strength of FOPTs;
- Different hydrodynamics;
- Reduced mean bubble separation.

The continuity equation of the background quantities

$$\dot{\rho}_b + 3H(\rho_b + p_b) = 0, \quad H^2 = \frac{1}{m_{pl}^2} \rho_b$$

gives  $w_b(t) \propto a(t)^{-4}$ . Therefore, we define the strength parameters as

$$\alpha(t) = \frac{4\epsilon}{3w_b(t)} = \alpha_0 a(t)^4$$

as the universe expands during the transition, the strength of the phase transition continuously grows.

# Effects of CE on FOPTs



- Growing strength of FOPTs;
- **Different** Hydrodynamics;
- Reduced mean bubble separation.

According to the conservation of the EMT

$$\nabla_\mu T^{\mu\nu} = 0$$

The fluid equations normalized by the background enthalpy is given by

$$\partial_t \tilde{E} + \frac{1}{a} \partial_i \tilde{Z}^i - H(\tilde{E} - 3\tilde{p} - \tilde{Z}_i v^i) = 0$$

$$\partial_t \tilde{Z}_i + \frac{1}{a} \partial_j (\tilde{Z}_i v^j) + \frac{1}{a} \partial_i \tilde{p} = 0$$

where  $\tilde{X} = X/w_b$ .

$$\tilde{p}(t, x) = \frac{3}{4} \frac{w(t, x)}{w_b(t)} - \frac{\epsilon}{w_b(t)} = \frac{3}{4} \tilde{w}(t, x) - \frac{3}{4} \alpha(t)$$

*The expanding bubbles are then simulated by incorporating a moving boundary with a wall velocity  $v_w$ , and the bag constant  $\epsilon$  or the strength parameter  $\alpha$  differs between the interior and exterior regions of the bubbles.*

# Effects of CE on FOPTs



- Growing strength of FOPTs;
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According to the conservation of the EMT

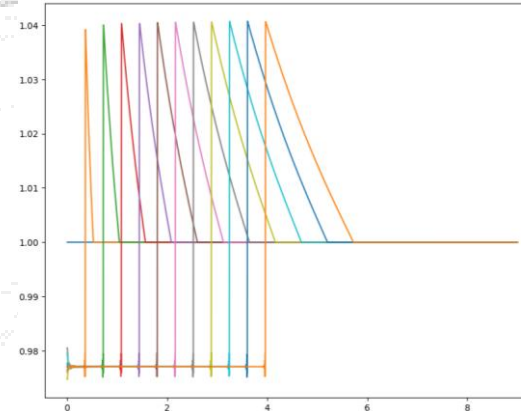
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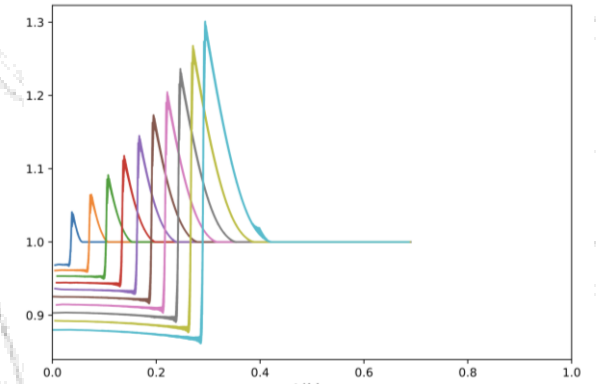
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where  $\tilde{X} = X/w_b$ .



*Non-expanding background*



*Expanding background*

**Self-similarity of fluid profile is broken in the expanding case**

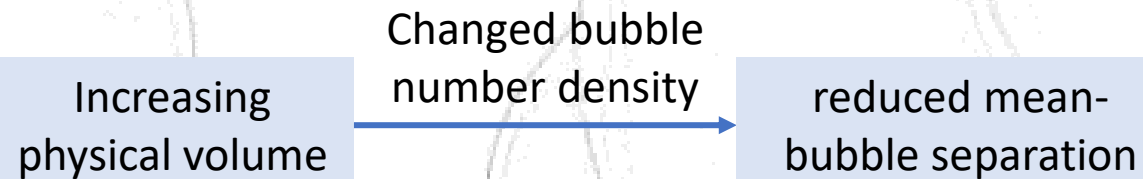
# Effects of CE on FOPTs



- Growing strength of FOPTs;
- Different hydrodynamics;
- **Reduced** mean bubble separation.

Single bubble → multiple bubbles

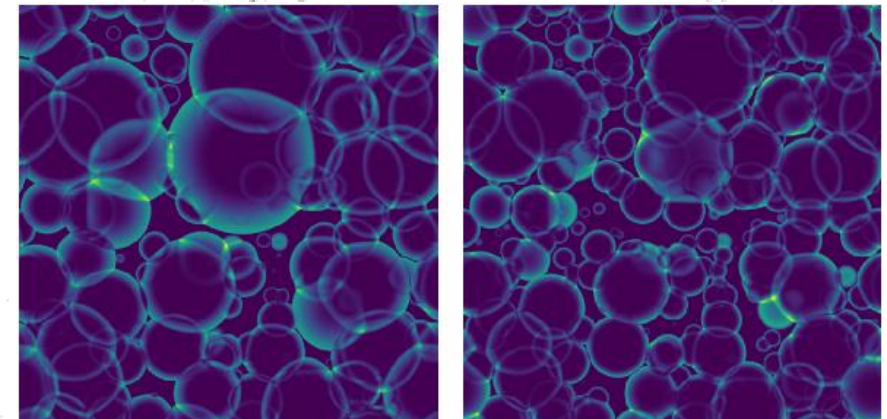
$$\Gamma = H_n^4 \exp \beta t$$



Number of bubbles with and without expansion

| $\beta/H_n$ | $v_w$ | No. bubbles without exp. | No. bubbles with exp. |
|-------------|-------|--------------------------|-----------------------|
| 15          | 0.3   | 2576                     | 11809                 |
|             | 0.6   | 2582                     | 10212                 |
|             | 0.9   | 2600                     | 9340                  |
| 20          | 0.3   | 2576                     | 9769                  |
|             | 0.6   | 2577                     | 8648                  |
|             | 0.9   | 2582                     | 7946                  |
| 25          | 0.3   | 2575                     | 8522                  |
|             | 0.6   | 2576                     | 7575                  |
|             | 0.9   | 2577                     | 7091                  |

Simulating nucleation history



# Impacts on GWs



Based on the definition of GW spectrum

$$\begin{aligned}\Omega_{\text{GW}}^*(t, k) &\equiv \frac{1}{\rho_{\text{tot}}} \frac{d\rho_{\text{GW}}(t)}{d \ln k} \\ &\simeq \frac{2}{3} \frac{H_n^2 a^4 k^3}{\pi^2 V} \int \frac{d\Omega_k}{4\pi} \dot{h}_{ij}(\mathbf{k}, t) \dot{h}_{ij}^*(\mathbf{k}, t) \\ \frac{d\rho_{\text{GW}}(t)}{d \ln k} &= \frac{m_p^2 k^3}{8\pi^2 V} \int \frac{d\Omega_k}{4\pi} \dot{h}_{ij}(\mathbf{k}, t) \dot{h}_{ij}^*(\mathbf{k}, t)\end{aligned}$$

To get  $h_{ij}$ , one needs to solve the EoM of GW

$$\ddot{h}_{ij}(t, \mathbf{k}) + 3H\dot{h}_{ij}(t, \mathbf{k}) + \frac{\mathbf{k}^2}{a^2} h_{ij}(t, \mathbf{k}) = \frac{2\Pi_{ij}^{\text{TT}}(t, \mathbf{k})}{m_p^2}$$

where  $\Pi_{ij}^{\text{TT}}$  is the transverse-traceless part of the anisotropic stress tensor

$$\Pi_{ij} \equiv (T_{ij} - pg_{ij})/a^2$$

## Impacts on GWs:

- **Nonlinear Growth** of GWs
- **Amplification** of GWs

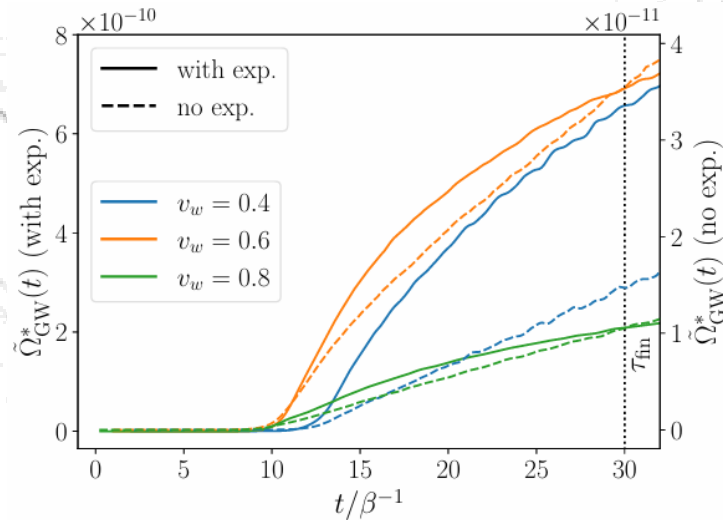
# Impacts on GWs



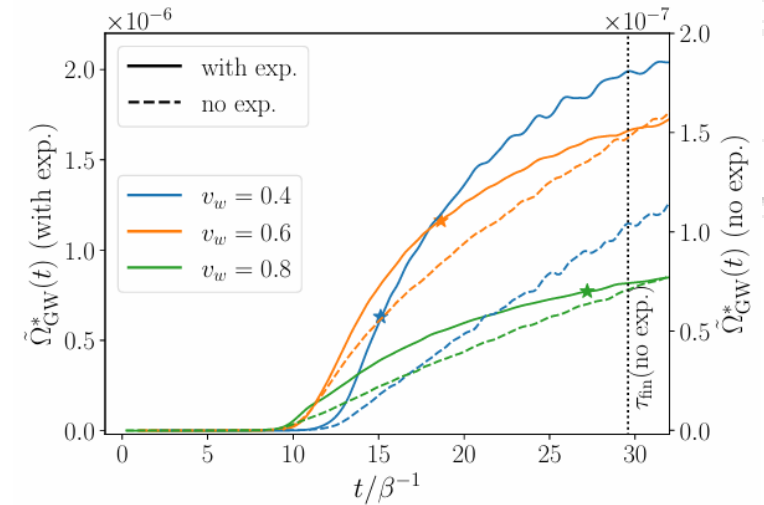
## ➤ Nonlinear Growth of GWs

## ➤ Amplification of GWs

Weak  $\alpha_0 = 0.005$

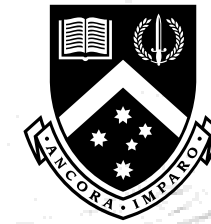


Intermediate  $\alpha_0 = 0.05$



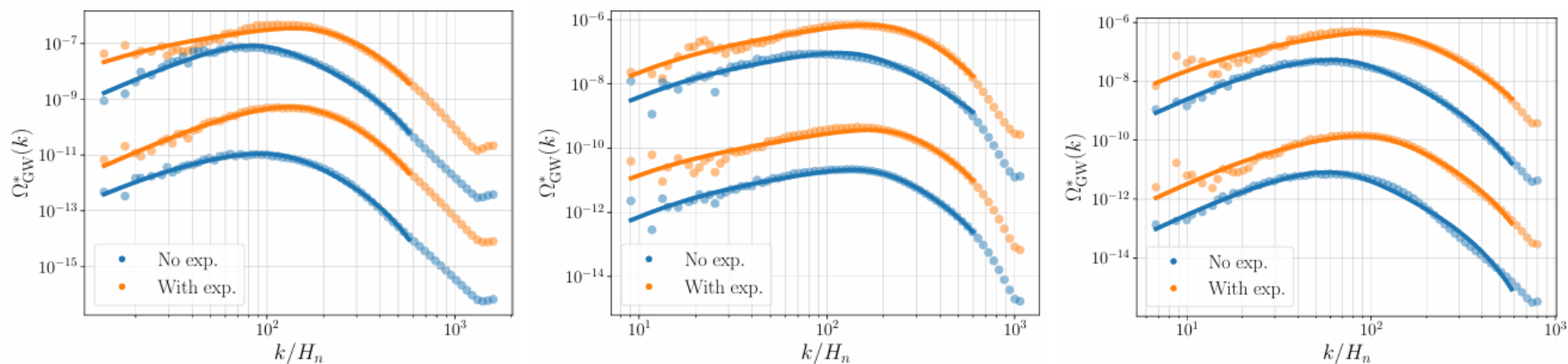
The time evolution of the GW energy fraction  $\tilde{\Omega}_{GW}^*(t) = \int \Omega_{GW}^*(t, k) d \ln k$  for various wall velocities  $v_w$  with  $\beta/H_n = 20$ .

# Impacts on GWs



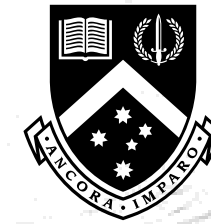
➤ Nonlinear Growth of GWs

➤ **Amplification** of GWs



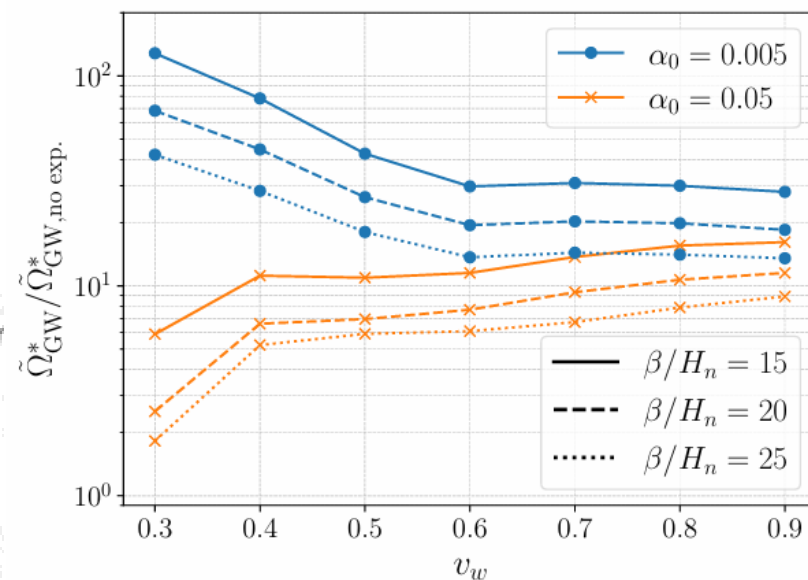
GW spectra for various bubble wall velocities ( $v_w = 0.4, 0.6, 0.8$  from left to right) with  $\beta/H_n = 20$ .

# Impacts on GWs



➤ Nonlinear Growth of GWs

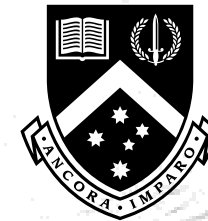
➤ **Amplification** of GWs



***But this is very dependent on the lifetime of the source.***

# Summary

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Cosmic expansion modifies GW production in 3 distinct ways:

- An increasing phase transition strength due to the dilution of the radiation fluid.
- The modification of hydrodynamics by the background expansion.
- A reduced comoving mean bubble separation, as the expanding physical volume effectively enhances the nucleation rate.

***Incorporating these effects results in a significant amplification of the GW signals generated by slow FOPTs!!!***