# Learning the latent structure of background distributions

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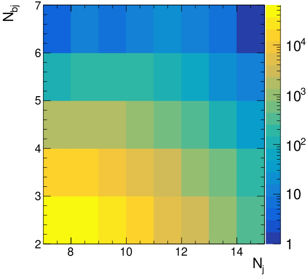
**How can we inject more information from known physics (multiple MC samples) into a data-driven inference pipeline?**

# A practical example: $pp \to t\bar{t}jj$

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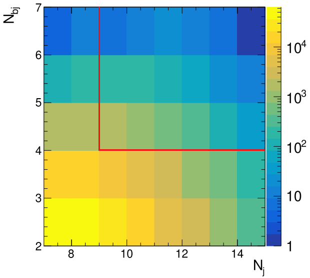


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Data-driven background estimation:

- The expected signal lies in a specific region (SR) in the space of 2 observables
- The **correlation** between observables is small

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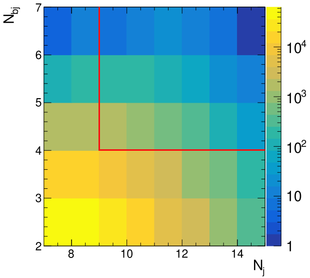


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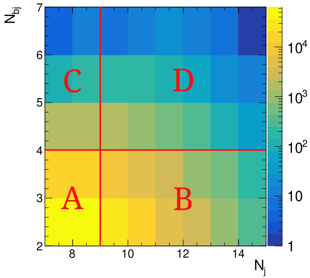


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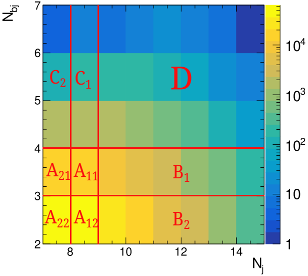


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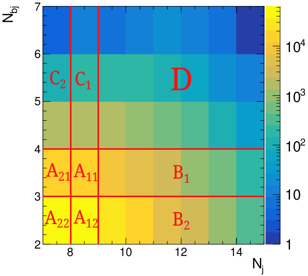


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| Extrapolation method | Prediction ($\hat{F}_D$) | $\hat{F}_D / F_D$ |
|---|---|---|
| ABCD (Eq. 12) | $3333 \pm 77$ | $0.684 \pm 0.015$ |
| Ext. ABCD (Eq. 14) | $4149 \pm 132$ | $0.851 \pm 0.027$ |
| Ext. ABCD (Eq. 15) | $4352 \pm 271$ | $0.893 \pm 0.056$ |
| Ext. ABCD (Eq. 16) | $4247 \pm 217$ | $0.871 \pm 0.045$ |

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- **Co-ocurrence** of words tend to group them by meaning $\Rightarrow$ **correlations ?**

# Using LDA in $t\bar{t}jj$

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| Corpus | $\Rightarrow$ | Set of simulations |
| Document | $\Rightarrow$ | Simulation with a given setup |
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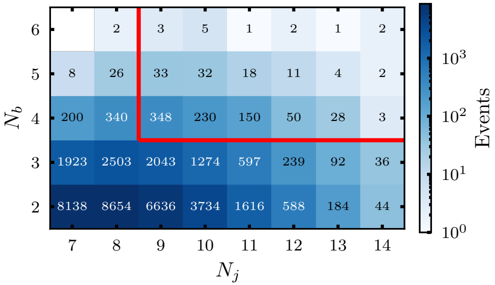
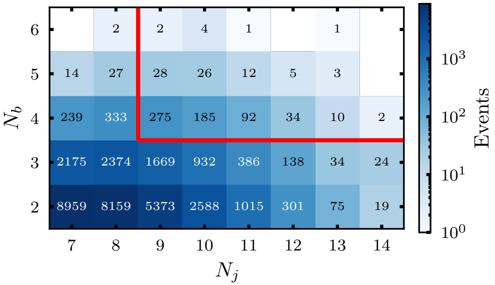
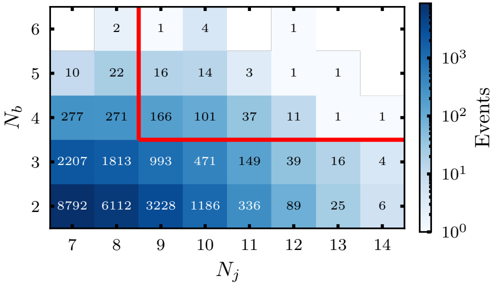
- Learn topics as underlying correlation structures between observations
- Dimensionality reduction
- Generate 70 simulations of $pp \to t\bar{t}jj$ varying factorization scale and $p_T^j$ cut

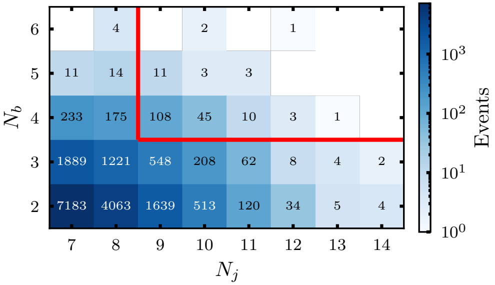
1. Learn the topics distributions from simulations
2. Once the topics are known, learn their mixing fractions in the target dataset

# Simulations of $pp \to t\bar{t}jj$



# Bayesian inference

Observations $\mathbf{W} = \{w_{dn}\}$, parameters of interest $\boldsymbol{\Theta} = \{\vec{\theta}^{(d)}\}$, $\boldsymbol{\Phi} = \{\vec{\varphi}^{(k)}\}$, probabilistic model

$$p\left(\mathbf{W} \,\middle|\, \boldsymbol{\Theta}, \boldsymbol{\Phi}\right)$$

Bayes' theorem allows us to find the **posterior** distribution,

$$\log p\left(\boldsymbol{\Theta}, \boldsymbol{\Phi} \,\middle|\, \mathbf{W}\right) = \log p\left(\mathbf{W} \,\middle|\, \boldsymbol{\Theta}, \boldsymbol{\Phi}\right) + \log p(\boldsymbol{\Theta}, \boldsymbol{\Phi}) - C$$

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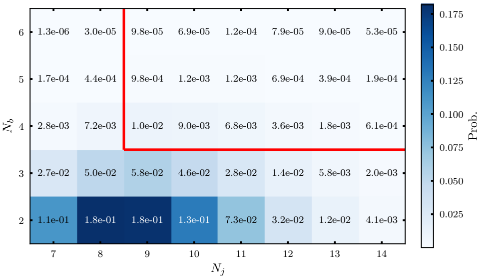
*Hamiltonian Monte Carlo* (HMC) to sample the posterior

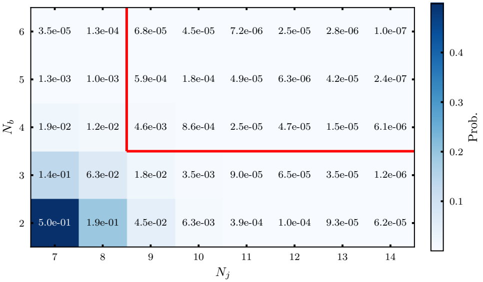
- Mapping the inference problem to a Hamiltonian dynamical system
- Parameters to infer $\longleftrightarrow$ generalized coordinates
- Auxiliary momenta $\boldsymbol{\rho}$

$$H(\boldsymbol{\rho}, \boldsymbol{\Theta}, \boldsymbol{\Phi}) = -\log p\left(\boldsymbol{\rho} \,\middle|\, \boldsymbol{\Theta}, \boldsymbol{\Phi}\right) - \log p\left(\boldsymbol{\Theta}, \boldsymbol{\Phi} \,\middle|\, \mathbf{W}\right) = T + V$$

# Topics ($K = 3$)



6 1.3e-06 3.0e-05 9.8e-05 6.9e-05 1.2e-04 7.9e-05 9.0e-05 5.3e-05
5 1.7e-04 4.4e-04 9.8e-04 1.2e-03 1.2e-03 6.9e-04 3.9e-04 1.9e-04
4 2.8e-03 7.2e-03 1.0e-02 9.0e-03 6.8e-03 3.6e-03 1.8e-03 6.1e-04
3 2.7e-02 5.0e-02 5.8e-02 4.6e-02 2.8e-02 1.4e-02 5.8e-03 2.0e-03
2 1.1e-01 1.8e-01 1.8e-01 1.3e-01 7.3e-02 3.2e-02 1.2e-02 4.1e-03
7 8 9 10 11 12 13 14
$N_b$
$N_j$
0.175
0.150
0.125
0.100
0.075
0.050
0.025
Prob.

6 3.5e-05 1.3e-04 6.8e-05 4.5e-05 7.2e-06 2.5e-05 2.8e-06 1.0e-07
5 1.3e-03 1.0e-03 5.9e-04 1.8e-04 4.9e-05 6.3e-06 4.2e-05 2.4e-07
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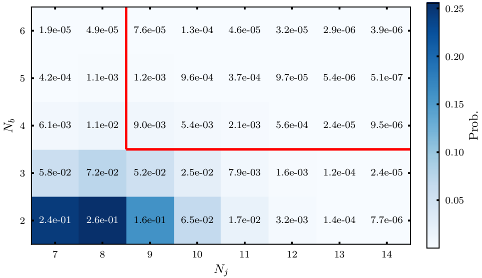

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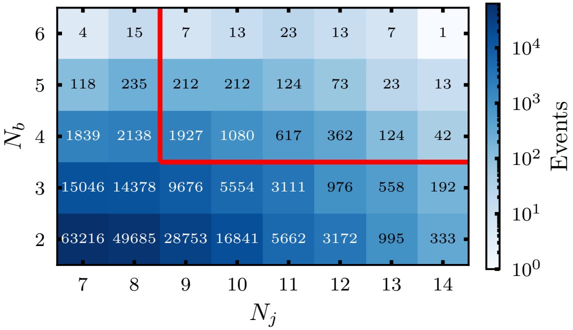
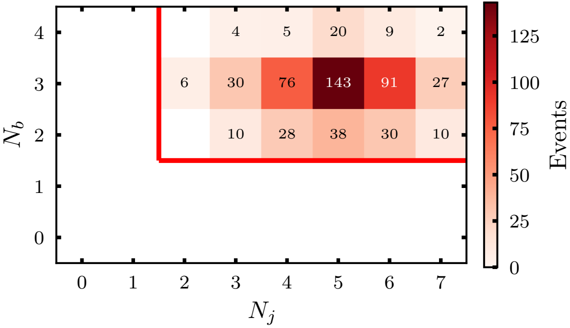
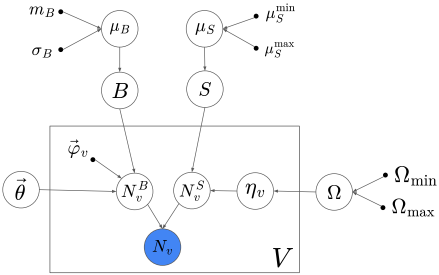
# Adding signal to target data



# Adding signal to the model

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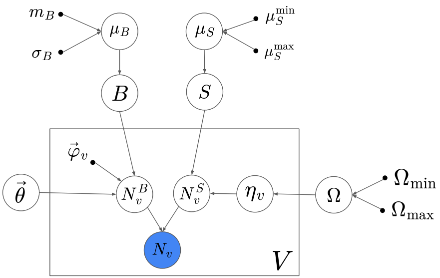


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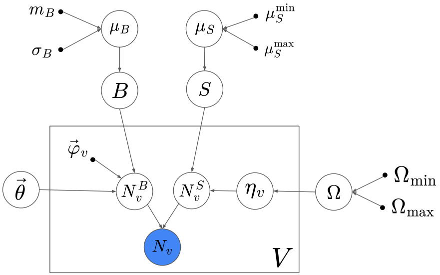


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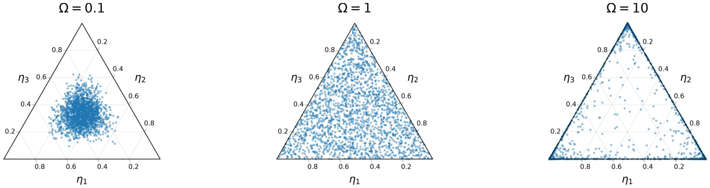
$$\{\eta_v\}_{v \in \text{SR}} \sim \text{Dir}\left(\frac{1}{\Omega}, \ldots, \frac{1}{\Omega}\right)$$



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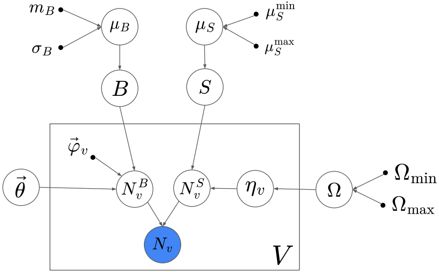


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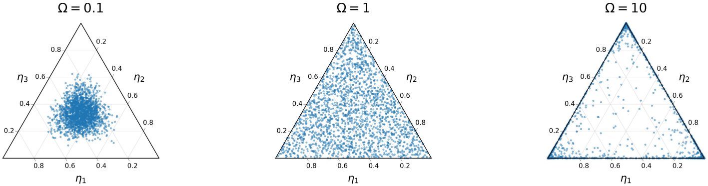
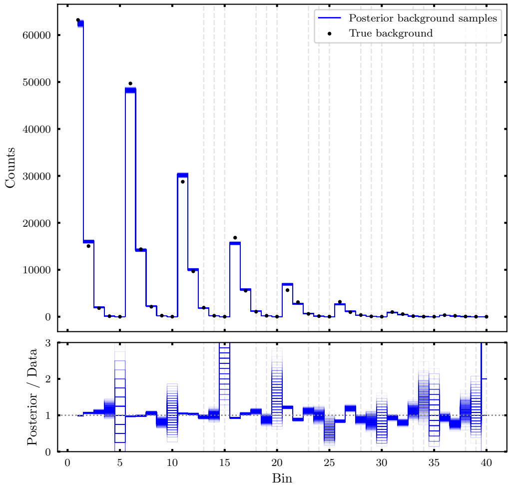
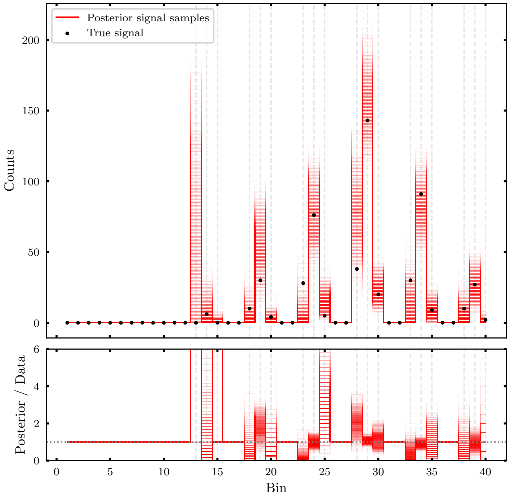



- Given the topics $\{\vec{\varphi}_v\}$, obtain posterior distributions of $\mu_B$, $\mu_S$, $\vec{\theta}$ and $\Omega$

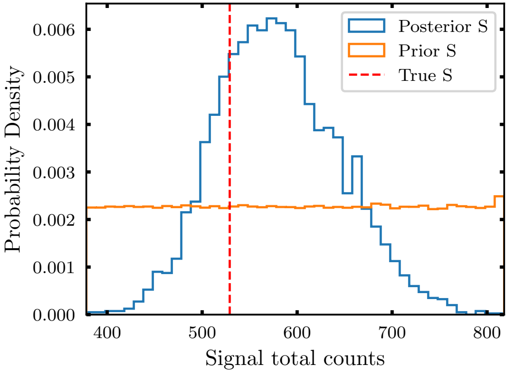
# Preliminary results

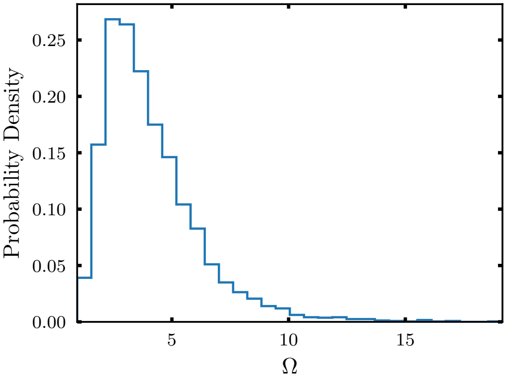
Sample new data from posterior distributions and compare with the training data



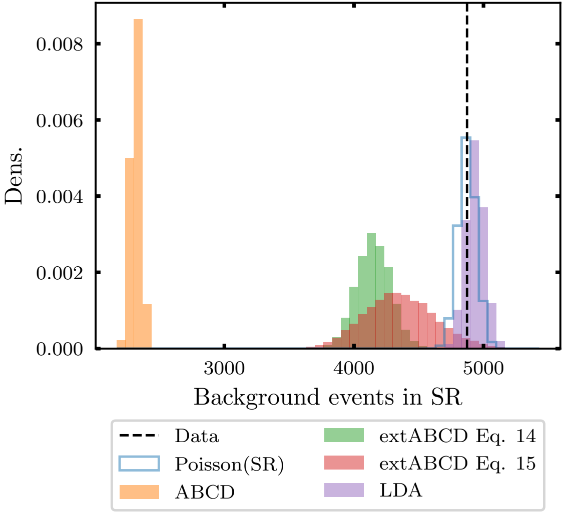



# Preliminary results



# Comparing with ABCDs



# Coverage of signal

Given the posterior of the number of signal events ($S$), the **smallest credible interval**

$$\mathrm{SCI}_{\alpha\%} = \underset{s_{\min}, s_{\max}}{\arg\min}\,(s_{\max} - s_{\min}) = [s_{\min}, s_{\max}], \qquad \int_{s_{\min}}^{s_{\max}} p(S|\mathbf{d})\,dS = \alpha$$

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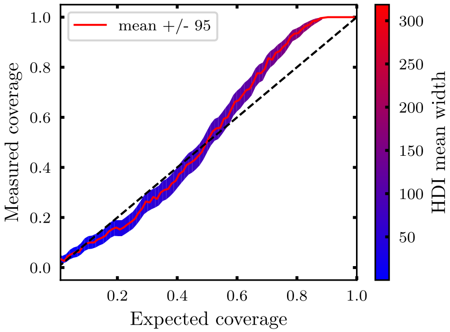


- Repeat the process for 500 independent signal samples
- Compute the fraction of times that the $\mathrm{SCI}_{\alpha\%}$ contains the true value of $S$
- Compare the measured coverage with the nominal $\alpha\%$

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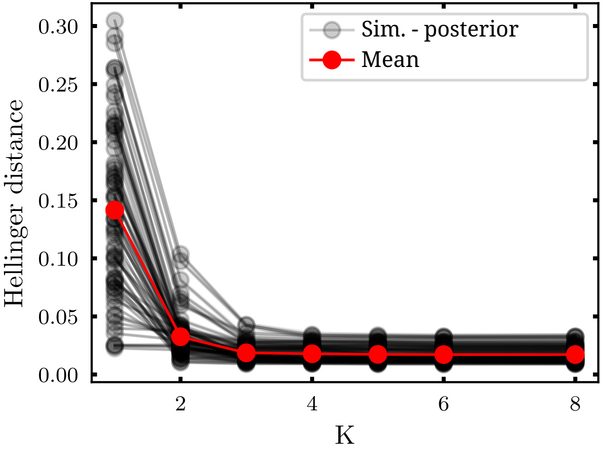
# Backup

# Choosing the number of topics

For each simulation, Hellinger distance from posterior to true

$$H(P,Q) = \frac{1}{\sqrt{2}}\sqrt{\sum_{v=1}^{V}\left(\sqrt{p_v} - \sqrt{q_v}\right)^2}$$



# Best topic mixture allowed by priors