

Gravitational Waves from Dimension-6 Assisted Peccei - Quinn Phase Transitions

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Kristjan Mürsepp

In collaboration with N. Benincasa

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Laboratori Nazionali di Frascati, (LNF-INFN),
National Institute of Chemical Physics and Biophysics (NICPB)

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Strong CP Problem & the Peccei–Quinn Solution

The Problem

- QCD admits $\mathcal{L}_\theta = \bar{\theta} \frac{g_s^2}{32\pi^2} G\tilde{G}$.
- $\bar{\theta} \equiv \theta_{\text{QCD}} + \arg \det(M_u M_d)$.
- Induces neutron EDM:
 $d_n \sim \bar{\theta} \times 10^{-16} \text{ e} \cdot \text{cm} \Rightarrow \bar{\theta} \lesssim 10^{-10}$.
- ⇒ Why so small? **Naturalness puzzle.**

Axion as a DM candidate

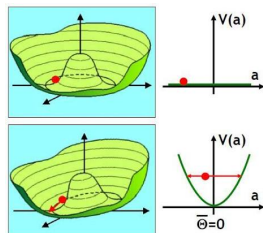
- For post-inflationary axion $\theta_{\text{eff}} \simeq 2.155$

$$\Omega_{\text{DM}}^a h^2 = 0.12 \left(\frac{f_a}{2 \times 10^{11} \text{ GeV}} \right)^{7/6},$$

neglecting contribution from topological defects.

The PQ Solution

- New global $U(1)_{PQ}$, broken at scale f_a .
- Goldstone boson: the **axion** $a(x)$.
- Anomaly shifts the term:
 $\bar{\theta} \rightarrow \bar{\theta} + a(x)/f_a$.
- Instantons tilt the potential:
 $m_a \sim \chi^{1/4}(T)/f_a$, with $\sqrt{\chi(0)} = f_\pi m_\pi$.



S. Hannestad, presentation at 5th Patras Workshop 2009

The KSVZ Realisation

- Complex scalar Φ (SM singlet) + fermion ψ : triplet of $SU(3)_c$, $Y = 0$, both PQ-charged.
- $\mathcal{L}_{\text{KSVZ}} \supset |\partial_\mu \Phi|^2 + \bar{\psi} i \not{D} \psi - (y_F \Phi \bar{\psi}_L \psi_R + \text{h.c.}) - V(|\Phi|^2)$.
- **Symmetry breaking**: $\Phi = \frac{\phi}{\sqrt{2}} e^{ia/f_a}$, $\langle \phi \rangle = f_a \Rightarrow m_\psi = y_F f_a / \sqrt{2}$.
- Chiral rotation removes the axion phase from m_ψ ; anomalous integral measure generates

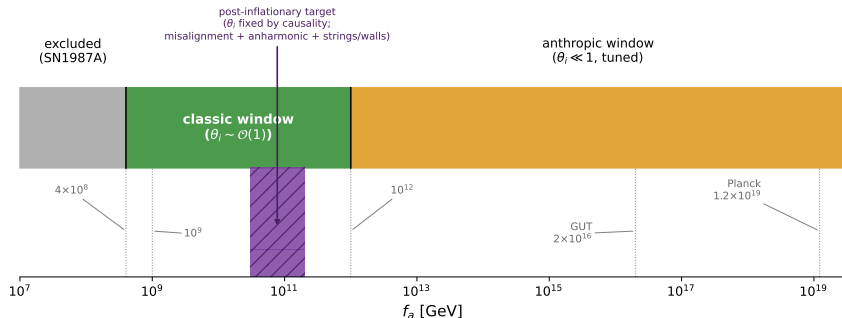
$$\delta \mathcal{L} \supset \frac{g_s^2}{32\pi^2} \frac{a}{f_a} G_{\mu\nu} \tilde{G}^{\mu\nu} \quad \Rightarrow \quad \text{ solves strong CP! }$$

- Tree-level (dimension-6 assisted) potential for the radial mode:

$$V_{\text{tree}}(\phi) = \frac{m^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4 + \frac{1}{8\Lambda^2} \phi^6.$$

$\Rightarrow$ **Tree-level barrier** needs $\lambda < 0$; **stability** requires $C_1 > 0$ (Here absorbed into Λ).

Constraints on the PQ-Breaking Scale

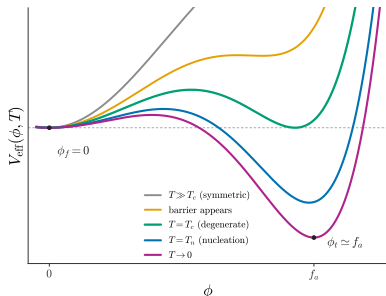


- $f_a \lesssim 4 \times 10^8$ GeV **excluded**: axion emission would over-cool SN1987A.
- **Post-inflationary PQ breaking**: θ_i is not free (averaged over causally disconnected patches) $\theta_{i,\text{eff}} \simeq 2.155$ including anharmonic effects.

Phase Transition: Key Quantities

Potential & Tunneling

- $V_{\text{eff}} = V_{\text{tree}} + V_{\text{CW}} + V_T(T)$, daisy-resummed (Parwani)
- **Milestones:**
- T_c : **degenerate minima**, $V(\phi_t, T_c) = V(0, T_c)$
- T_n : **nucleation**, $\Gamma/H^4 \sim 1$ (for us $S_3/T \sim 90$)
- T_p : **percolation**, false-vacuum fraction $P_t(T_p) \simeq 0.29$
- T_{reh} : **reheating**, from released vacuum energy



Bounce Action & Tunneling Rate

- Nucleation rate per volume: $\Gamma \simeq A(T) e^{-S/T}$ with,

$$A_3(T) \sim T^4 \left(\frac{S_3}{2\pi T} \right)^{3/2}, \quad \text{for } R_b^{-1} \ll T$$

$$A_4(T) \sim \frac{1}{R_b^4} \left(\frac{S_4}{2\pi} \right)^2, \quad \text{for } R_b^{-1} \gg T.$$

- $O(d)$ -symmetric bounce action:

$$S_d[\phi] = \mathcal{A}_d \int_0^\infty dr r^{d-1} \left[\frac{1}{2} \left(\frac{d\phi}{dr} \right)^2 + V_{\text{eff}} \right]$$

- $\phi(r)$ solves $\phi'' + \frac{d-1}{r} \phi' = \partial_\phi V_{\text{eff}}$, with $\phi'(0) = 0$, $\phi(\infty) = \phi_f$

PT Strength & Duration

- Strength $\alpha = \Delta\theta(T_p)/\rho_{\text{rad}}(T_p)$: **latent heat**.
- **Duration** β/H_* : from mean bubble separation $R_*(T_p)$.

KSVZ + Dim-6: Tree-Level Estimates

- **Focus on scenarios where the barrier survives to $T = 0$** (strong supercooling)
 $\Rightarrow$ **large latent heat released**, best prospects for GW detection.
- Tree-level **PQ vacuum** and peak at the barrier:

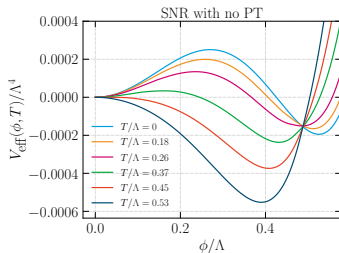
$$f_a = \sqrt{\frac{2}{3}} \sqrt{-\lambda \Lambda^2 + \Lambda \sqrt{\lambda^2 \Lambda^2 - 3m^2}}, \quad \phi_{\max} = \sqrt{\frac{2}{3}} \sqrt{-\lambda \Lambda^2 - \Lambda \sqrt{\lambda^2 \Lambda^2 - 3m^2}}$$

- Barrier exists (real $f_a, \phi_{\max}$) $\iff \lambda < -\sqrt{3} m/\Lambda$.
 - f_a is the **global** minimum (deeper than origin) $\iff \lambda < -\frac{2m}{\Lambda}$ (stronger condition).
 - Barrier height above the false vacuum: $V(\phi_{\max}) \simeq -\frac{m^4}{4\lambda} \lesssim \frac{m^3 \Lambda}{8}$.
- $\Rightarrow$ Barrier grows as $\lambda \rightarrow -2m/\Lambda$ (**saturation**):
high barrier, large $S(T)$ down to low $T \implies$ **strong FOPT with significant supercooling**.

One-Loop Bounds & Non-Standard Thermal Histories

One-Loop y_F Window ($T = 0$)

- $\lambda < -2m/\Lambda$ changes into $y_F^{\min} < y_F < y_F^{\max}$.
- Origin stays a local min. (barrier exists) $\Rightarrow$ **upper** bound $y_F < y_F^{\max}(m, \lambda, \Lambda, \mu)$
- f_a stays the global min. $\Rightarrow$ **lower** bound $y_F > y_F^{\min}(m, \lambda, \Lambda, \mu)$.
- Perturbativity: set $\mu = \Lambda$ (highest EFT scale)
 $\Rightarrow$ **no UV Landau pole**; running of λ, y_F spans $\lesssim 1$ decade below Λ , so **no IR Landau poles either**.



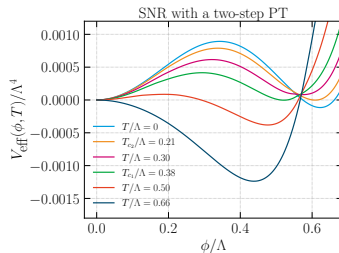
Non-Standard Thermal Histories (SNR)

- High - T potential keeps the tree-level sextic form, with effective couplings

$$m_{\text{eff}}^2(T) = m^2 + \left(\frac{y_F^2}{4} + \frac{\lambda}{3} \right) T^2,$$

$$\lambda_{\text{eff}}(T) = \lambda + \frac{3}{4} \frac{T^2}{\Lambda^2}.$$

- If $m_{\text{eff}}^2(T) < 0$ up to $T_{\text{SNR}} \lesssim \Lambda$: PQ symmetry is **not restored** (SNR), with $T_{\text{SNR}}^2 = -\frac{12m^2}{3y_F^2 + 4\lambda}$.



Zero-Temperature Quantum Tunneling: $O(4)$ Bounce

- Barrier persists to $T = 0$ by construction $\Rightarrow$ vacuum decay via $O(4)$ bounce remains available once thermal ($O(3)$) nucleation stalls.

- At low T , temperature-independent rate: $\Gamma_0 = \frac{1}{R_b^4} \left(\frac{S_4}{2\pi} \right)^2 e^{-S_4}$.

- Extreme supercooling $\Rightarrow$ vacuum energy dominates, de Sitter expansion:

$$H_V^2 = \frac{8\pi}{3M_{\text{Pl}}^2} \Delta V_0, \quad \Delta V_0 \equiv V_{\text{eff}}(0) - V_{\text{eff}}(f_a)$$

- Percolation integral solvable in closed form $\Rightarrow$ Guth–Weinberg criterion:

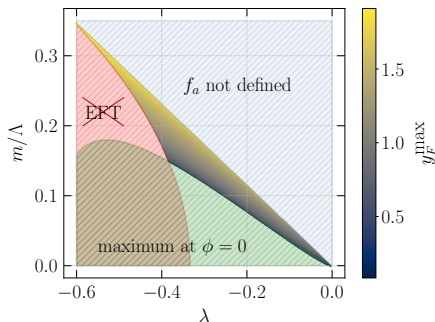
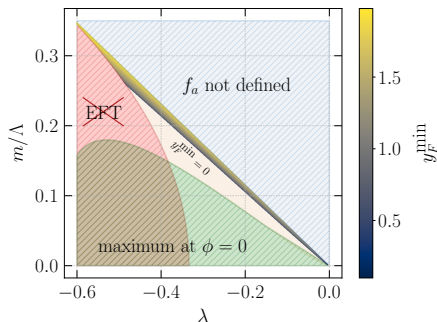
$$\frac{\Gamma_0}{H_V^4} > \frac{9}{4\pi v_w^3} \simeq 0.72 \quad (v_w \rightarrow 1) \quad \Longleftrightarrow \quad S_4 < S_4^{\text{crit}}(R_b H_V, v_w)$$

Guth, Weinberg Phys. Rev. D **23** (1981)

- If satisfied, T_p follows analytically: $T_p = T_V e^{-x_p}$, $T_V = (30 \Delta V_0 / \pi^2 g_*)^{1/4}$.
- $\Rightarrow$ Completion window is **narrow**: supercooling extends only $\lesssim 1.1$ e-folds past vacuum domination — **percolates almost immediately, or never**.

Parameter Space & Constraints

- Scan: $\Lambda \in \{10^{10}, 10^{11}, 5 \times 10^{11}, 10^{12}\}$ GeV;
for each Λ : $m \in [0, 35] \times \frac{\Lambda}{100}$, $\lambda \in [-0.6, 0]$, $y_F \in [0.1, 2]$



Where FOPTs Happen: Parameter Correlations

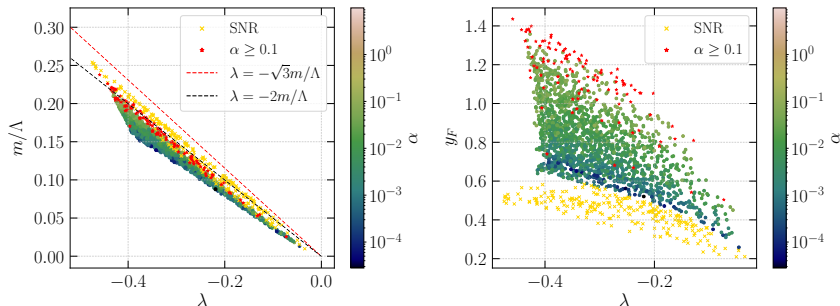
- Numerical scan with CosmoTransitions, up to $T = \Lambda$.

C.Wainwright, Comput. Phys. Comm. **183** (2012)

- PT strength α grows with the slope $\lambda/(m/\Lambda)$; strong points ($\alpha \geq 0.1$, red stars) cluster around $\lambda = -2m/\Lambda$.
- PT strength grows with y_F , while SNR favours **smaller** y_F .

$$\Delta V^\psi = \frac{3y_F^4 f_a^4}{128\pi^2} > 0, \quad m_{\text{eff}}^2(T) = m^2 + \left(\frac{y_F^2}{4} + \frac{\lambda}{3}\right)$$

- SNR points (yellow crosses): bounded to $\alpha \lesssim 10^{-2}$.



GW Spectrum vs. Cutoff Scale (set-up)

- Refined scan near $\lambda \simeq -2m/\Lambda$, using `TransitionListener`.

J.Matuszak and C.Tasillo 2605.15259 [hep-ph]

- Thermodynamic quantities at T_p , check completion : $P_t(T_f) = 0.99$.
- v_w approximated by v_w^{LTE} . (upper bound)
- Broken power law GW spectrum

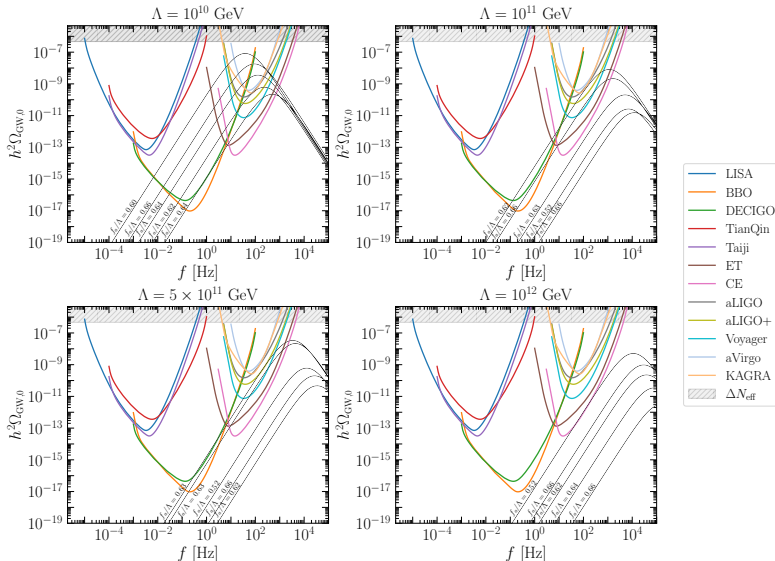
$$h^2\Omega_{\text{GW},0} = \mathcal{R} \left(h^2\Omega_{\text{col}} + h^2\Omega_{\text{sw}} + h^2\Omega_{\text{turb}} \right). \quad (1)$$

- $f^{\text{peak}} \sim 10^{-5} \text{ Hz} \times \frac{T_{\text{reh}}}{100 \text{ GeV}} \times \frac{\beta}{H(T_p)}$, with $T_{\text{reh}} \sim \Lambda/10$

$\Rightarrow \Lambda = 10^{10} \text{ GeV}$: $f^{\text{peak}} \sim 10^2 \text{ Hz} \times \beta/H(T_p)$
 $\Rightarrow$ **reachable** by aLIGO/aVirgo/KAGRA **today** with $(\beta/H \sim \mathcal{O}(1))$.

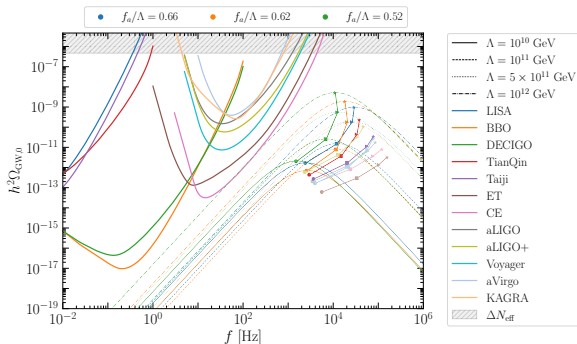
$\Rightarrow \Lambda \geq 10^{12} \text{ GeV}$ would need $\beta/H(T_p) \sim 10^{-2}$, **excluded by causality**
 $(\beta/H(T_p) \gtrsim 4.43)$ — **unreachable** by increasing the PT duration.

GW Spectrum vs Cutoff Scale (in action)



Same Benchmark Points, Varying Λ

- Track $h^2\Omega_{\text{GW},0}^{\text{peak}}$ for fixed $(m/\Lambda, \lambda, y_F)$ as Λ is varied across the four benchmark values.
 - Increasing Λ always increases the peak amplitude; the peak frequency first **increases**, then **decreases** — the signal moves up-right, then up-left.
- ⇒ A point too weak to see at $\Lambda = 10^{10}$ GeV **may become observable at higher Λ , provided the PT still completes.**



Punchline: Narrow f_a/Λ Window & Axion DM

- $O(4)$ **rescue check**: non-completing points have $S_4 \in [296, \infty)$ vs. $S_4^{\text{crit}} \in [70, 110]$ across all $\Lambda \Rightarrow$ **No point is rescued by quantum tunneling.**
 - **Completion requires** $0.42 \lesssim f_a/\Lambda \lesssim 2/3$ (only 3% of scanned points complete). Completion fraction falls from $\sim 70\%$ near $f_a/\Lambda \in (0.6, 2/3)$ to $\sim 5\%$ at $f_a/\Lambda \in (0.42, 0.5)$.
- $\Rightarrow$ **f_a is no longer free**: fixed by Λ to within a factor $\simeq 1.6$ (post-inflationary breaking, misalignment + anharmonic corrections).

Λ [GeV]	f_a [GeV]	$\Omega_{\text{DM}}^a h^2$
10^{10}	$(4.2\text{--}6.7) \times 10^9$	0.0013–0.0022
10^{11}	$(4.2\text{--}6.7) \times 10^{10}$	0.019–0.033
5×10^{11}	$(2.1\text{--}3.3) \times 10^{11}$	0.13–0.22
10^{12}	$(4.2\text{--}6.7) \times 10^{11}$	0.29–0.49

- **Observed** $\Omega_{\text{DM}} h^2 = 0.12$ **reproduced** at $\Lambda \simeq (3\text{--}5) \times 10^{11}$ GeV.
- $\Rightarrow$ **Multimessenger picture**: GW detectable *today* at low Λ (axion **subdominant DM**). **Full DM abundance** needs higher Λ , typically pushing the signal *beyond* current detectors.

UV Completions: The Common Mechanism

- The dim-6 operator $\phi^6/8\Lambda^2$ **is not put in by hand**, but **arises from integrating out a heavy state in a renormalisable UV theory**

- **General recipe:** couple Φ to a heavy real scalar S (mass $\mu_S \equiv \Lambda$) via a portal,

$$\mathcal{L}_{\text{UV}} \supset |\partial_\mu \Phi|^2 + \frac{1}{2}(\partial_\mu S)^2 - \mu_\Phi^2 |\Phi|^2 - \frac{\mu_S^2}{2} S^2 - \kappa_{\Phi S} |\Phi|^2 S - \lambda_\Phi |\Phi|^4 - \frac{\lambda_{\Phi S}}{2} |\Phi|^2 S^2 - \frac{\kappa}{3} S^3$$

- Solve the S equation of motion **in the Φ background** and expand for $\mu_S^2 \gg \square, |\Phi|^2$:
 $\hat{S} \simeq -\kappa_{\Phi S} |\Phi|^2 / \mu_S^2 + \dots$

- Plugging $\hat{S}(\Phi)$ back into $\mathcal{L}_{\text{UV}}$ generates $|\Phi|^6/\Lambda^2$ (**plus derivative operators, removed by a field redefinition**) $\Rightarrow$ **closed-form matching of (m^2, λ, Λ) onto $(\mu_\Phi, \lambda_\Phi, \kappa_{\Phi S}, \lambda_{\Phi S}, \kappa, \mu_S)$**

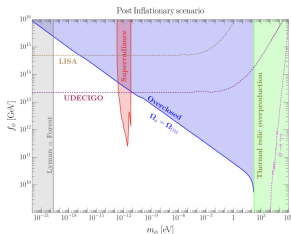
$\Rightarrow$ Minimal proof-of-concept: **a single heavy real singlet suffices.**

- The next two completions embed this same mechanism in physically motivated sectors — each also supplying its own DM candidate

Motivated UV Completions & Their DM Candidates

Type-I Seesaw / Majoron

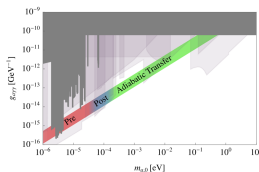
- Add right-handed neutrinos N_R + complex scalar Φ_M
 $y_M \bar{N}_R^c N_R \Phi_M \Rightarrow M_M = \sqrt{2} y_M f_\phi$, $m_\nu \simeq m_D^2 / M_M$.
 - Portal $\lambda_\rho |\Phi|^2 |\Phi_M|^2$: integrating out the heavy radial mode of Φ_M generates the dim-6 operator exactly as for the singlet, with $\Lambda \propto f_\phi$; needs $\tilde{\lambda} < 0$.
 - Angular mode = massless **Majoron**; explicit $U(1)_L$ breaking gives it mass m_ϕ .
- ⇒ Viable **DM candidate** (misalignment, + cosmic strings if post-inflationary). Also for $f_a \lesssim 10^{11}$ GeV where **axion misalignment alone underproduces Ω_{DM}**



B.Batell et.al. 2606.02706 [hep-ph]

Two-Axion System

- Extend $\Phi \rightarrow (\Phi, \chi)$, gauge group $SU(3)_c \times SU(3)_D$: hidden sector confines at $\Lambda_D \gg \Lambda_{\text{QCD}}$, giving axions a (QCD) and b (dark) with masses $m_a(T)$, m_b .
 - m_b is T -independent; $m_a(T)$ grows as $T \rightarrow \Lambda_{\text{QCD}} \Rightarrow$ **adiabatic level crossing** at $m_a(T_*) = m_b$ transfers energy between mass eigenstates.
- ⇒ Modifies the QCD axion relic abundance, allowing a **lower f_a** (down to $\sim 10^9$ GeV) to still match Ω_{DM} — **exactly the window with the best GW prospects.**
- Same trick: integrating out χ 's heavy radial mode generates the dim-6 operator, with Λ set by the heavy axion scale.
 - For $f_b \gg f_a$ we need $\lambda_\chi \ll \lambda_{\Phi\chi}$.



D.Cyncynates and O.Thompson, Phys.Rev.D. **108** (2023)

Conclusion

- A single PQ-invariant **dimension-6 operator** added to the minimal KSVZ potential can turn the PQ transition into FOPT.
- One can have **different thermal histories** : a single FOPT, two-step transition and SNR.
- Numerical scan: FOPTs **strongest near $\lambda = -2m/\Lambda$** ; low Λ gives GW signals reachable by aLIGO/aVirgo/KAGRA **today**, with a **non-monotonic peak frequency** as Λ grows
- ⇒ **Multimessenger sweet spot**: $\Lambda \simeq (3-5) \times 10^{11}$ GeV reproduces the observed axion DM abundance *and* a GW signal within reach of future interferometers
- Renormalisable **UV completions** (real singlet, Type-I seesaw/Majoron, two-axion system) generate the operator dynamically — the latter two also modifying the minimal misalignment DM prediction.
- **Outlook**: RG-improved potentials, 3-D EFT, higher-dim. operators, GW from multi-step, $O(4)$ transitions elsewhere in parameter space.

Higher-dimensional operators turn the KSVZ axion into a genuine multimessenger target: GWs complementing direct axion searches.