



Pulsar Timing Arrays as Probe of Massive Gravity

Tina Kahniashvili
Carnegie Mellon University, USA · Ilia State University, Georgia

September 1, 2026



Outline

Why Gravitational Waves Based Tests?

Why New Physics?

Maybe Massive Gravity?

Pulsar Timing Arrays – Theory & Data

Interpretation – What Did We Learn

Results

Take Home Thoughts & Future Work

Collaboration



Marcus Bosca



Chris Choi

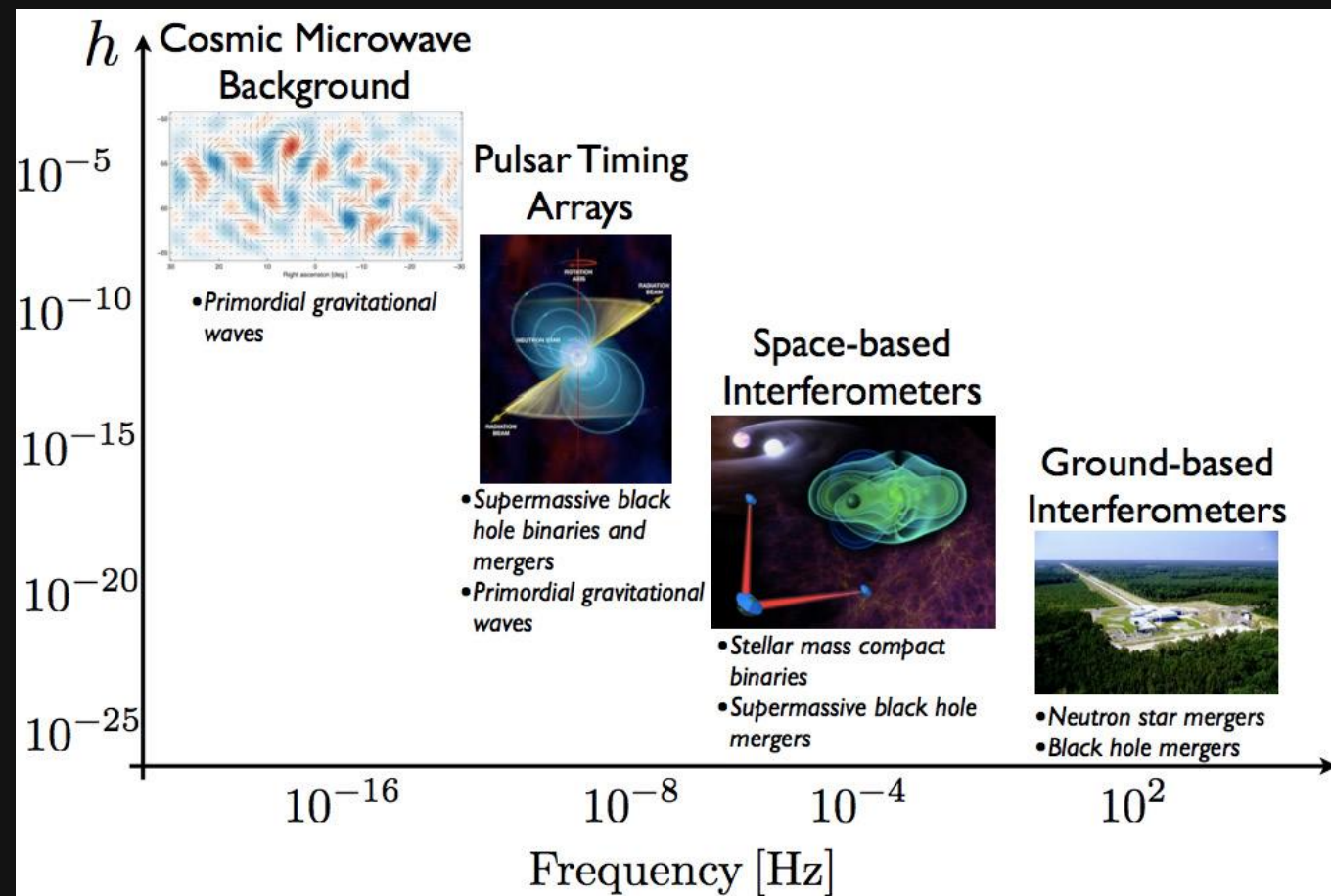
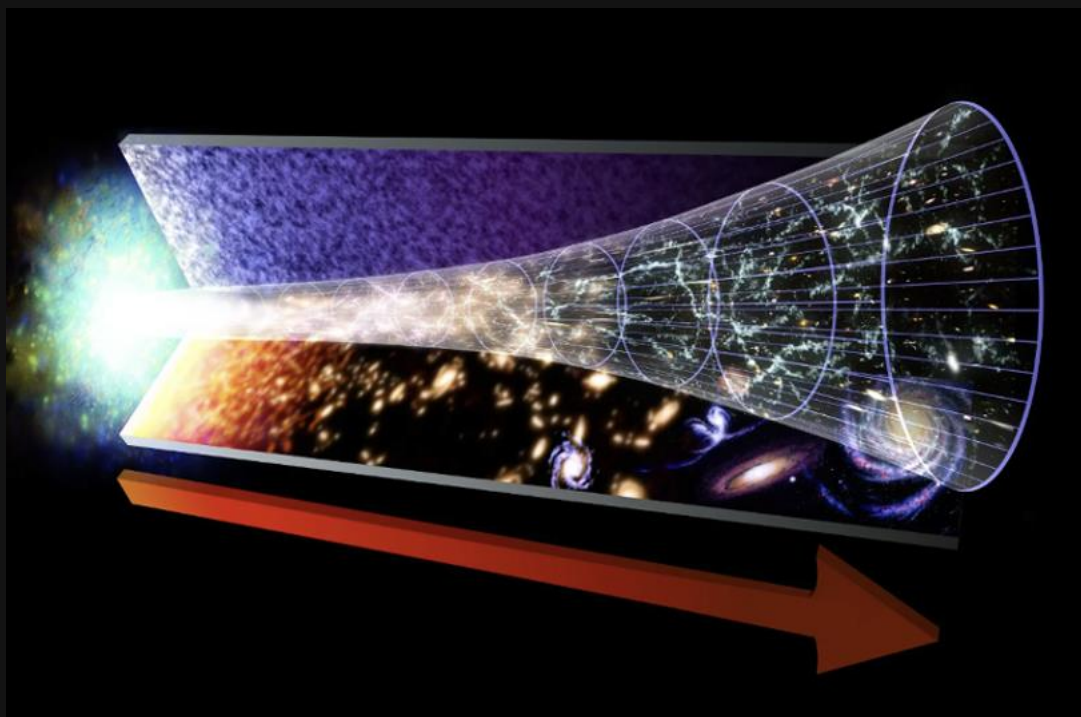


Murman Gurenidze

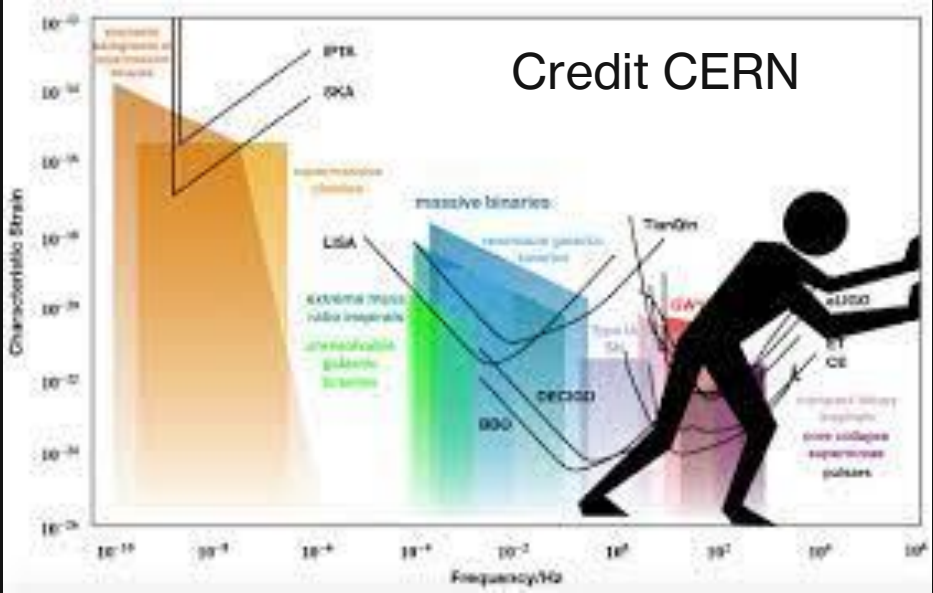
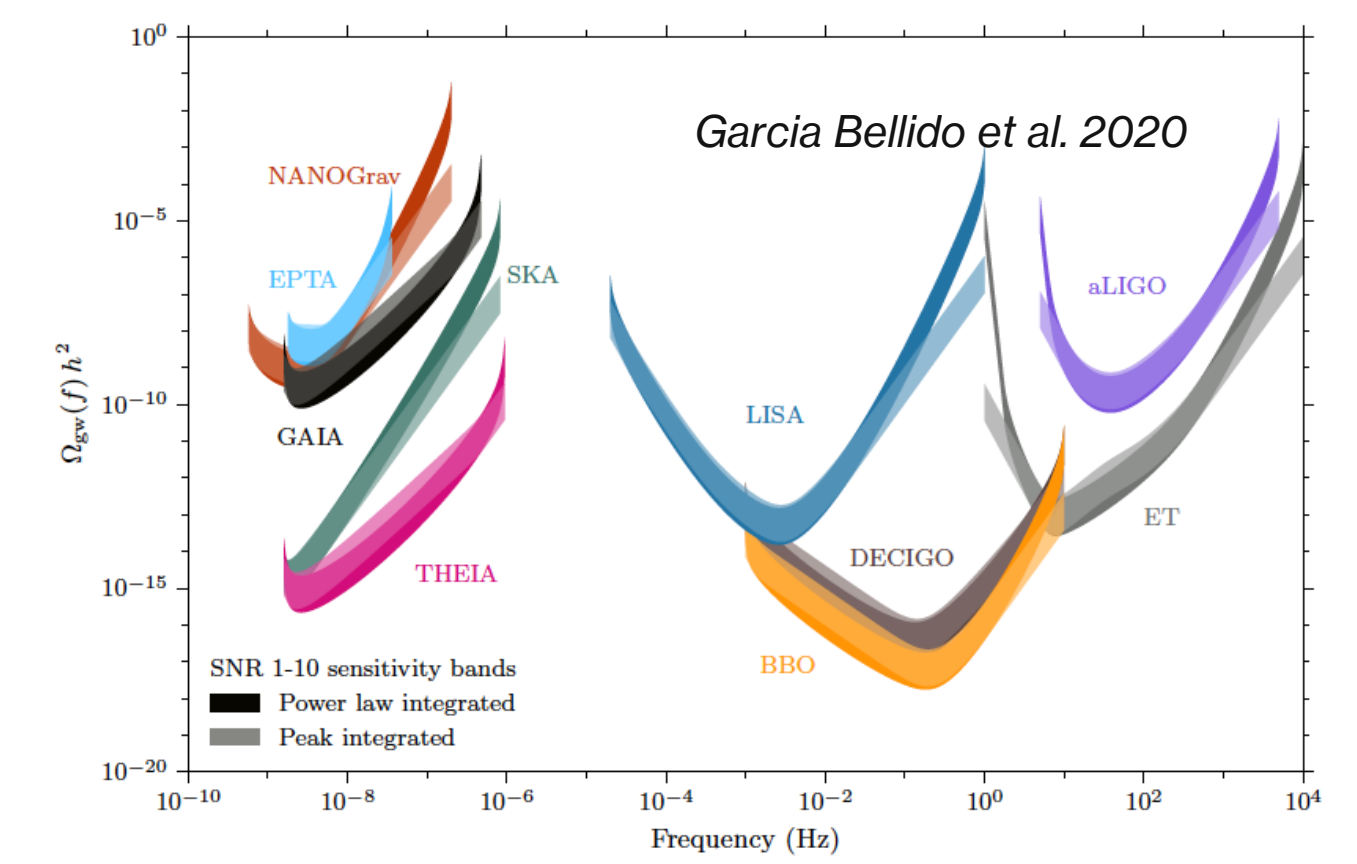


Barnabas P2oczoz

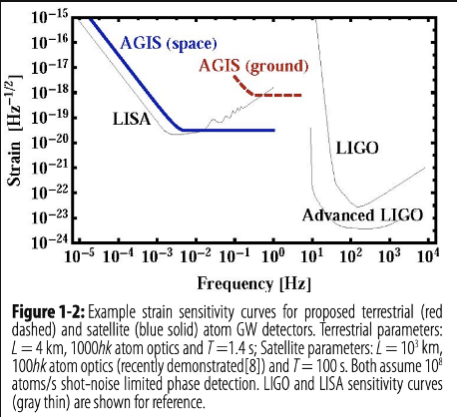
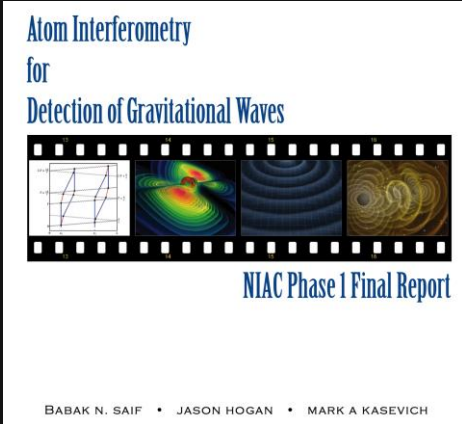
Why Gravitational Waves ?



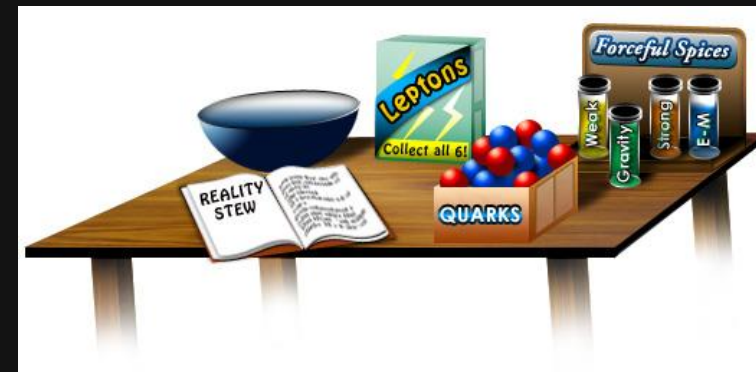
What is next?



Connection with High Energy Particle Physics – the best laboratory to test the energy scales EVEN near the Planck scale



Why NEW Physics



Standard Model of Elementary Particles

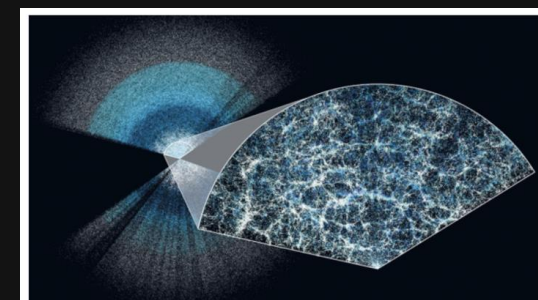
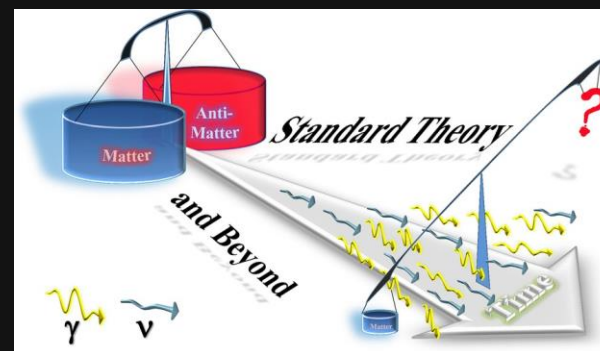
three generations of matter (fermions)			interactions / force carriers (bosons)	
I	II	III		
mass charge spin			0 0 1	$\approx 124.97 \text{ GeV}/c^2$ 0 0 0
$\approx 2.2 \text{ MeV}/c^2$ $\frac{2}{3}$ $\frac{1}{2}$	$\approx 1.28 \text{ GeV}/c^2$ $\frac{2}{3}$ $\frac{1}{2}$	$\approx 173.1 \text{ GeV}/c^2$ $\frac{2}{3}$ $\frac{1}{2}$	g gluon	H higgs
u up	c charm	t top		
$\approx 4.7 \text{ MeV}/c^2$ $-\frac{1}{3}$ $\frac{1}{2}$	$\approx 96 \text{ MeV}/c^2$ $-\frac{1}{3}$ $\frac{1}{2}$	$\approx 4.18 \text{ GeV}/c^2$ $-\frac{1}{3}$ $\frac{1}{2}$	0 0 1	
d down	s strange	b bottom	γ photon	
$\approx 0.511 \text{ MeV}/c^2$ -1 $\frac{1}{2}$	$\approx 105.66 \text{ MeV}/c^2$ -1 $\frac{1}{2}$	$\approx 1.7768 \text{ GeV}/c^2$ -1 $\frac{1}{2}$	0 1	
e electron	μ muon	τ tau	Z Z boson	
$< 1.0 \text{ eV}/c^2$ 0 $\frac{1}{2}$	$< 0.17 \text{ MeV}/c^2$ 0 $\frac{1}{2}$	$< 18.2 \text{ MeV}/c^2$ 0 $\frac{1}{2}$	$\approx 80.39 \text{ GeV}/c^2$ -1 1	
ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	

QUARKS

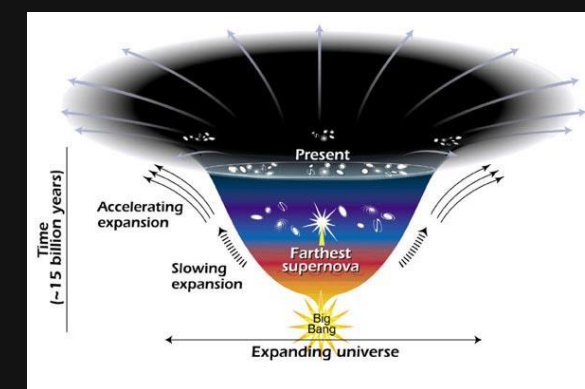
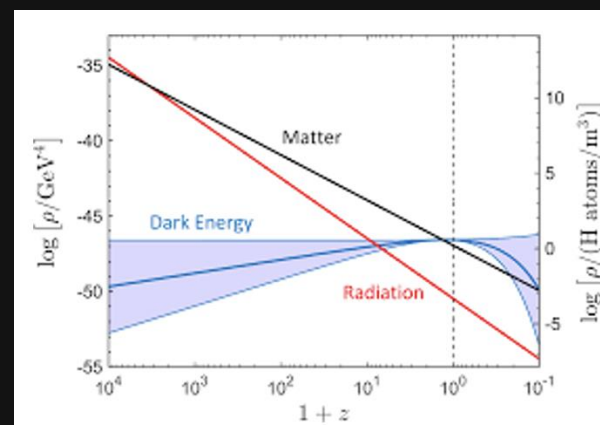
LEPTONS

SCALAR BOSONS

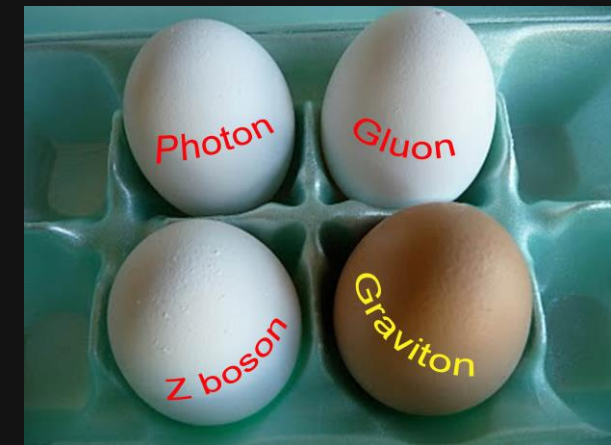
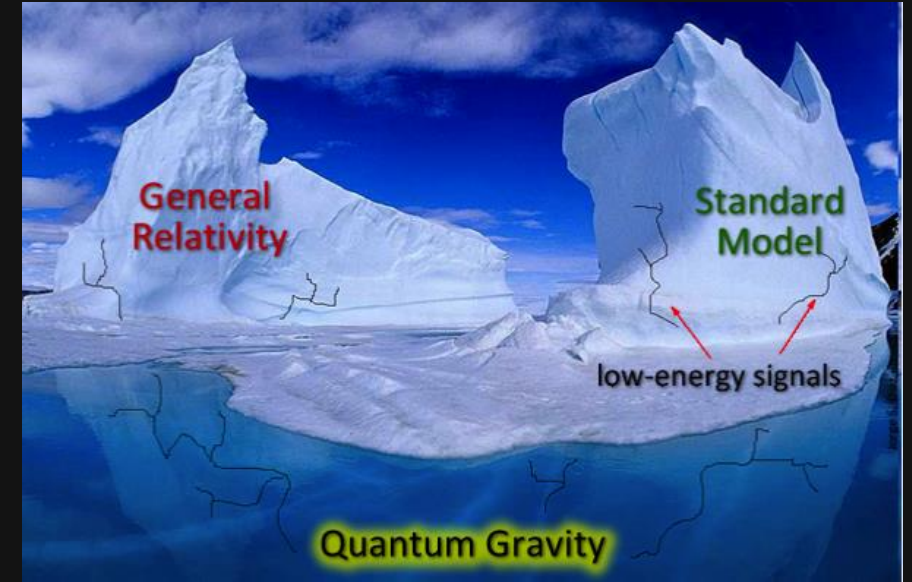
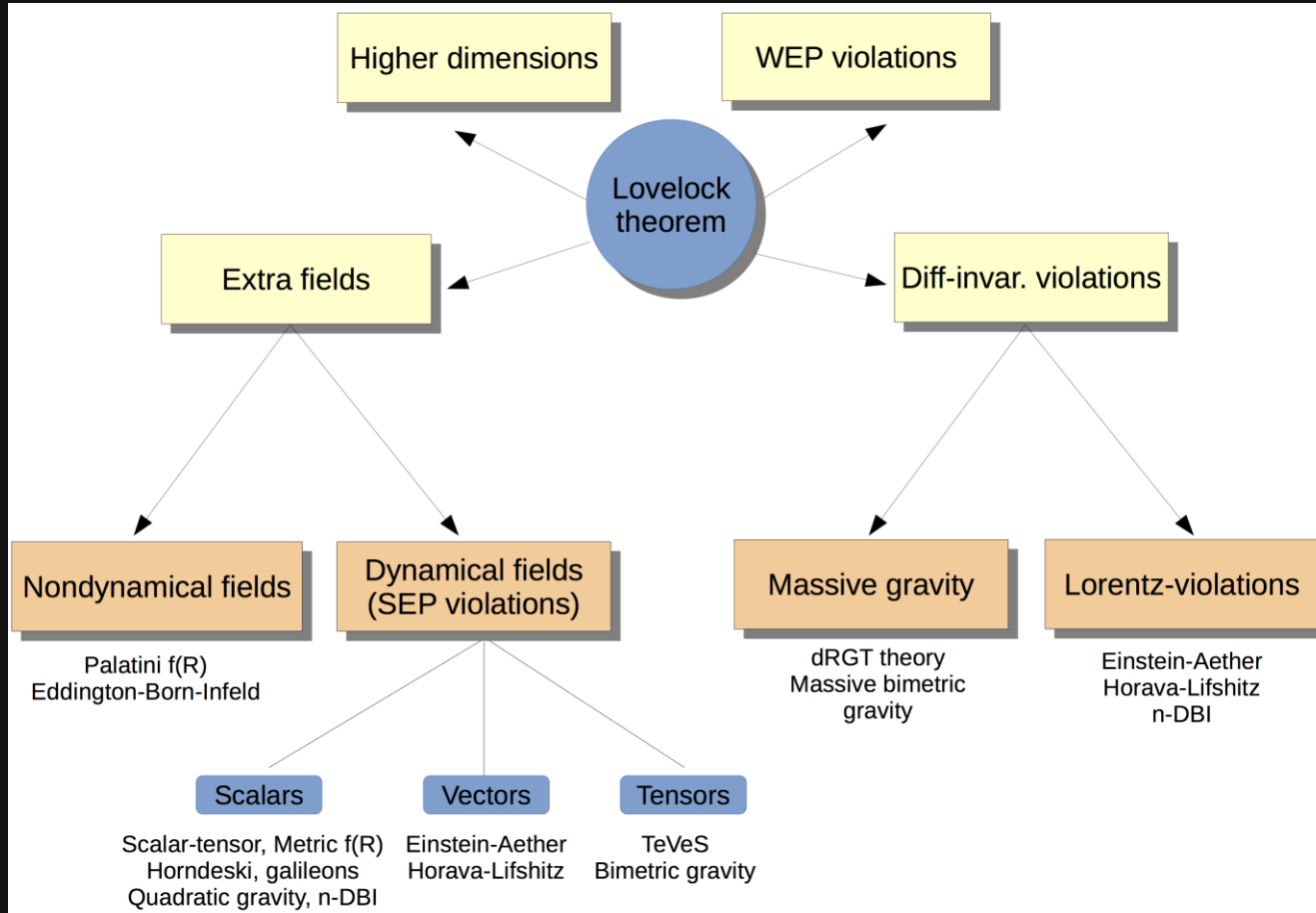
GAUGE BOSONS
VECTOR BOSONS



Wide open DESI's map of the universe is the largest to date, showing delicate bubble-like structures in the distribution of galaxies (inset) that contain clues to the expansion history of the universe. Credit: C. Lamman/DESI/cmastr



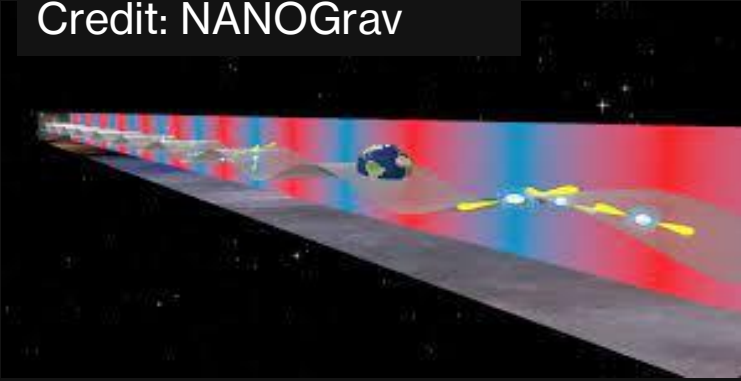
Modifications of General Relativity



Berti et al.

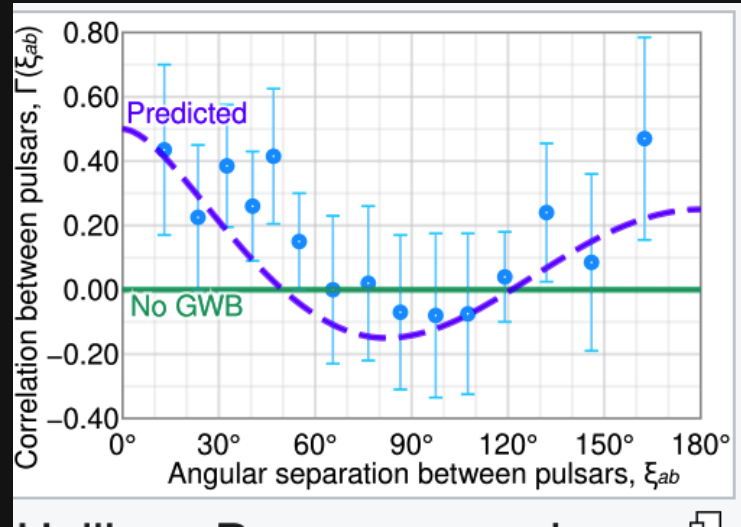
Pulsar Timing Arrays

Credit: NANOGrav



The proposal to use pulsars as GW detectors was originally made by Sazhin (1978) and Detweiler (1979). The idea is to treat the solar system barycenter and a galactic pulsar as opposite ends of an imaginary arm in space.

Assumptions: Isotropic GW that obeys Einstein's general relativity. It is valid for "long-arm" detectors like pulsar timing arrays, where the wavelengths of typical GWs are much shorter than the "long-arm" distance between Earth and typical pulsars.

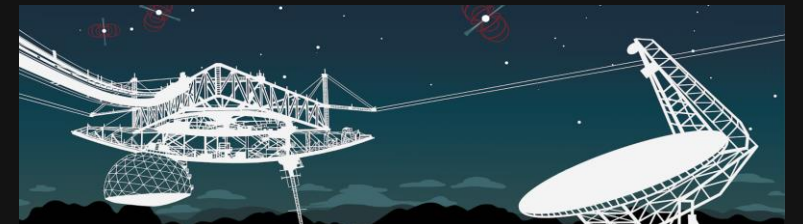


$$\Gamma_{ab} = \frac{1}{2}\delta_{ab} + \frac{1}{2} - \frac{x_{ab}}{4} + \frac{3}{2}x_{ab} \ln x_{ab}$$

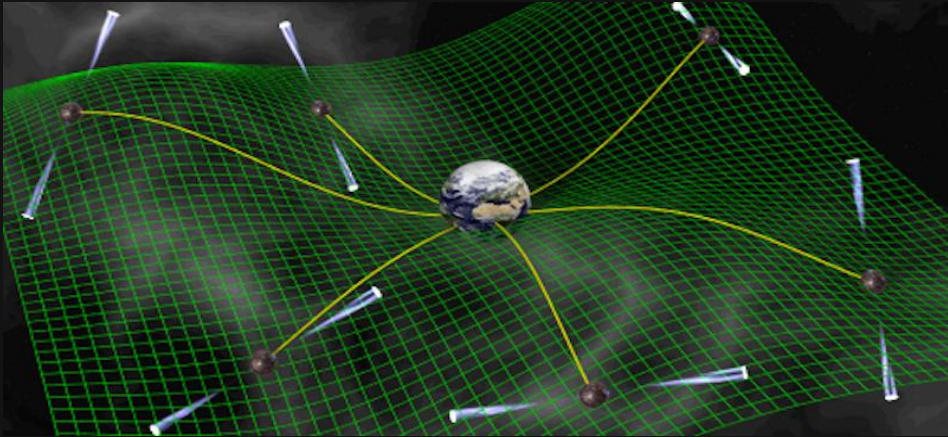
$$x_{ab} = (1 - \cos \zeta_{ab})/2$$

overlap reduction function

In 1983, Hellings and Downs[[] extended this idea to an array of pulsars and found that a stochastic background of GWs would produce a distinctive GW signature: a quadrupolar spatial correlation between arrival times of pulses emitted by different millisecond pulsar pairings that depends only on the pairing's angular separation in the sky as viewed from Earth (actually the solar system barycenter). The key property of a pulsar timing array is that the signal from a stochastic GW background will be correlated across the sightlines of pulsar pairs, while that from the other noise processes will not.



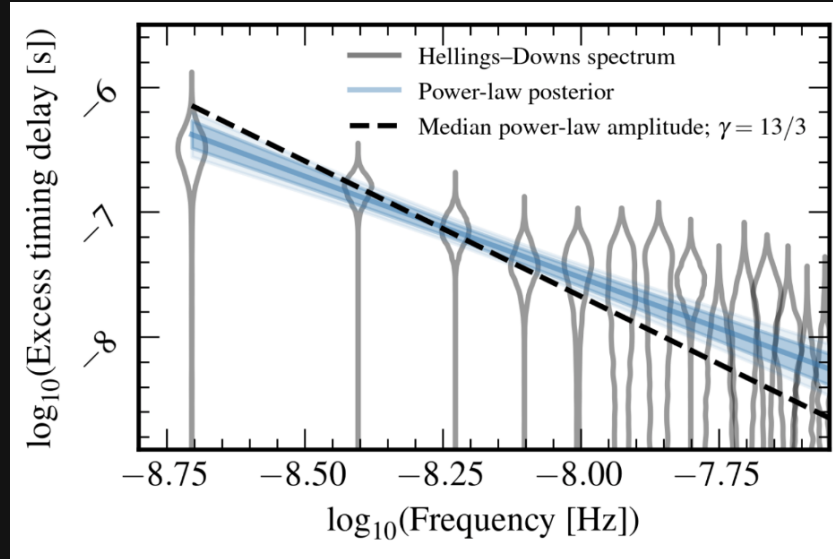
Stochastic Gravitational Waves Detection



The pulsar acts as the reference clock at one end of the arm sending out regular signals which are monitored by an observer on Earth. The effect of a passing long-wavelength GW would be to perturb the galactic spacetime and cause a small change in the observed time of arrival of the pulses.

Credit: NANOGrav

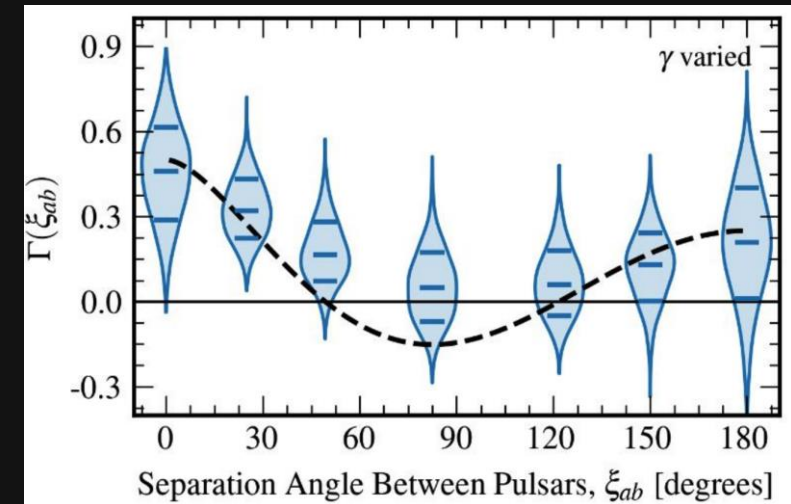
$$\Omega_{\text{GW}}(f) = \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f) = \Omega_{\text{GW}}^{\text{yr}} \left(\frac{f}{f_{\text{yr}}} \right)^{5-\gamma_{\text{CP}}}$$



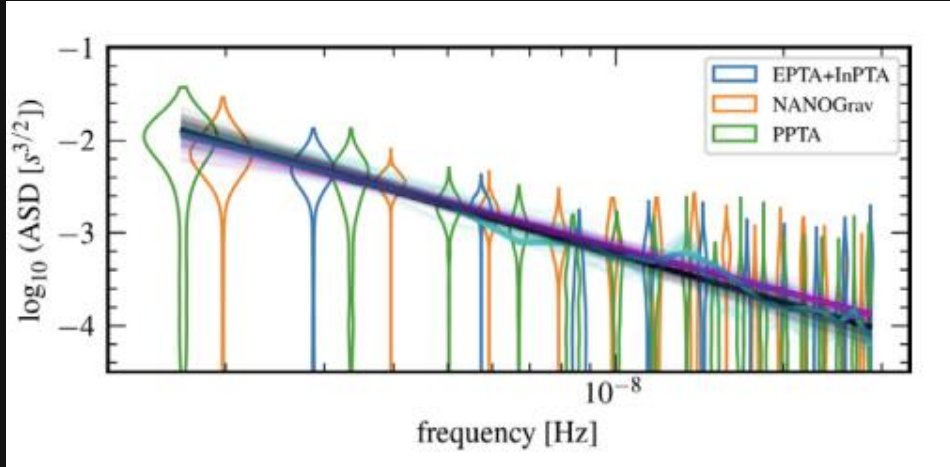
$$R(t) \equiv \int_0^t dt' \left(\frac{\nu_0 - \nu(t')}{\nu_0} \right)$$

$$\frac{1}{T_{\text{obs}}} < f < \frac{1}{\delta t}$$

$$h_c(f) = A_{\text{CP}} \left(\frac{f}{f_{\text{yr}}} \right)^{\alpha_{\text{CP}}},$$



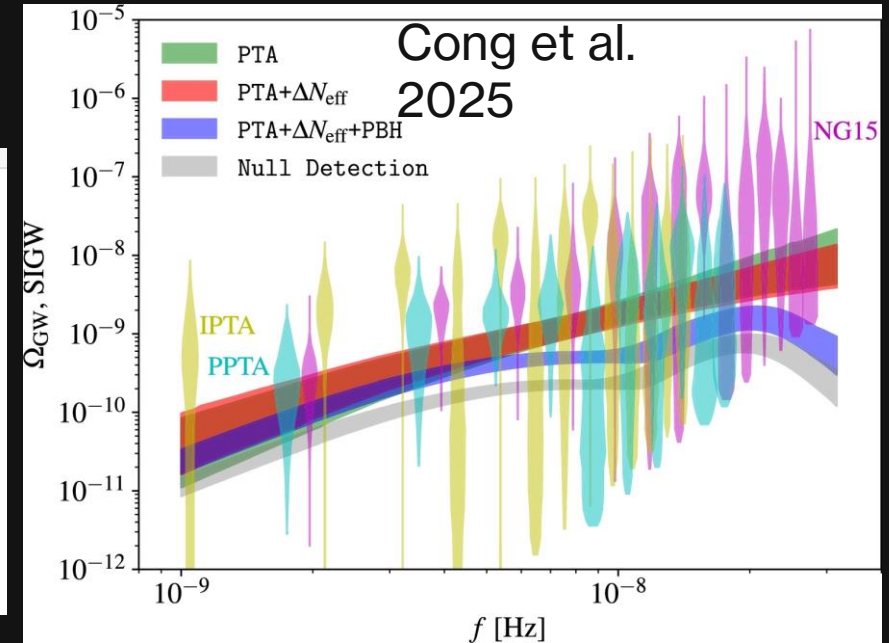
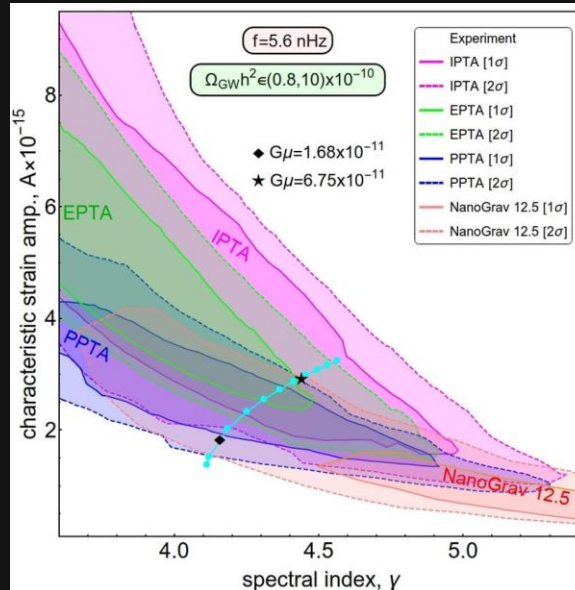
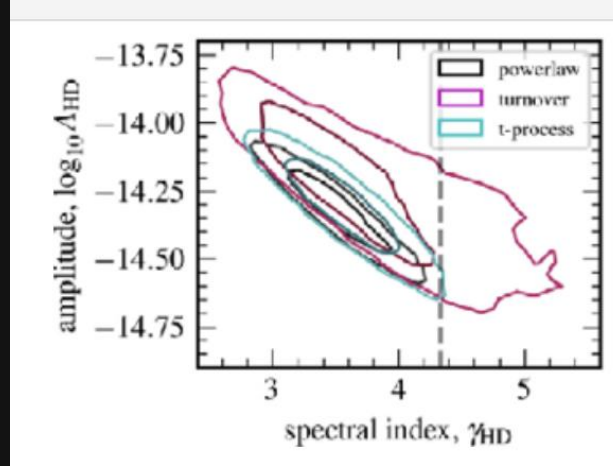
Combined PTA Results



Agazie et al. 2024

A combined fit (cyan curve) to NANOGrav, PPTA, EPTA, and IPTA data in the (A, γ) plane. The diamond and the star shaped points are the two chosen benchmark scenarios of the proposed model.

Saad 2022



Posterior distributions of SIGW and comparison with experimental data. The magenta, yellow and cyan regions show the measured Ω_{GW} posterior from NANOGrav 15 years result (NG15), IPTA and PPTA respectively. The green, red, and blue contours show our 68% C.L. posterior for SIGW

Stochastic Gravitational Waves Detection

$$h_{\mu\nu}(k) = \sum_i h^{(i)}(k) \epsilon_{\mu\nu}^{(i)},$$

$\epsilon_{\mu\nu}^i$ polarization tensor

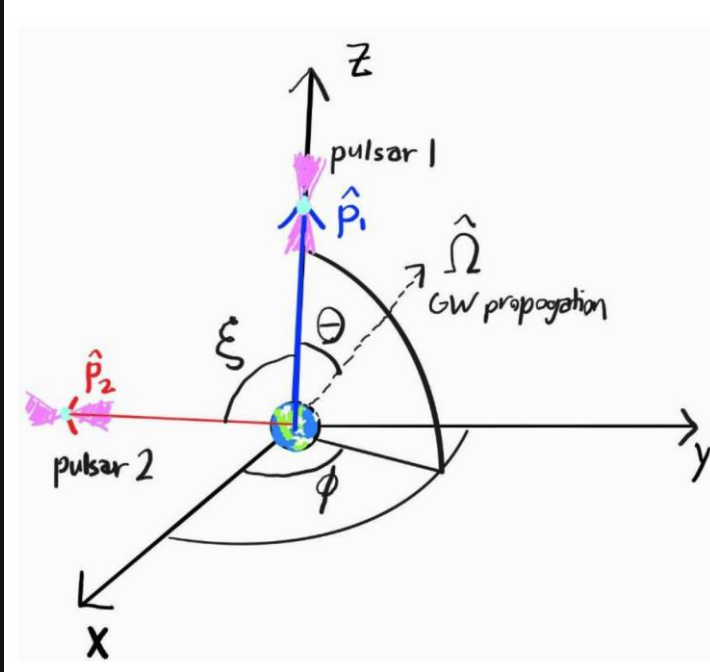
$$\langle h^{(i)*}(f, \hat{\Omega}) h^{(i')}(f', \hat{\Omega}') \rangle = \frac{3H_0^2}{32\pi^3} \delta^2(\hat{\Omega}, \hat{\Omega}') \delta_{ii'} \delta(f - f') \times |f|^{-3} \Omega_{\text{gw}}(|f|).$$

In the barycenter reference frame of the solar system, the frequency shift is characterized by the redshift $z(t) = \frac{\nu_0 - \nu(t)}{\nu_0}$ and averaged redshift:
 $\tilde{z}(f) \equiv \int d^2\hat{\Omega} z(f, \Omega)$

$$\langle \tilde{z}(f) \tilde{z}(f') \rangle = \frac{3H_0^2 \delta^2(\hat{\Omega}, \hat{\Omega}') \delta_{ii'} \delta(f - f')}{32\beta\pi^3 |f|^3} \Omega_{\text{gw}}(|f|) \Gamma(|f|)$$

β is the normalization factor introduced so that $\Gamma(|f|) = 1/2$ for coaligned coincident pulsars to match the Hellings-Downs (HD) correlation

$$\Gamma_I(|f|) = \sum_{i \in I} \int_{S^2} d^2\hat{\Omega} \mathcal{E}_1(-f, \hat{\Omega}) \mathcal{E}_2(f, \hat{\Omega}) F_1^{(i)}(\hat{\Omega}) F_2^{(i)}(\hat{\Omega}),$$



Massive Gravity

Shankaranarayanan & Johnson

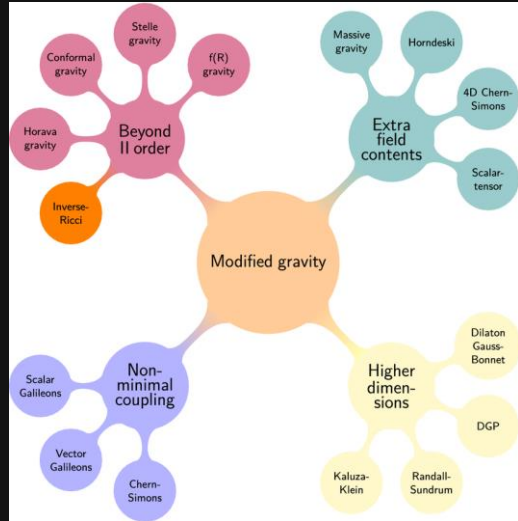
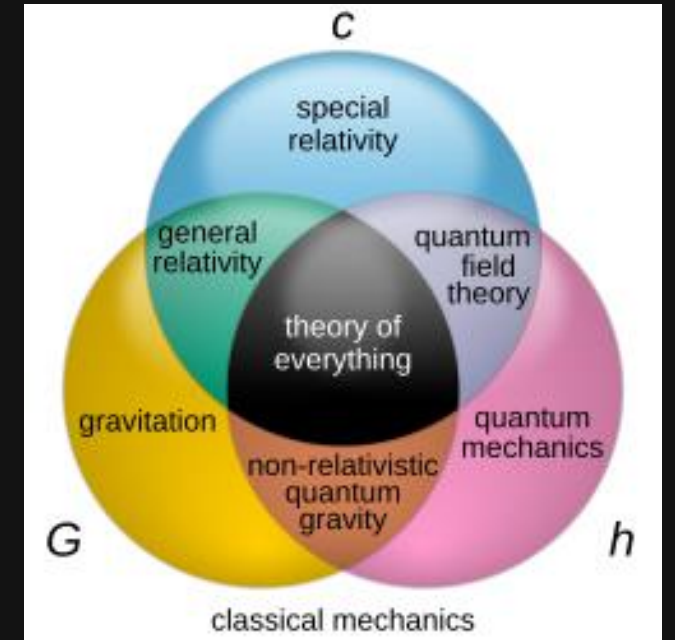
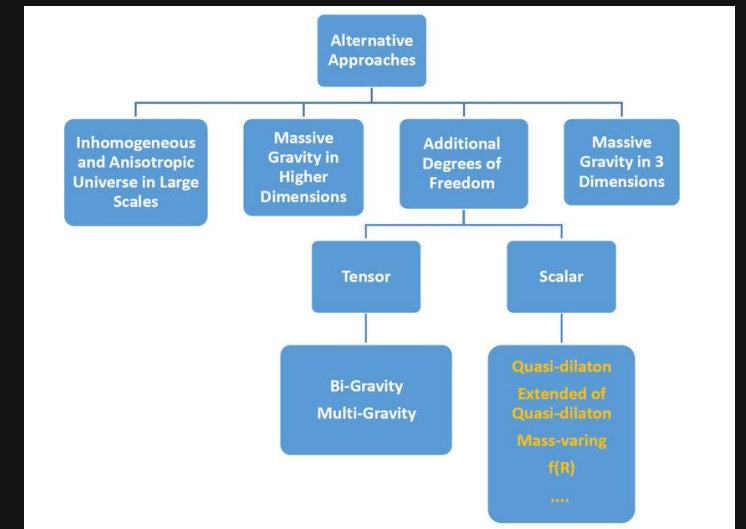


Figure 1. A modified gravity architecture, which gives a representation of some possible ways of modifying GR through breaking the Lovelock's theorem along with some examples. Theories are classified in order to avoid Lovelock's restrictions. In the green region we have theories as de Rham-Gabadadze-Tolley (dRGT) massive gravity theories, and cases with $m_g > 0$. In the red region we have theories with additional field as scalar (Horndeski) which includes cases as Galileon, Quintessence, Brans–Dicke and $f(R)$. Moreover, Vector theories such as Proca and Tensor–Vector–Scalar (TeVes) are presented and, finally, bigravity theories. In the olive region we have an example of theories that are constructed in the break assumption formalism as non-local ones, extra dimensions and Lorentz violating. The main gravitational wave (GW) test of each theory is framed at the top, where the following was considered: GW speed (blue color) and dispersion (red color) refers to constraints from late time GW sources. GW oscillations (yellow color) refer to these oscillations during the propagation introducing a modulation of the GW amplitude. GW damping (green color) refers to probing the damping of GWs using standard sirens from d_L^{GW} (luminosity distance of GWs) and d_L^{EM} (electro-magnetic luminosity distance). Theories that are constrained by these GW observables are enclosed by a dashed square of each GW color, respectively. This figure was designed based on the ideas and discussions behind the construction of theoretically sensible modified gravity theories [10,11,12]. Classification of parameterised post-Newtonian (PPN) tests was based on approximations to obtain the PPN parameters at 1, 2, 4 PN level [13,14,15]. Moreover, a PPN extension for a five-dimensional metric was included [16]. Notice that theories that are in agreement with this PPN test are filled in their respective colors: 2PN level (solid green color), unconstrained (solid red color) and 4PN level/PPN extra-dimensional (olive color).

Yukawa			
m_g (eV)	λ_g (km)	Eq.	
7.2×10^{-23}	2.8×10^{12}	(49)	A 2σ bound from the precession of Mercury (Talmadge <i>et al.</i> , 1988; Will, 1998).
6×10^{-32}	3×10^{21}	(53)	A 1σ bound from weak lensing of a cluster at $z = 1.2$ (Choudhury <i>et al.</i> , 2004). Sensitive to the dark matter distribution and cosmological model.
10^{-29}	10^{19}	(52)	From observations of gravitationally bound clusters of 0.5 Mpc (Goldhaber and Nieto, 1974; Hare, 1973). Sensitive to the dark matter distribution.
Dispersion Relation			
m_g (eV)	λ_g (km)	Eq.	
1.2×10^{-22}	1.7×10^{12}	(58)	A 90% confidence bound two $30 M_\odot$ bh-bh merger (GW150916) (Abbott <i>et al.</i> , 2016; Will, 1998).
7.6×10^{-20}	2.6×10^9	(65)	From pulsar timing of PSR B1913+16 and PSR B1534+12 (Finn and Sutton, 2002).
10^{-30}	10^{20}	(63)	Observations of power in B-mode polarization in CMB at low ℓ (Dubovsky <i>et al.</i> , 2010; Gumrukcuoglu <i>et al.</i> , 2012; Raveri <i>et al.</i> , 2015).
10^{-26}	10^{16}	(59)	A 10^4 to $10^7 M_\odot$ merger by eLISA type experiment (Will, 1998).
10^{-24}	10^{14}	(60)	A dual messenger observation of IBWD by eLISA type experiment (Cooray and Seto, 2004; Cutler <i>et al.</i> , 2003; Larson and Hiscock, 2000).
10^{-23}	10^{13}	(66)	Pulsar timing array of 100ns accuracy with 10 year observation (Lee <i>et al.</i> , 2010).
10^{-20}	10^{10}	(61)	Dual messenger observation of SNe gamma ray burst and gravitational waves (Nishizawa and Nakamura, 2014).
Fifth Force			
m_g (eV)	λ_g (km)	Eq.	
10^{-32}	10^{22}	(77)	From earth-moon precession for cubic Galileon theories (Dvali <i>et al.</i> , 2003).
10^{-32}	10^{22}	(84)	From precession in full 5D DGP in the Solar System (Gruzinov, 2005; Lue and Starkman, 2003).
10^{-30}	10^{20}	(81)	From earth-moon precession for quartic Galileon theories (dRGT-like) (de Rham, 2014).
10^{-27}	10^{17}	(86)	From PSR B1913+16 pulsar in cubic Galileon theories (DGP) (de Rham <i>et al.</i> , 2013d).
10^{-33}	10^{23}	(89)	A prospective 4σ bound from weak lensing on next-gen surveys (Park and Wyman, 2015; Wyman, 2011). Sensitive to alternative DM halo profiles.
10^{-32}	10^{22}	(90)	Observations of altered structure formation from fifth force (Khouri and Wyman, 2009; Park and Wyman, 2015; Wyman, 2011; Zu <i>et al.</i> , 2014). Sensitive to the particular theory of massive gravity.



Sobhan Kazempour



Escamilla-Rivera and Fabris 2021

Tests of Massive Gravity

$$S = \int d^4x \left[\frac{1}{2} \partial_\lambda h_{\mu\nu} \partial^\lambda h^{\mu\nu} - \partial_\mu h_{\nu\lambda} \partial^\nu h^{\mu\lambda} + \partial_\mu h^{\mu\nu} \partial_\nu h - \frac{1}{2} \partial_\lambda h \partial^\lambda h + \frac{1}{2} m^2 (h_{\mu\nu} h^{\mu\nu} - h^2) \right] \quad \text{Fierz-Pauli 1937}$$

Yukawa-like exponential suppression is expected when the force carrier possesses a nonzero mass. In the case of MG, suppression of the gravitational potential is of the form

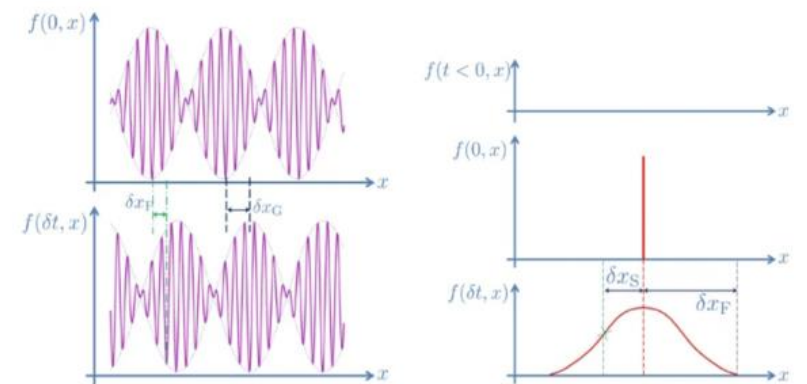
$$\Phi \propto \exp[-m_g R/R]$$

There is also a suppression of GWs at wavelengths larger than the Compton length scale of the graviton $R_g \sim (m_g)^{-1}$

Modification of the dispersion relation gains a massive term, implying a difference between the GW propagation speed c_g and the speed of light c , which can be used to constrain m_g .

Additional degrees of polarization are characterized by the additional helicity ± 1 and 0 modes. Certain theories of MG, such as the minimal theory of MG, only have two tensor modes, as in GR, but a general theory of MG that assumes Lorentz invariance will have five in total, De Rham-Gabadadze-Tolley model

Figure 5



Difference between phase, group, signal and front velocities. At $t = \delta t$, the phase and group velocities are represented on the left and given respectively by $v_{\text{phase}} = \delta x_P / \delta t$ and $v_{\text{group}} = \delta x_G / \delta t$ (in the limit $\delta t \rightarrow 0$). The signal and front velocity represented on the right are given by $v_{\text{signal}} = \delta x_S / \delta t$ (where δx_S is the point where at least half the intensity of the original signal is reached.) The front velocity is given by $v_{\text{front}} = \delta x_F / \delta t$.

De Rham 2014, Living Review

Pulsar Timing Arrays Data and Additional Polarizations

THE ASTROPHYSICAL JOURNAL LETTERS, 923:L22 (18pp), 2021 December 20

© 2021. The Author(s). Published by the American Astronomical Society.

OPEN ACCESS

<https://doi.org/10.3847/2041-8213/ac401c>



The NANOGrav 12.5-year Data Set: Search for Non-Einsteinian Polarization Modes in the Gravitational-wave Background

Zaven Arzoumanian¹, Paul T. Baker², Harsha Blumer^{3,4}, Bence Bécsey⁵, Adam Brazier^{6,7}, Paul R. Brook^{3,4}, Sarah Burke-Spolaor^{3,4,8}, Maria Charisi⁹, Shami Chatterjee⁶, Siyuan Chen^{10,11,12}, James M. Cordes⁶, Neil J. Cornish⁵, Fronefield Crawford¹³, H. Thankful Cromartie⁶, Megan E. DeCesar^{14,15,48}, Dallas M. DeGan¹⁶, Paul B. Demorest¹⁷, Timothy Dolch^{18,19}, Brendan Drachler^{20,21}, Justin A. Ellis²², Elizabeth C. Ferrara^{23,24,25}, William Fiore^{3,4}, Emmanuel Fonseca²⁶, Nathan Garver-Daniels^{3,4}, Peter A. Gentile^{3,4}, Deborah C. Good²⁷, Jeffrey S. Hazboun^{28,48}, A. Miguel Holgado^{29,30}, Kristina Islo³¹, Ross J. Jennings⁶, Megan L. Jones³¹, Andrew R. Kaiser^{3,4}, David L. Kaplan³¹, Luke Zoltan Kelley³², Joey Shapiro Key²⁸, Nima Laal¹⁶, Michael T. Lam^{20,21}, T. Joseph W. Lazio³³, Duncan R. Lorimer^{3,4}, Tingting Liu³¹, Jing Luo³⁴, Ryan S. Lynch³⁵, Dustin R. Madison^{3,4,48}, Alexander McEwen³¹, Maura A. McLaughlin^{3,4}, Chiara M. F. Mingarelli^{36,37}, Cherry Ng³⁸, David J. Nice¹⁴, Ken D. Olum³⁹, Timothy T. Pennucci^{40,41,48}, Nihan S. Pol^{3,4,9}, Scott M. Ransom⁴⁰, Paul S. Ray⁴², Joseph D. Romano⁴³, Shashwat C. Sardesai³¹, Brent J. Shapiro-Albert^{3,4}, Xavier Siemens^{16,31}, Joseph Simon^{33,44}, Magdalena S. Siwek⁴⁵, Renée Spiewak⁴⁶, Ingrid H. Stairs²⁷, Daniel R. Stinebring⁴⁷, Kevin Stovall¹⁷, Jerry P. Sun¹⁶, Joseph K. Swiggum^{14,48}, Stephen R. Taylor⁹, Jacob E. Turner^{3,4}, Michele Vallisneri³³, Sarah J. Vigeland³¹, Haley M. Wahl^{3,4}, and Caitlin A. Witt^{3,4}

The NANOGrav Collaboration

The analyses in this Letter, as well as those in Bernardo & Ng (2023c) and Chen et al. (2023), do not rule out the possibility of ST correlations in our data. However, our analysis also shows no statistical need for an additional stochastic process with ST correlations.

THE ASTROPHYSICAL JOURNAL LETTERS, 923:L22 (18pp), 2021 December 20

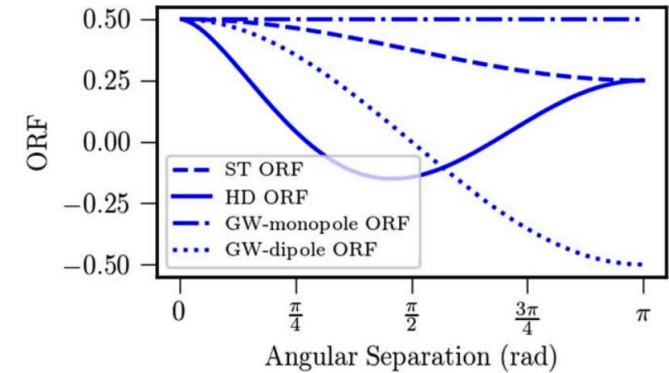


Figure 4. The solid line is the HD correlation curve, the dashed line is the ST ORF, the dashed-dotted line is the GW-monopole ORF, and the dotted line is the GW-dipole ORF. These four ORFs can be hard to distinguish if the uncertainties in the timing residual cross-correlations are sufficiently large.

THE ASTROPHYSICAL JOURNAL LETTERS, 964:L14 (10pp), 2024 March 20

© 2024. The Author(s). Published by the American Astronomical Society.

OPEN ACCESS

<https://doi.org/10.3847/2041-8213/ad2a51>



The NANOGrav 15 yr Data Set: Search for Transverse Polarization Modes in the Gravitational-wave Background

Gabriella Agazie¹, Akash Anumalapudi¹, Anne M. Archibald², Zaven Arzoumanian³, Jeremy Baier⁴, Paul T. Baker⁵, Bence Bécsey⁴, Laura Blecha⁶, Adam Brazier^{7,8}, Paul R. Brook⁹, Sarah Burke-Spolaor^{10,11}, Rand Burnette⁴, Robin Case⁴, J. Andrew Casey-Clyde¹², Maria Charisi¹³, Shami Chatterjee⁷, Tyler Cohen¹⁴, James M. Cordes⁷, Neil J. Cornish¹⁵, Fronefield Crawford¹⁶, H. Thankful Cromartie^{7,64}, Kathryn Crowder¹⁷, Megan E. DeCesar¹⁸, Dallas DeGan⁴, Paul B. Demorest¹⁹, Timothy Dolch^{20,21}, Brendan Drachler^{22,23}, Elizabeth C. Ferrara^{24,25,26}, William Fiore^{10,11}, Emmanuel Fonseca^{10,11}, Gabriel E. Freedman¹, Nate Garver-Daniels^{10,11}, Peter A. Gentile^{10,11}, Joseph Glaser^{10,11}, Deborah C. Good^{12,27}, Kayhan Gültekin²⁸, Jeffrey S. Hazboun⁴, Ross J. Jennings^{10,11,65}, Aaron D. Johnson^{1,29}, Megan L. Jones¹, Andrew R. Kaiser^{10,11}, David L. Kaplan⁷, Luke Zoltan Kelley³⁰, Matthew Kerr³¹, Joey S. Key³², Nima Laal⁴, Michael T. Lam^{22,23}, William G. Lamb¹³, T. Joseph W. Lazio³³, Natalia Lewandowska³⁴, Tingting Liu^{10,11}, Duncan R. Lorimer^{10,11}, Jing Luo^{35,66}, Ryan S. Lynch³⁶, Chung-Pei Ma^{30,37}, Dustin R. Madison³⁸, Alexander McEwen¹, James W. McKee^{39,40}, Maura A. McLaughlin^{10,11}, Natasha McMann¹³, Bradley W. Meyers^{17,41}, Chiara M. F. Mingarelli^{12,27,42}, Andrea Mitridate⁴³, Priyamvada Natarajan^{44,45}, Cherry Ng⁴⁶, David J. Nice⁴⁷, Stella Koch Ocker⁷, Ken D. Olum⁴⁸, Timothy T. Pennucci⁴⁹, Benetge B. P. Perera⁵⁰, Nihan S. Pol¹³, Henri A. Radovan⁵¹, Scott M. Ransom⁵², Paul S. Ray³¹, Joseph D. Romano⁵³, Alexander Saffer^{10,11}, Shashwat C. Sardesai¹, Ann Schmiedekamp⁵⁴, Carl Schmiedekamp⁵⁴, Kai Schmitz⁵⁵, Brent J. Shapiro-Albert^{10,11,56}, Xavier Siemens^{1,4}, Joseph Simon^{57,67}, Magdalena S. Siwek⁵⁸, Ingrid H. Stairs¹⁷, Daniel R. Stinebring⁵⁹, Kevin Stovall¹⁹, Jerry P. Sun⁴, Abhimanyu Susobhanan¹, Joseph K. Swiggum^{47,65}, Jacob A. Taylor⁴, Stephen R. Taylor¹³, Jacob E. Turner^{10,11}, Caner Unal^{60,61}, Michele Vallisneri^{29,33}, Sarah J. Vigeland¹, Haley M. Wahl^{10,11}, Caitlin A. Witt^{62,63}, Olivia Young^{22,23}, and The NANOGrav Collaboration

**PTAs are sensitive to both: modified propagation
and
the additional polarization sectors.**

$$\omega^2 = k^2 + m_g^2 \rightarrow v_g = \sqrt{(1 - m_g^2/\omega^2)}$$



low frequency cut-off,
no signal below $\omega = m_g$



$$m_g \lesssim 10^{-24} \text{ eV}$$

**5 polarization degrees of
freedom**

- tensor: helicity ± 2
- vector: helicity ± 1
- scalar: helicity 0

$$\Omega_S \quad \Omega_V \quad \Omega_T$$

Results does not depend if the source is astrophysical or cosmological

Modification of ORF

$$h_{\mu\nu}(x) = \frac{1}{2\pi} \int d^4k \frac{2\delta(\omega^2 - |\mathbf{k}|^2 - m_g^2)}{|\mathbf{k}|} e^{ikx} h_{\mu\nu}(k),$$

$$\langle \tilde{z}_a^*(f) \tilde{z}_b(f') \rangle = \frac{3H_0^2}{32\pi^3} \delta(f - f') |f|^{-3} \times \sum_{I \in \{T, V, S\}} \Omega_I(|f|) \frac{\tilde{\Gamma}_{ab}^I(\xi_{ab}, |f|)}{\beta_I(f)}.$$

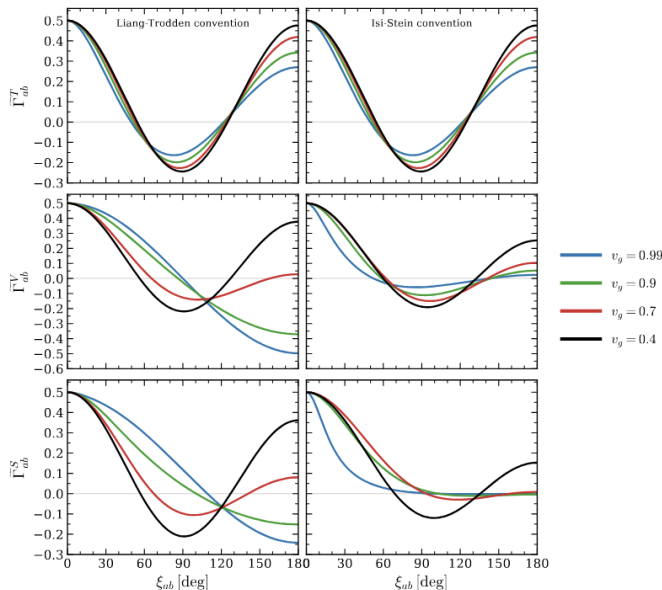


FIG. 1: Comparison of the ORFs without pulsar terms in the Liang-Trodden and Isi-Stein polarization basis and vector conventions. Rows show the normalized tensor, vector, and scalar ORFs, respectively. The left column shows the Liang-Trodden expressions as derived in Ref. [39], while the right column shows our PTA calculation using the polarization basis defined by Isi-Stein; Isi-Stein do not explicitly calculate these PTA ORFs. Each curve is normalized to $\tilde{\Gamma}_{ab}^I(\xi_{ab} = 0) = 1/2$. Colors indicate different values of v_g .

PTA collaborations

PTA directly measure the times of arrival of pulsar signals, which correspond to the integral of $z(t) = (v_0 - v(t))/v_0$ - time residual

$$\tilde{z}_a(f, \hat{\Omega}) = \sum_{I \in \{T, V, S\}} \sum_{i \in I} \left[e^{-i2\pi f L_a (1 + v_g \hat{\Omega} \cdot \hat{\mathbf{p}}_a)} - 1 \right] \times h^{(i)}(f, v_g \hat{\Omega}) F_a^{(i)}(\hat{\Omega}),$$

Liang & Trodden, 2021

Anholm, et al. 2008

$$F_a^{(i)}(\hat{\Omega}) = \frac{\hat{p}_a^\mu}{2} \left(-\frac{\hat{p}_a^\nu}{1 + v_g \hat{\Omega} \cdot \hat{\mathbf{p}}_a} \epsilon_{\mu\nu}^{(i)} + \epsilon_{0\mu}^{(i)} \right).$$

$$\begin{aligned} \tilde{\Gamma}_{ab}^I(\xi_{ab}, f) &\equiv \beta_I \Gamma_{ab}^I \\ &= \beta_I \sum_{i \in I} \int_{S^2} d^2\hat{\Omega} \left[e^{i2\pi f L_a (1 + v_g \hat{\Omega} \cdot \hat{\mathbf{p}}_a)} - 1 \right] \\ &\times \left[e^{-i2\pi f L_b (1 + v_g \hat{\Omega} \cdot \hat{\mathbf{p}}_b)} - 1 \right] F_a^{(i)*}(\hat{\Omega}) F_b^{(i)}(\hat{\Omega}), \end{aligned}$$

$$\tilde{\Gamma}_{ab}^{\text{eff}} = \beta_T \left[\Gamma_{ab}^T + \frac{\Omega_V}{\Omega_T} \Gamma_{ab}^V + \frac{\Omega_S}{\Omega_T} \Gamma_{ab}^S \right]$$

$$\Omega_I(|f|) \equiv \frac{1}{\rho_{\text{crit}}} \frac{d\rho_I}{d \ln f}$$

$$\begin{aligned} \frac{\Omega_V}{\Omega_T} &= \left(\frac{m_g}{\omega} \right)^{2+n_V} \\ \frac{\Omega_S}{\Omega_T} &= \left(\frac{m_g}{\omega} \right)^{4+n_S} \end{aligned}$$

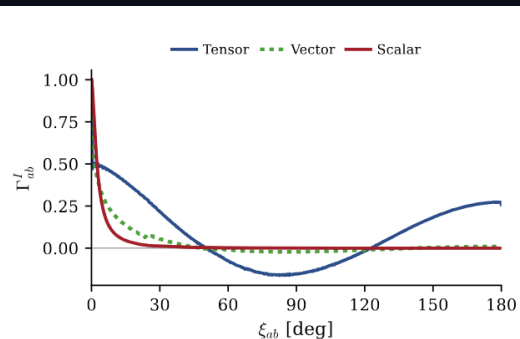


FIG. 5: ORFs with pulsar terms in the massless limit, normalized to 1 at $\xi_{ab} = 0$ to show their angular dependence. The explicit divergent prefactors are removed from the vector and scalar ORFs before plotting. We set $f = 1/T_{\text{obs}}$ with $T_{\text{obs}} = 16.03$ yr, and take $L_a = L_b = 4500.95$ ly, the median pulsar distance in the NANOGrav 15-year data set, giving $fL_a = fL_b = 280.78$.

Choi, TK, Poczos 2026 (in preparation)

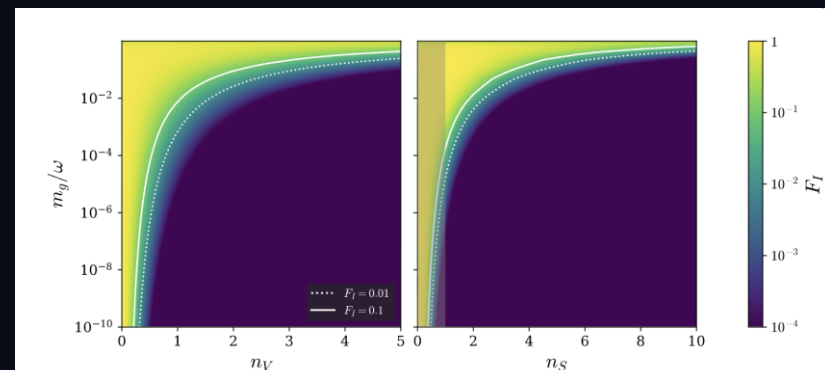


FIG. 8: Absolute nontensorial fractions for the vector and scalar modes in the left and right subplots, respectively. The solid white contour is $F_I = 0.1$ and the dotted white contour is $F_I = 0.01$. The gray shaded region in the scalar subplot denotes $n_S \leq 1$, which is excluded by the massless-limit scaling of the scalar autocorrelation. We use $f = 1/T_{\text{obs}}$ and pulsar distances $L_a = 500$ ly and $L_b = 10000$ ly.

Preliminary Results: NanoGrav

Choi, Bosca, TK, Poczos, et al. 2026 (in preparation)

$$A_{gw} = (4.8^{+5.2}_{-4.3}) \times 10^{-15}, \quad \gamma_{gw} = 3.01^{+1.04}_{-0.66}$$

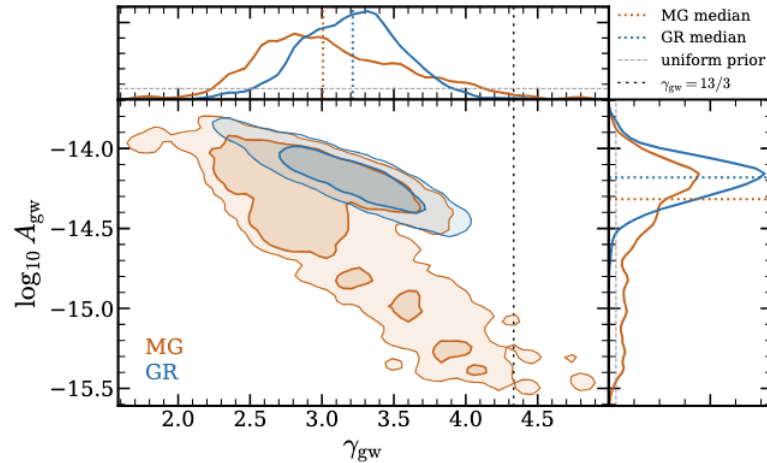
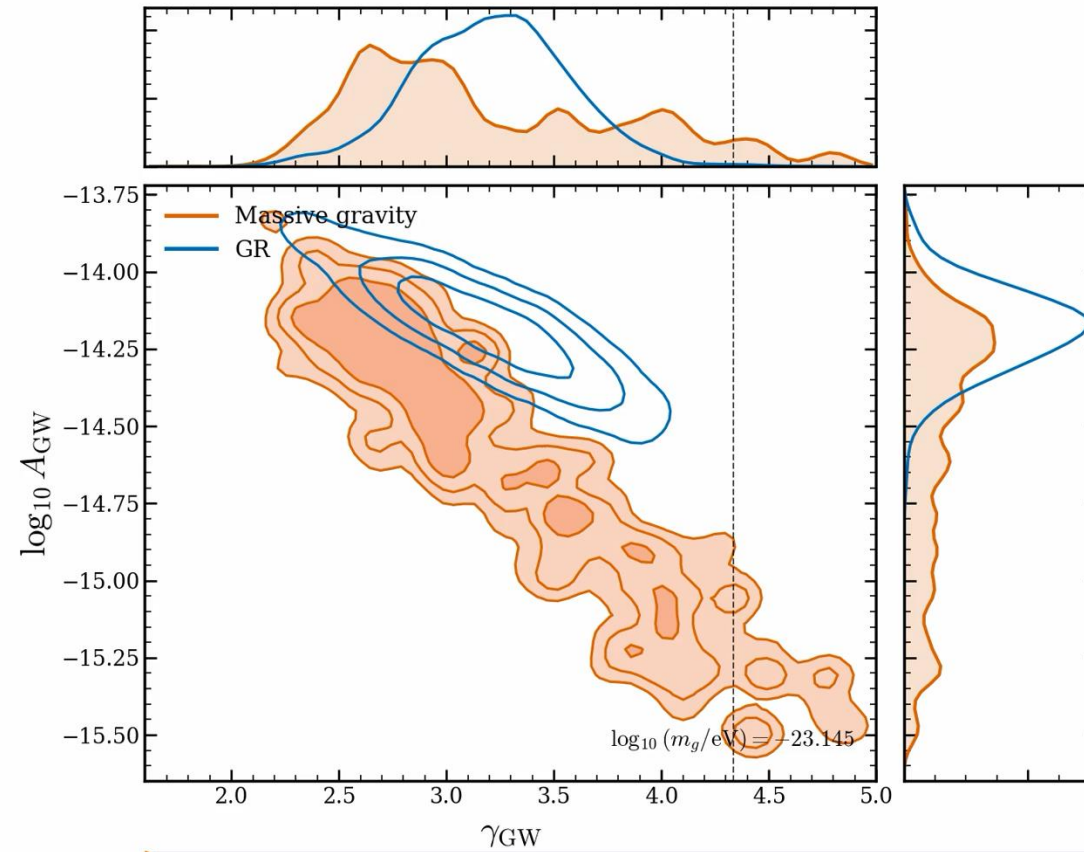


FIG. 1: Joint Posterior distributions of the GW amplitude A_{gw} and the spectral index γ_{gw} for the GR branch (blue) and the massive gravity branch (orange). The two branches favor partially different regions of the A_{gw} - γ_{gw} parameter space. The massive-gravity posterior exhibits more structure, including additional modes and extended degeneracy directions. This behavior likely reflects correlations between A_{gw} , γ_{gw} , and the additional Xparameters, as well as the use of modified ORFs

Unlike in Wu et al. 2023, the posteriors distributions of A_{gw} and γ_{gw} inferred for massive gravity are not necessarily consistent with those of GR



Credit: Chris Choi

Preliminary Results: NanoGrav

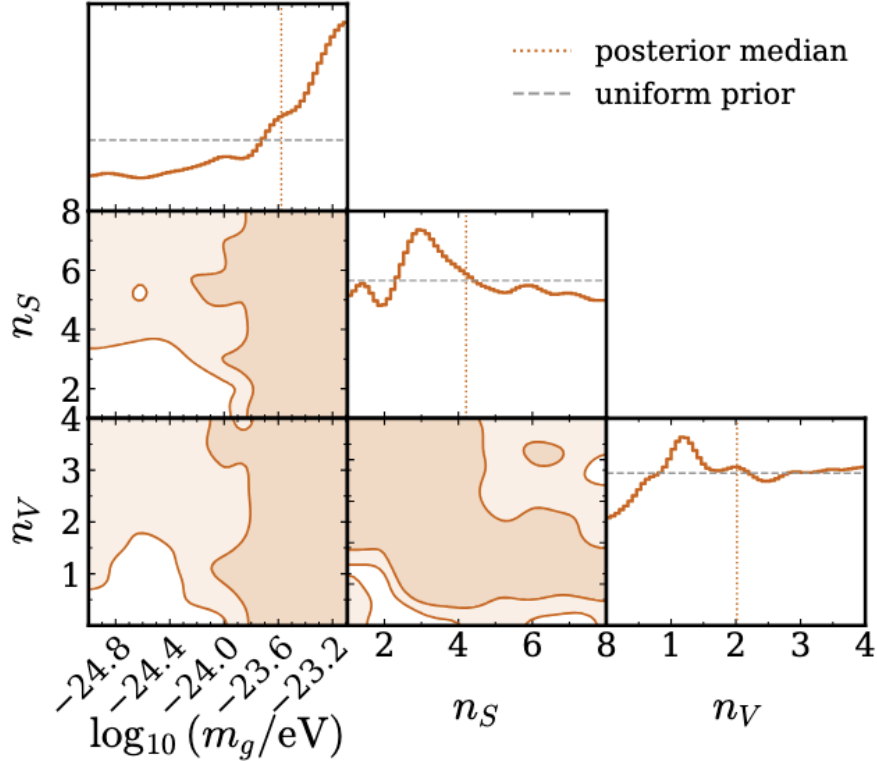


FIG. 2: Posterior distributions of the three massive gravity parameters: m_g , n_V , and n_S .

$$\begin{aligned} m_g &= (2.6_{-2.5}^{+4.9}) \times 10^{-24} eV, \\ n_V &= 2.0_{-1.8}^{+1.8} \quad \& \quad n_S = 4.2_{-2.8}^{+3.4}, \\ BF &= 1.238 \pm 0.015. \end{aligned}$$

The distribution of the parameters in massive gravity, while they do overlap substantially with the ones for GR, contain a richer structure. The distribution of A_{gw} for massive gravity is broader, with a lower median and a wider tail for smaller values of A_{gw} compared to GR. The spectral index shows a similar pattern: the median is lower for massive gravity and the distribution is broader.

These differences can be attributed to the behavior observed in the effective ORF. Since both of the models are being tested against the same set of data, they should predict a similar signal in order to produce maximal agreement with observation.

Preliminary Results: NanoGrav

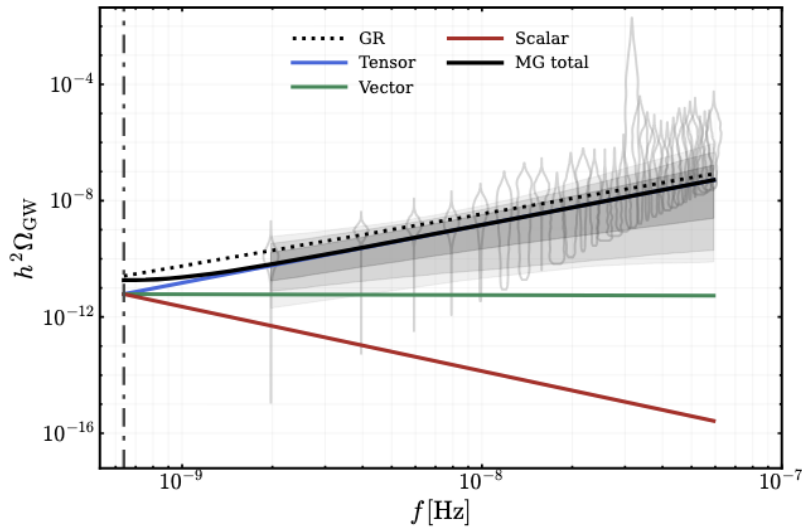
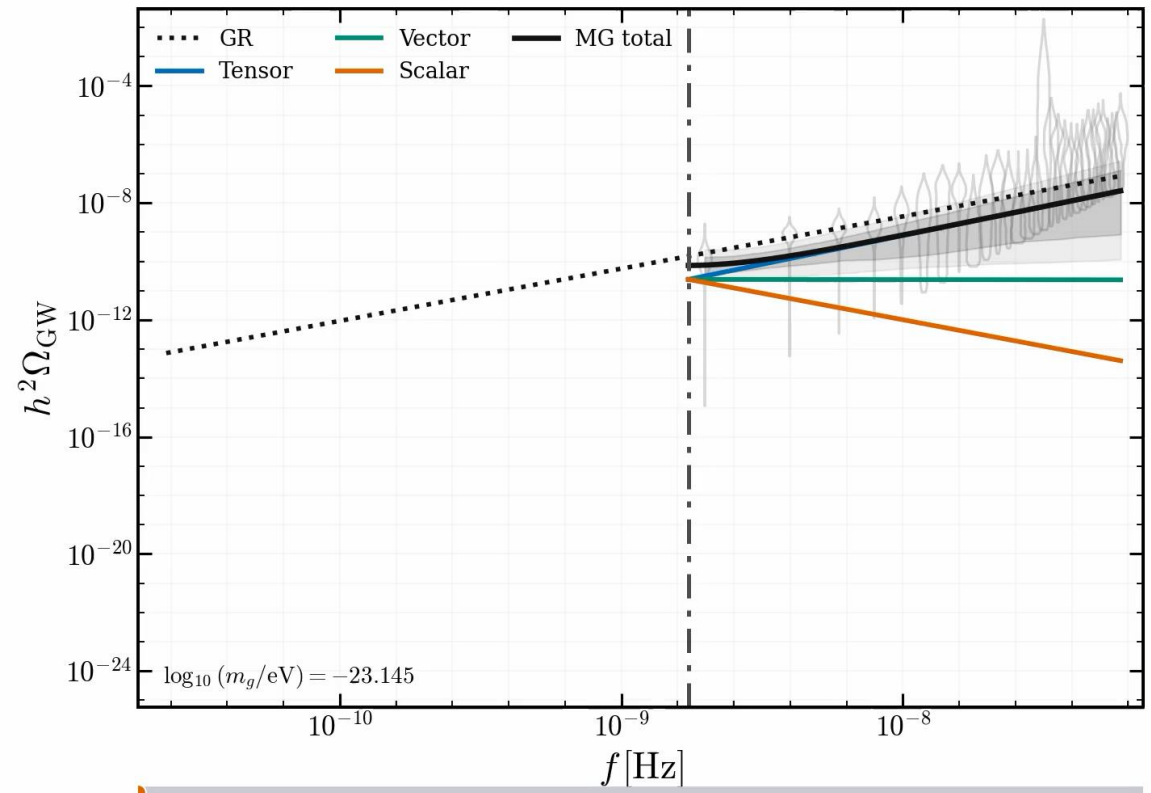


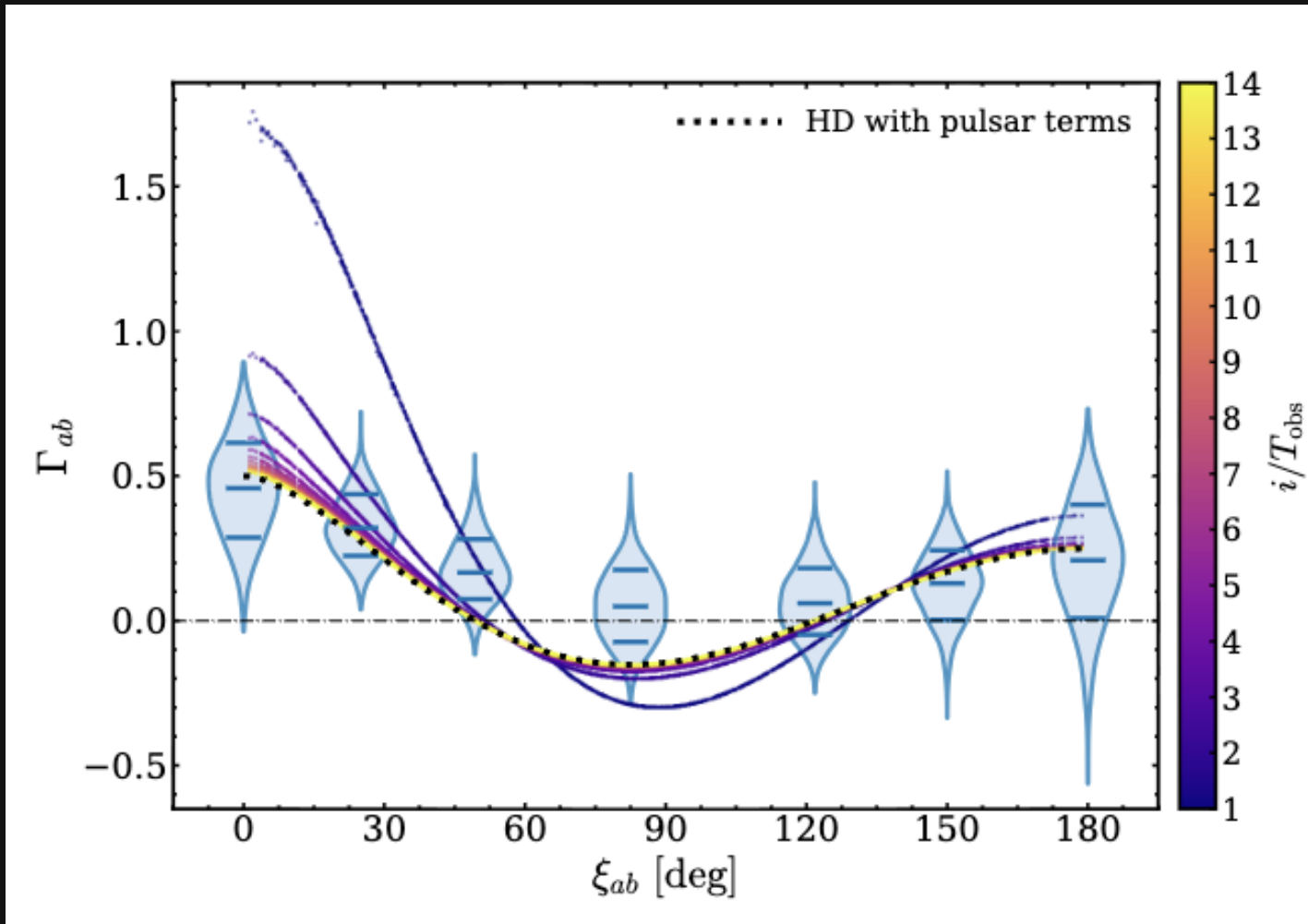
FIG. 3: Energy-density spectrum for the massive gravity analysis. The gray violins show the NANOGrav HD-correlated free-spectrum reconstruction, while the nested gray bands show the $1\text{--}3\sigma$ range of the total massive gravity spectrum (black). The solid curves show the tensor (blue), vector (green), and scalar (red) contributions. The dotted black curve shows the corresponding GR spectrum. The vertical dashed line marks the median graviton cut-off frequency, $f_g = m_g/(2\pi) \simeq 0.64$ nHz, corresponding to $m_g \simeq 2.64 \times 10^{-24}$ eV. The massive gravity spectrum lies within the broad range favored by the NANOGrav free-spectrum reconstruction and is slightly below the GR median. The tensor contribution dominates away from the cut-off, while the vector and scalar contributions become larger toward low frequencies.

The total spectrum of massive gravity energy density lies in the range preferred by the PTA common-process reconstruction. A concrete observable of massive gravity in an extended PTA campaign would be a low-frequency enhancement of the energy-density spectrum, driven by the increasing relative contribution of the scalar and vector modes.



Credit: Chris Choi

Preliminary Results: NanoGrav

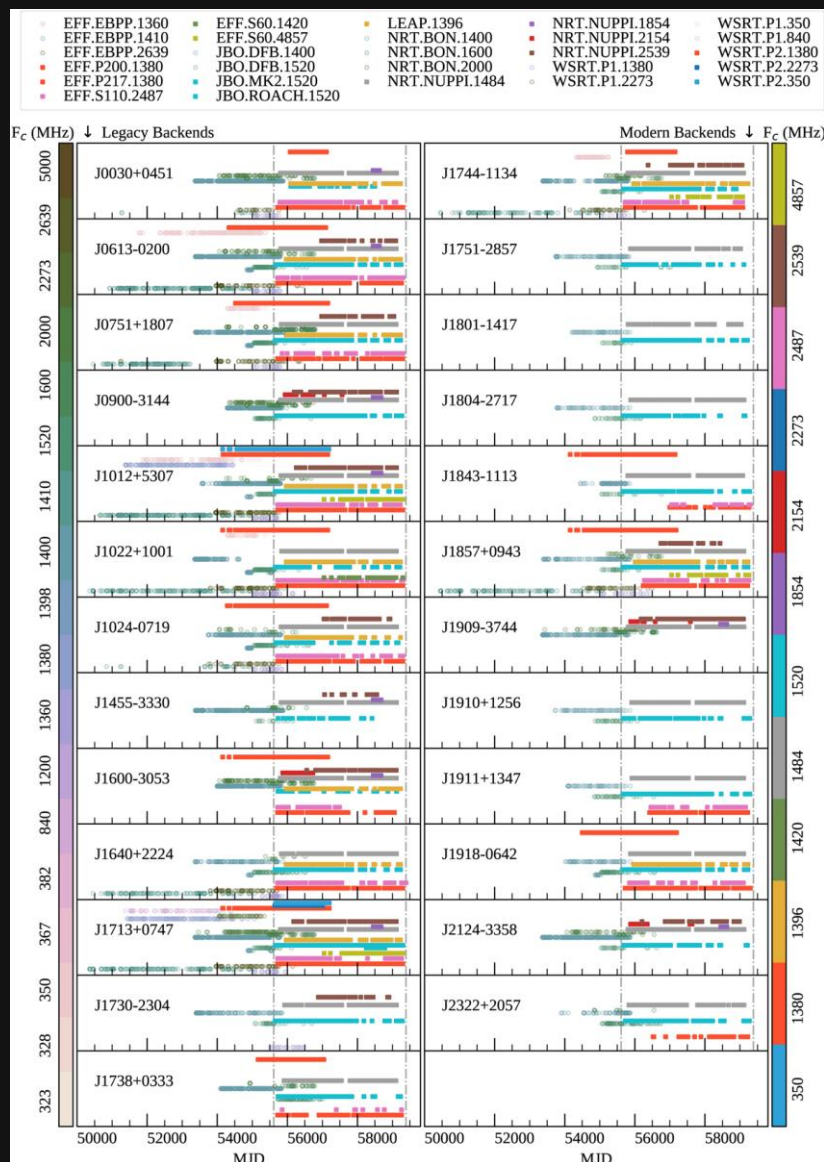


Effective ORFs from the massive gravity (MG) analysis, compared with the HD correlation. The blue violins show the NANOGrav frequency-independent spline reconstruction; the horizontal bars mark the 16th, 50th, and 84th percentiles at each angular knot. Colored points show the effective ORF for every pulsar pair, at the 14 Fourier frequencies $f_i = i/T_{obs}$; the color bar gives the Fourier index i . The black dotted curve shows the HD correlation. The largest departures from HD occur in the lowest-frequency bins, where the MG prediction exhibits enhanced correlations at small angular separations. At higher frequencies, the MG predictions converge toward HD.

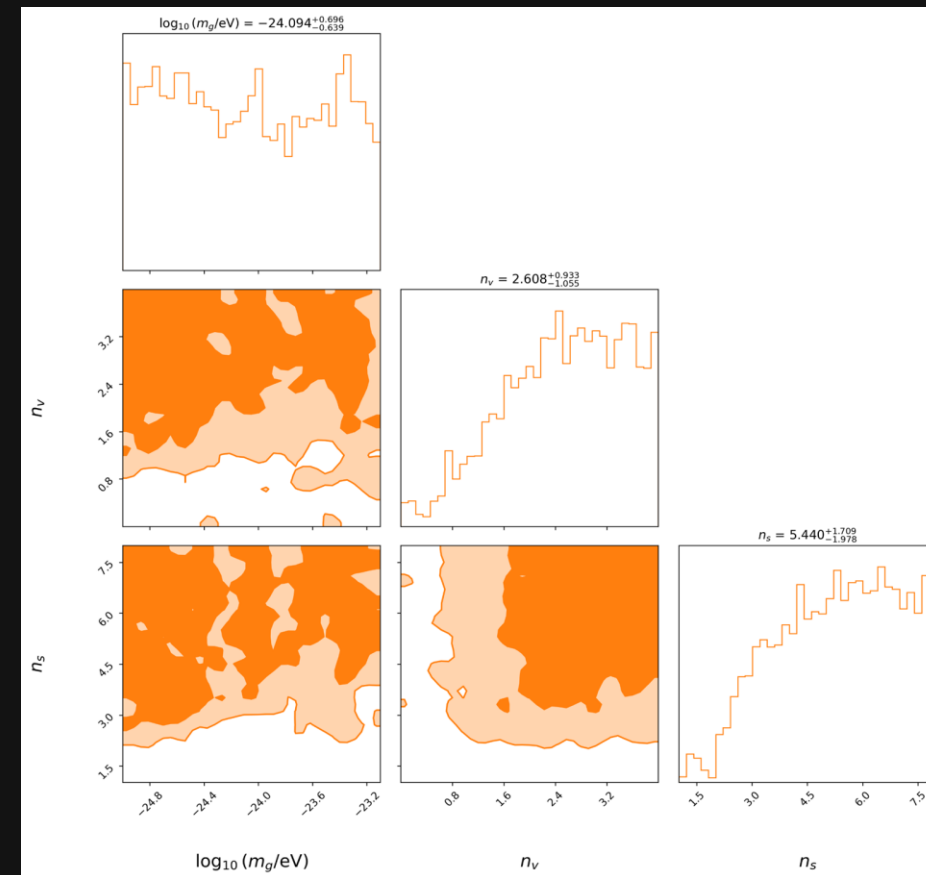
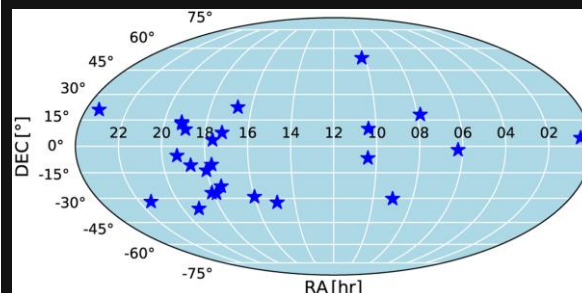
In addition, the modified dispersion relation implies a propagation threshold: modes with $f < f_g = m_g / (2\pi)$ have imaginary k and are therefore non-propagating.

Preliminary Results: EPTA

Credit Marcus Bosca



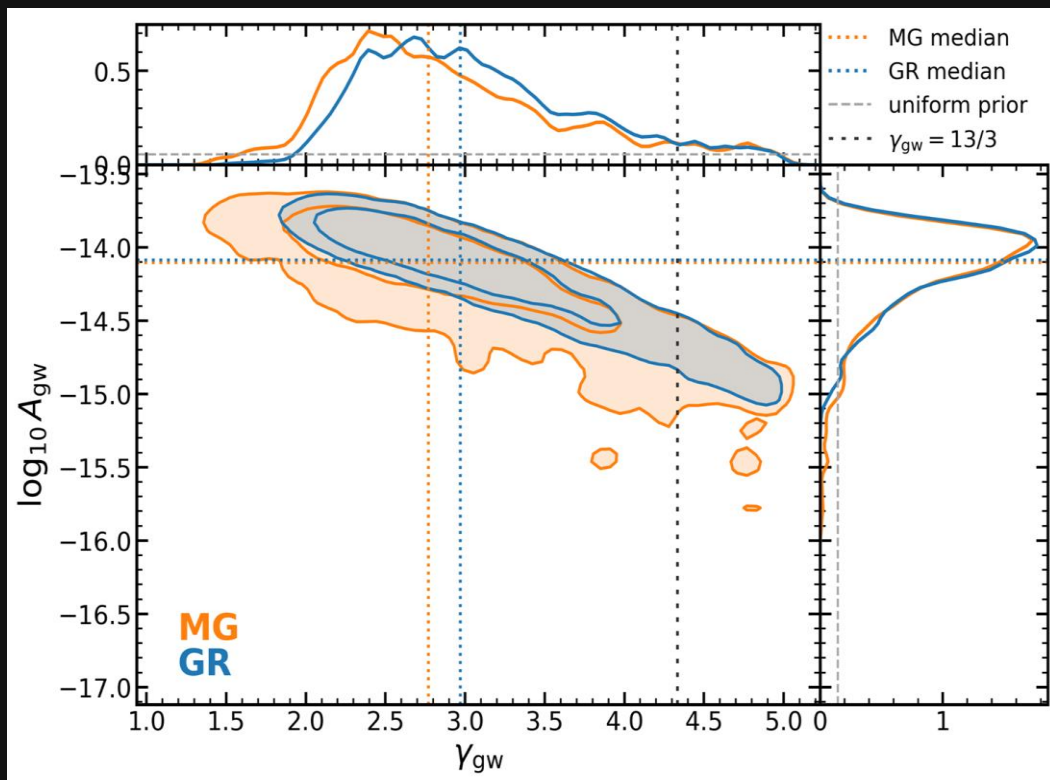
Overview of the full EPTA DR2 dataset. Empty circles denote legacy data, while filled squares show new EPTA backends. Vertical lines bound the range of the 'DR2new' dataset.



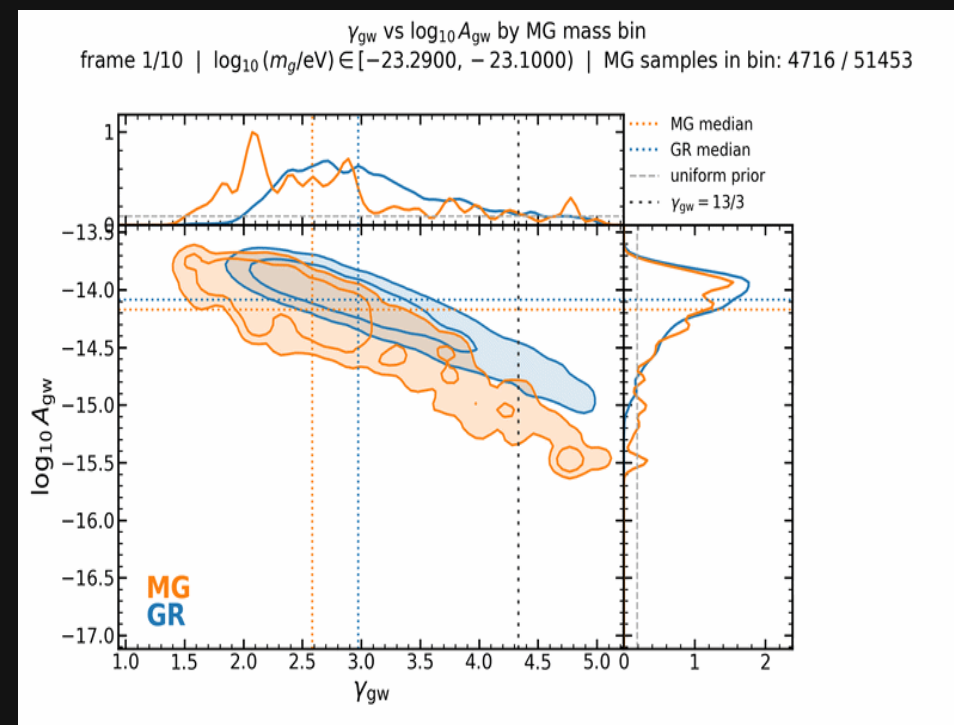
Joint Posterior distributions for the graviton mass, m_g , and the vector/scalar suppression indices, n_V and n_S . Distributions show that each of these parameters are poorly constrained due to the MCMC being prior-dependent. This fundamentally exemplifies that the EPTA dataset, like the NANOGrav dataset, also cannot probe SGWB frequencies low enough to differentiate the MG model from GR, and to constrain graviton mass and suppression.

Credit: EPTA

Preliminary Results: EPTA



Joint Posterior distribution for the GW amplitude, A_{GW} and the spectral index, γ_{GW} for a GR model (blue) and a Massive Gravity model (orange) for the EPTA DR2new+dataset. The branches occupy a very similar area in the $A_{GW} - \gamma_{GW}$ parameter space. The strongest deviations of the MG branch from the GR branch occur at large masses, as expected. The use of modified ORF still provides more complex structure for the MG branch, like for the NANOGrav posteriors.



Joint Posterior distribution organized and evolving by mass bins from highest mass to lowest mass. The lowest mass bin approaches the GR case, which exemplifies the effective behavior in the limit of masslessness.

Credit Marcus Bosca

The Hellings–Downs correlation tells us that the signal is gravitational-wave-like, but it does not tell us whether the background is astrophysical or primordial. For that, we need spectral, statistical, anisotropy, and polarization information.

Living Rev Relativ (2017) 20:2
<https://doi.org/10.1007/s41114-017-0004-1>

 CrossMark

REVIEW ARTICLE

Detection methods for stochastic gravitational-wave backgrounds: a unified treatment

Joseph D. Romano¹ · Neil. J. Cornish²

For a PTA, separating an astrophysical stochastic gravitational-wave background – mainly supermassive black-hole binaries (SMBHBs) – from a cosmological primordial background is difficult because PTA measures the *combined* stochastic signal. The discrimination comes from looking for differences in spectral shape, angular correlations, statistics, and polarization.

Astrophysical vs Cosmological SGWB		
Observable	Astrophysical (SMBHB)	Cosmological
Spectrum	$h_c \sim f^{-2/3*}$	model dependent
HD correlation	✓	✓
Isotropy	approximate	very high
Gaussianity	potentially non-Gaussian	often Gaussian
Resolvable sources	possible	no
Anisotropy / galaxy correlation	expected	very small
Chirality	not expected generically	possible
Non-GR polarizations	possible only beyond GR	possible new physics

LETTERS
<https://doi.org/10.1038/s41550-021-01489-8>




Ultra-low-frequency gravitational waves from cosmological and astrophysical processes

Christopher J. Moore[✉] and Alberto Vecchio[✉]

Kinematic anisotropy

PRL 106, 191301 (2011)
PHYSICAL REVIEW LETTERS
week ending
13 MAY 2011

Signature of Local Motion in the Microwave Sky

Arthur Kosowsky*

Department of Physics and Astronomy, University of Pittsburgh, 3941 O'Hara Street, Pittsburgh, Pennsylvania 15260, USA

Tina Kahniashvili

The McWilliams Center for Cosmology and Department of Physics, Carnegie Mellon University, 5000 Forbes Avenue, Pittsburgh, Pennsylvania 15213, USA, Department of Physics, Laurentian University, Ramsey Lake Road, Sudbury, Ontario P3E 2C6, Canada, and Abastumani Astrophysical Observatory, Ilia Chavchavadze State University, 2A Kazbegi Avenue, Tbilisi, GE-0160, Georgia

(Received 2 July 2010; revised manuscript received 18 March 2011; published 9 May 2011)

For observers moving with respect to the cosmic rest frame, the microwave background temperature fluctuations will no longer be statistically isotropic. Aside from the familiar temperature dipole, an observer's velocity will also induce changes in the temperature angular correlation function and create nonzero off-diagonal correlations between multipole moments. We show that both of these effects should be detectable in future full-sky maps from the Planck satellite, and can constrain modifications of the standard cosmological model proposed to explain anomalous current observations.

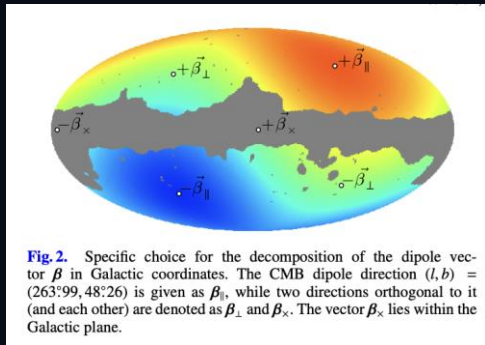


Fig. 2. Specific choice for the decomposition of the dipole vector β in Galactic coordinates. The CMB dipole direction $(l, b) = (263.99, 48.26)$ is given as β_l , while two directions orthogonal to it (and each other) are denoted as $\beta_\perp$ and $\beta_\times$. The vector $\beta_\times$ lies within the Galactic plane.

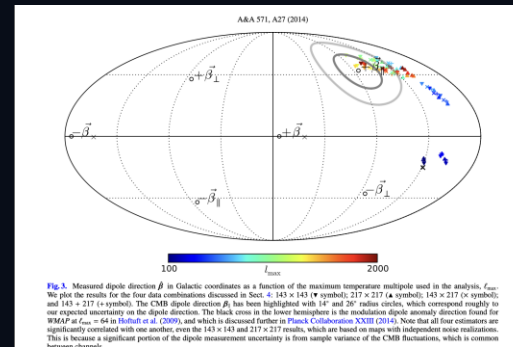


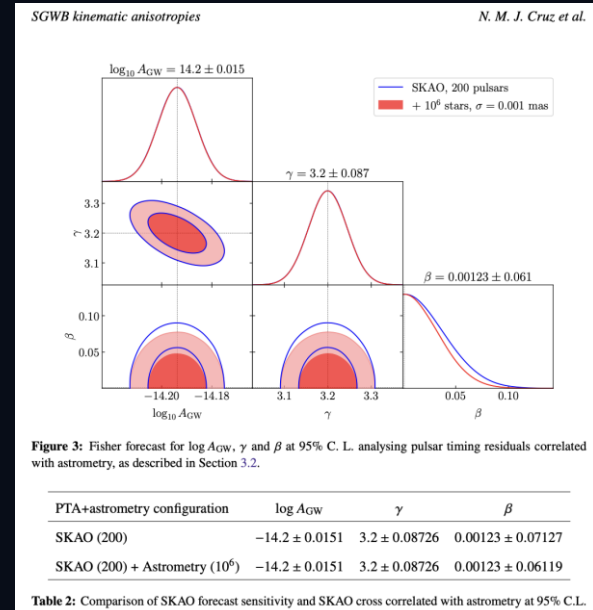
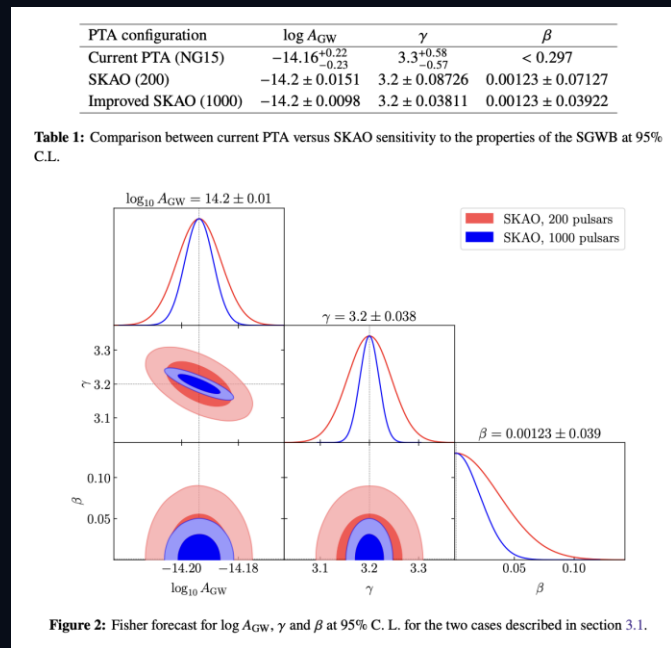
Fig. 3. Measured dipole direction β in Galactic coordinates as a function of the maximum temperature multipole used in the analysis, l_{max} . We plot the results for the four data combinations discussed in Sect. 4: 143×143 (x symbol), 217×217 (o symbol), 143×217 (v symbol), and 143×217 (x symbol). The CMB dipole direction β has been highlighted with 14° and 26° radius circles, which correspond roughly to our expected uncertainty on the dipole direction. The black cross in the lower hemisphere is the modulation dipole anomaly direction found by WMAP at $l_{\text{max}} = 64$ in Huftli et al. (2009), and which is discussed further in Planck Collaboration XXII (2014). Note that all four estimates are significantly correlated with one another, even the 143×143 and 217×217 results, which are based on maps with independent noise realizations. This is because a significant portion of the dipole measurement uncertainty is from sample variance of the CMB fluctuations, which is common to all channels.

$$\Delta\beta \propto \sqrt{\frac{1}{T_{\text{obs}} N_{\text{psr}}}}$$

Status and outlook:

Current PTA and SKAO data sensitivity is not enough to constrain dipole in any significant capacity [Cruz et al. 2606.25845]

Extensions: astrometric observables alongside PTAs, and spectra departing from a pure power law can be pass forward to detect kinematic dipole.



To Do:

Redefine receiving functions based on Liang & Troddden formalism and re-do complete analysis

Submission history

From: Qiuyue Liang [[view email](#)]

[v1] Wed, 11 Aug 2021 17:53:08 UTC (434 KB)

[v2] Fri, 17 Sep 2021 17:47:52 UTC (433 KB)

[v3] Fri, 24 Jul 2026 01:40:05 UTC (430 KB)

$$F^{(i)}(\hat{\Omega}) \equiv -\frac{\hat{p}^\mu \hat{p}^\nu}{2 \left(1 + \frac{|\mathbf{k}|}{k_0} \hat{\Omega} \cdot \hat{\mathbf{p}}\right)} \epsilon_{\mu\nu}^{(i)} + \frac{\hat{p}^\mu}{2} \epsilon_{0\mu}^{(i)}$$



$$F^{(i)}(\hat{\Omega}) \equiv -\frac{\hat{p}^\mu \hat{p}^\nu}{2 \left(1 + A \hat{\Omega} \cdot \hat{\mathbf{p}}\right)} \epsilon_{\mu\nu}^{(i)} + \hat{p}^\mu \epsilon_{0\mu}^{(i)} - \frac{1}{2} \epsilon_{00}^{(i)} (1 + A \hat{\Omega} \cdot \hat{\mathbf{p}})$$

Main difference in derivation

- v2 follows the perturbed photon trajectory but does not consistently include the perturbed four-velocities of the emitter and receiver.
- The perturbation of the emitter's and receiver's four-velocities.
- v3 adds the timelike geodesics of the pulsar and Earth observers.

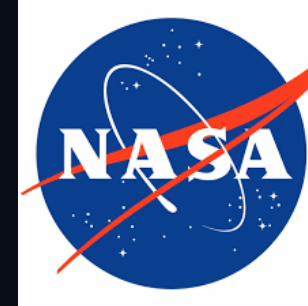
Take-home message

PTAs probe gravity on the largest accessible scales

- Massive gravity leaves characteristic frequency- and angle-dependent signatures in pulsar timing correlations.
- The strongest departures from GR occur at the lowest frequencies, where the graviton-mass cutoff and additional polarizations become important.
- Current PTA data do not yet significantly favor massive gravity over GR and provide only weak constraints on the graviton mass and extra-mode suppression.
- Kinematic anisotropy provides a complementary observable for testing the nature and propagation of the gravitational-wave background; however, PTA and SKA are not sensitive enough to provide detection of our proper motion.

PTAs are not only gravitational-wave detectors — they are precision tests of gravity.

Acknowledgements



PPC2026 Organizers

Collaborators: Barnabas Poczos (and on future project Alberto Roper Pol and Hippolyte Jean Hugo Quelquejay-Leclere)

Students: Marcus Bosca, Chris Choi, Murman Gurgenidze

Partial support from:

- Shota Rustaveli National Science Foundation of Georgia FR24-901
- NASA Astrophysics Theory Program: ATP80NSSC22K0825
- NSF Astrophysics and Astronomy Research Grant Program: AST2307698 & AST2408411