UNIVERSITÄT
HEIDELBERG
ZUKUNFT
SEIT 1386

IMPRS
for Precision Tests of
Fundamental Symmetries
INTERNATIONAL MAX PLANCK
RESEARCH SCHOOL

Generative Amplification with Surrogate Monte Carlo

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with Henning Bahl, Tilman Plehn

ML4Jets 2026 - Vienna - 15/09/2026

Why amplitude surrogates?

Challenge

High-precision amplitudes are computationally expensive

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The cost grows rapidly with final state multiplicity and perturbative order

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Exact training data are limited

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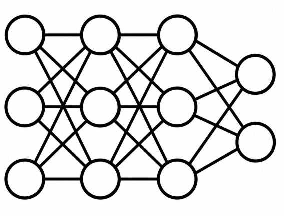
Exact training data are limited

Strategy

Exact amplitudes

Surrogate



Fast evaluations

From generative models to Surrogate Monte Carlo

z_i → Generative model → $x_i \sim p_{\text{gen}}(x)$

Learn the event distribution to **sample** new phase-space points

From generative models to Surrogate Monte Carlo

$z_i \longrightarrow$ Generative model $\longrightarrow x_i \sim p_{\mathrm{gen}}(x)$

Learn the event distribution to **sample** new phase-space points

$x_i \longrightarrow$ Amplitude surrogate / Exact amplitude $\longrightarrow$

$$w_i = \frac{|\mathcal{M}|^2_{\mathrm{surr}}(x_i)}{|\mathcal{M}|^2_{\mathrm{true}}(x_i)}$$

Learn only the amplitude to **reweight** existing phase-space points

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$x_i \rightarrow$ Amplitude surrogate

$x_i \rightarrow$ Exact amplitude

$$w_i = \frac{|\mathcal{M}|^2_{\text{surr}}(x_i)}{|\mathcal{M}|^2_{\text{true}}(x_i)}$$

Learn only the amplitude to **reweight** existing phase-space points

Keep the standard MC chain — replace only the expensive amplitude

What are we comparing?

$q\bar{q} \;\rightarrow\; Z + n_g\, g, \qquad n_g = 1, \ldots, 4$

Focus in this talk: $\boxed{q\bar{q} \;\rightarrow\; Z + 4\, g}$

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Training sample

- Exact amplitude evaluations
- Used to train the surrogate

$n_{\text{train}} = 10^4, 10^5, 10^6$

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Surrogate-weighted sample

- Independent truth sample
- Reweighted with the surrogate

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Reference sample

- Independent truth sample
- Used as reference

$n_{\text{ref}} = 10^6$

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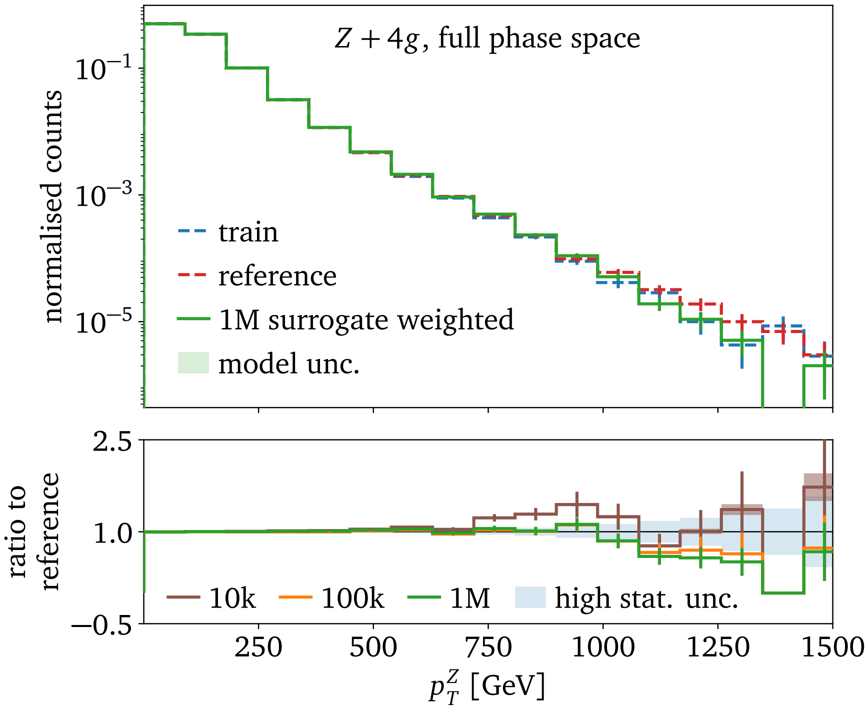
Statistically independent samples

Surrogate accuracy even in the tails

Full phase space



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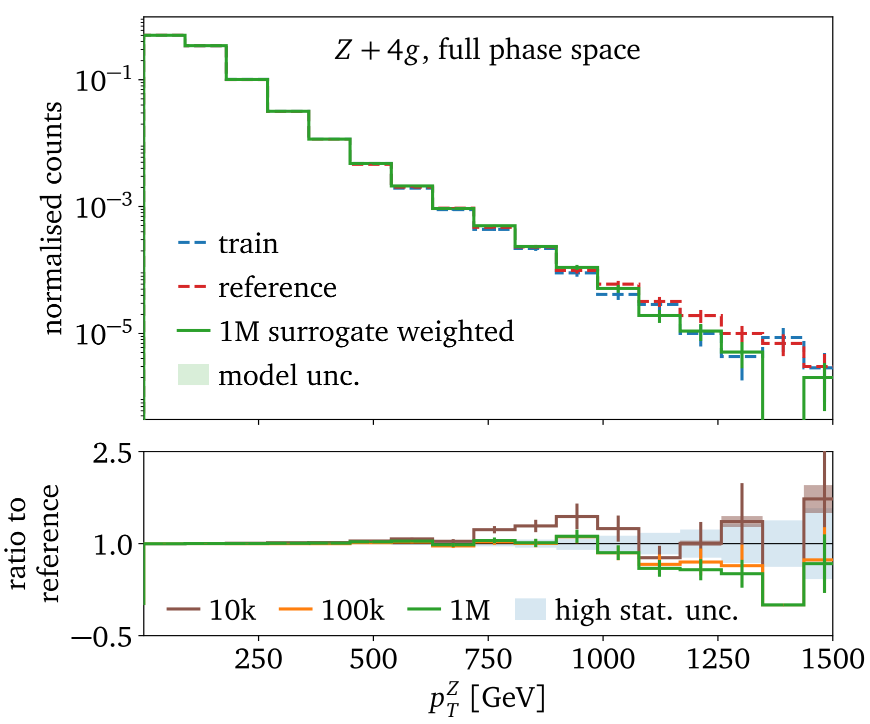
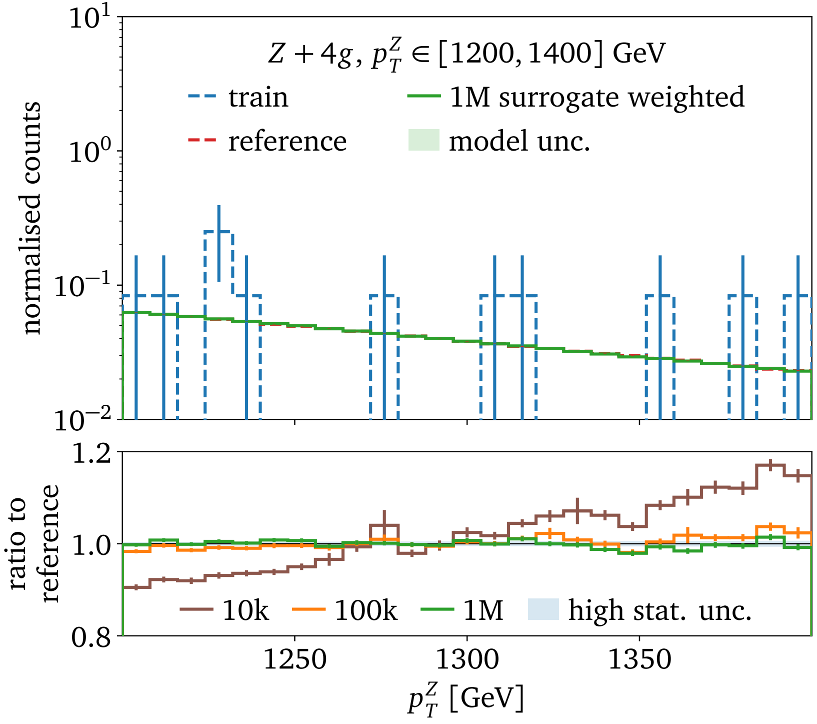


Tail



Averaging amplification

Do we reproduce an average observable in V?

True observable in a region V

$$I(p_{\mathrm{true}}) = \int_V \mathrm{d}x\, p_{\mathrm{true}}(x)$$

Estimated observable from surrogate weights

$$\bar{I} = \frac{\sum_{i \in V} w_i}{\sum_i w_i}$$

Statistical uncertainty

$$\sigma^2_{\mathrm{stat}}(n_{\mathrm{eff}}) \simeq \frac{\bar{I}(1-\bar{I})}{\boxed{n_{\mathrm{eff}}}}$$

Effective sample size

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Effective sample size

How many truth events are needed to match the surrogate accuracy? $\longrightarrow$ $\sigma_{\text{stat}}(n_{\text{equiv}}) = \sigma_{\text{model}}$

$$G = \frac{n_{\text{equiv}}}{n_{\text{train}}}$$ $\longrightarrow$ statistical **amplification** over the training sample

Averaging amplification

$$M_I = \left\langle \left[I(p_{\text{true}}) - \bar{I}(D_{\text{surr}}^{n_{\text{gen}}}) \right]^2 \right\rangle = \sigma_{\text{stat}}^2 + \sigma_{\text{model}}^2$$

- The statistical uncertainty decreases as

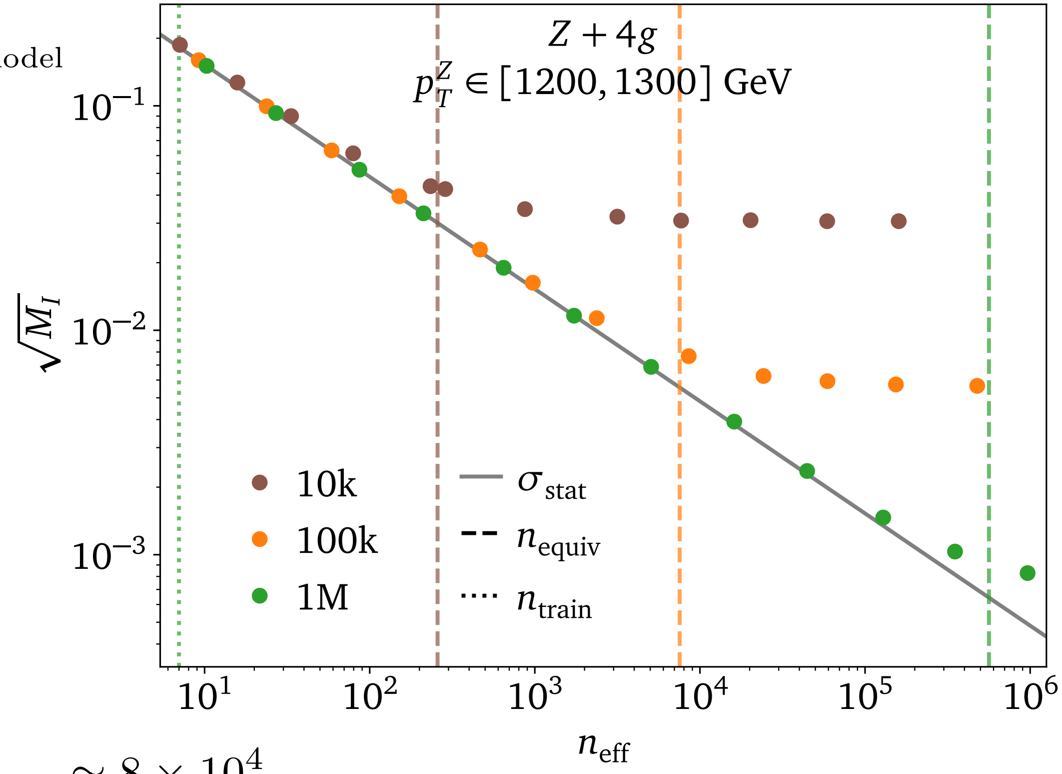
  $\sigma_{\text{stat}} \propto 1/\sqrt{n_{\text{eff}}}$

- The surrogate reaches a model-limited plateau

  $\sigma_{\text{stat}} \sim \sigma_{\text{model}}$

- The intersection defines

  n_{equiv}



$$n_{\text{train}} = 7 \longrightarrow n_{\text{equiv}} \simeq 5.6 \times 10^5 \longrightarrow G_{\text{avg}} \simeq 8 \times 10^4$$

Differential amplification

Do we reproduce the distribution within V?

Optimal test statistic

$$T(x) = \log \frac{p_{\mathrm{surr}}(x)}{p_{\mathrm{true}}(x)} = \log w(x)$$

Cumulative distributions

$$F_{\mathrm{surr}}(t)$$
$$F_{\mathrm{true}}(t)$$

Kolmogorov-Smirnov distance

$$M_{\mathrm{KS}} = \sup_t \left| F_{\mathrm{surr}}(t) - F_{\mathrm{true}}(t) \right|$$

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How many truth events are needed to match the surrogate accuracy? → $M_{\text{KS}}^{\text{stat}}(n_{\text{equiv}}) = M_{\text{KS}}^{\text{surr}}$

$G = \dfrac{n_{\text{equiv}}}{n_{\text{train}}}$ → statistical **amplification** over the training sample

Differential amplification

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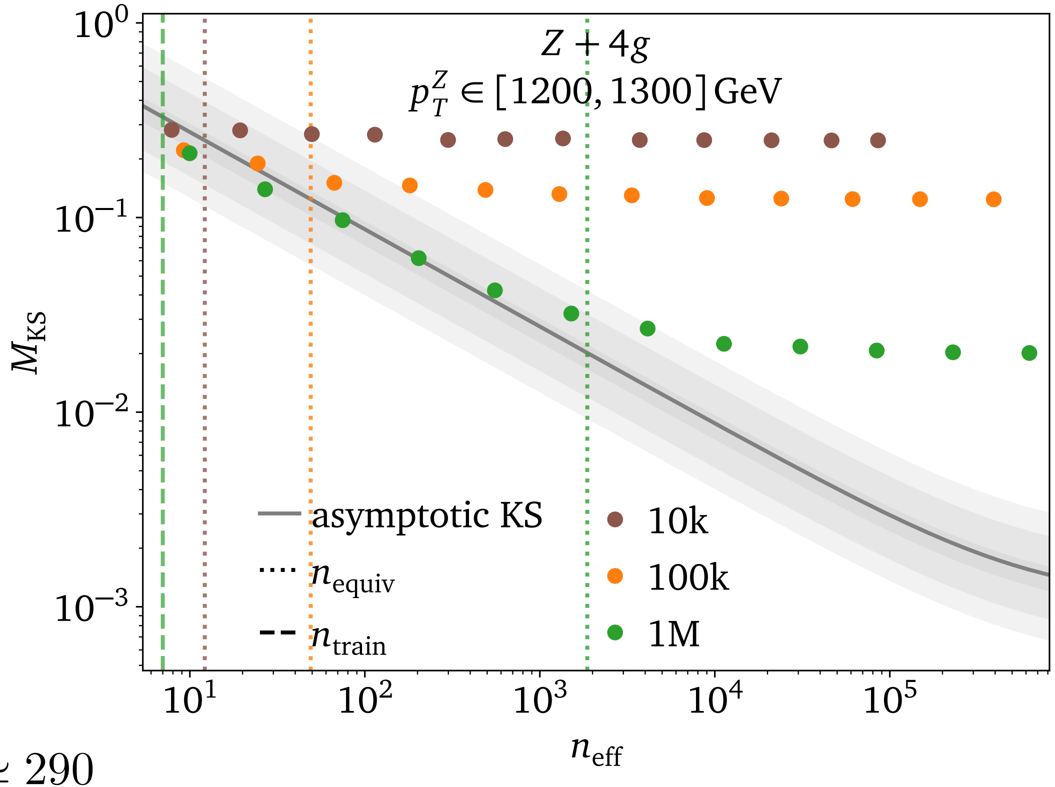
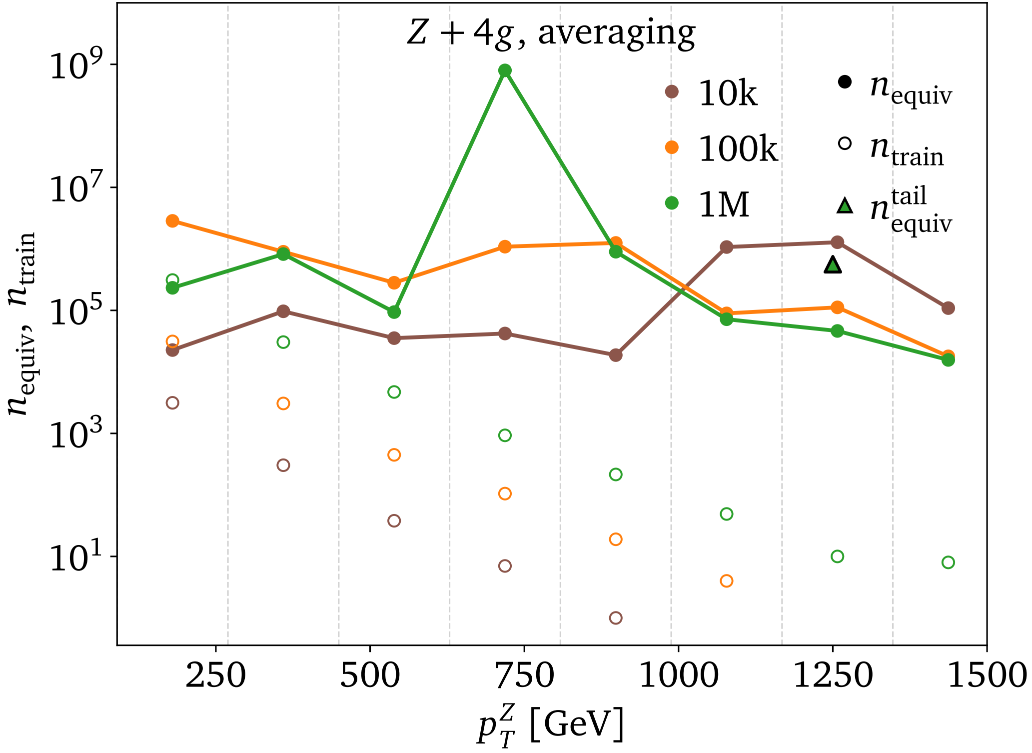


$$n_{\mathrm{train}} = 7 \longrightarrow n_{\mathrm{equiv}} \simeq 2000 \longrightarrow G_{\mathrm{KS}} \simeq 290$$

Amplification across phase space

- Averaging amplification provides a robust measure
- Large equivalent statistics across the full spectrum
- Training statistics drop in sparsely populated regions
- Amplification increases where training data are scarce

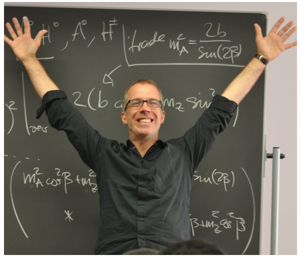


Take-home message

- Amplitude surrogates provide **statistical amplification** beyond their limited training statistics
- **Amplification** is strongest in sparsely populated **tails**
- Surrogate Monte Carlo preserves the standard simulation chain while replacing the **expensive amplitude**

Team

Henning



Tilman

Rebecca

Paper

Poster