Testing Cosmic Anisotropy and Modified Gravity: $f(R)$ Equivalences, Moving Fields, and Bianchi-I Dark Energy Models

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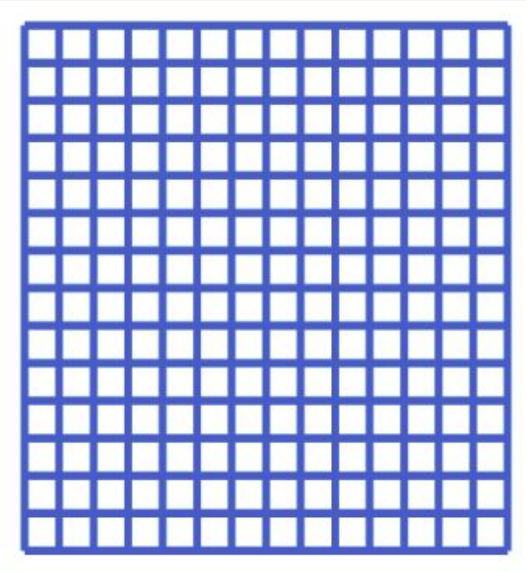
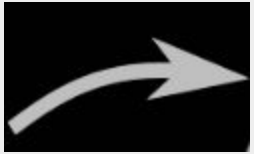
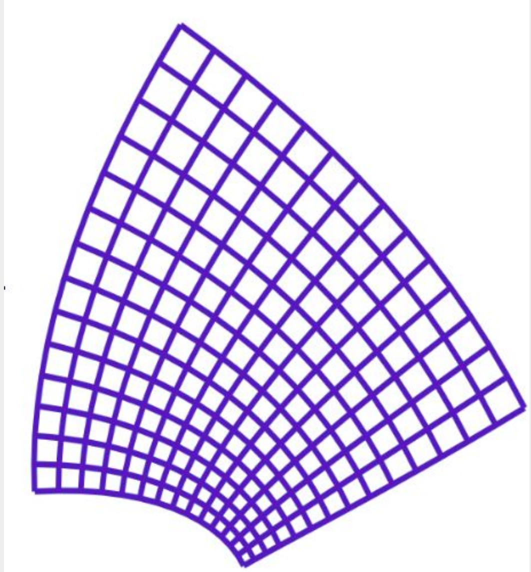
Cosmological Observations in *f(R)* Theories and Their Conformal Equivalence





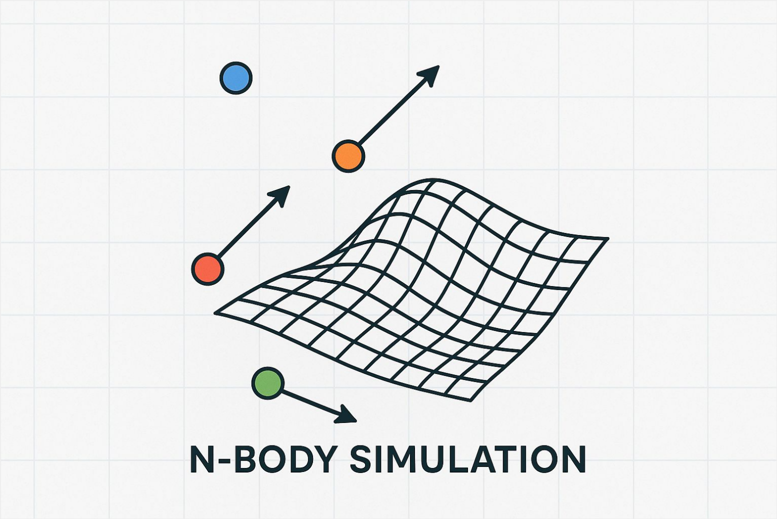


$$\mathcal{S} = \int \sqrt{-g} \left[\frac{1}{2\kappa} f(R) + \mathcal{L}_{\mathrm{m}} \left(g_{\mu\nu}, \psi_{\mathrm{m}} \right) \right]$$

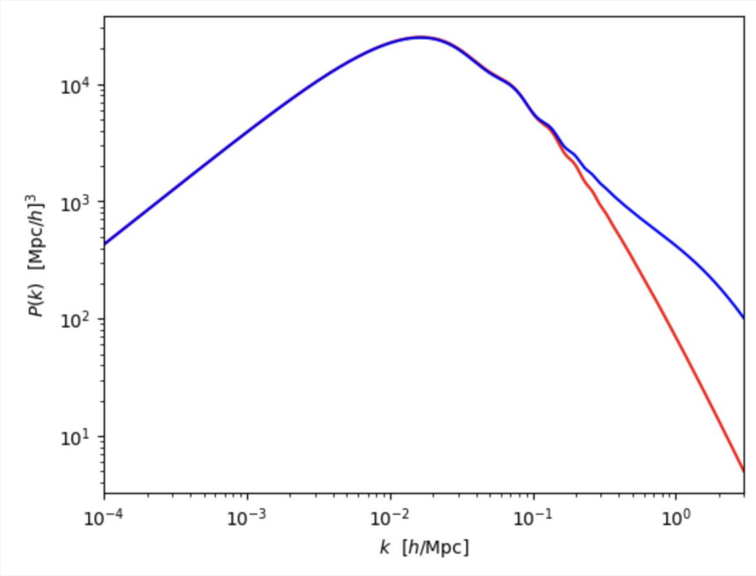
$$\mathcal{S} = \int \sqrt{-\tilde{g}} \left[\frac{1}{2\kappa} \tilde{R} - \frac{1}{2} \tilde{\nabla}_{\mu} \phi \tilde{\nabla}^{\mu} \phi - V(\phi) + \mathcal{L}_{\mathrm{m}} \left(F^{-1} \tilde{g}_{\mu\nu}, \psi_{\mathrm{m}} \right) \right]$$

Background, Linear and Beyond Linear Order Perturbations

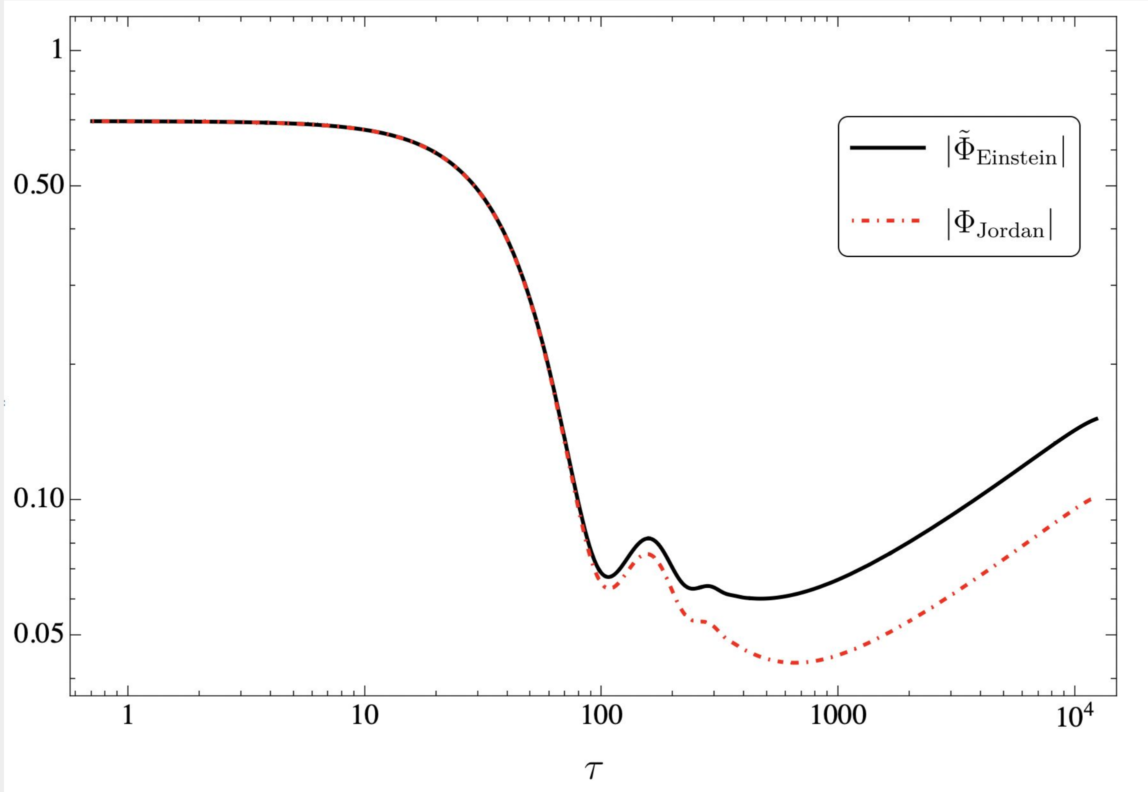
MG-PICOLA



MG-CLASS



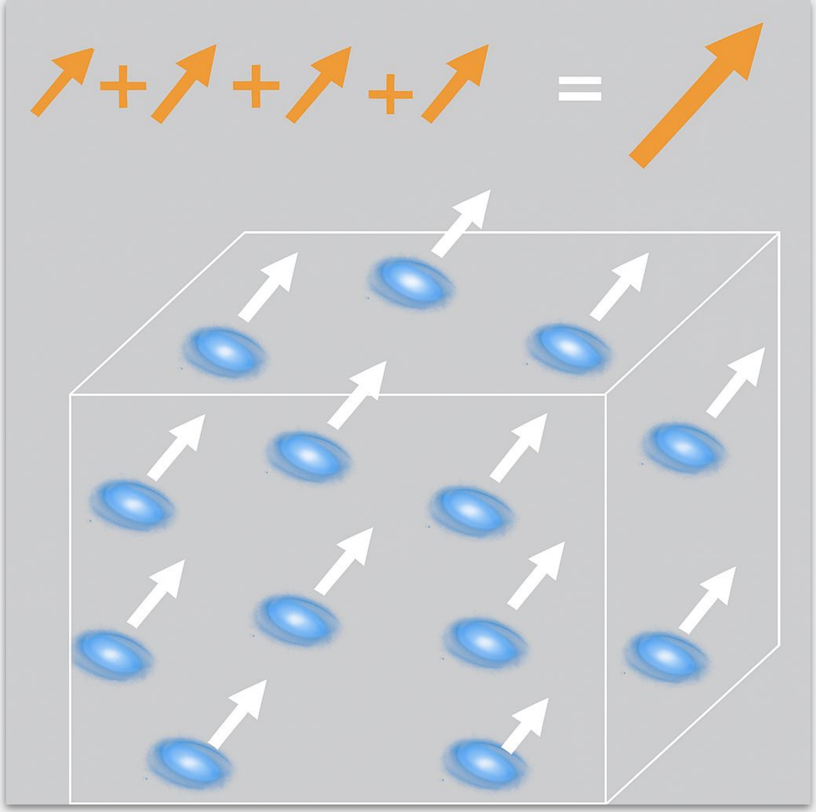
Gravitational potentials

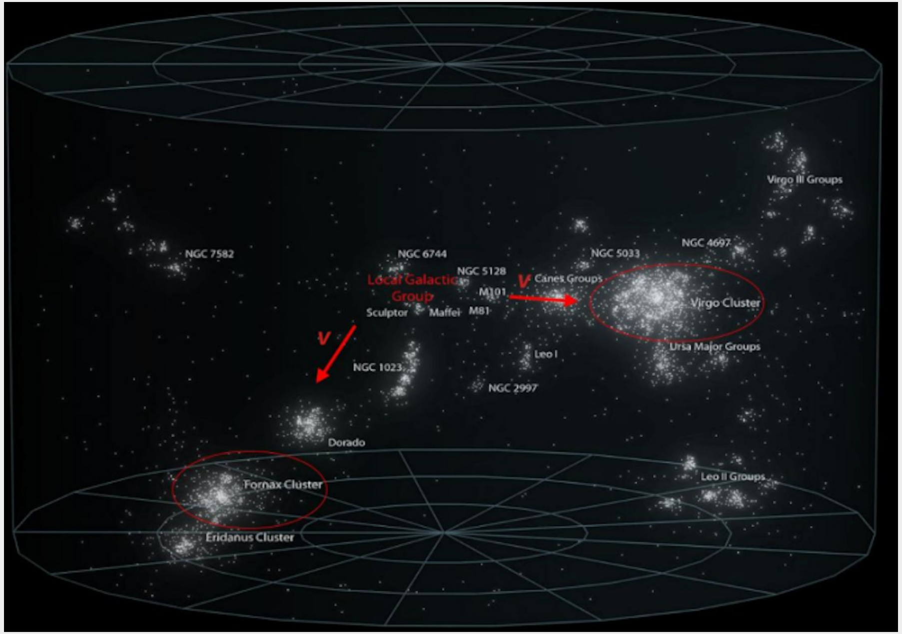


$$\Psi = \tilde{\Psi} + \frac{1}{2}\frac{\delta F}{F}$$

$$\Phi = \tilde{\Phi} - \frac{1}{2}\frac{\delta F}{F}$$

Moving KGB





Virgo III Groups
NGC 7582
NGC 6744
NGC 5033
NGC 4697
NGC 5128
Local Galactic Group
M101
V
Canes Groups
Virgo Cluster
Sculptor
Maffei
M81
V
Leo I
Ursa Major Groups
NGC 1023
NGC 2997
Dorado
Fornax Cluster
Leo II Groups
Eridanus Cluster

Moving Dark Energy

$$ds^2 = a^2\left[-d\tau^2 - 2S_i d\tau dx^i + \delta_{ij}dx^i dx^j\right]$$

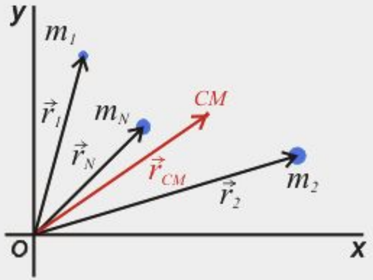
$$T^{\mu\nu}_{(\alpha)} = \left(\rho_{(\alpha)} + p_{(\alpha)}\right) u^{\mu}_{(\alpha)} u^{\nu}_{(\alpha)} + p_{(\alpha)} g^{\mu\nu}$$

$$\sum_{\alpha}\left(\rho_{(\alpha)} + p_{(\alpha)}\right) v_i^{(\alpha)} = 0$$

$$u^{\mu}_{(\alpha)} = \frac{1}{a}\left(-1, \vec{v}_{(\alpha)}\right)$$

Cosmic center of mass

$$S_i = \frac{\sum_\alpha \left(\rho_{(\alpha)} + p_{(\alpha)}\right) v_i^{(\alpha)}}{\sum_\alpha \left(\rho_{(\alpha)} + p_{(\alpha)}\right)}$$



Horndeski in motion

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$$\mathcal{S} = \int d^4x\sqrt{-g}\left\{\frac{1}{16\pi}R + G_2(X) - G_3(X)\Box\phi\right\}$$

$$X = -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi.$$

$$ds^2 = a^2\left[-d\tau^2 + e^{2\sigma}\left(dx^2 + dy^2\right) + e^{-4\sigma}dz^2\right]$$

$$\langle\phi\rangle = \bar{\phi} + \lambda_i x^i \qquad \vec{\lambda} = \lambda\hat{e}_z$$

$$-\frac{\delta^{\mu}{}_{\nu}\,\mathcal{H}^2}{a^2}-\frac{2\,\delta^{\mu}{}_{\nu}\,\mathcal{H}'}{a^2}-\frac{\delta^{\mu}{}_{\nu}\,\sigma_{\alpha\beta}\,\sigma^{\alpha\beta}}{2\,a^2}+\frac{2\,\mathcal{H}\,\sigma^{\mu}{}_{\nu}}{a^2}-\frac{2\,\sigma^{\mu\alpha}\,\sigma_{\nu\alpha}}{a^2}+\frac{\bar{h}^{\mu\alpha}\,\sigma'_{\alpha\nu}}{a^2}+\frac{2\,\mathcal{H}^2\,\bar{n}^{\mu}\,\bar{n}_{\nu}}{a^2}-\frac{2\,\mathcal{H}'\,\bar{n}^{\mu}\,\bar{n}_{\nu}}{a^2}-\frac{\sigma_{\alpha\beta}\,\sigma^{\alpha\beta}\,\bar{n}^{\mu}\,\bar{n}_{\nu}}{a^2}+$$

ϵ

$$\left(\frac{\left({}^{(1)}E''_{\alpha\nu}\right)\bar{h}^{\mu\alpha}}{a^2}+\frac{2\left({}^{(1)}E'_{\alpha\nu}\right)\bar{h}^{\mu\alpha}\,\mathcal{H}}{a^2}-\frac{\delta^{\mu}{}_{\nu}\left({}^{(1)}E'_{\alpha\beta}\right)\sigma^{\alpha\beta}}{a^2}+\frac{2\,\delta^{\mu}{}_{\nu}\left({}^{(1)}E^{\alpha\beta}\right)\sigma_{\alpha}{}^{\gamma}\,\sigma_{\beta\gamma}}{a^2}-\frac{2\left({}^{(1)}E'_{\nu\alpha}\right)\sigma^{\mu\alpha}}{a^2}-\frac{4\left({}^{(1)}E^{\mu\alpha}\right)\mathcal{H}\,\sigma_{\nu\alpha}}{a^2}+\right.$$

$$\frac{4\left({}^{(1)}E^{\alpha\beta}\right)\sigma^{\mu}{}_{\alpha}\,\sigma_{\nu\beta}}{a^2}-\frac{2\left({}^{(1)}E'_{\alpha\beta}\right)\bar{h}^{\mu\alpha}\,\sigma_{\nu}{}^{\beta}}{a^2}+\frac{4\left({}^{(1)}E^{\mu\alpha}\right)\sigma_{\alpha\beta}\,\sigma_{\nu}{}^{\beta}}{a^2}-\frac{2\left({}^{(1)}E^{\mu\alpha}\right)\sigma'_{\nu\alpha}}{a^2}-\frac{2\left({}^{(1)}B^{\mu}\right)\mathcal{H}^2\,\bar{n}_{\nu}}{a^2}+$$

$$\frac{2\left({}^{(1)}B^{\mu}\right)\mathcal{H}'\,\bar{n}_{\nu}}{a^2}+\frac{\left({}^{(1)}B^{\mu}\right)\sigma_{\alpha\beta}\,\sigma^{\alpha\beta}\,\bar{n}_{\nu}}{a^2}-\frac{2\left({}^{(1)}B^{\alpha}\right)\mathcal{H}\,\sigma^{\mu}{}_{\alpha}\,\bar{n}_{\nu}}{a^2}+\frac{2\left({}^{(1)}B^{\alpha}\right)\sigma_{\alpha\beta}\,\sigma^{\mu\beta}\,\bar{n}_{\nu}}{a^2}-\frac{\left({}^{(1)}B^{\alpha}\right)\bar{h}^{\mu\beta}\,\sigma'_{\beta\alpha}\,\bar{n}_{\nu}}{a^2}-$$

$$\left.\frac{2\left({}^{(1)}E'_{\alpha\beta}\right)\sigma^{\alpha\beta}\,\bar{n}^{\mu}\,\bar{n}_{\nu}}{a^2}+\ldots\right)$$

DE Models in Bianchi I Universes

1. *Scalar field coupled to a vector field.*
2. *Inhomogeneous scalar field.*
3. *Scalar field with gauge symmetry.*
4. *2-form coupled to an scalar field.*
5. *2-form coupled to cold dark matter.*
6. *Vector field.*
7. *Scalar field.*

Anisotropic universe with anisotropic dark energy

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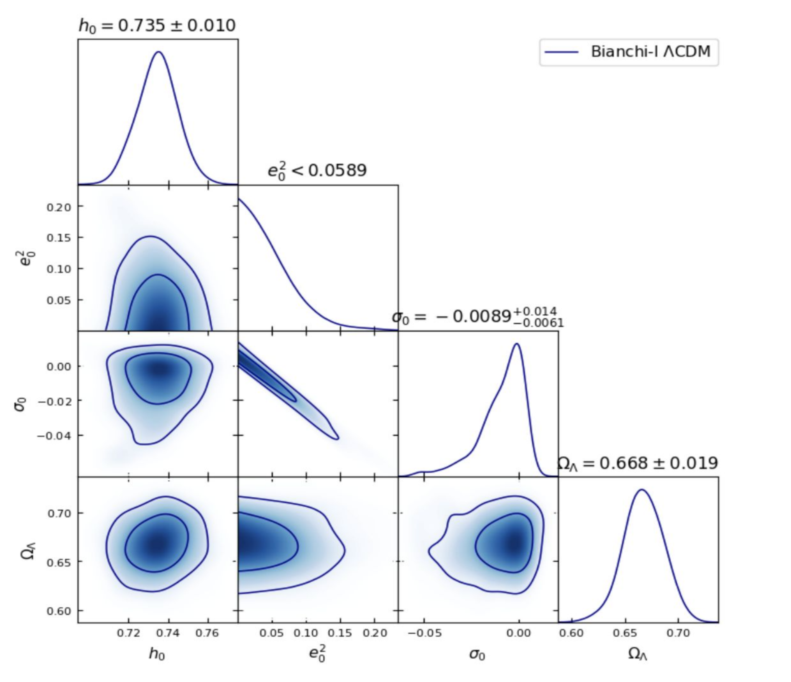
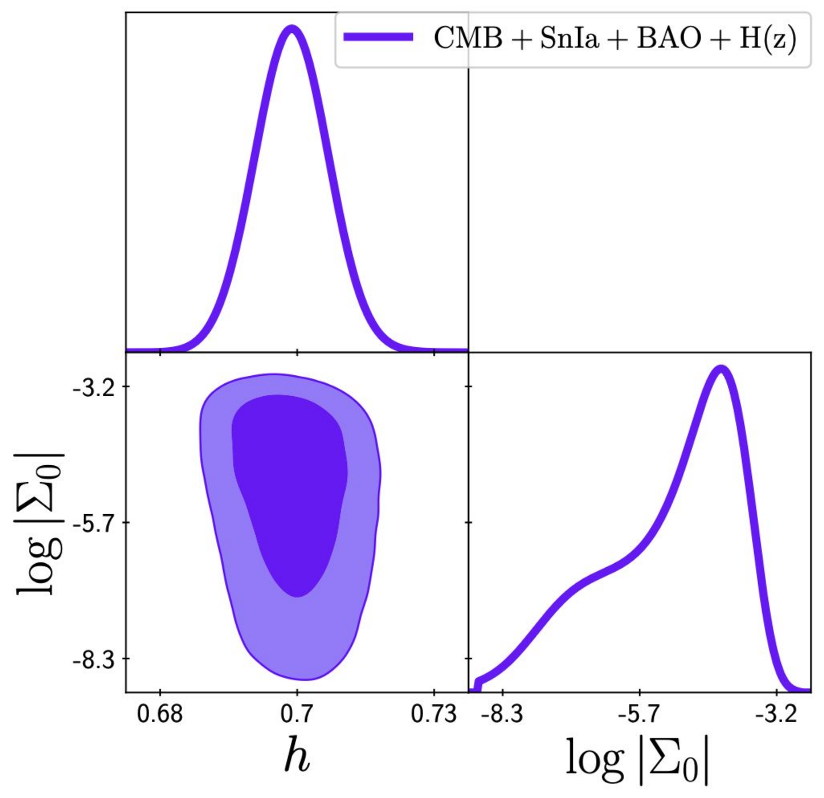


2-Form Coupled to Scalar Field



$$\mathcal{L}_{\rm DE} = -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{12}f_{\rm FQ}(\phi)H_{\mu\nu\alpha}H^{\mu\nu\alpha} - V_{\rm FQ}(\phi)$$

$$|\Sigma_0| = 0.530^{+2.37}_{-0.285} \times 10^{-2}$$

Thanks