

Thermal evolution of neutron stars in linear $f(R, \mathcal{T})$ gravity

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Why cooling is a complementary gravity test

Static information

- ▶ Mass-radius relation and central density
- ▶ Compactness, redshift, and crust thickness
- ▶ Composition-dependent emission thresholds

Time-domain information

- ▶ Thermal inertia and heat diffusion
- ▶ Neutrino cooling and photon emission
- ▶ Pairing transitions and envelope response

At fixed microphysics, does the matter–geometry coupling produce a repeatable shift in the cooling clock?

1 Radial branch

Modified hydrostatic equilibrium
and the M–TOV background

2 Timelike branch

Leading correction to the
effective thermal inertia

3 Controlled scan

EoS, pairing, envelope, and
neutrino microphysics

Linear $f(R, \mathcal{T})$ gravity: one balance law, two projections

Model

$$f(R, \mathcal{T}) = R + 2\lambda\kappa\mathcal{T}, \quad \lambda = 0 \implies \text{GR},$$
$$G_{\mu\nu} = \kappa(1 + 2\lambda)T_{\mu\nu} + \lambda\kappa(\rho + 3P)g_{\mu\nu}.$$

Ordinary matter is not conserved separately

$$\nabla^\mu T_{\mu\nu} = -\frac{\lambda}{1 + 2\lambda} \nabla_\nu(\rho + 3P) \equiv S_\nu^{(\lambda)}.$$

The complete effective source remains covariantly conserved.

modified matter-balance equation



radial component



modified hydrostatic equilibrium



same fixed
background

projection along u^ν



modified thermal inertia

This equation is the bridge between the structural and cooling parts of the calculation.

Harko et al., Phys. Rev. D 84, 024020 (2011).

Radial projection: gravity changes the star we cool

Modified TOV Equations

$$\frac{dm}{dr} = 4\pi r^2 \rho_{\text{eff}},$$

$$\frac{d\Phi}{dr} = \frac{m + 4\pi r^3 P_{\text{eff}}}{r(r - 2m)},$$

$$\frac{dP}{dr} = -\frac{\rho_{\text{eff}} + P_{\text{eff}}}{1 + 5\lambda + \lambda d\rho/dP} \frac{d\Phi}{dr}.$$

$$\rho_{\text{eff}} = (1 + \lambda)\rho - 3\lambda P, \quad P_{\text{eff}} = \lambda\rho + (1 + 5\lambda)P.$$

Local EoS dependence

$$D(r) = 1 + 5\lambda + \lambda \frac{d\rho}{dP} > 0.$$

- ▶ Not a constant rescaling of G .
- ▶ $D(r)$ varies with density and with the EoS.
- ▶ Each λ produces a distinct density, metric, and crust profile.

Fixed-mass construction: adjust P_c for each EoS and λ so that $M = 1.4 M_\odot$ and $P_{\text{eff}}(R) = 0$.

The cooling calculation is performed on a self-consistent M–TOV background, not on a common GR star.

Timelike projection: a correction to thermal inertia

Cold background + thermal perturbation

$$\rho = \rho_0(n_b) + \rho_{\text{th}}(n_b, T), \quad P = P_0(n_b) + P_{\text{th}}(n_b, T),$$

$$c_{V,\lambda}^{\text{eff}} = c_V - \frac{\lambda}{1 + 2\lambda} \left[\left(\frac{\partial \rho_{\text{th}}}{\partial T} \right)_{n_b, \{Y_i\}} + 3 \left(\frac{\partial P_{\text{th}}}{\partial T} \right)_{n_b, \{Y_i\}} \right].$$

Leading-order sensitivity closure

$$\frac{\partial \rho_{\text{th}}}{\partial T} = c_V, \quad \frac{\partial P_{\text{th}}}{\partial T} \simeq \frac{2}{3} c_V \quad \Rightarrow \quad \frac{c_{V,\lambda}^{\text{eff}}}{c_V} = 1 - \frac{3\lambda}{1 + 2\lambda}.$$

0.2994%

reduction of the adopted local heat capacity at $\lambda = 10^{-3}$

Physical interpretation

- ▶ Changes local thermal inertia.
- ▶ No additional neutrino emissivity.
- ▶ No imposed heat source.

The timelike matter-balance term has the same temperature-derivative structure as the ordinary heat-capacity term.

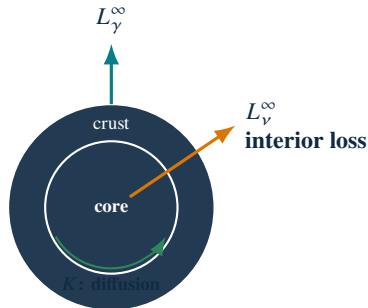
Thermal evolution on fixed M-TOV background

Local relativistic cooling equations

$$\begin{aligned}\tilde{T} &= T e^{\Phi}, & \tilde{L}_r &= e^{2\Phi} L_r, \\ \frac{\partial \tilde{L}_r}{\partial r} &= 4\pi r^2 e^{\Lambda} \left[e^{2\Phi} (Q_h - Q_v) - c_{V,\lambda}^{\text{eff}} \frac{\partial \tilde{T}}{\partial t} \right], \\ \tilde{L}_r &= -4\pi r^2 K e^{\Phi-\Lambda} \frac{\partial \tilde{T}}{\partial r}.\end{aligned}$$

Passive, redshift-isothermal limit

$$C_{\lambda}^{\text{eff}}(T_b^{\infty}) \frac{dT_b^{\infty}}{dt} = -L_v^{\infty} - L_{\gamma}^{\infty}, \quad Q_h = 0.$$



$$L_{\gamma}^{\infty} = 4\pi\sigma_{\text{SB}}R_{\infty}^2 (T_{\text{eff}}^{\infty})^4.$$

The background enters through $\rho(r)$, composition, $\Phi(r)$, $\Lambda(r)$, and the envelope boundary.

Thermal storage is modified through $c_{V,\lambda}^{\text{eff}}$; radial heat transport retains its standard NSCool form.

What actually cools the star?

Process	Schematic role	Region
Modified Urca	Baseline neutrino emission	Core
<i>NN</i> bremsstrahlung	Baseline emission; pairing suppression included	Core + inner crust
Direct Urca	Active only above the composition threshold	Core
Pair break- ing/formation	Activated near $T \approx T_c$	Paired crust + core
Electron bremsstrahlung	Crustal neutrino emission	Crust
Plasmon decay	Crustal neutrino emission	Crust

Fixed $1.4 M_\odot$ models

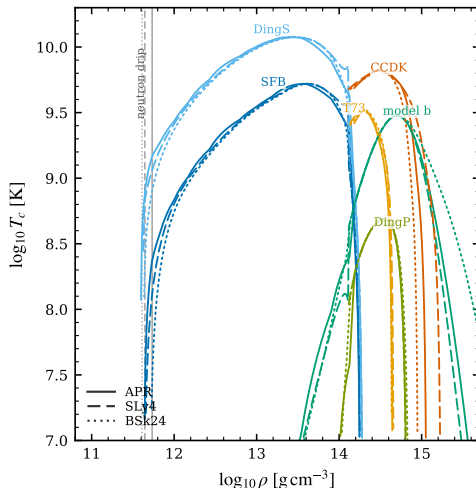
Direct Urca is absent

Controlled comparison

- ▶ Same neutrino setup for every λ .
- ▶ No exotic fast-cooling channel.
- ▶ Rapid declines arise from crust relaxation and pairing physics.

λ changes the stellar background and the timing of known cooling processes, not the reaction set.

Nucleon pairing and its role in cooling



Two pairing prescriptions

SFB-b-T73

n^1S_0 : SFB

n^3P_2 : model b

p^1S_0 : T73

DingS-DingP-CCDK

n^1S_0 : DingS

n^3P_2 : DingP

p^1S_0 : CCDK

Physical effect

- ▶ Pairing suppresses the nucleonic heat capacity.
- ▶ Standard neutrino-emission rates are modified.
- ▶ PBF becomes important when $T \sim T_c$.

Numerical setup and controlled parameter space

Gravity-dependent background

For each EoS and coupling λ , an independent M–TOV background is constructed at

$$M = 1.4 M_{\odot}.$$

$$\mathcal{B}_{\lambda}(r) = \{\rho(r), P(r), m(r), \Phi(r), R, R_{\text{cc}}, R_b\}.$$

Cooling calculation

Each background is supplied directly to NSCool,

$$\mathcal{B}_{\lambda}(r) \xrightarrow{\text{NSCool}} T_{\text{eff}}^{\infty}(t), \quad L_{\gamma}^{\infty}(t).$$

Passive cooling:

$$Q_h = 0, \quad c_V \rightarrow c_{V,\lambda}^{\text{eff}}.$$

Important: different values of λ are not evolved on a common GR background.

Controlled parameter space

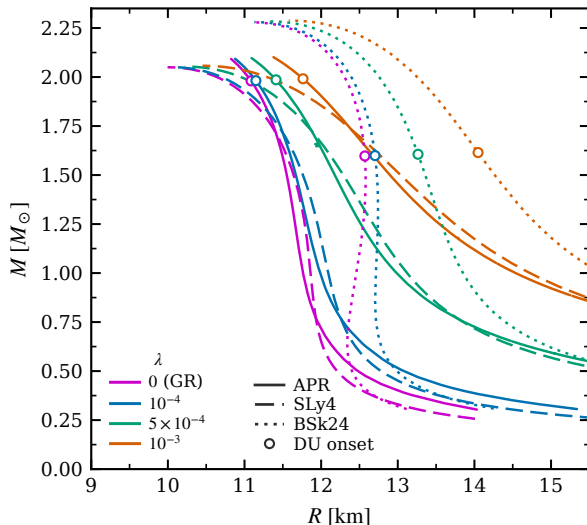
Input	Adopted values
EoS	APR, SLy4, BSk24
λ	0, 10^{-4} , 5×10^{-4} , 10^{-3}
Mass	$1.4 M_{\odot}$
Pairing	SFB-b-T73 DingS-DingP-CCDK
Envelope	Fe Light ($\eta = 10^{-7}$)
Neutrinos	Standard NSCool channels DU inactive
Heating	$Q_h = 0$

Comparison strategy

EoS, pairing, envelope and neutrino setup are held fixed when varying λ .

Gravity-induced cooling changes are therefore identified at fixed microphysics.

Modified gravity changes the stellar background



Positive matter–geometry coupling

At fixed gravitational mass,

$$\lambda \uparrow \Rightarrow R \uparrow, \quad \rho_c \downarrow.$$

Positive λ shifts the APR, SLy4 and BSk24 sequences toward larger radii.

Maximum-mass response

Across $0 \leq \lambda \leq 10^{-3}$,

$$\Delta M_{\max} < 0.01 M_\odot.$$

Thus, the dominant response in this coupling range is structural rather than a large change in $M_{\max}$.

Direct-Urca regime

Open circles mark the direct-Urca threshold.

All selected $1.4 M_\odot$ backgrounds remain below this threshold.

Fixed $1.4 M_{\odot}$ backgrounds: structural origin of delayed relaxation

Evolved crust thickness

$$\Delta R_{\text{evol}} = R_b - R_{\text{cc}}$$

EoS	GR	$\lambda = 10^{-3}$	Change
APR	0.880 km	1.293 km	$\approx 47\%$
SLy4	0.776 km	1.203 km	$\approx 55\%$
BSk24	0.868 km	1.363 km	$\approx 57\%$

ΔR_{evol} increases by approximately 47% – 57% from GR to $\lambda = 10^{-3}$.

The corresponding central baryon density decreases by approximately 1%.

Why does this matter for cooling?

$$\lambda \uparrow \implies \Delta R_{\text{evol}} \uparrow$$

$$\implies t_{\text{diff}} \uparrow$$

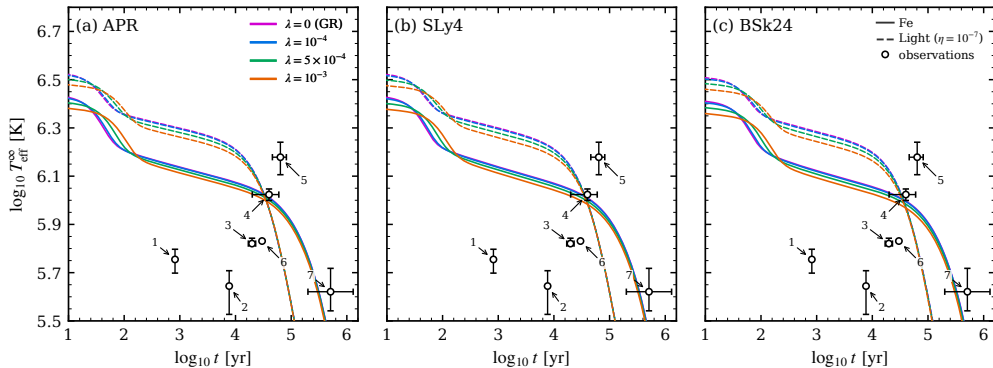
$$\implies t_{\text{relax}} \uparrow$$

A thicker evolved crust increases the thermal-diffusion distance between the crust–core boundary and the base of the heat-blanketing envelope.

Physical interpretation

The structural effect of positive λ therefore appears primarily as a **delay of crust–core thermal relaxation**, rather than as a change of the neutrino-emission regime.

SFB-b-T73 pairing: positive λ delays early crust-core relaxation



Gravity-dependent timing

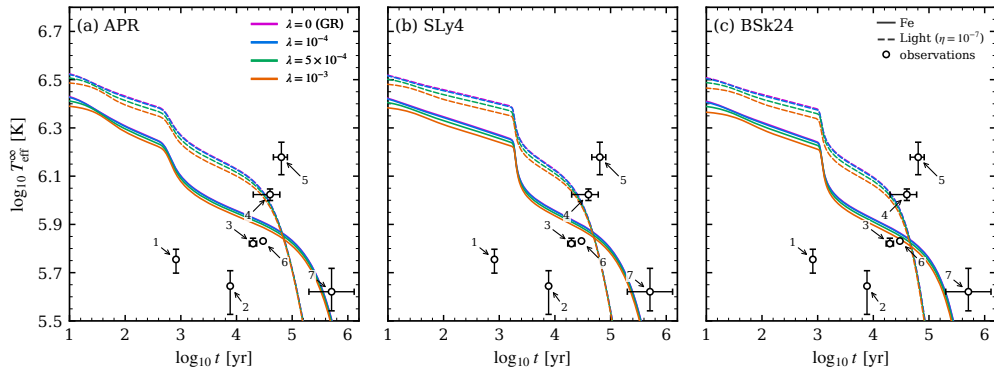
Increasing λ shifts the early relaxation feature systematically to **later ages** for all three EoSs.

Envelope response

The light-element envelope mainly shifts the cooling tracks to **higher surface temperatures**.

Gravity primarily shifts the timing; the envelope primarily shifts the temperature level.

DingS–DingP–CCDK pairing: EoS-dependent cooling feature



Pairing-driven feature

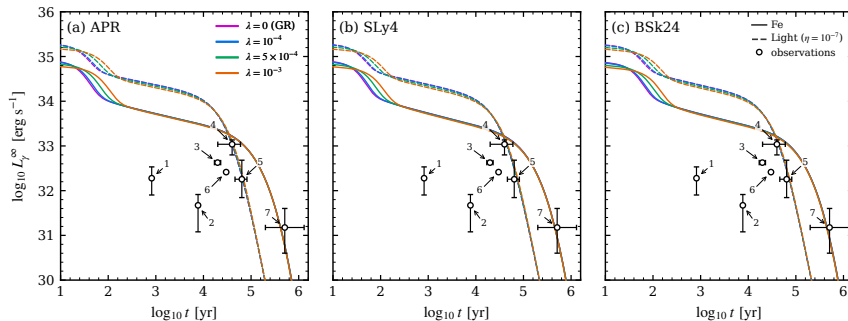
With the **DingS–DingP–CCDK** prescription, an additional intermediate-age decline appears.

DingS–DingP–CCDK determines the intermediate-age feature; λ shifts its timing.

Gravity-induced displacement

At fixed EoS and pairing prescription, increasing λ produces an additional **timing shift** of the same feature.

SFB-b-T73 pairing: delayed relaxation in photon luminosity



Gravity-dependent timing

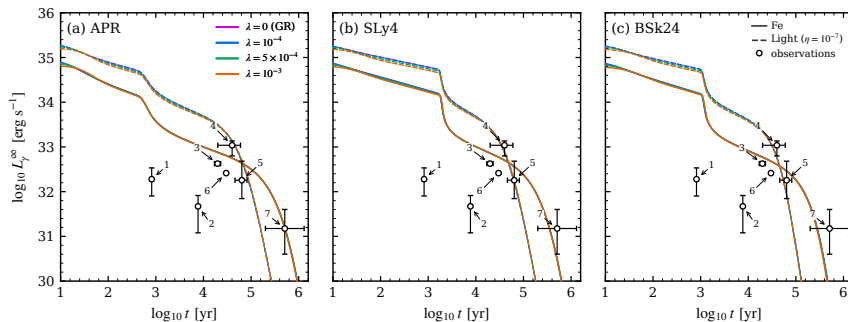
Increasing λ shifts the early luminosity decline to **later** ages for APR, SLy4, and BSk24.

Luminosity response

Because $L_\gamma^\infty \propto R_\infty^2 (T_{\text{eff}}^\infty)^4$, the same thermal transition appears more prominently in photon luminosity.

The gravity-dependent timing shift is reproduced in both $T_{\text{eff}}^\infty(t)$ and $L_\gamma^\infty(t)$.

DingS–DingP–CCDK pairing: intermediate-age feature in photon luminosity



Pairing-dependent feature

The intermediate-age decline remains strongly **EoS dependent**, reflecting the different realization of the pairing groups inside each star.

Additional gravity shift

At fixed EoS and pairing prescription, changing λ produces a further **shift in the timing** of the same feature.

Pairing determines the intermediate-age structure; modified gravity provides an additional timing displacement.

Two thermal imprints of the matter–geometry coupling

Radial branch: structural response

The modified M–TOV background gives

$$\Delta R_{\text{evol}} \uparrow \quad \text{by} \quad \approx 47\% - 57\%,$$

while the central baryon density decreases by approximately 1%.

$$\Delta R_{\text{evol}} \uparrow \implies t_{\text{relax}} \uparrow$$

This produces the systematic shift of the crust–core relaxation feature to later ages.

Timelike branch: thermal inertia

For the adopted leading-order closure,

$$\frac{c_{V,\lambda}^{\text{eff}}}{c_V} = 1 - \frac{3\lambda}{1+2\lambda}.$$

At $\lambda = 10^{-3}$,

$$\frac{c_{V,\lambda}^{\text{eff}}}{c_V} \approx 0.9970$$

corresponding to only a

$$\approx 0.30\%$$

reduction in the local thermal inertia.

It modifies the timing of thermal evolution; it does **not** introduce a new heating or neutrino-emission channel.

Structural modification provides the dominant global timing shift, while the timelike correction is smaller and can be locally amplified near a sharp pairing transition.

Observational comparison: viability, not a constraint on λ

Comparison strategy

Seven isolated, non-magnetar pulsars with independently constrained ages are considered.

Model compatibility is assessed in the

$$t - L_{\gamma}^{\infty} \text{ plane}$$

rather than simultaneously using T_{eff}^{∞} and L_{γ}^{∞} .

For each EoS, pairing prescription, and λ ,

$$\text{Fe} \longleftrightarrow \text{light envelope}$$

defines the range associated with envelope uncertainty.

Compatibility criterion

A source is regarded as compatible when its age–luminosity uncertainty region intersects the calculated cooling range.

No statistical fit is performed.

Outcome of the comparison

Status	Sources
Compatible across the displayed λ grid	J0538, B1951, Morla
Outside the calculated ranges	J0205, B2334, Vela, J0617

Main implication

The coverage of the observational sample does **not change across the tested coupling range**.

Therefore,

no preferred value of λ

is selected by the present fixed-mass comparison.

The nonzero- λ models remain **observationally viable** within the adopted microphysical framework.

Conclusions: what does the cooling calculation tell us?

1. Modified stellar structure

For fixed $M = 1.4 M_{\odot}$, positive λ produces a coherent structural response:

$$n_c \downarrow \sim 1\%, \quad \Delta R_{\text{evol}} \uparrow \sim 47\% - 57\%.$$

The thicker evolved crust shifts crust–core thermal relaxation to **later** ages.

2. Timelike contribution

The timelike matter-balance term enters through $c_{V,\lambda}^{\text{eff}}$.

$$\lambda = 10^{-3} \quad \Rightarrow \quad \frac{c_{V,\lambda}^{\text{eff}}}{c_V} \simeq 0.997.$$

It modifies the thermal inertia and transition timing, but introduces **no additional heating or neutrino-emission channel**.

3. Gravity versus microphysics

The controlled comparison separates the dominant roles:

Gravity: shifts the timing of cooling features.

Pairing: controls the occurrence and shape of the intermediate-age decline.

Envelope: mainly sets the vertical surface-temperature level.

4. Observational interpretation

Direct Urca remains absent from every selected $1.4 M_{\odot}$ background.

Three of the seven comparison sources remain compatible throughout the tested coupling range.

The present data therefore support **model viability**, but do not select a preferred value of λ .

Thank You

Neutron-star cooling provides a time-domain probe of matter–geometry coupling, complementary to the static M – R relation.

Questions?

Backup: EoS constructions and model grid

EoS	Uniform core	Inner crust	Outer crust / construction
APR	A18+ δv +UIX*	NV	HZD; matched, non-unified
SLy4	SLy4	SLy4	BPS/HP; unified inner crust–core
BSk24	BSk24	BSk24 (ETFSI)	BSk24; unified above ρ_{match}

Gravity and mass

$\lambda \in \{0, 10^{-4}, 5 \times 10^{-4}, 10^{-3}\}$, with $M = 1.4 M_{\odot}$ fixed independently for every EoS and coupling.

Thermal controls

Two pairing prescriptions; Fe and light-element envelopes; nonmagnetized passive cooling; common low-density extension.

Backup: direct-Urca threshold test

Kinematic conditions

$$k_{F,n} \leq k_{F,p} + k_{F,e}, \quad k_{F,n} \leq k_{F,p} + k_{F,\mu}.$$

The first density satisfying either condition defines n_{DU} and is mapped onto each M–TOV sequence.

Threshold masses

- ▶ APR: $M_{\text{DU}} = 1.980\text{--}1.991 M_{\odot}$.
- ▶ BSk24: $M_{\text{DU}} = 1.597\text{--}1.615 M_{\odot}$.
- ▶ SLy4: threshold not reached on the stable branch.

All fixed- $1.4 M_{\odot}$ stars satisfy $n_c < n_{\text{DU}}$.

Backup: origin of the effective heat capacity

$$\begin{aligned}\nabla^\mu T_{\mu\nu} &= S_\nu^{(\lambda)}, \\ q_Q^{(\lambda)} \equiv -u^\nu S_\nu^{(\lambda)} &= \frac{\lambda}{1+2\lambda} \frac{d}{d\tau} (\rho + 3P), \\ \partial_t(\rho + 3P) &= \left[\left(\frac{\partial \rho_{\text{th}}}{\partial T} \right)_{n_b, \{Y_i\}} + 3 \left(\frac{\partial P_{\text{th}}}{\partial T} \right)_{n_b, \{Y_i\}} \right] \partial_t T.\end{aligned}$$

Moving the term proportional to $\partial_t T$ to the left-hand side of the local energy equation gives $c_{V,\lambda}^{\text{eff}}$. This rearrangement does not restore separate conservation of $T_{\mu\nu}$; it identifies the common temperature-derivative structure under the adopted closure.

Backup: neutrino processes retained

Process	Schematic reaction	Region
Direct Urca	$n \rightarrow p + \ell + \bar{\nu}_\ell$	Core
Modified Urca	$n + N \rightarrow p + N + \ell + \bar{\nu}_\ell$	Core
Nucleon bremsstrahlung	$N + N \rightarrow N + N + \nu + \bar{\nu}$	Core + inner crust
PBF	$N + N \rightleftharpoons [NN] + \nu + \bar{\nu}$	Paired crust + core
Electron bremsstrahlung	$e + X \rightarrow e + X + \nu + \bar{\nu}$	Crust
Plasmon decay	$\gamma^* \rightarrow \nu + \bar{\nu}$	Crust

Backup: independently aged pulsars used in the comparison

ID	Source	Age [kyr]	$L_{\gamma}^{\infty} / 10^{32}$	$k_B T_{\text{eff}}^{\infty}$ [eV]
1	J0205	0.819	$1.9^{+1.5}_{-1.1}$	49^{+5}_{-6}
2	B2334	~ 7.7	0.47 ± 0.35	38^{+6}_{-9}
3	Vela	$19.8^{+3.2}_{-2.8}$	4.24 ± 0.12	57^{+3}_{-1}
4	J0538	40 ± 20	$10.9^{+2.7}_{-4.6}$	91 ± 5
5	B1951	64 ± 18	$1.8^{+3.0}_{-1.1}$	130 ± 20
6	J0617	~ 30	2.6 ± 0.1	$58.4^{+0.6}_{-0.4}$
7	Morla	510^{+790}_{-310}	$0.15^{+0.25}_{-0.11}$	36^{+9}_{-6}

Data adopted in the manuscript from Potekhin et al., MNRAS 496,

5052 (2020).

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