

Space-Borne Atomic Clocks for Gravitational Detection of Local Dark Matter Overdensities

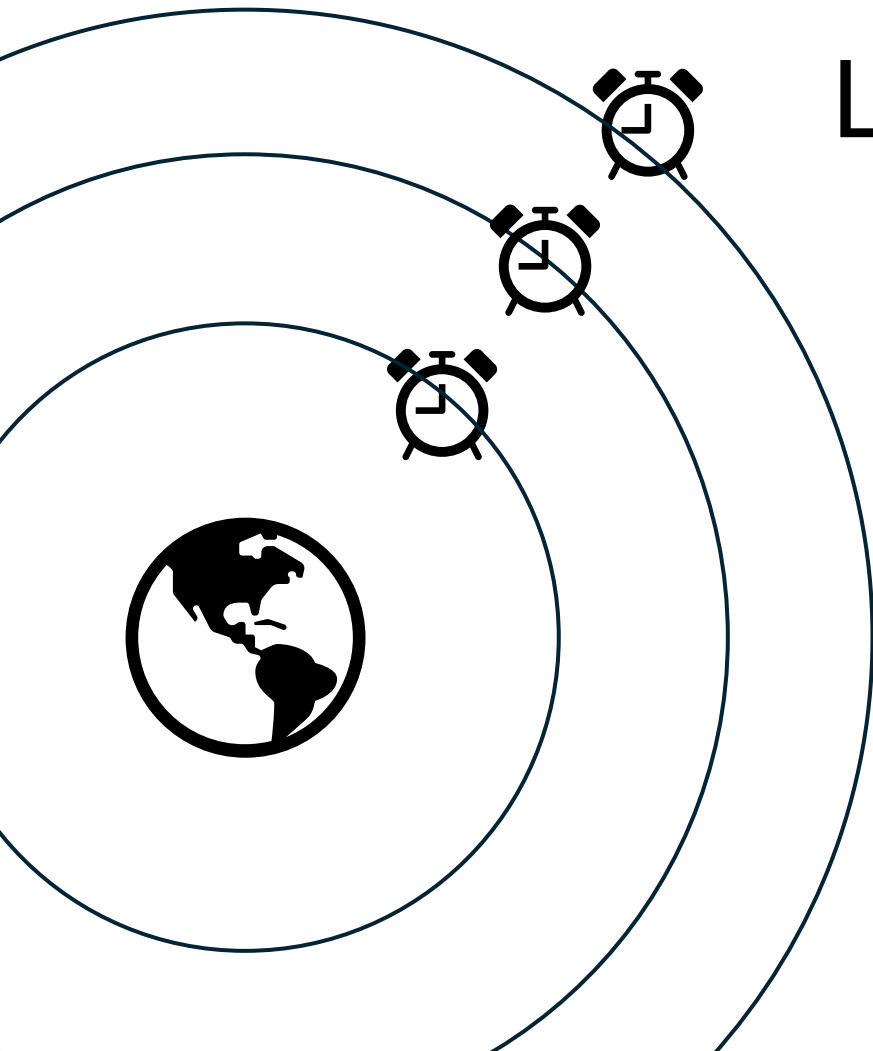
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PPC conference

UNSW, 3 Sep 2026



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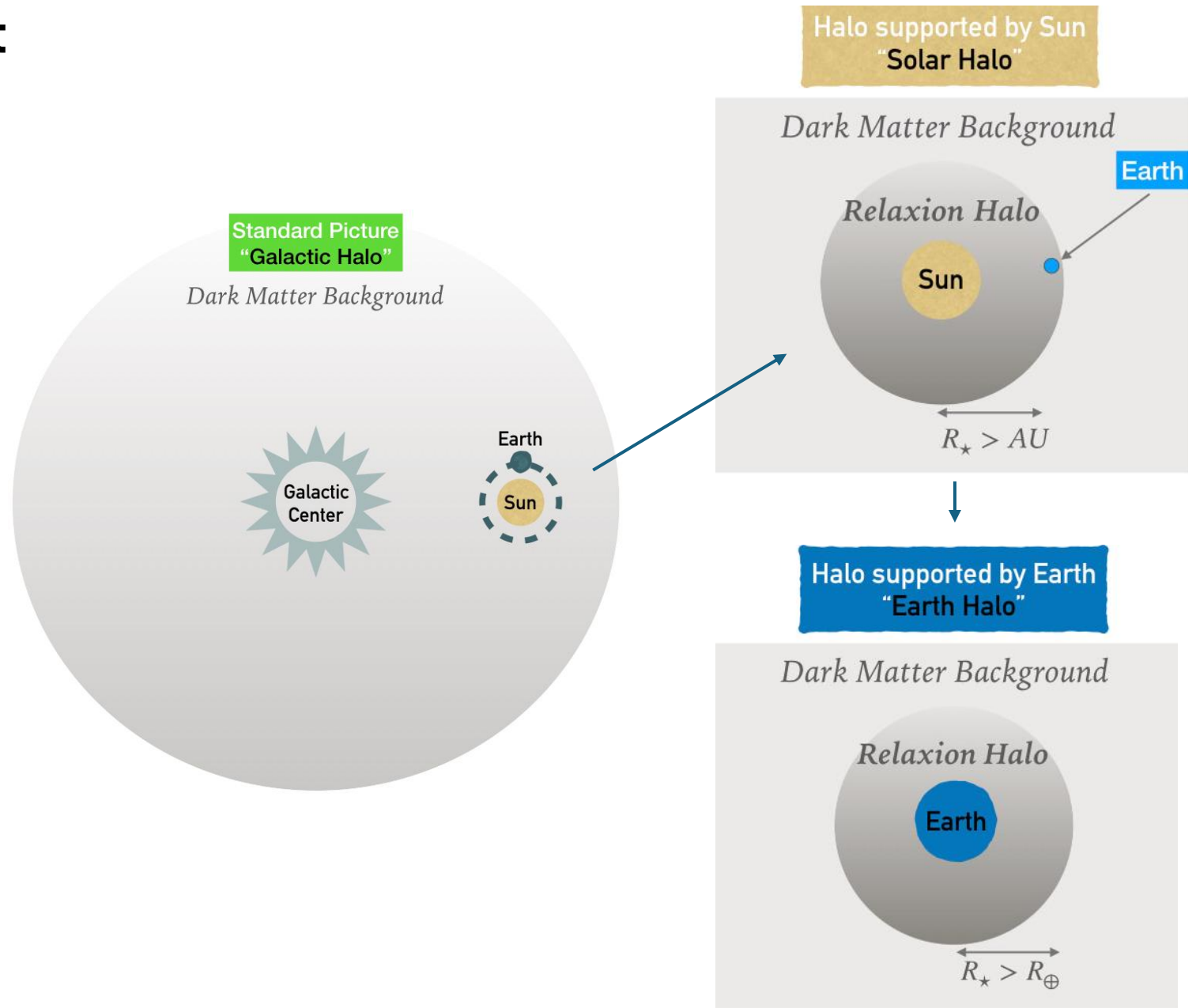
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Why should we care about the local DM density?

- It could affect the interpretation of terrestrial **direct-detection experiments**
- It gives us a way to test **models of DM capture**

Current bounds

- **0.4 GeV/cm^3** on **galactic scale**
- **Earth-bound DM** upper limit from **LLR** (Lunar Laser Ranging):
 $4 \times 10^{-9} M_{\oplus}$ between the LAGEOS satellite orbit and the moon's orbit
(**$\sim 10^{10} \text{ GeV/cm}^3$** if we assume uniform density)



Example for a Relaxion Halo

Why atomic clocks?

Atomic clocks are among the **most precise instruments ever made** and are **rapidly advancing**.

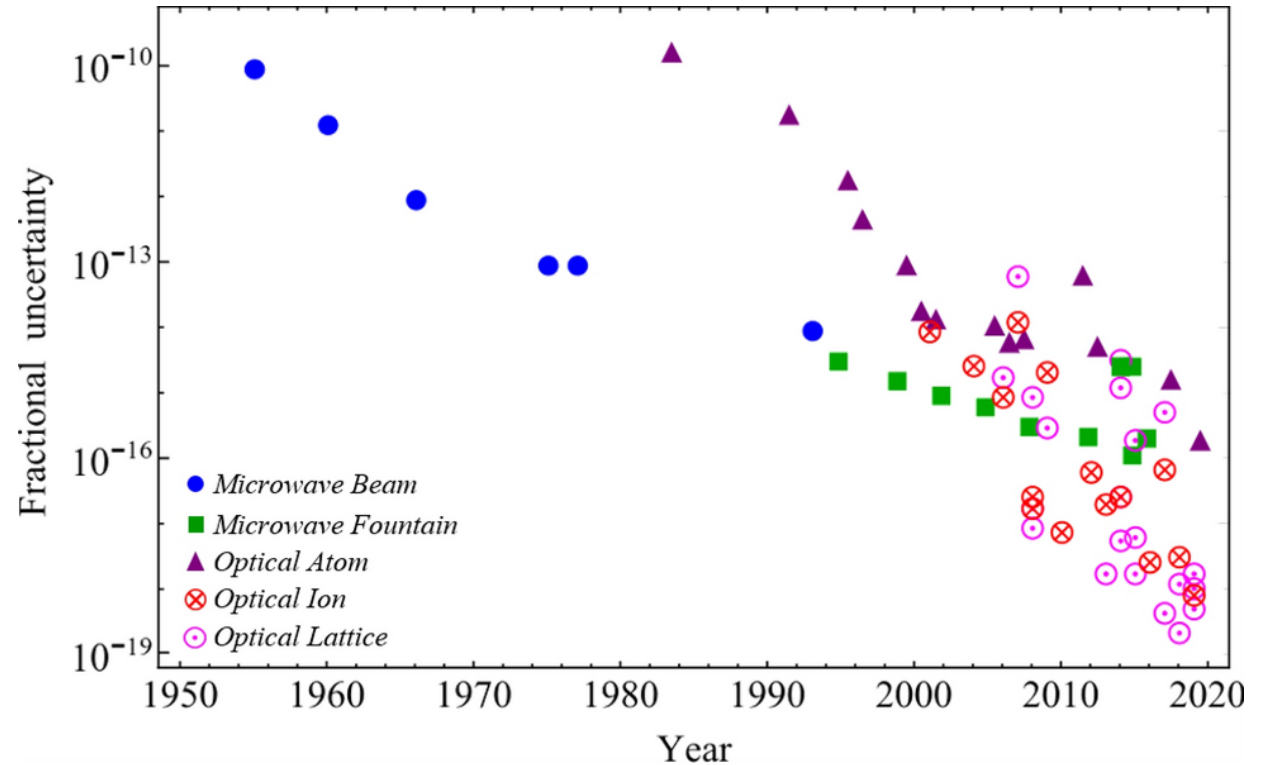
They work by tuning a laser/oscillator to the transition frequency between **atomic energy levels**, providing an extremely stable **frequency reference**.

- **Previous studies** on using clocks for DM detection mainly focus on changes in transition frequency due to **interactions**

This work

- We investigate the viability of a purely **gravitational** detection by sending clocks to space (placing direct limits like the LLR, without DM model assumptions)

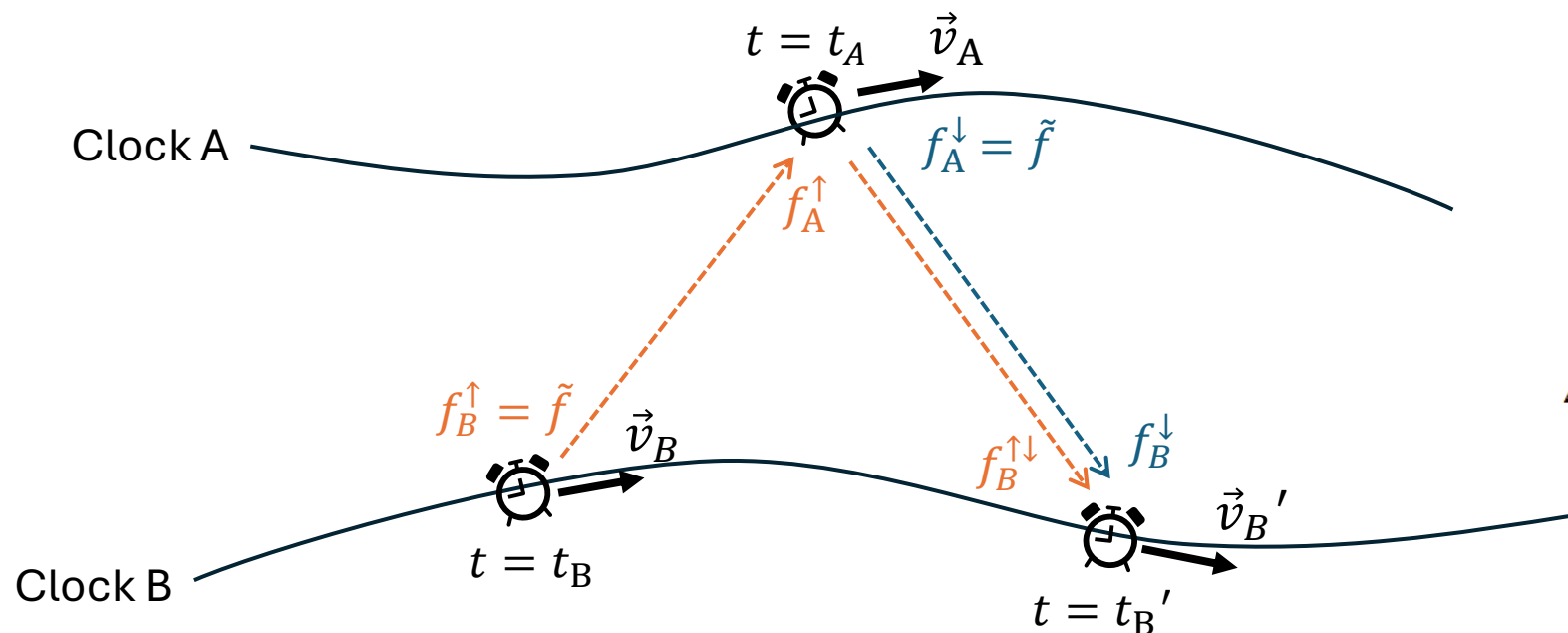
Evolution of atomic clocks in fractional uncertainty
<https://doi.org/10.1007/s12647-020-00397-y>



A commercially available chip-scale atomic clock



Two-way frequency transfer



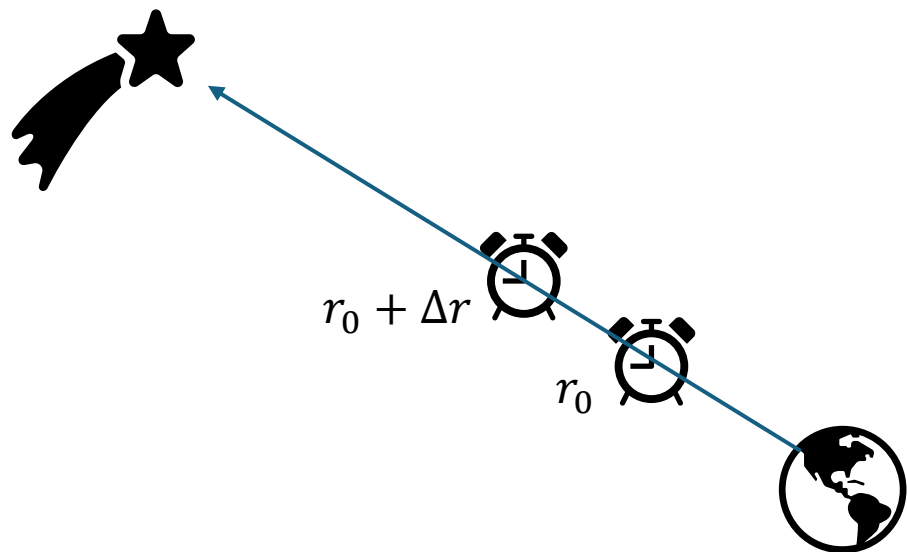
Doppler-cancelling technique
[Vessot et al. 1979]

$$\Delta_{AB} = \frac{f_B^\downarrow - \tilde{f}}{\tilde{f}} - \frac{1}{2} \frac{f_B^{\uparrow\downarrow} - \tilde{f}}{\tilde{f}}$$

Doppler-cancelled frequency shift formula, up to order $1/c^2$ [Blanchet et al. 2001]:

$$\Delta_{AB} = \frac{\phi_A - \phi_B}{c^2} - \frac{|\vec{v}_B - \vec{v}_A|^2}{2c^2} - \frac{\vec{r}_{AB} \cdot \vec{a}_B}{c^2}$$

A clock pair for DM clumps and PBHs



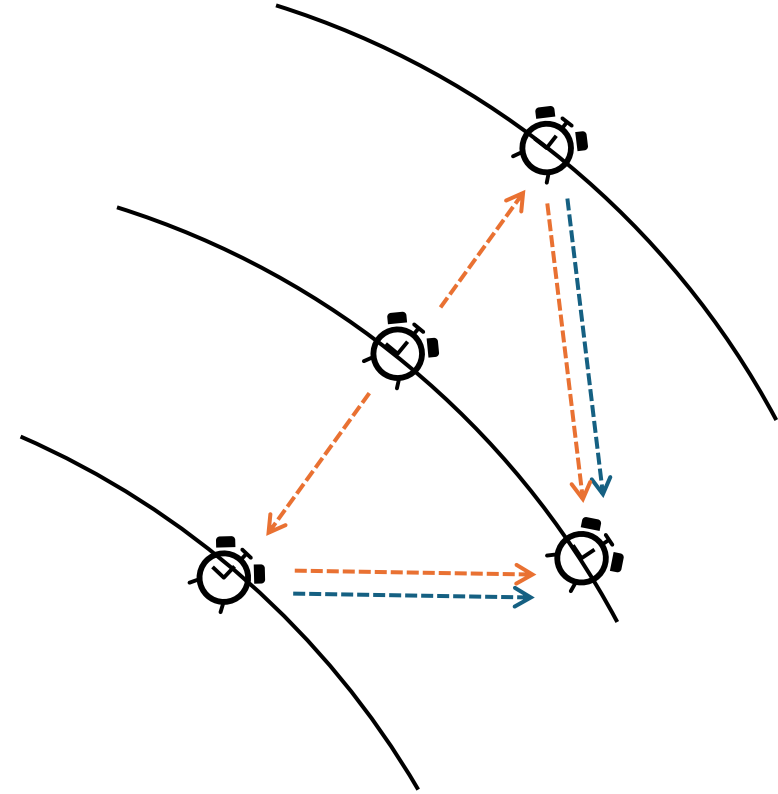
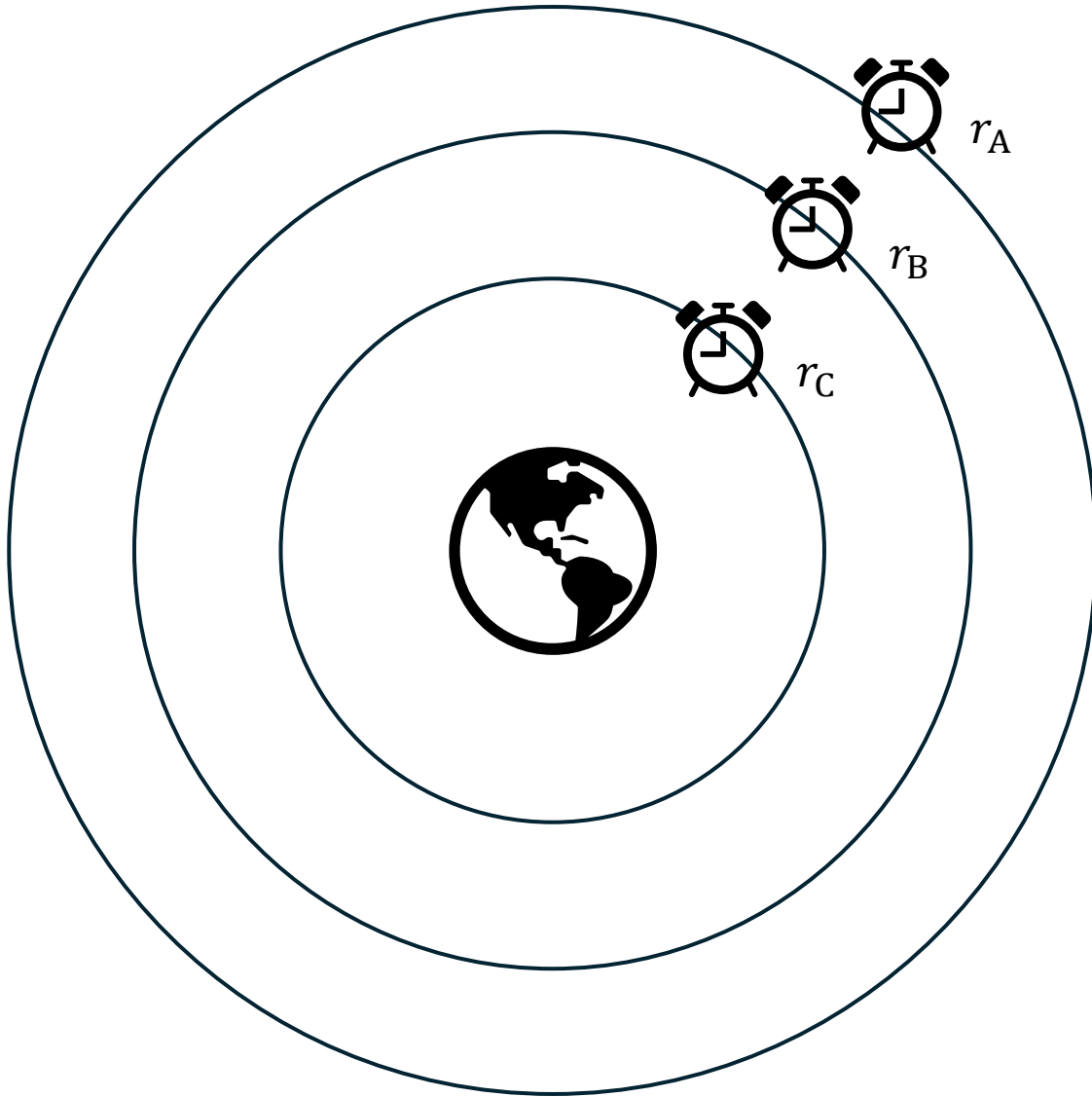
Massive objects outside the Earth-clock system induce extra **tidal frequency shift** between clocks

$$\Delta_{\text{tidal}} = \frac{1}{c^2} [V(\chi_A) - V(\chi_B) - (\chi_A - \chi_B)^i \partial_i V(\chi_E)]$$

(linear term removed by the equivalence principle due to clocks freely falling towards the Earth) [1307.6525]

→ **Transient** clock shifts could **signal DM clump/PBH encounter**
(after removing backgrounds)

3-clock system for density measurement



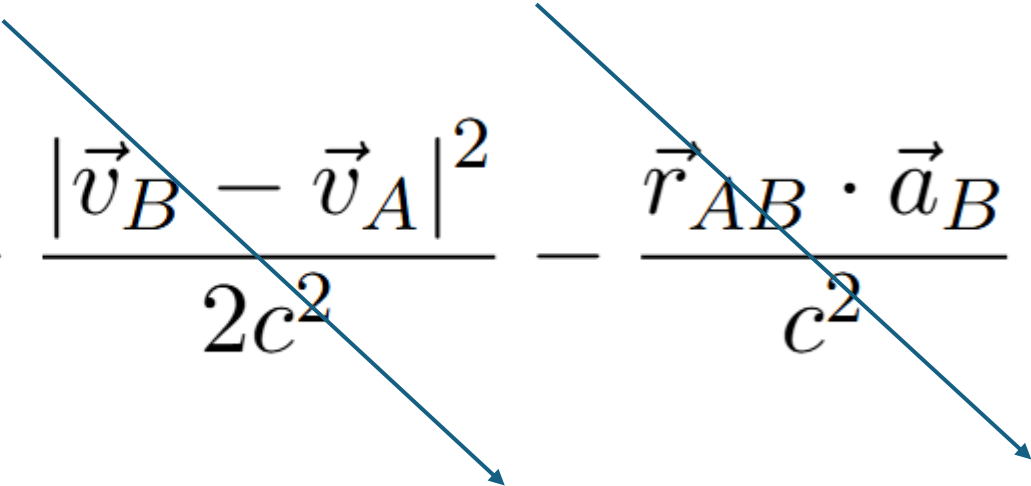
Assume:

- Spherically symmetric potential
- Synchronous orbits

STEP 1:

Assume **velocity** and **acceleration** are measured onboard

(Doppler tracking, accelerometer, uncertainties will be discussed later)

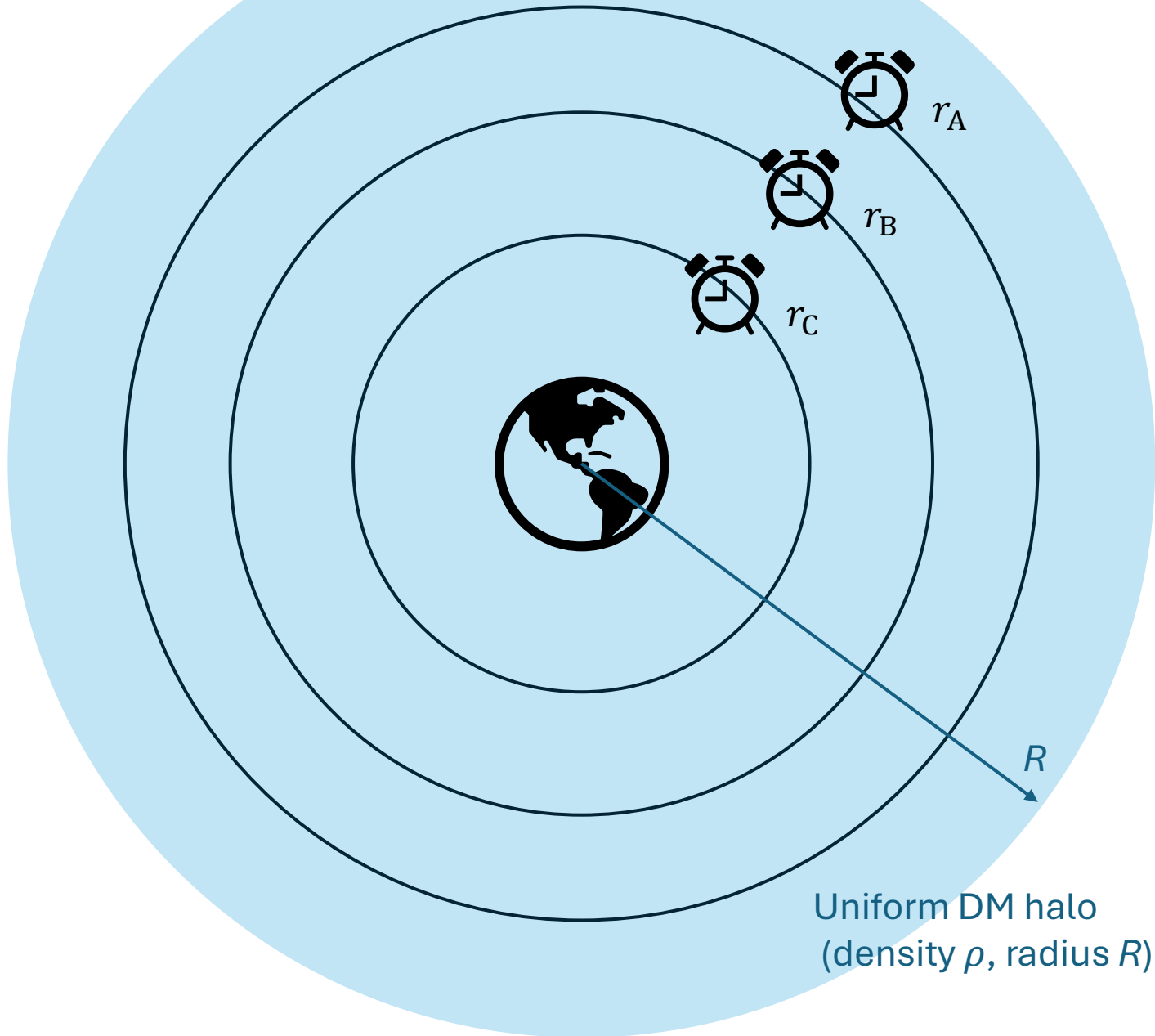
$$\Delta_{AB} = \frac{\phi_A - \phi_B}{c^2} - \frac{|\vec{v}_B - \vec{v}_A|^2}{2c^2} - \frac{\vec{r}_{AB} \cdot \vec{a}_B}{c^2}$$


we get measurements of the **potential differences** from frequency shift measurements:

$$\Delta_{AB} \rightarrow \phi_{AB}$$

$$\Delta_{CB} \rightarrow \phi_{CB}$$

STEP 2: Express **potential differences** in terms of **density**



Uniform halo Model:

Point mass + uniform sphere

$$\phi(r) = -\frac{GM_{\oplus}}{r} - \frac{2G\pi\rho}{3}(3R^2 - r^2)$$

$\phi_A = \phi(r_A)$ can be expressed in $M_{\oplus}$, ρ , r_A and R

Similarly for all clocks

Then we get expressions for ϕ_{AB} and ϕ_{CB} (independent of R)

STEP 3: Solve for **density** in terms of potential differences

$$\left\{ \begin{array}{l} \phi_{AB} = \frac{G(r_A - r_B)(3M_{\oplus} + 2\pi r_A r_B (r_A + r_B)\rho)}{3r_A r_B} \\ \phi_{CB} = \frac{G(r_C - r_B)(3M_{\oplus} + 2\pi r_C r_B (r_C + r_B)\rho)}{3r_C r_B} \end{array} \right.$$

2 equations with 2 unknowns

--> Can solve for $M_{\oplus}$ and ρ (exact single solution)

$$\left\{ \begin{array}{l} M_{\oplus} = \frac{r_A r_B r_C (r_A^2 (-\phi_{CB}) + r_B^2 (\phi_{CB} - \phi_{AB}) + r_C^2 \phi_{AB})}{G(r_A - r_B)(r_A - r_C)(r_B - r_C)(r_A + r_B + r_C)} \\ \rho = \frac{3(r_A \phi_{AB}(r_B - r_C) + r_C \phi_{CB}(r_A - r_B))}{2\pi G(r_A - r_B)(r_A - r_C)(r_B - r_C)(r_A + r_B + r_C)} \end{array} \right.$$

$$\rho = \frac{3(r_A \phi_{AB}(r_B - r_C) + r_C \phi_{CB}(r_A - r_B))}{2\pi G(r_A - r_B)(r_A - r_C)(r_B - r_C)(r_A + r_B + r_C)}$$

Density bound depends on the **uncertainty** in density measurement:

- Current limit for potential difference measurement $\sim \delta\phi \sim 10^{-20}$

Taking the current limit for velocity accuracy to be 1 micron/s [1], acceleration accuracy to be 10^{-10} m/s² [2], and laser ranging to be at the 1 mm level [3], and a clock accuracy of 10^{-20} (near future projection), using absolute values $|v_A - v_B| \sim 10$ m/s and $a_B \sim 10^{-4}$ m/s² for orbits near L2 ($r_B = 10^6$ km, $r_A = 1.01 r_B$)

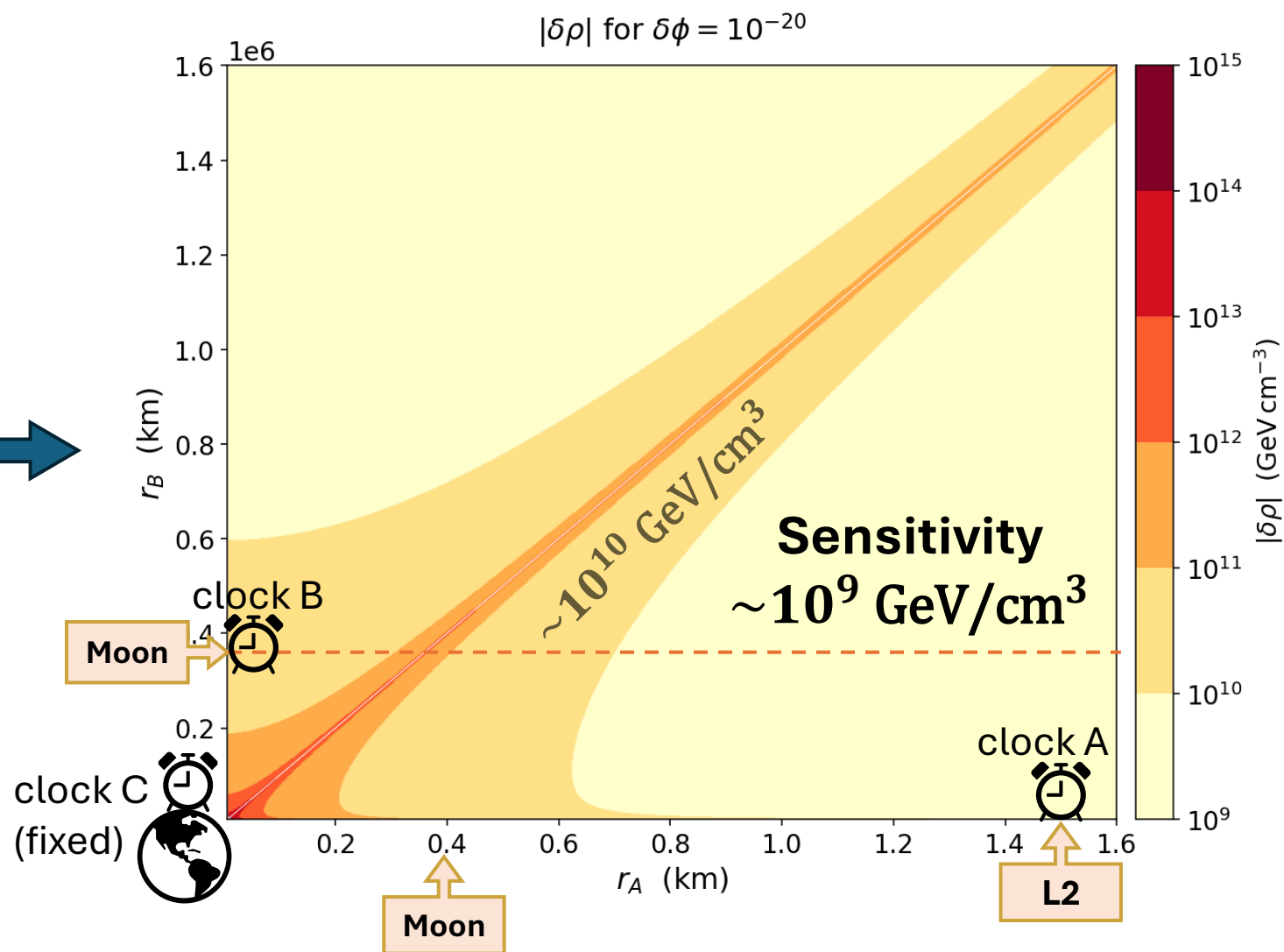
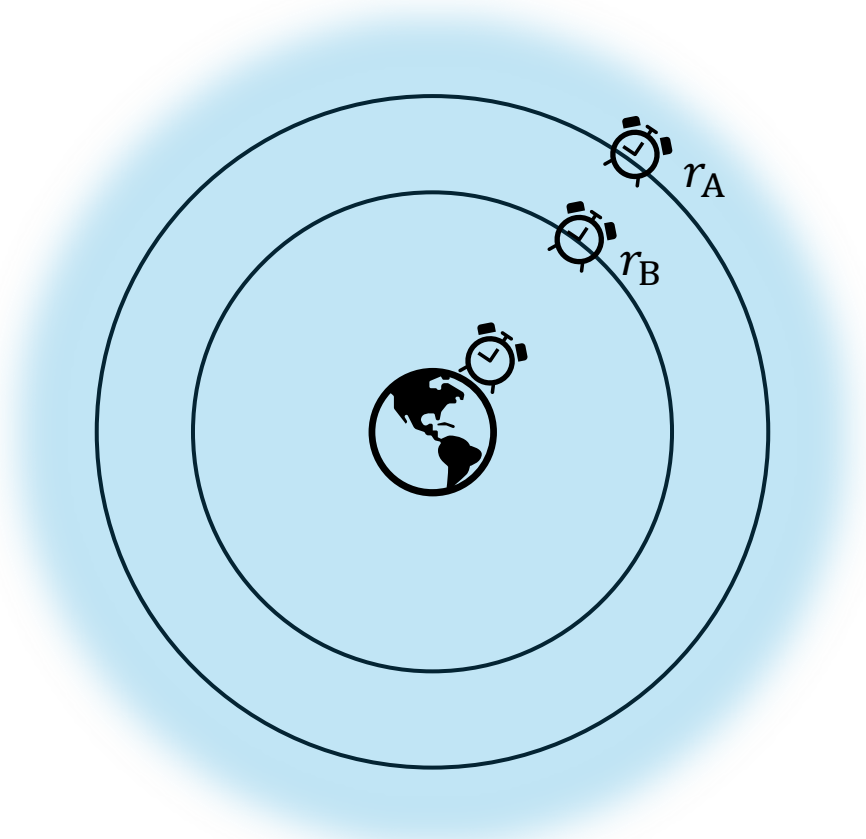
- Assume fractional errors in distances are much smaller than frequency shifts: $\delta r/r \ll \delta\phi/\phi$

$$\delta\rho \approx \frac{3 \delta\phi [r_A(r_B - r_C) + r_C(r_A - r_B)]}{2\pi G(r_A - r_B)(r_A - r_C)(r_B - r_C)(r_A + r_B + r_C)}$$

For $\delta\phi = 10^{-20}$ and 3 clocks on the **Earth's surface**, the **moon**, and **L2**, the uncertainty in the local density would be of order $\delta\rho \sim 10^9$ GeV/cm³.

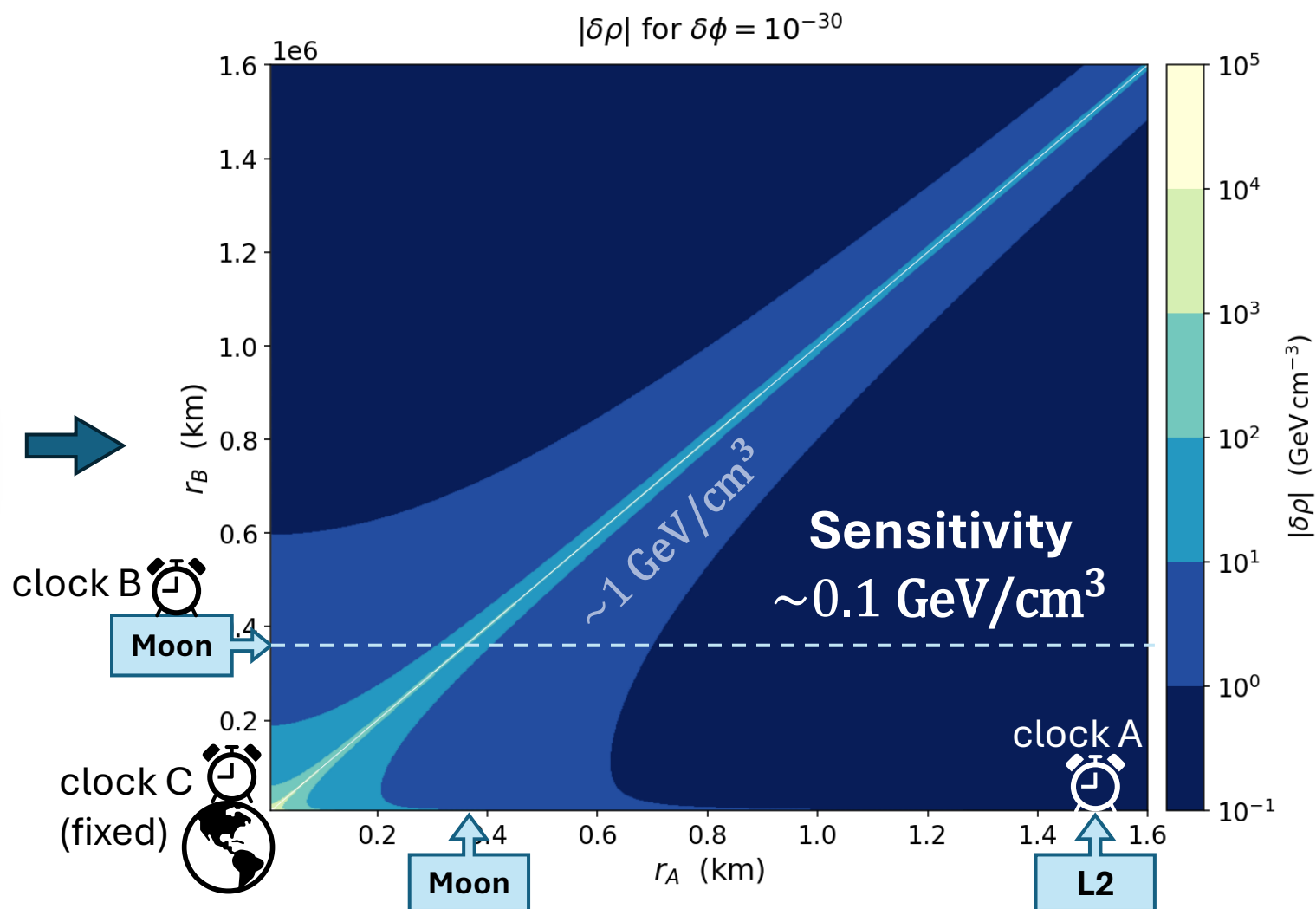
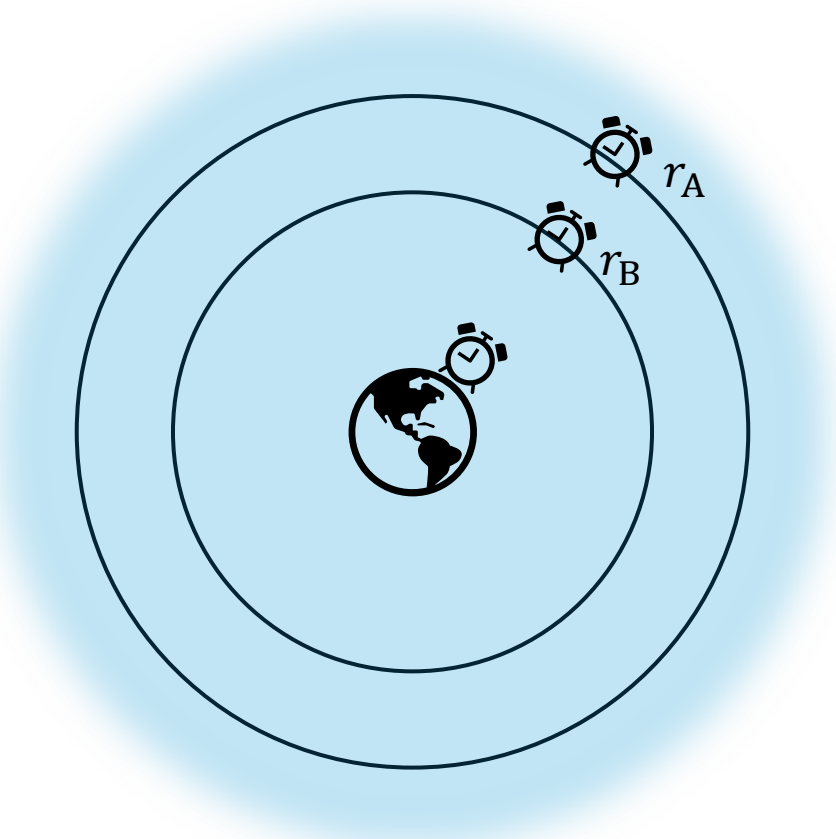
(the LLR limit was $\sim 10^{10}$ GeV/cm³ !)

Uncertainty in density measurement for a uniform halo model,
with clock C fixed on the Earth's surface



With $\delta\phi = 10^{-20}$ and $\sim \text{L2}$ orbits, we can push the bound to $\delta\rho \sim 10^9 \text{ GeV/cm}^3$.

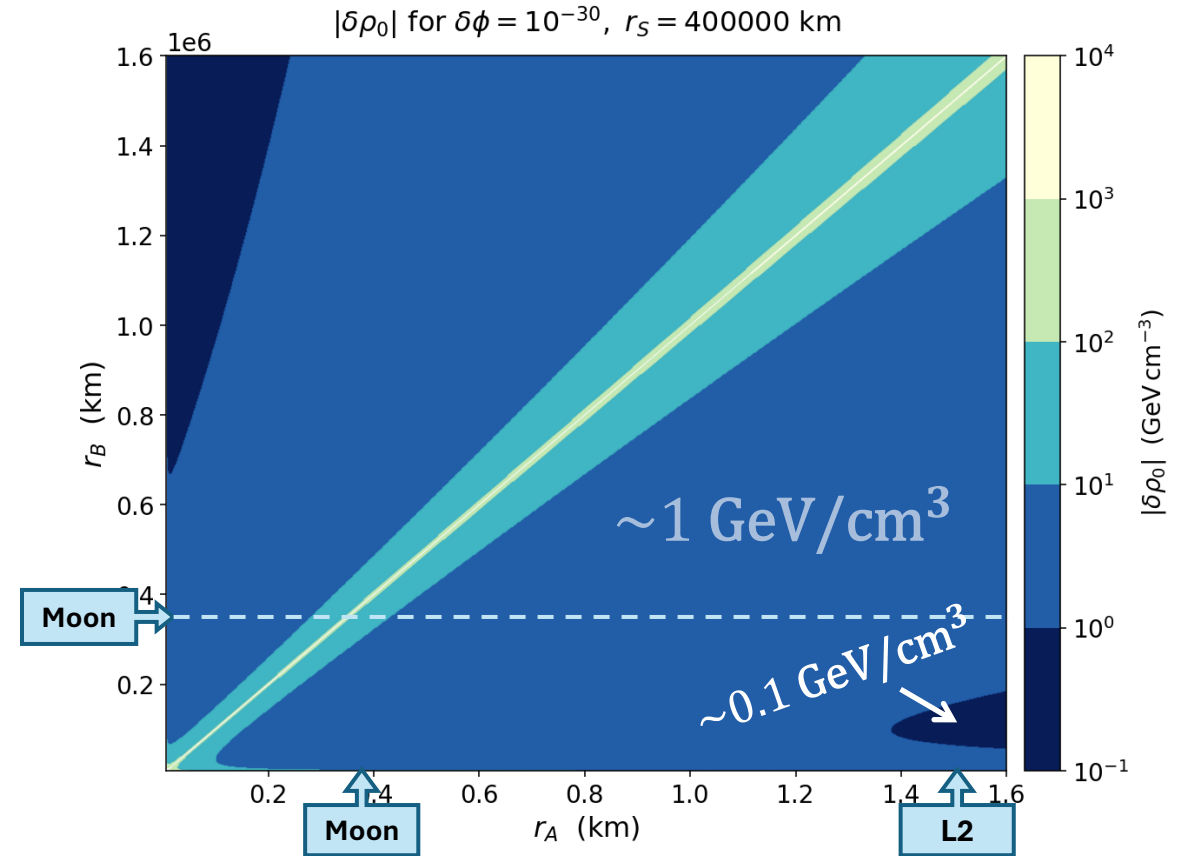
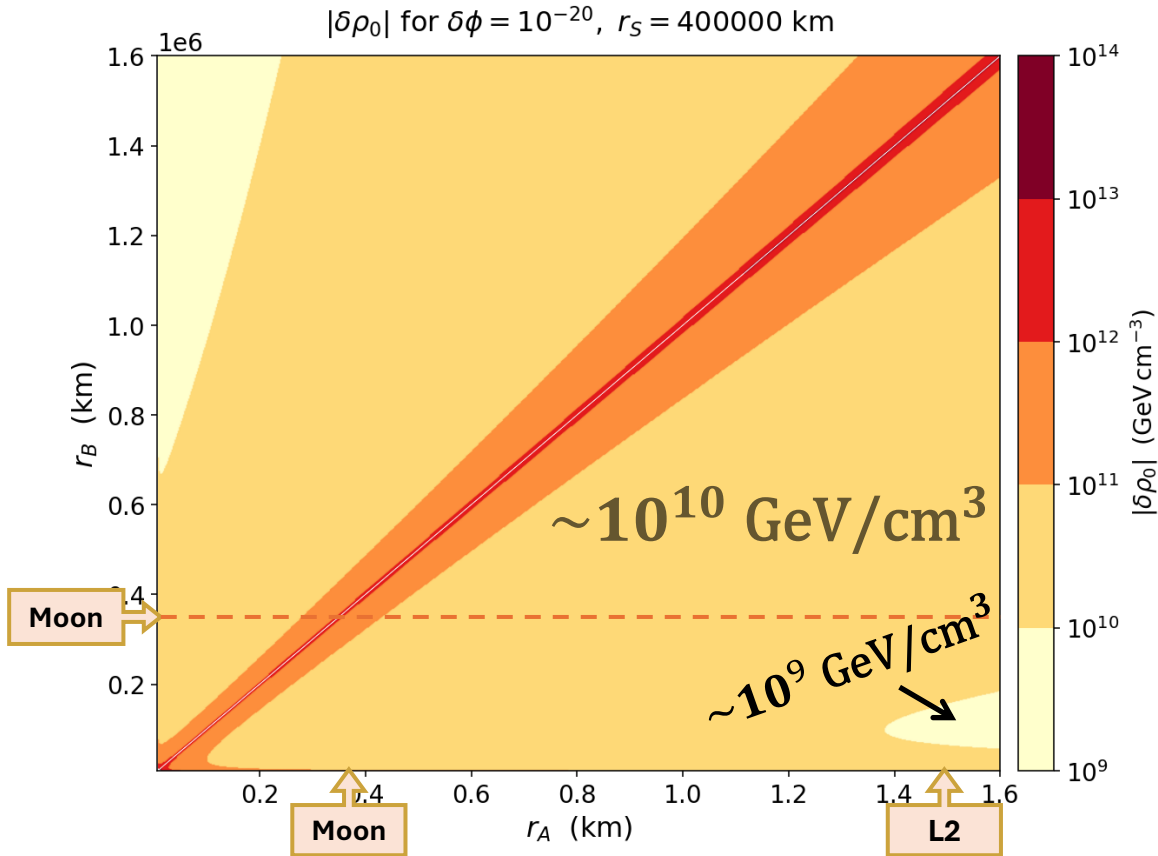
Uncertainty in density measurement for a uniform halo model,
with clock C fixed on the Earth's surface



A future precision of $\delta\phi \sim 10^{-30}$ will allow us to see the **0.4 GeV/cm^3** densities for **$\sim$ L2** orbits!

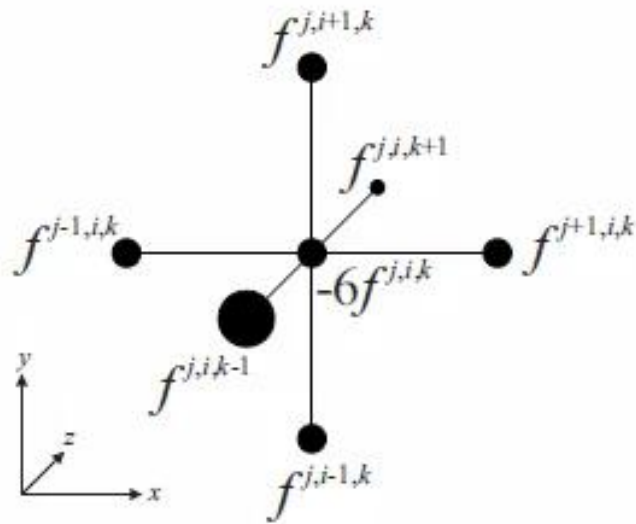
Similarly, for a NFW-like halo centred at Earth

$$\phi(r) = -\frac{GM_{\oplus}}{r} - \frac{4\pi G\rho_0 r_s^3 \log\left(\frac{r+r_s}{r_s}\right)}{r} \quad \longrightarrow \quad \rho_0 = \frac{r_B r_C \phi_{CB} - r_A (r_B \phi_{AB} + r_C (\phi_{CB} - \phi_{AB}))}{4\pi G r_s^3 \left((r_B - r_C) \log\left(\frac{r_A + r_s}{r_s}\right) + (r_C - r_A) \log\left(\frac{r_B + r_s}{r_s}\right) + (r_A - r_B) \log\left(\frac{r_C + r_s}{r_s}\right) \right)}$$



Uncertainty in ρ_0 measurement for a NFW halo model,
with clock C fixed on the Earth's surface, r_s set to $\sim$ moon distance

7-clock system for arbitrary density measurement: (Finite difference method)



$$\begin{aligned}\nabla^2 \phi(x, y, z) &= 4\pi G \rho(x, y, z) \\ &\approx \frac{1}{h^2} \left[\phi(x+h, y, z) + \phi(x-h, y, z) + \phi(x, y+h, z) \right. \\ &\quad \left. + \phi(x, y-h, z) + \phi(x, y, z+h) + \phi(x, y, z-h) - 6\phi(x, y, z) \right] \\ &= \frac{1}{h^2} \sum_{i=1}^6 \phi_{i0}\end{aligned}$$

- No symmetry assumption for the profile
- Need to **subtract** the Earth's potential along the radial direction (as well as other **backgrounds**)
- Uncertainty depends on clock precision and choice of separation h
- Valid if the potential is not changing rapidly within h

For $\delta\phi = 10^{-20}$, $h = 1000$ km, the uncertainty in the local density would be of order $\delta\rho \sim 10^{15}$ GeV/cm³.

For $\delta\phi = 10^{-30}$, $h = 1000$ km, the uncertainty in the local density would be of order $\delta\rho \sim 10^5$ GeV/cm³.

Take-home Messages

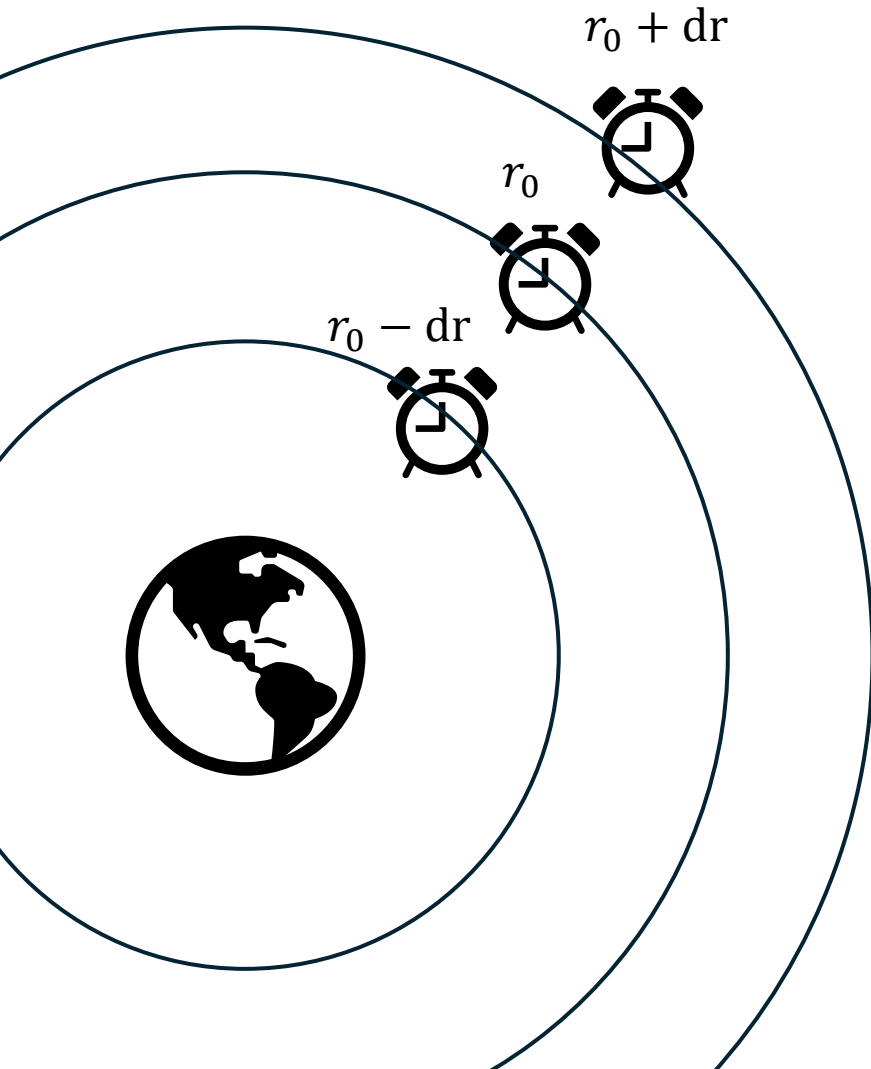
- We can use an ensemble of atomic clocks in space to detect local DM density
- A clock precision of 10^{-20} with lower than L2 orbits can push the upper bound for Earth-bound DM density to around $\sim 10^9 \text{ GeV/cm}^3$
(This is already competitive with the current LLR bound $\sim 10^{10} \text{ GeV/cm}^3$)
- To place bounds with accuracy $0.1 \sim 1 \text{ GeV/cm}^3$, need precision 10^{-30} for the combined uncertainty in clock frequency, velocity and acceleration measurements

Outlook

- Sending clocks into deep space (e.g. Alpha Centauri with chip-scale clocks) will allow us to measure the 0.4 GeV/cm^3 galactic halo density with 10^{-20} precision and clock separation of $\sim 100 \text{ AU}$
- A system of many clocks to perform finite difference method

Thank you!

Generic potential:
(spherically symmetric)
only works for $dr \ll r_0$



- Expand potential around r_0 to order dr^3
- Apply Poisson equation,
- Assume 2nd and higher-order derivatives of $\rho(r_0)$ vanish

$$\rho(r_0) = -\frac{3 \left(dr^2 (7\phi_{AB} + \phi_{CB}) + dr r_0 (3\phi_{AB} + \phi_{CB}) - r_0^2 (\phi_{AB} + \phi_{CB}) \right)}{4\pi dr^2 G (2dr^2 + 6dr r_0 + 3r_0^2)}$$

For $\delta\phi = 10^{-20}$, $r_0 = 10^6$ km, $dr = 1000$ km, the uncertainty in the local density would be of order $\delta\rho \sim 10^{15}$ GeV/cm³.

For $\delta\phi = 10^{-30}$, $r_0 = 10^6$ km, $dr = 1000$ km, the uncertainty in the local density would be of order $\delta\rho \sim 10^5$ GeV/cm³.

Method 2: (won't include in talk)

Express **velocity** and **acceleration** in terms of the potential and solve for full Δ_{AB} and Δ_{CB}

$$\Delta_{AB} = -\frac{1}{2} \left(\sqrt{\left| (dr + r_0) \left(\frac{GM}{(dr + r_0)^2} + \frac{4}{3} G\pi (dr + r_0) \rho \right) \right|} - \sqrt{\left| r_0 \left(\frac{GM}{r_0^2} + \frac{4}{3} G\pi r_0 \rho \right) \right|} \right)^2 + \frac{2}{3} G\pi (3R^2 - r_0^2) \rho - \frac{2}{3} G\pi (3R^2 - (dr + r_0)^2) \rho - dr \left(\frac{GM}{r_0^2} + \frac{4}{3} G\pi r_0 \rho \right) + \frac{GM}{r_0} - \frac{GM}{dr + r_0}$$

$$\Delta_{CB} = -\frac{1}{2} \left(\sqrt{\left| (r_0 - dr) \left(\frac{GM}{(r_0 - dr)^2} + \frac{4}{3} G\pi (r_0 - dr) \rho \right) \right|} - \sqrt{\left| r_0 \left(\frac{GM}{r_0^2} + \frac{4}{3} G\pi r_0 \rho \right) \right|} \right)^2 + \frac{2}{3} G\pi (3R^2 - r_0^2) \rho - \frac{2}{3} G\pi (3R^2 - (r_0 - dr)^2) \rho + dr \left(\frac{GM}{r_0^2} + \frac{4}{3} G\pi r_0 \rho \right) + \frac{GM}{r_0} - \frac{GM}{r_0 - dr}$$

2 equations with 2 unknowns

- Can't solve for $M_{\oplus}$ and ρ exactly (highly non-linear)
- Solving the first two orders in (dr/r_0) gives 2 roots that both give wrong values for ρ when checked backwards
- Can't solve numerically either.. (gives wrong numbers)