

Aspects of a Five-Dimensional $U(1)_{L\mu-L\tau}$ Model at Future Muon-Based Colliders

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Based on works with Arindam Chatterjee, Asesh Krishna Datta, Ayushi Kaushik, and Kenji Nishiwaki

arXiv: 2604.05041

Talk @ PPC-2026,

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Why $U(1)_{L\mu-L\tau}$?

- **Minimal and predictive:** Anomaly-free with SM fermions only
- **Massive Z' after symmetry breaking:** Couples to μ, τ and their neutrinos
- **Leptophilic:** Evades many electron- and quark-based constraints at tree level
- **Beyond $(g-2)_\mu$:** Testable via collider and neutrino probes

Why Extra Dimensions?

Possible Solutions to Open Problems:

- Addressing the hierarchy problem

L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999), hep-ph/9905221

- New dark matter candidates and cosmological implications

T. Lee, Y.-D. Tsai, Phys. Rev. D114, L011701 (2026)

Rich and tastable phenomenology:

Compactification of the extra dimension $\rightarrow$ an infinite tower of Kaluza–Klein (KK) modes

Broad KK mass spectrum from the tower

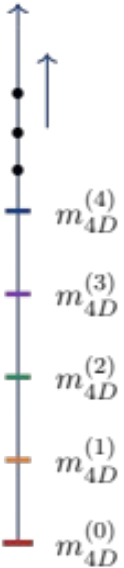
[Sub-GeV]

[TeV]

=====>

Low energy experiments
[e.g. beam-dump]

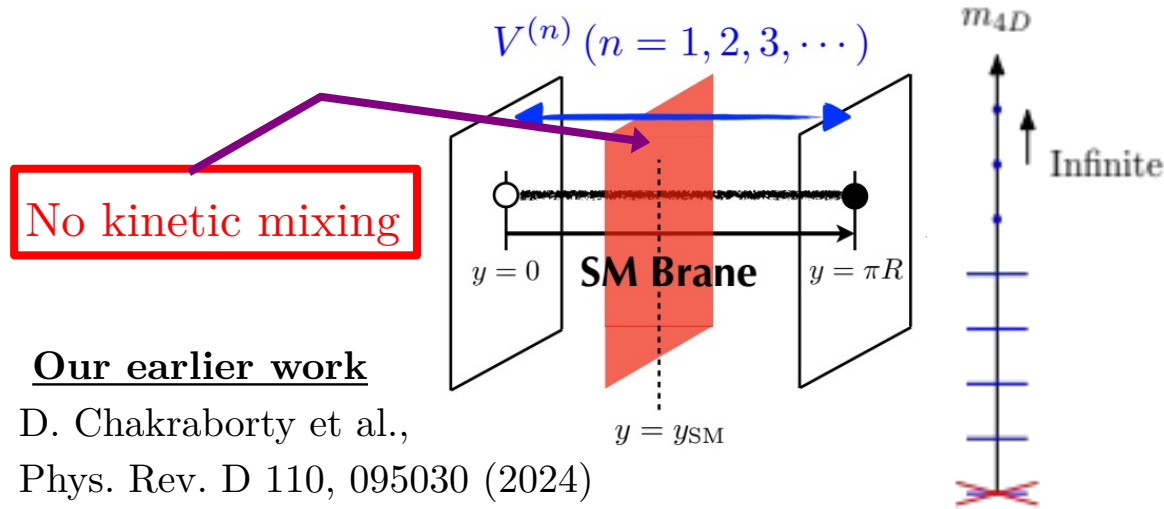
Next gen. High-energy colliders
[e.g. future muon collider]



5D $U(1)_{L\mu-L\tau}$ Model [Flat Geometry]

Gauge group	(Q_L^i, u_R^i, d_R^i)	(L_L^e, e_R)	(L_L^μ, μ_R)	(L_L^τ, τ_R)	ϕ_h
$U(1)_{L\mu-L\tau}$	0	0	1	-1	0

- Twisted (Neumann, Dirichlet) boundary conditions $\implies$ no **massless zero mode**



Mode function:

$$f_V^{(n)}(y_{SM}) = \sqrt{2} \cos \left[\left(n - \frac{1}{2} \right) \frac{y_{SM}}{R} \right]$$

$M_n = (2n - 1)m_{KK}$

M_4
 M_3
 M_2
 M_1

$m_{KK} \rightarrow$ lowest order KK mass $\rightarrow M_1$

☼ **Note:** One 5D gauge boson $\implies$ **an infinite tower of KK states**

The 4D Effective Lagrangian

$$\mathcal{L}_{4D}^{\text{eff}} \supset \sum_n \left[\underbrace{-\frac{1}{4} V_{\mu\nu}^{(n)} V^{(n)\mu\nu} + \frac{1}{2} M_n^2 V_\mu^{(n)} \boxed{V^{(n)\mu}}}_{\text{Free part}} \right] \quad \text{Under the assumption of zero kinetic mixing}$$

$$+ \boxed{g_D} \sum_{a=e,\mu,\tau} Q_{L_\mu - L_\tau}^a \underbrace{\sum_n f_n V_\mu^{(n)} (\bar{l}^a \gamma^\mu l^a + \bar{\nu}_L^a \gamma^\mu \nu_L^a)}_{\text{Interaction part}}.$$

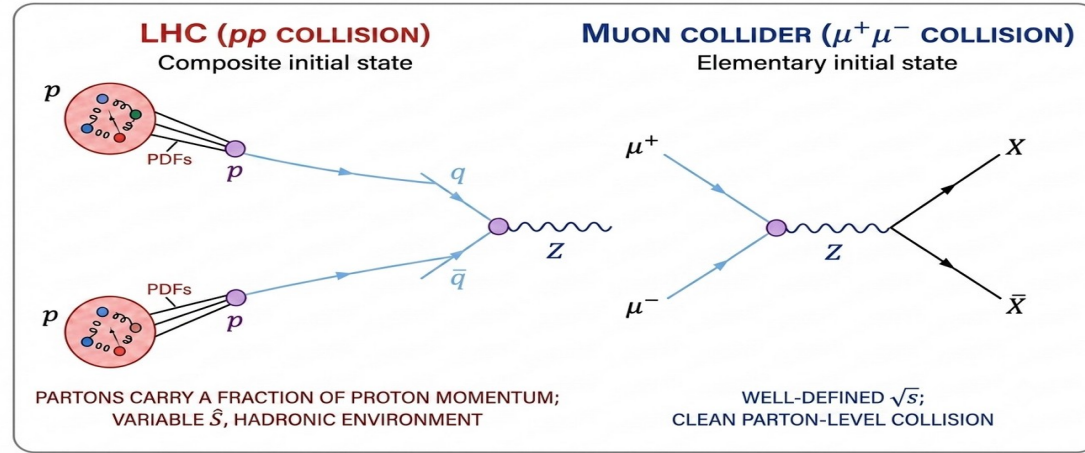
Effective 4D gauge coupling

Interaction of KK modes ==>

- With electron type; no interaction
- With muon type; [summed over KK tower]
- ✓ [proposed muon-based machines]
- With tau type; [summed over KK tower]

Extremely short τ lifetime → difficult to produce and manipulate τ beams!!!

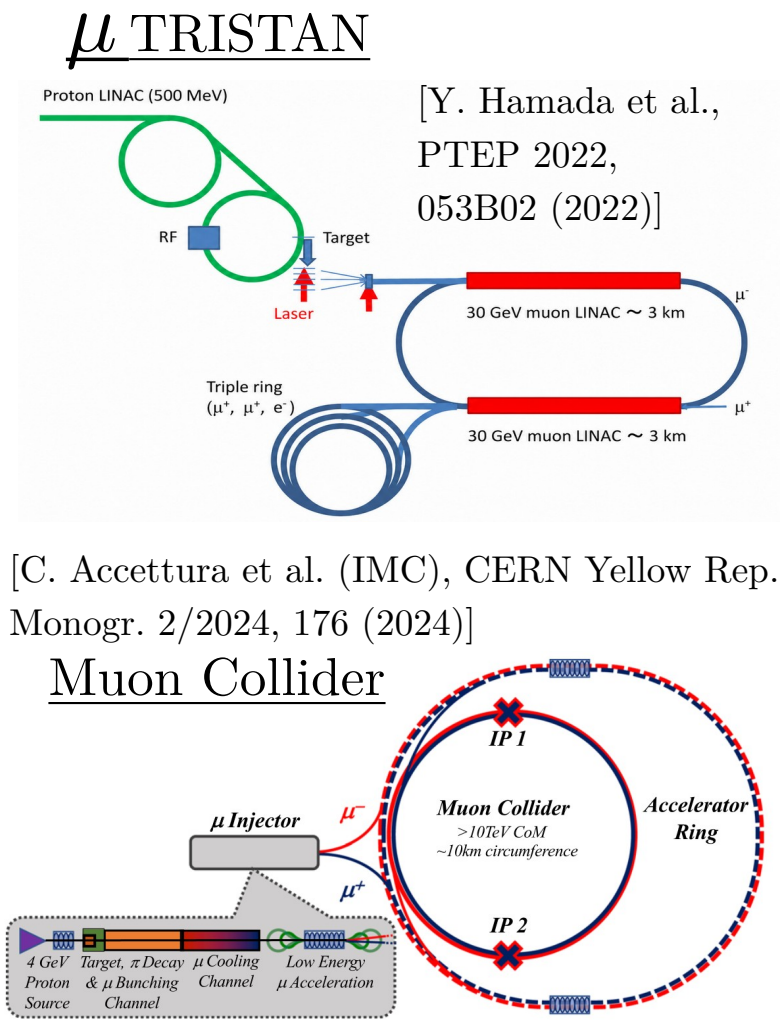
Why Muon-Based Colliders for Our Model?



- ✓ Can probe the gauge interaction **directly at tree level**
- ✓ Muons are much heavier than electrons thus **reduce the synchrotron radiation**
- ✓ Low-energy experiments (e.g. beam-dump exp.) can access only a **limited number of KK modes [upto $\mathcal{O}(10)$ GeV]**
- ✓ **KK enhancement in the signal ^{probe} $\Rightarrow$ both KK modes of mass MeV as well as TeV scale**

Future Muon-Based Colliders

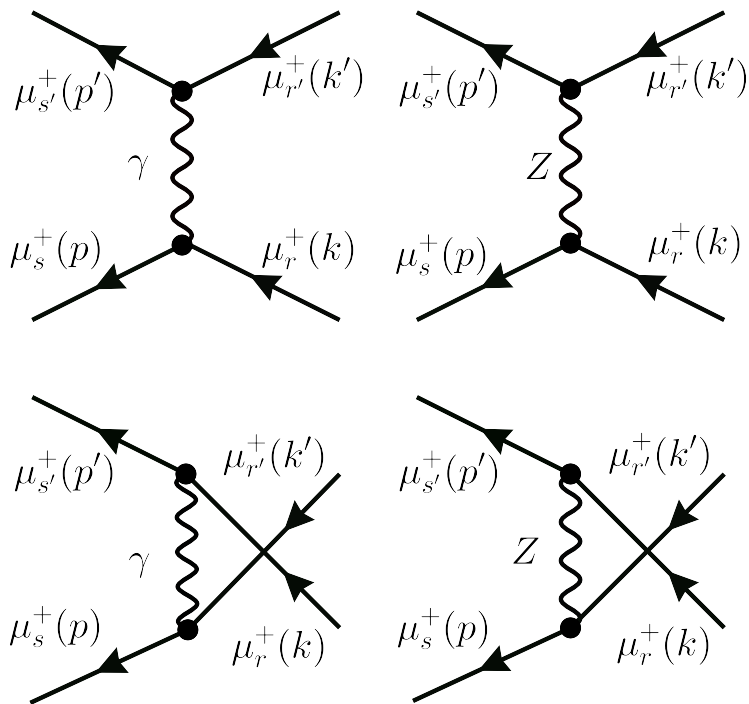
	μ TRISTAN	Muon Collider
Collision	$\mu^+ \mu^+$	$\mu^+ \mu^-$
$\sqrt{s}$	2-20 TeV	3 TeV
Integrated Luminosity	1, 10 ab ⁻¹	1, 10 ab ⁻¹
Key channel	2→2 scattering + associated KK production	Resonant KK production
Physics Role	Indirect + on-shell KK sensitivity	Direct s-channel KK resonances
Proposed Timeline	No concrete timeline yet	~2045 (possible start)



μ -TRISTAN Collider

Measuring $\mu^+ \mu^+ \rightarrow \mu^+ \mu^+$ deviation from Standard Model

Contribution from SM

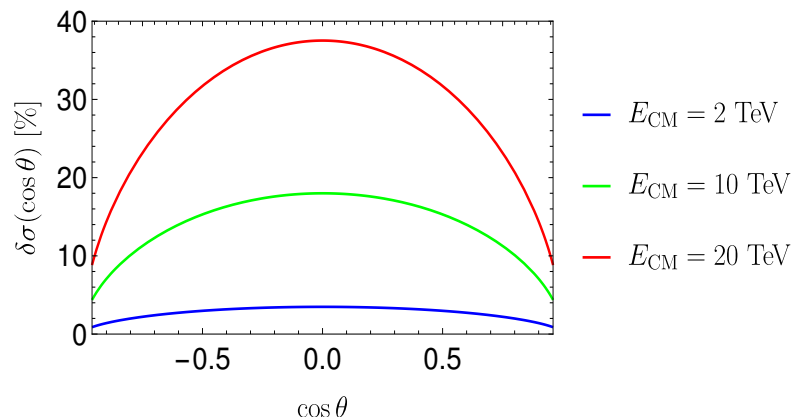


Deviation parameter,

$$\delta\sigma(\cos\theta) \equiv \frac{\text{Model} - \text{SM}}{\text{SM}} \times 100$$

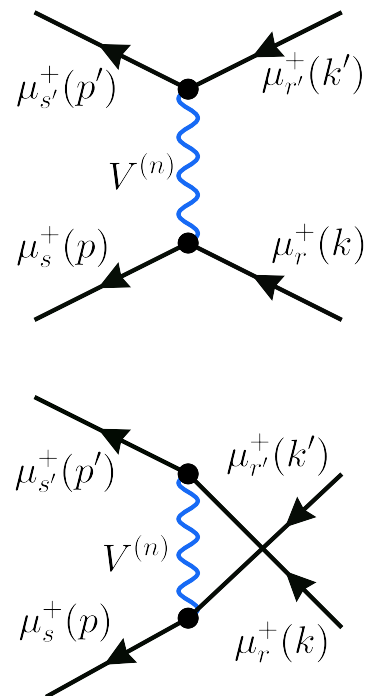
$\theta \rightarrow$ Scattering angle

$$m_{\text{KK}} = 500 \text{ MeV}, g_D = 10^{-3}, \tilde{y}_{\text{SM}} = \frac{\pi}{2}$$



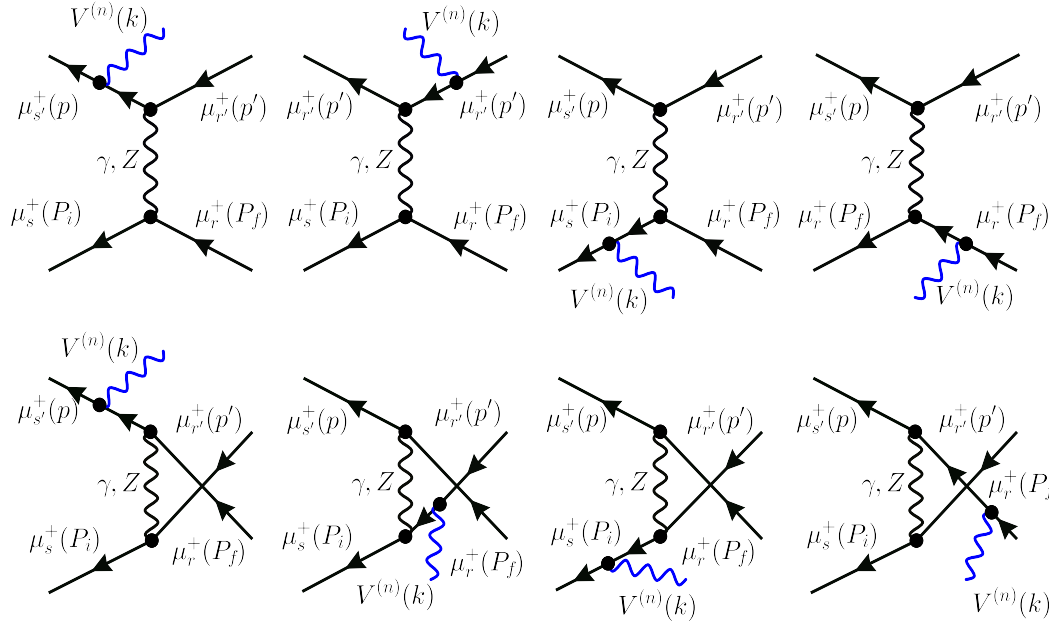
Clean muon interaction $\Rightarrow$ measure angular deviations in distributions

From KK tower



KK Bremsstrahlung in μ -TRISTAN Collider

Feynman diagrams for bremsstrahlung like processes:-



Production of the n th KK mode

$$\mu^+ \mu^+ \rightarrow \mu^+ \mu^+ V^{(n)}, \quad n = 1, 2, 3, \dots$$

Semi-visible final state

$$V^{(n)} \rightarrow \nu \bar{\nu}$$

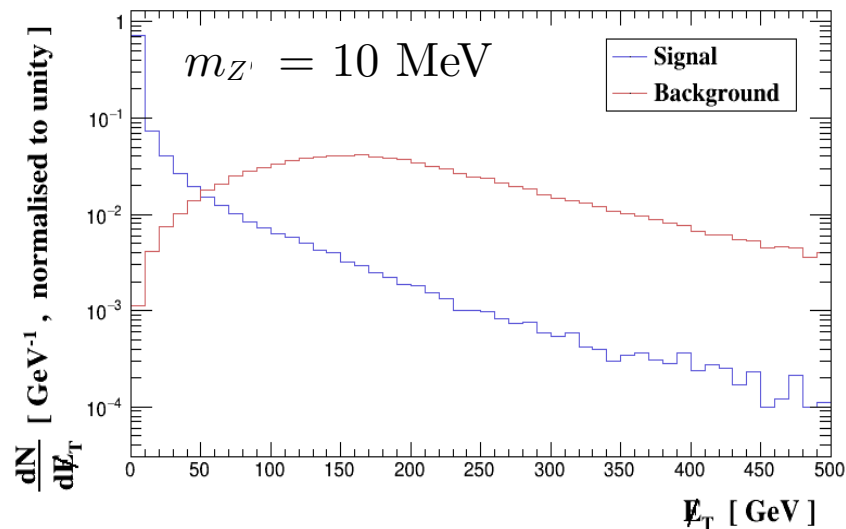
All-visible final state

$$V^{(n)} \rightarrow \mu^+ \mu^-$$

- ✓ A clean MET signature with reduced SM background
- ✓ The dimuon invariant-mass distribution can reconstruct the resonant KK modes

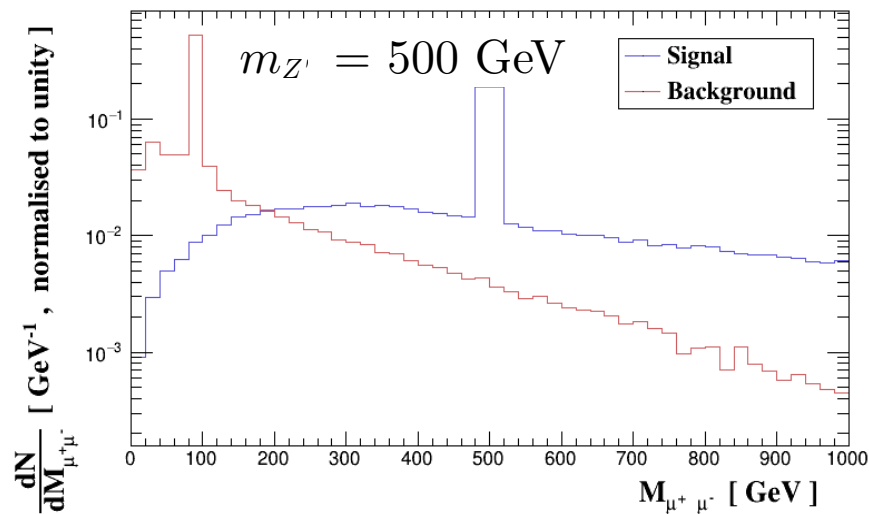
Event Distributions

Semi-visible final state: $[V^{(n)} \rightarrow \nu \bar{\nu}]$



$E_T \rightarrow$ Missing Transverse Energy (MET)

All-visible final state: $[V^{(n)} \rightarrow \mu^+ \mu^-]$



$M_{\mu^+ \mu^-} \rightarrow$ dimuon invariant mass

Following the MET distribution we choose the cut:

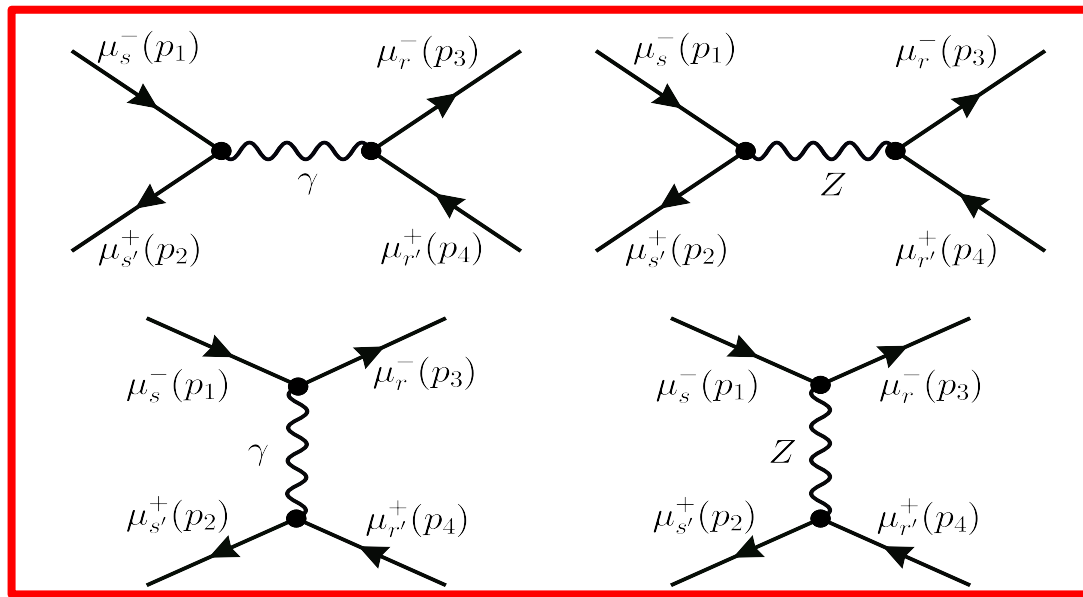
$$1 \text{ GeV} < \text{Reject MET} < 10 \text{ GeV}$$

Cut: Decided by seeing the invariant mass peak and optimising the sensitivity (not universal)

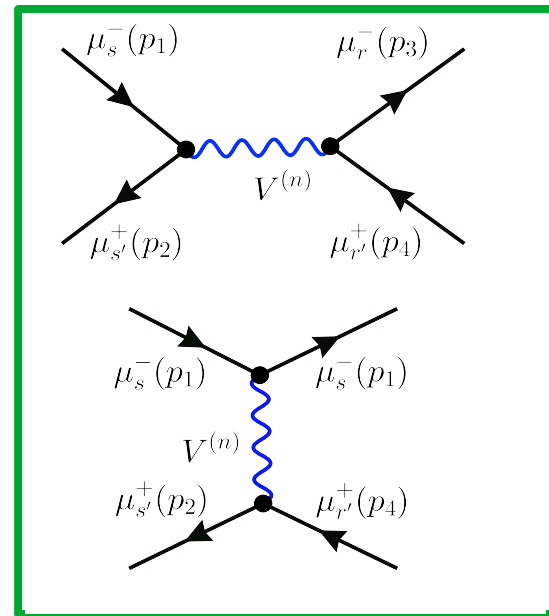
Resonant Production at Muon Collider

Resonant production including interference effect $\mu^+ \mu^- \rightarrow \mu^+ \mu^-$

SM contribution



Contribution from KK tower



KK mass --> hit the CM energy --> cross-section increases (via resonance)--> most dominant one!

CM energy distribution --> multiple KK modes from tower can hit res.--> multiple resonant peaks!

CM Energy Modelling

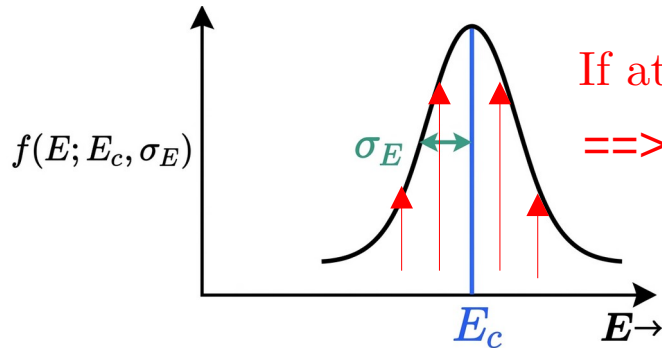
- Due to ISR, beamstrahlung, and synchrotron radiation, CM energy is not fixed
- A Gaussian profile models the energy distribution more realistic than a delta function

Gaussian Beam Energy Distribution:

$$f(E; E_c, \sigma_E) = \frac{1}{\sqrt{2\pi}\sigma_E} \exp \left[-\frac{(E - E_c)^2}{2\sigma_E^2} \right]$$

E_c : Central (peak) beam energy of CM energy

σ_E : Energy spread (e.g., $\sigma_E = 0.1\% \times E_c$) of CM energy



If atleast one KK mode lie within $5\sigma_E$ window:

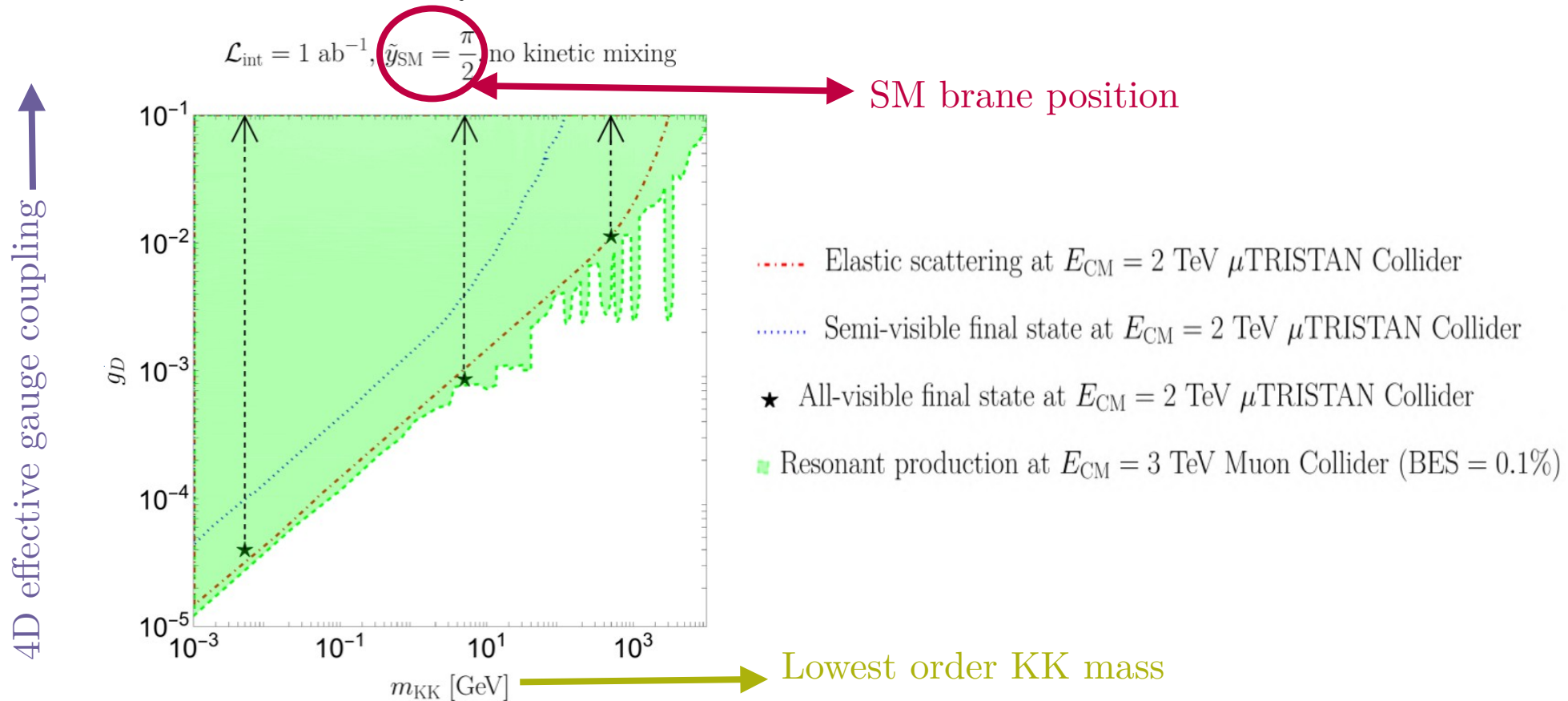
~~==> infinite KK tower sum~~ ==> summed over only those KK modes

If no KK mode lies within the window:

==> infinite KK tower sum [analytical form possible]

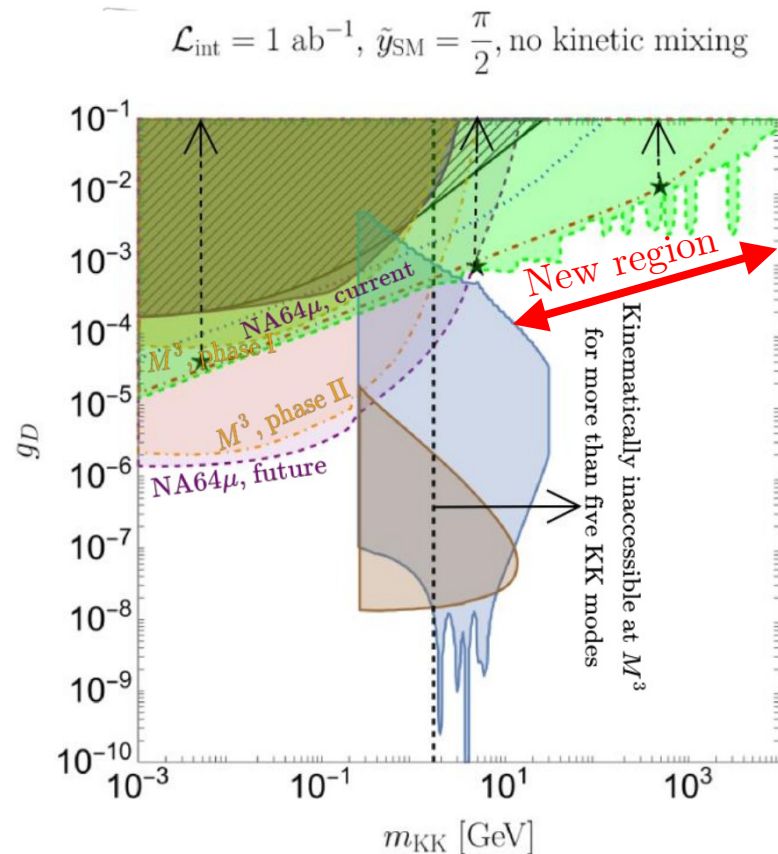
Comparison of the Results

Results from statistical analysis



- ▶ The above shaded regions are shown for future 2σ exclusion
- ▶ The four channels provide a very good complementary results to each other 13

Comparing Two Muon-Based Probes



- Elastic scattering at $E_{\text{CM}} = 2 \text{ TeV}$ $\mu\text{TRISTAN}$ Collider
- Semi-visible final state at $E_{\text{CM}} = 2 \text{ TeV}$ $\mu\text{TRISTAN}$ Collider
- ★ All-visible final state at $E_{\text{CM}} = 2 \text{ TeV}$ $\mu\text{TRISTAN}$ Collider
- Resonant production at $E_{\text{CM}} = 3 \text{ TeV}$ Muon Collider (BES = 0.1%)
- NA64 μ , current bound
- NA64 μ , future bound
- M^3 , phase I
- M^3 , phase II
- MuSIC
- Future muon beam dump
- Muon $g - 2$ (2σ exclusion)

This work!

D. Chakraborty, A. Chatterjee,
A. Kaushik, K. Nishiwaki,
Phys. Rev. D 113, 115038 (2026)

- ✓ Covers the parameter space beyond the reach of beam-dump experiments
- ✓ New region of parameter space explored with $10^{-3} \leq g_D \leq 10^{-1}$, above mass scale 10 GeV

Conclusion

- ✓ Muon-based colliders offer unique sensitivity to five-dimensional $U(1)_{L\mu-L\tau}$ extensions, providing complementary probes across **elastic, semi-visible, all-visible and resonant production**

Expected sensitivity (shown as 2σ exclusion reach):

Max reach :) $g_D \gtrsim 10^{-5}, \quad m_{KK} \sim 10^{-3} - 10^4 \text{ GeV}.$

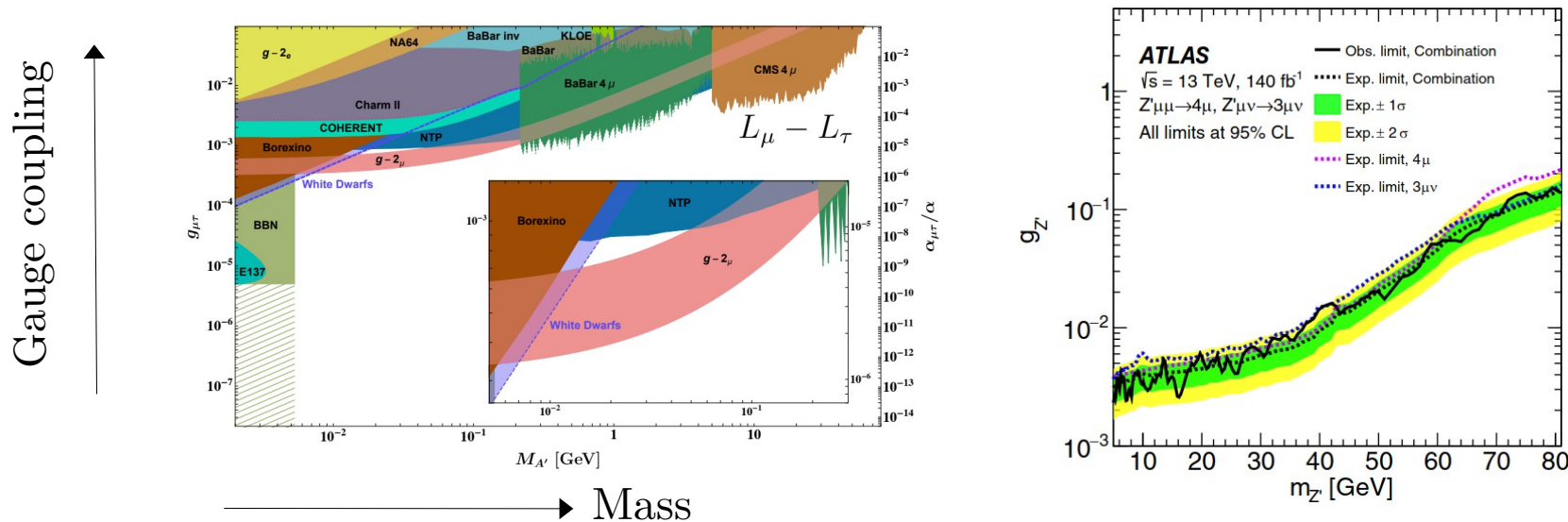
- ✓ By combining the four final state analysis, the muon-based collider setup covers a broad region of the 5D $U(1)_{L\mu-L\tau}$ parameter space
- ✓ Probes a **wide range of m_{KK} (MeV to TeV)** values even those are inaccessible to **low-energy or beam-dump experiments** due to kinematic and lifetime limitations

Thank you!

Backup Slides

Why Collider Phenomenology for Our Model?

- We can not constrain our model through the low-energy experiments
When the masses of the extra gauge bosons are above GeV scale
- High-energy collider experiments are essential to probe the higher mass region



Above plots tell that above GeV scale mass collider constraints are dominating

Hunting All the Hidden Photons, M. Bauer et al, JHEP 07 (2018) 094

ATLAS Collaboration, Phys. Rev. D 110, 072008 (2024)

The 5D action:

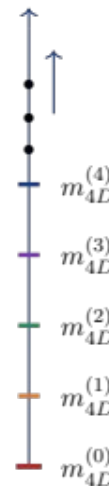
$$S_5 \supset \int d^4x \int_0^L dy \left[-\frac{1}{4} V_{AB} V^{AB} \right], \text{ where } A, B = \mu, 5 \quad [L \text{ is the compactification length}]$$

To obtain the 4D Lagrangian, we use the Kaluza-Klein (KK) decomposition:

$$V_\mu(x, y) = \frac{1}{\sqrt{L}} \sum_n V_\mu^{(n)}(x) f_A^n(y) \quad \leftarrow \text{Mode function}$$

After integrating out the extra spatial part we get effective 4D action

$$S_4 \supset \sum_n \int d^4x \left[-\frac{1}{4} V_{\mu\nu}^{(n)} V^{\mu\nu(n)} \right]$$



Infinite tower of KK modes

Infinite KK Tower in Flat Space Geometry

- The metric in flat space is,

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu - dy^2, \text{ where } \eta_{\mu\nu} = \text{diag}(+, -, -, -), y \in [0, \pi R]$$

- The mode functions take sine function form using (Neumann, Dirichlet) b.c.s

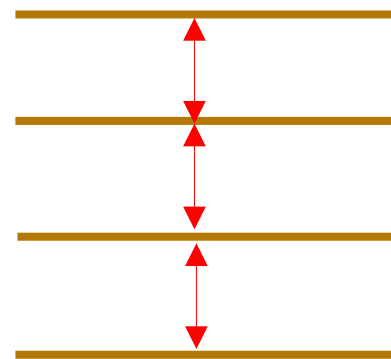
$$f_V^{(n)}(\tilde{y}_{\text{SM}}) = \sqrt{2} \cos \left[\left(n - \frac{1}{2} \right) \tilde{y}_{\text{SM}} \right], \quad \tilde{y}_{\text{SM}} \equiv \frac{y_{\text{SM}}}{R},$$

Mass of the nth KK mode:

$M_n = (2n - 1)m_{\text{KK}}$ where, $n = 1, 2, 3, \dots$ and m_{KK} is lowest order KK mass

$$\mathcal{S}(a, \tilde{y}_{\text{SM}}) = \sum_{n=1}^{\infty} \frac{\left[f_V^{(n)}(\tilde{y}_{\text{SM}}) \right]^2}{a - M_n^2}, \quad a = t, u \text{ Mandelstam variables}$$

$$S\left(a, \frac{\pi}{2}\right) = -\frac{\pi}{2m_{\text{KK}}\sqrt{a}} \tan\left(\frac{\pi\sqrt{a}}{4m_{\text{KK}}}\right)$$



Uniform KK splitting

The metric of the 5D model is given by, $ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$ where $\eta_{\mu\nu} = \text{diag}(+, -, -, -)$

5D Action is :
$$S_5 = \int d^4x \int_0^L dy \left[\sqrt{|g|} \left\{ -\frac{1}{4} g^{MN} g^{KL} \hat{V}_{MK} \hat{V}_{NL} \right\} \right. \\ \left. + \left(-\frac{1}{4} \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} + \frac{\epsilon_5}{2c_W} \hat{V}^{\mu\nu} \hat{B}_{\mu\nu} + \text{other part} \right) \delta(y - y_{SM}) \right] + S_{G.F}$$

Mode function:
$$f_{A,\text{warped}}^n(y) = N_n e^{ky} \left[J_1 \left(\frac{\sqrt{e^{2ky}} m_n}{k} \right) - \frac{J_0\left(\frac{m_n}{k}\right)}{Y_0\left(\frac{m_n}{k}\right)} Y_1 \left(\frac{\sqrt{e^{2ky}} m_n}{k} \right) \right]$$

Note: In this case, the normalization constant N_n and KK mass m_n are complex and determined numerically, considering boundary conditions.

The 4D Effective Lagrangian

Assumption of a vanishing kinetic mixing:

For nth KK mode, $\kappa_n \rightarrow 0$

$$\begin{aligned}
 \mathcal{L}_{\text{int}}^{\text{eff}} = & \sum_{a=e,\mu,\tau} \left\{ \bar{l}_R^a i \partial l_R^a + e \bar{l}_R^a \gamma^\mu l_R^a A_\mu + g_1 s_W \bar{l}_R^a \gamma^\mu l_R^a Z_\mu \right. \\
 & + \bar{l}_R^a \gamma^\mu \left\{ g_1 \sum_n \left(\frac{\kappa_n}{c_W} + s_W t_W \frac{\kappa_n m_Z^2}{M_n^2 - m_Z^2} \right) V_\mu^{(n)} + g_D Q_{L_\mu - L_\tau}^a \sum_n f_n \left(V_\mu^{(n)} - t_W \frac{\kappa_n m_Z^2}{M_n^2 - m_Z^2} Z_\mu \right) \right\} l_R^a \\
 & + \bar{l}_L^a i \partial l_L^a + \bar{\nu}_L^a i \partial \nu_L^a + \frac{g_2}{\sqrt{2}} (\bar{\nu}_L^a \gamma^\mu l_L^a W_\mu^+ + \bar{l}_L^a \gamma^\mu \nu_L^a W_\mu^-) \\
 & + \bar{l}_L^a \gamma^\mu \left[e A_\mu + \frac{1}{2} (-g_2 c_W + g_1 s_W) Z_\mu + \sum_n \left\{ (-g_2 c_W + g_1 s_W) t_W \frac{\kappa_n m_Z^2}{M_n^2 - m_Z^2} + g_1 \frac{\kappa_n}{c_W} \right\} V_\mu^{(n)} \right. \\
 & \left. + g_D Q_{L_\mu - L_\tau}^a \sum_n f_n \left(V_\mu^{(n)} - t_W \frac{\kappa_n m_Z^2}{M_n^2 - m_Z^2} Z_\mu \right) \right] l_L^a \\
 & + \bar{\nu}_L^a \gamma^\mu \left[\frac{1}{2} (g_2 c_W + g_1 s_W) Z_\mu + \sum_n \left\{ (g_2 c_W + g_1 s_W) t_W \frac{\kappa_n m_Z^2}{M_n^2 - m_Z^2} + g_1 \frac{\kappa_n}{c_W} \right\} V_\mu^{(n)} \right. \\
 & \left. + g_D Q_{L_\mu - L_\tau}^a \sum_n f_n \left(V_\mu^{(n)} - t_W \frac{\kappa_n m_Z^2}{M_n^2 - m_Z^2} Z_\mu \right) \right] \nu_L^a \left. \right\} + \mathcal{O}(\kappa_n^2).
 \end{aligned}$$

Photon

Z-boson

Nth KK mode

4D eff. gauge coup.

we define $c_W(s_W) := \cos \theta_W(\sin \theta_W)$ and $t_W := \tan \theta_W$, where θ_W is the weak (Weinberg) mixing angle

Effective Lagrangian in the **mass eigenbasis**:

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4} \sum_n V_{\mu\nu}^{(n)} V^{(n)\mu\nu} - \frac{1}{4} A_{\mu\nu} A^{\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} \\ & + \frac{1}{2} m_Z^2 Z_\mu Z^\mu + \frac{1}{2} \sum_n m_n^2 V_\mu^{(n)} V^{(n)\mu} + \text{Other part}\end{aligned}$$

Feynman Rules:

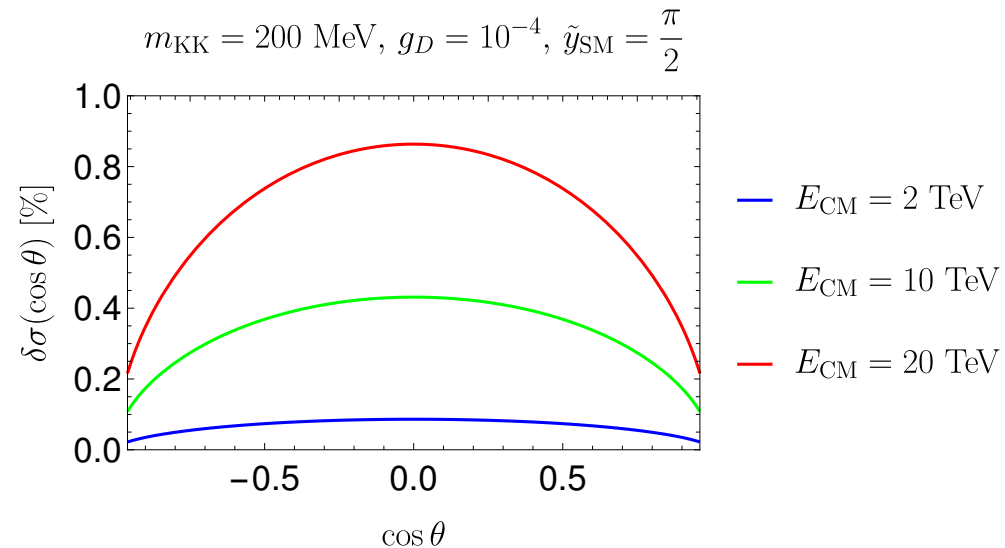
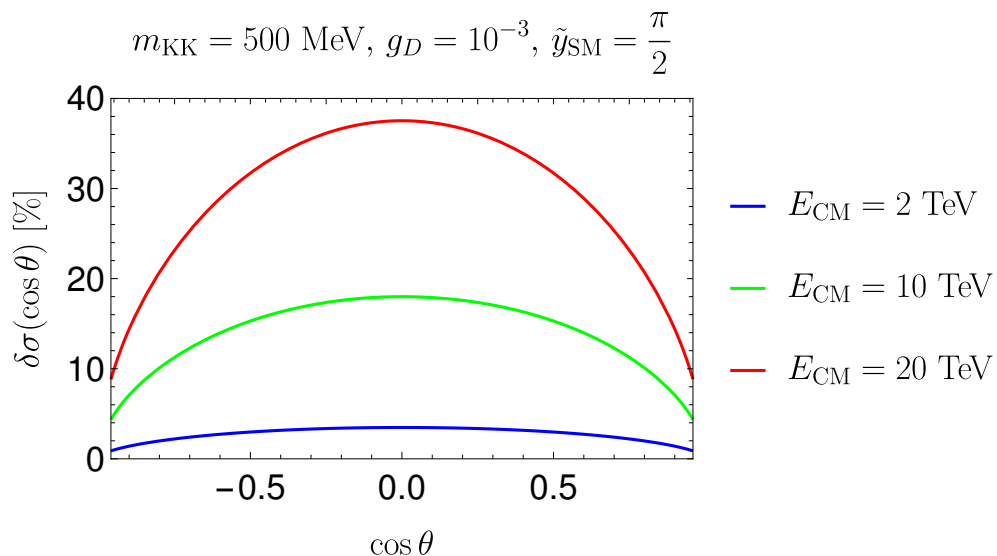
$$\begin{aligned}\Gamma_A^\mu &= ie\gamma^\mu, \\ \Gamma_W^\mu &= i\gamma^\mu \frac{g_2}{\sqrt{2}} P_L, \\ \Gamma_{\nu_\alpha Z}^\mu &= i\gamma^\mu \left[\frac{1}{2} (g_2 c_W + g_1 s_W) - g' Q_{L_\mu - L_\tau}^{\nu_\alpha} t_W \sum_n \frac{f_A^n(y_{SM}) \epsilon_n m_Z^2}{m_n^2 - m_Z^2} \right] P_L\end{aligned}$$

Other vertices

$$\begin{aligned}
 \Gamma_{l_\alpha Z}^\mu &= i\gamma^\mu \left\{ P_R \left[g_1 s_W - g' Q_{L_\mu - L_\tau}^{l_\alpha} t_W \sum_n \frac{f_A^n(y_{SM}) \epsilon_n m_Z^2}{m_n^2 - m_Z^2} \right] \right. \\
 &\quad \left. + P_L \left[\frac{1}{2} (-g_2 c_W + g_1 s_W) - g' Q_{L_\mu - L_\tau}^{l_\alpha} t_W \sum_n \frac{f_A^n(y_{SM}) \epsilon_n m_Z^2}{m_n^2 - m_Z^2} \right] \right\}, \\
 \Gamma_{\nu_\alpha V(n)}^\mu &= i\gamma^\mu \left[\frac{1}{2} \left((g_2 c_W + g_1 s_W) t_W \frac{\epsilon_n m_Z^2}{m_n^2 - m_Z^2} + g_1 \frac{\epsilon_n}{c_W} \right) \right. \\
 &\quad \left. + g' f_A^n(y_{SM}) Q_{L_\mu - L_\tau}^{\nu_\alpha} \right] P_L, \\
 \Gamma_{l_\alpha V(n)}^\mu &= i\gamma^\mu \left\{ P_R \left[g_1 \left(\frac{\epsilon_n}{c_W} + s_W t_W \frac{\epsilon_n m_Z^2}{m_n^2 - m_Z^2} \right) + g' f_A^n(y_{SM}) Q_{L_\mu - L_\tau}^{l_\alpha} \right] \right. \\
 &\quad \left. + P_L \left[\frac{1}{2} \left((g_2 c_W + g_1 s_W) t_W \frac{\epsilon_n m_Z^2}{m_n^2 - m_Z^2} + g_1 \frac{\epsilon_n}{c_W} \right) + g' f_A^n(y_{SM}) Q_{L_\mu - L_\tau}^{l_\alpha} \right] \right\}
 \end{aligned}$$

KK mode induced deviations

- The deviation parameter, $\delta\sigma(\cos\theta) \equiv \frac{\text{Model} - \text{SM}}{\text{SM}} \times 100$
- Where, θ is the scattering angle



Note:

- KK modes distort the angular distributions from SM
- Hence hint a new physics indirectly

Goal: Quantify BSM sensitivity via angular deviations in $\mu^+\mu^- \rightarrow \mu^+\mu^-$.

- Divide angular range: $\cos\theta \in [\cos(164^\circ), \cos(16^\circ)]$ into 10 uniform bins.
- Apply binned χ^2 statistic:

$$\chi^2 = \sum_{i=1}^{10} \frac{(N_i^{\text{BSM}})^2}{N_i^{\text{SM}} + (\epsilon_i N_i^{\text{SM}})^2}$$

- $N_i^{\text{BSM}} = \mathcal{L}_{\text{int}} \int_{\text{bin } i} \left(\frac{d\sigma^{\text{total}}}{d\cos\theta} - \frac{d\sigma^{\text{SM}}}{d\cos\theta} \right)$
- $N_i^{\text{SM}} = \mathcal{L}_{\text{int}} \int_{\text{bin } i} \frac{d\sigma^{\text{SM}}}{d\cos\theta}$

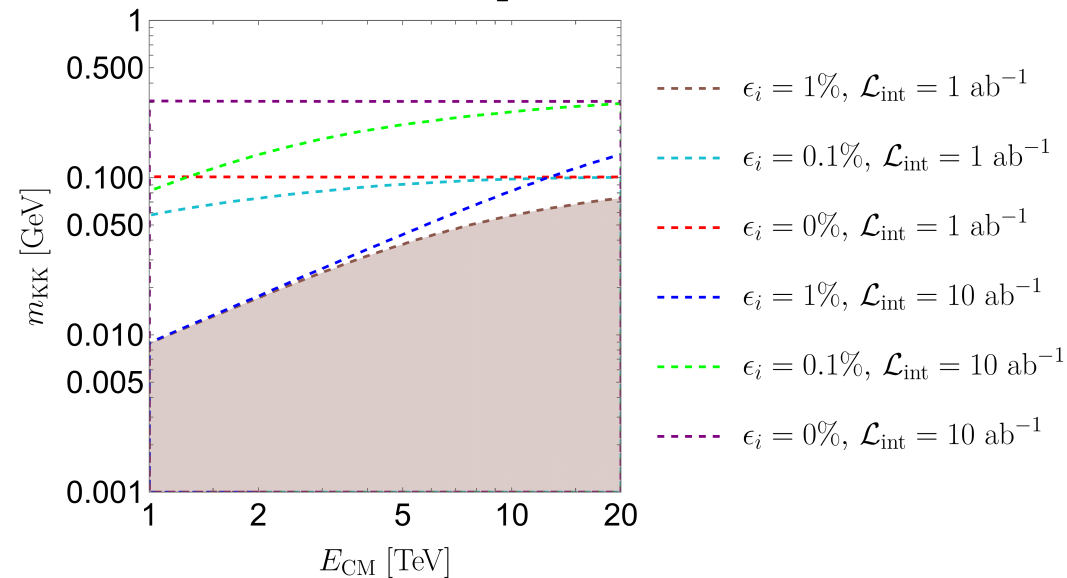
Parameters:

- Integrated luminosity: $\mathcal{L}_{\text{int}} = 1 \text{ ab}^{-1}$ and 10 ab^{-1}
- Systematics: $\epsilon_i = 0, 0.1\%, 1\%$

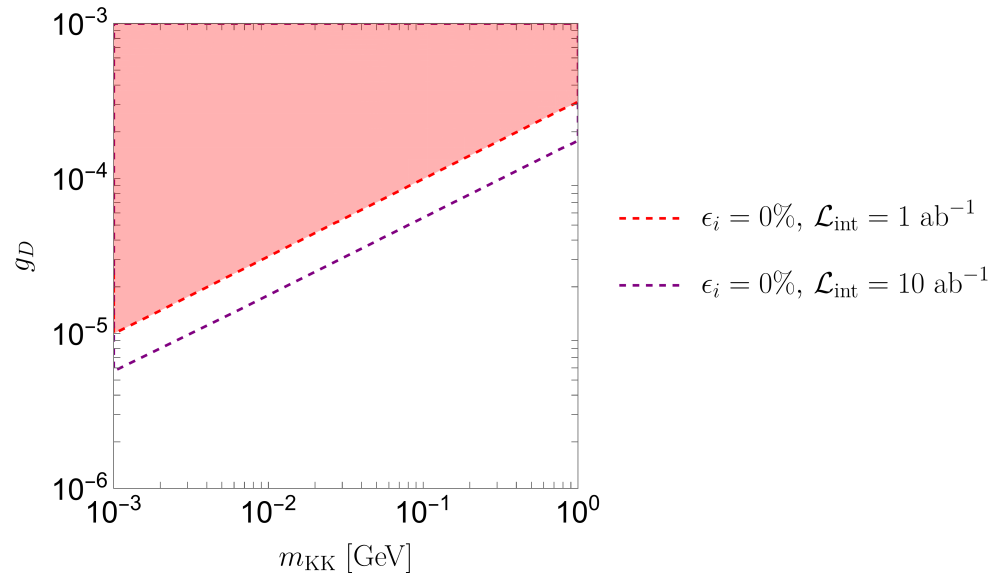
Exclusion Threshold (95% CL, 2 d.o.f.): $\chi^2 > 6.18$

Results from Elastic Scattering

$$g_D = 10^{-4}, \tilde{y}_{\text{SM}} = \frac{\pi}{2}$$



$$E_{\text{CM}} = 2 \text{ TeV}, \tilde{y}_{\text{SM}} = \frac{\pi}{2}$$



Comments:

- ▶ The above shaded regions are future 2σ exclusion achieved after a χ^2 analysis
- ▶ ϵ_i is systematic uncertainty, E_{CM} and $\mathcal{L}_{\text{int}}$ is the centre of mass energy and integrated luminosity of the collider respectively.

Goal: Incorporate the full tower of KK gauge bosons to compute the BSM signal rate.

- The total signal cross section is approximated by a continuum integral:

$$\sigma_{\text{signal}} = \int_{m=m_{\text{KK}}}^{1 \text{ TeV}} \frac{\sigma_{\text{eff}}(m)}{2m_{\text{KK}}} dm$$

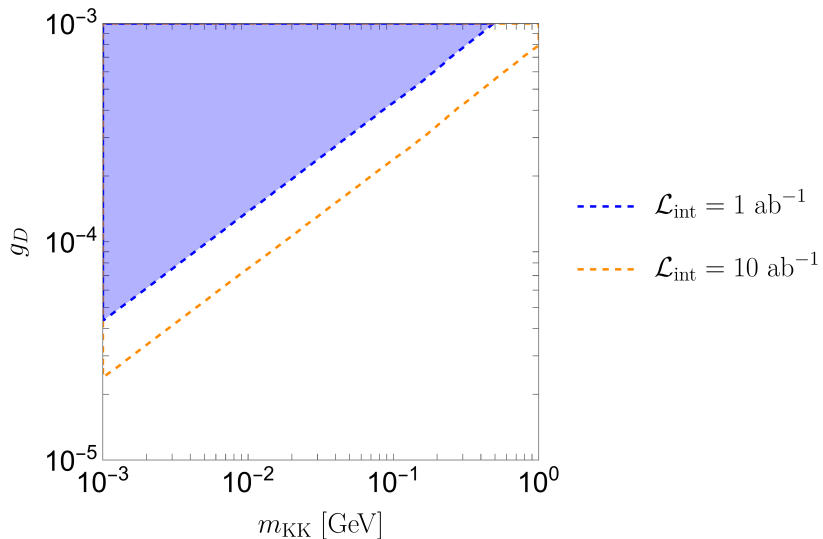
- Components of the integrand:
 - $\sigma_{\text{eff}}(m) = \sigma_n(m) \cdot \epsilon_n(m)$: interpolated product of cross section and efficiency,
 - $\sigma_n \propto g_D^2$: mode-wise signal scales with the squared coupling,
 - $\frac{1}{2m_{\text{KK}}}$: density of KK states in the flat extra-dimensional setup.
- Integration performed numerically using simulated data grids.

Comment: The KK tower sum effectively resums multiple narrow resonances—crucial for accurately capturing the signal yield.

Results

MET Channel

$$E_{\text{CM}} = 2 \text{ TeV}, \tilde{y}_{\text{SM}} = \frac{\pi}{2}$$



Dimuon Channel

BP	$m_{\text{KK}} [\text{GeV}]$	g_D	Kinematic window [GeV]	Significance [σ]
BP1	5×10^{-3}	4.01222×10^{-5}	$M_{\mu^+\mu^-} \in [1, 9]$	2
BP2	5	8.6474×10^{-4}	$M_{\mu^+\mu^-} \in [4.5, 5.5]$	2
BP3	500	1.60083×10^{-2}	$M_{\mu^+\mu^-} \in [499.5, 500.5]$	2

- Shaded regions and the table of three benchmark points are future 2σ exclusion

Resonant Mode Selection:

$$\Delta E = 3\sigma_E, \quad m_n = (2n - 1)m_{\text{KK}}, \quad |m_n - \sqrt{s}| \leq \Delta E$$

$$n \in [n_{\min}, n_{\max}]$$

Resonant Sum (Breit-Wigner):

$$\sum_{n=n_{\min}}^{n_{\max}} \frac{f_A^n(\pi/2)^2}{s - m_n^2 + im_n\Gamma_n}, \quad f_A^n(\pi/2) = \sqrt{2} \cos \left[\frac{(n - \frac{1}{2})\pi}{2} \right]$$

Captures resonance enhancement near on-shell KK modes accurately.

Off-shell Regime:

- Applies when no KK mode lies within the resonance window:

$$|m_n - \sqrt{s}| > \Delta E \quad \forall n$$

- Infinite sum admits analytic continuation:

$$\sum_{n=1}^{\infty} \frac{f_A^n(\pi/2)^2}{s - m_n^2} = -\frac{\pi}{4\sqrt{s}m_{\text{KK}}} \tan\left(\frac{\pi\sqrt{s}}{2m_{\text{KK}}}\right)$$

Effective Cross Section:

- Realistic collider beams have finite energy spread:

$$\sigma_{\text{eff}} = \int (f(E, E_c, \sigma_E))^2 \sigma(4E^2) dE$$

- $f(E, E_c, \sigma_E)$: normalized Gaussian centered at E_c , with width σ_E

This hybrid treatment ensures accurate modelling across both resonant and non-resonant parameter regions.

Goal: Quantify BSM sensitivity via angular deviations in $\mu^+\mu^- \rightarrow \mu^+\mu^-$.

- Divide angular range: $\cos\theta \in [\cos(164^\circ), \cos(16^\circ)]$ into 10 uniform bins.
- Apply binned χ^2 statistic:

$$\chi^2 = \sum_{i=1}^{10} \frac{(N_i^{\text{BSM}})^2}{N_i^{\text{SM}} + (\epsilon_i N_i^{\text{SM}})^2}$$

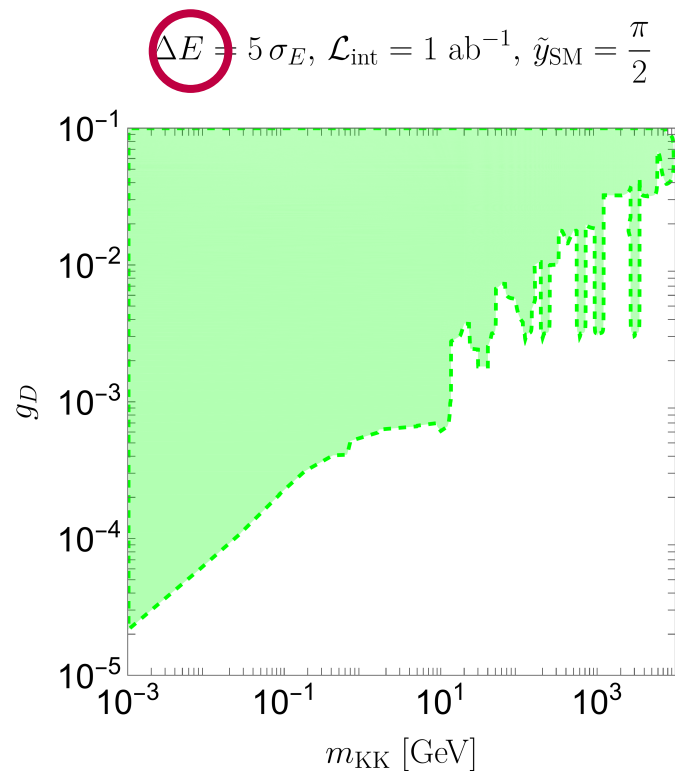
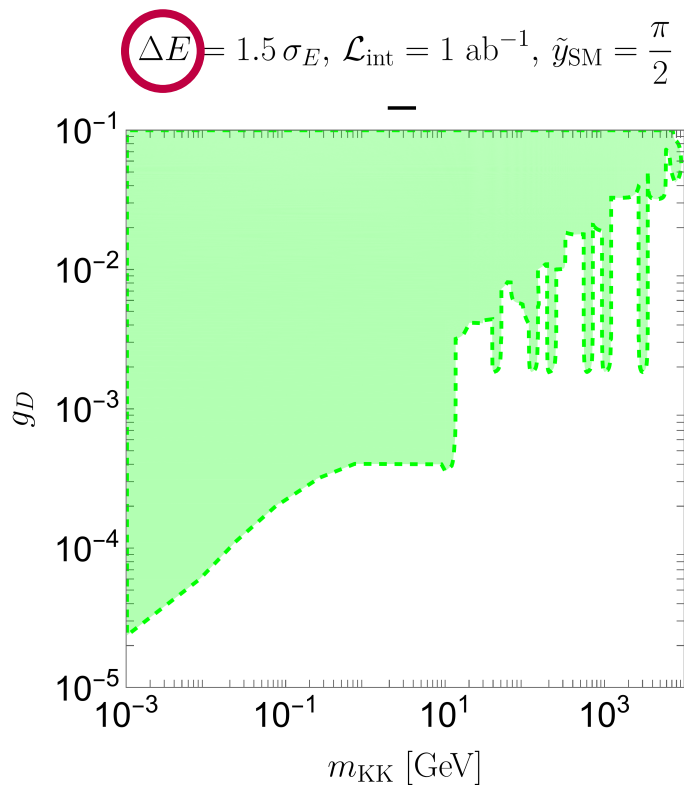
- $N_i^{\text{BSM}} = \mathcal{L}_{\text{int}} \int_{\text{bin } i} \left(\frac{d\sigma^{\text{total}}}{d\cos\theta} - \frac{d\sigma^{\text{SM}}}{d\cos\theta} \right)$
- $N_i^{\text{SM}} = \mathcal{L}_{\text{int}} \int_{\text{bin } i} \frac{d\sigma^{\text{SM}}}{d\cos\theta}$

Parameters:

- Integrated luminosity: $\mathcal{L}_{\text{int}} = 1 \text{ ab}^{-1}$ and 10 ab^{-1}
- Systematics: $\epsilon_i = 0, 0.1\%, 1\%$

Exclusion Threshold (95% CL, 2 d.o.f.): $\chi^2 > 6.18$

Results from Resonant Production



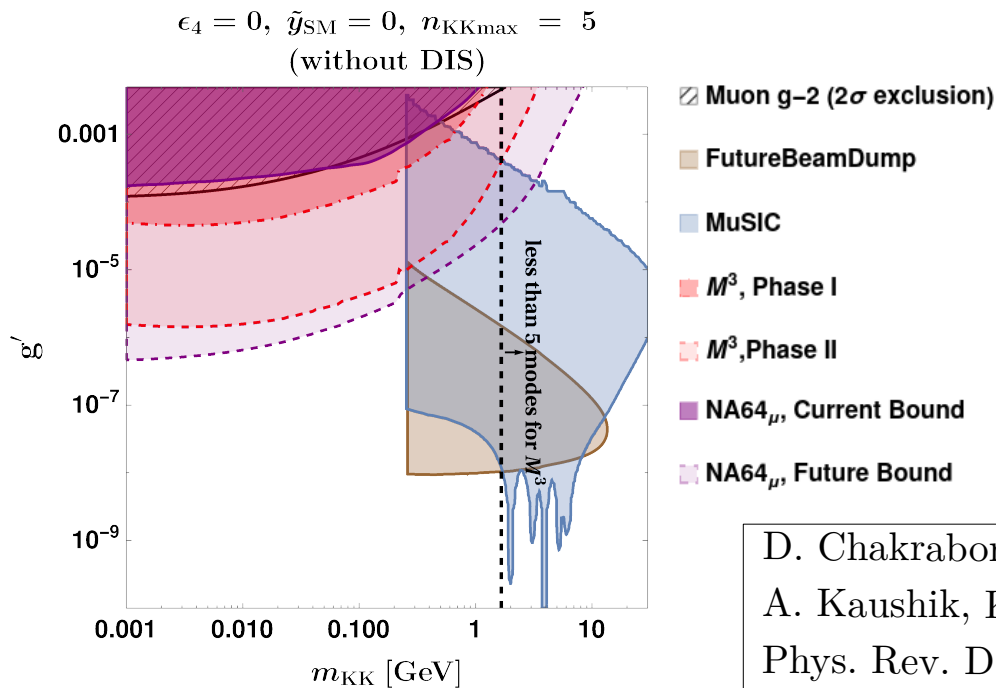
Comments:

- The above shown regions are future 2σ exclusion
- For two different energy peak window (ΔE) results are almost similar

Why High Energy Muon-Based Colliders?

➤ Can probe the gauge interaction directly at tree level **without relying on kinetic mixing**

- m_{KK} : lowest KK mode mass
- g' : effective 4D gauge coupling
- $\tilde{y}_{\text{SM}}$: SM brane position
- ϵ_4 : 4D kinetic mixing
- n_{KKmax} : maximum number of KK modes



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➤ Low-energy experiments (e.g. beam-dump exp.) can access only a **limited number of KK modes**

➤ **Due to KK enhancement in the signal, can probe both KK modes of mass MeV as well as TeV**

Summary of Constraints from 4D $U(1)_{L\mu-L\tau}$ Model

