

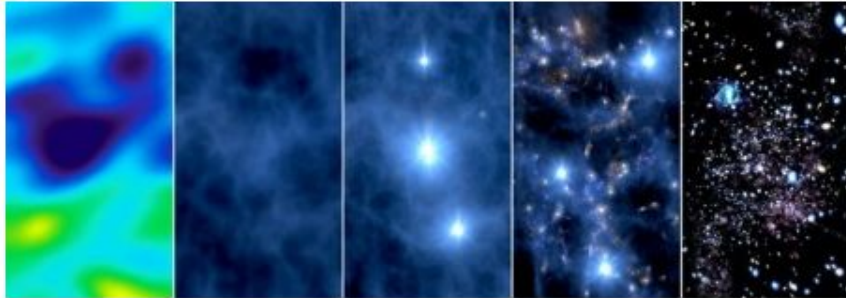


Funciones de Filtrado de Jeans: una herramienta para la formación de estructuras a gran escala, con inclusión de materia bariónica

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Gravitación y Cosmología

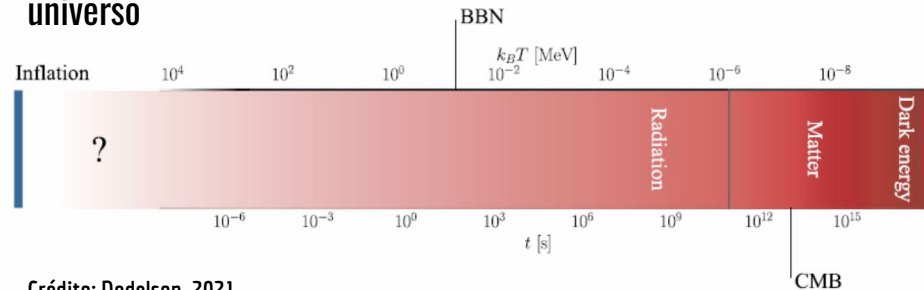
Contenido

1. Motivaciones cosmológicas y breve historia del universo.
2. Irregularidades en la distribución de materia (Aproximación Newtoniana).
3. Teoría de perturbaciones cosmológicas estándar.
4. Ecuaciones de movimiento en el espacio de Fourier.
5. Aproximación analítica a primer orden.
6. Aproximación analítica a segundo orden.
7. Descripción analítica de fluctuaciones bariónicas.
8. Consideraciones finales.
9. Referencias.



Crédito: Nasa

Breve historia del universo



Crédito: Dodelson, 2021

Modelo cosmológico estándar

ECUACIONES FUNDAMENTALES
EN COSMOLOGÍA

GRAVEDAD
Ecuaciones de Einstein

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\frac{H^2}{H_0^2} = \sum_{s=r,m,v,DE} \frac{\Omega_s}{[a(t)]^{3(1+\omega_s)}} + \frac{\Omega_k}{[a(t)]^2}$$

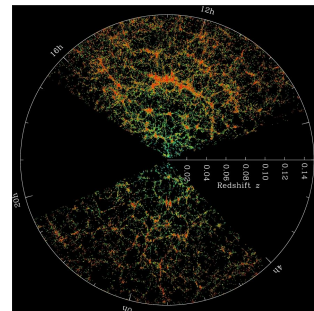
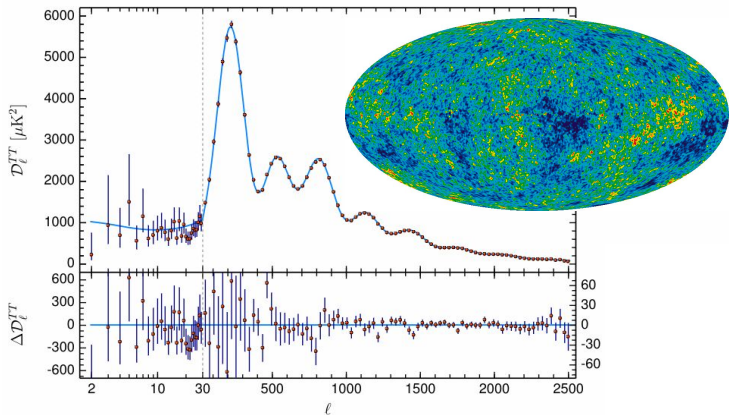
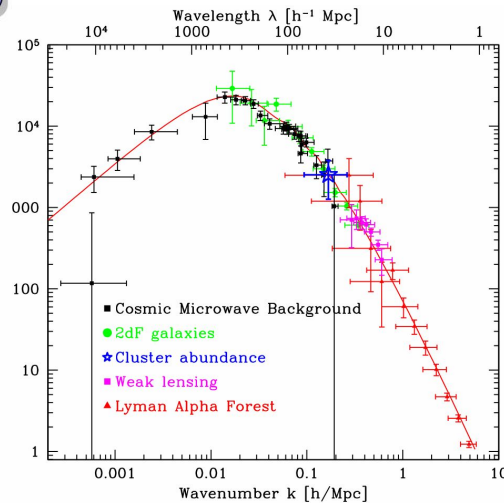
MECÁNICA ESTADÍSTICA
Ecuaciones de Boltzmann

$$\frac{\partial f}{\partial t} + \frac{p}{E} \frac{\hat{p}^i}{a} \frac{\partial f}{\partial x^i} - H p \frac{\partial f}{\partial p} = C[f]$$

$$\frac{\partial f}{\partial t} + \frac{\hat{p}^i}{a} \frac{\partial f}{\partial x^i} - H p \frac{\partial f}{\partial p} = C[f]$$

$$\frac{\partial f}{\partial t} + \frac{p}{m} \frac{\hat{p}^i}{a} \frac{\partial f}{\partial x^i} - H p \frac{\partial f}{\partial p} = C[f]$$

Tegmark & Zaldarriaga, 2002.



ΛCDM

Irregularidades en la distribución de materia

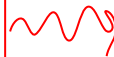


Siguiendo a Peebles (1980)

"Discussion of how irregularities in the matter distribution behave in an expanding universe is greatly simplified by the fact a limiting approximation of general relativity, Newtonian mechanics, applies in a region small compared to the Hubble length cH^{-1} " (and large compared the Schwarzschild radii of any collapsed objects). The rest of the universe can affect the region only through a tidal field".



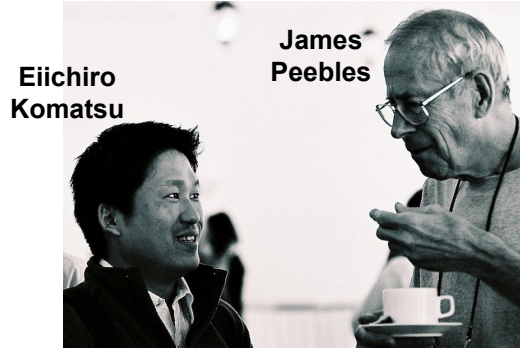
Es posible simplificar la descripción de irregularidades en el campo de materia utilizando una región del espacio-tiempo que satisfaga el rango de escalas:

 Radio de Schwarzschild ($\sim 10^{14}$ cm) $< R <$ Radio de Hubble ($\sim 10^{28}$ cm)



Se describe la distribución de materia y su movimiento en términos de la desviación respecto a un universo homogéneo e isotrópico (modelo de concordancia).

Siguiendo a Peebles (1980) e inspirados por Shoji y Komatsu (2009)



Eiichiro Komatsu

James Peebles

Credits: LeCosPA

Función de distribución

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{ma^2(t)} \cdot \nabla_{\mathbf{x}} f - m \nabla_{\mathbf{x}} \phi \cdot \nabla_{\mathbf{p}} f = 0$$

Potencial generalizado

Coordenadas Eulerianas!

$$\mathbf{r} = a(t)\mathbf{x}$$

$$\frac{m}{a^3(t)} \int d^3p f(t, \mathbf{x}, \mathbf{p}) = \rho(t, \mathbf{x}) \equiv \rho_B(t) \left[1 + \delta(t, \mathbf{x}) \right]$$

Densidad de contraste

$$\frac{1}{a^4(t)} \int d^3p f(t, \mathbf{x}, \mathbf{p}) \mathbf{p} \equiv \rho(t, \mathbf{x}) \mathbf{u}(t, \mathbf{x})$$

Velocidad peculiar

$$\frac{1}{ma^5(t)} \int d^3p f(t, \mathbf{x}, \mathbf{p}) p^i p^j \equiv \rho(t, \mathbf{x}) u^i(t, \mathbf{x}) u^j(t, \mathbf{x})$$

$$\nabla_{\mathbf{x}}^2 \phi = \frac{3}{2} \mathcal{H}(\tau) \Omega(\tau) \delta(\tau, \mathbf{x})$$

Ecuación de Poisson (GR)

$$\frac{\partial}{\partial \tau} \delta_C(\tau, \mathbf{x}) + \nabla_{\mathbf{x}} \cdot \left[\left(1 + \delta_C(\tau, \mathbf{x}) \right) \mathbf{u}_C(\tau, \mathbf{x}) \right] = 0$$

$$\frac{\partial}{\partial \tau} \delta_B(\tau, \mathbf{x}) + \nabla_{\mathbf{x}} \cdot \left[\left(1 + \delta_B(\tau, \mathbf{x}) \right) \mathbf{u}_B(\tau, \mathbf{x}) \right] = 0$$

Ecuaciones de Continuidad

Ecuaciones de Euler

Efectos de presión

$$\frac{\partial}{\partial \tau} \mathbf{u}_C(\tau, \mathbf{x}) + \mathcal{H}(\tau) \mathbf{u}_C(\tau, \mathbf{x}) + \left(\mathbf{u}_C(\tau, \mathbf{x}) \cdot \nabla_{\mathbf{x}} \right) \mathbf{u}_C(\tau, \mathbf{x}) = -\nabla_{\mathbf{x}} \phi(\tau, \mathbf{x})$$

$$\frac{\partial}{\partial \tau} \mathbf{u}_B(\tau, \mathbf{x}) + \mathcal{H}(\tau) \mathbf{u}_B(\tau, \mathbf{x}) + \left(\mathbf{u}_B(\tau, \mathbf{x}) \cdot \nabla_{\mathbf{x}} \right) \mathbf{u}_B(\tau, \mathbf{x}) + \frac{\nabla_{\mathbf{x}} [P_B(\rho_B)]}{\rho_B(\tau, \mathbf{x})} = -\nabla_{\mathbf{x}} \phi(\tau, \mathbf{x})$$

Explorando el campo de densidad de contraste a primer orden (materia oscura y bariónica)

$$\frac{\partial^2}{\partial \tau^2} \delta_C(\tau, \mathbf{x}) + \mathcal{H}(\tau) \frac{\partial}{\partial \tau} \delta_C(\tau, \mathbf{x}) - \frac{3}{2} \mathcal{H}^2 \Omega(\tau) \delta_C(\tau, \mathbf{x}) = 0$$

$$\frac{d^2}{d\tau^2} D(\tau) + \mathcal{H}(\tau) \frac{d}{d\tau} D(\tau) - \frac{3}{2} \mathcal{H}^2(\tau) \Omega(\tau) D(\tau) = 0$$

$$(z+1)P(z) \frac{d^2}{dz^2} D(z) + Q(z) \frac{d}{dz} D(z) - \frac{3}{2} \Omega_{m,0} (z+1)^3 D(z) = 0$$

$$D^+(z) = CP^{1/2}(z) \int_z^\infty \frac{s+1}{P^{3/2}(s)} ds$$

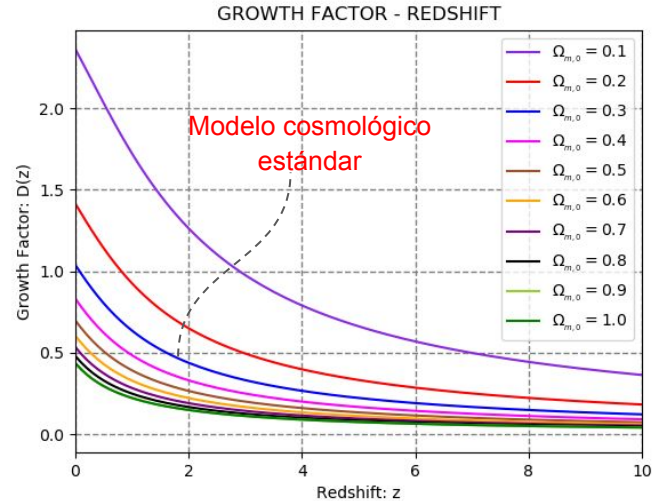
con $P(z) = \Omega_{m,0} z^3 + \left[\frac{3}{2} \Omega_{m,0} + q_0 + 1 \right] z^2 + 2[1 + q_0] z + 1$

$$A = \frac{D^+(z_f)}{D^+(z_b)} = \frac{D^+(2)}{D^+(1000)} = \frac{0.4235}{0.001293} = 327.5$$

$$z_f \geq \frac{1}{\Omega_{m,0}} - 1 \quad 1000 \leq z_b \leq 1500$$

$$\delta = \frac{1}{A} = 0.003053 \sim 10^{-3}$$

$$\delta_C(\tau, \mathbf{x}) = D(\tau) \delta_C(\tau_i, \mathbf{x})$$



Fonseca & Castañeda, 2023.

- D. Edwards and D. Heat (1975).
- F. Bernardeau, S. Colombi, E. Gaztañaga, and R. Scoccimarro, (2001).
- Dodelson and Schmidt (2021).
- Huterer (2023).

Trabajando en el espacio de Fourier (Shoji and Komatsu, 2009)

$$\frac{\partial}{\partial \tau} \tilde{\delta}_C(\mathbf{k}, \tau) + \tilde{\theta}_C(\mathbf{k}, \tau)$$

Divergencia de la velocidad para CDM

$$= -\frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^3 \mathbf{k}_1 d^3 \mathbf{k}_2 \frac{\mathbf{k} \cdot \mathbf{k}_2}{k_2^2} \delta^D(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}) \tilde{\delta}_C(\mathbf{k}_1, \tau) \tilde{\theta}_C(\mathbf{k}_2, \tau)$$

$$\frac{\partial}{\partial \tau} \tilde{\delta}_B(\mathbf{k}, \tau) + \tilde{\theta}_B(\mathbf{k}, \tau)$$

Divergencia de la velocidad para materia bariónica

$$= -\frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^3 \mathbf{k}_1 d^3 \mathbf{k}_2 \frac{\mathbf{k} \cdot \mathbf{k}_2}{k_2^2} \delta^D(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}) \tilde{\delta}_B(\mathbf{k}_1, \tau) \tilde{\theta}_B(\mathbf{k}_2, \tau)$$

Función delta Dirac

$$\frac{\partial}{\partial \tau} \tilde{\theta}_C(\mathbf{k}, \tau) + \mathcal{H}(\tau) \tilde{\theta}_C(\mathbf{k}, \tau) + \frac{6}{\tau^2} \tilde{\delta}(\mathbf{k}, \tau)$$

$$= -\frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^3 \mathbf{k}_1 d^3 \mathbf{k}_2 k_2^2 \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{2k_1^2 k_2^2} \delta^D(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}) \tilde{\theta}_C(\mathbf{k}_1, \tau) \tilde{\theta}_C(\mathbf{k}_2, \tau)$$

$$\frac{\partial}{\partial \tau} \tilde{\theta}_B(\mathbf{k}, \tau) + \mathcal{H}(\tau) \tilde{\theta}_B(\mathbf{k}, \tau) + \frac{6}{\tau^2} \tilde{\delta}(\mathbf{k}, \tau)$$

$$= -\frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^3 \mathbf{k}_1 d^3 \mathbf{k}_2 k_2^2 \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{2k_1^2 k_2^2} \delta^D(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}) \tilde{\theta}_B(\mathbf{k}_1, \tau) \tilde{\theta}_B(\mathbf{k}_2, \tau) + c_s^2(\tau) k^2$$

$$\times \left[\tilde{\delta}_B(\mathbf{k}, \tau) - \frac{1}{2(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^3 \mathbf{k}_1 d^3 \mathbf{k}_2 \delta^D(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}) \tilde{\delta}_B(\mathbf{k}_1, \tau) \tilde{\delta}_B(\mathbf{k}_2, \tau) + \frac{1}{3(2\pi)^6} \right.$$

$$\left. \times \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^3 \mathbf{k}_1 d^3 \mathbf{k}_2 d^3 \mathbf{k}_3 \delta^D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 - \mathbf{k}) \tilde{\delta}_B(\mathbf{k}_1, \tau) \tilde{\delta}_B(\mathbf{k}_2, \tau) \tilde{\delta}_B(\mathbf{k}_3, \tau) \right]$$

Interacciones!

Velocidad del sonido

Aproximación analítica a primer orden

$$\frac{\partial}{\partial \tau} \delta_C(\tau, \mathbf{x}) + \nabla_{\mathbf{x}} \cdot \mathbf{u}_C(\tau, \mathbf{x}) = 0$$

$$\frac{\partial}{\partial \tau} \delta_B(\tau, \mathbf{x}) + \nabla_{\mathbf{x}} \cdot \mathbf{u}_B(\tau, \mathbf{x}) = 0$$

$$\frac{\partial}{\partial \tau} \mathbf{u}_C(\tau, \mathbf{x}) + \mathcal{H}(\tau) \mathbf{u}_C(\tau, \mathbf{x}) + \nabla_{\mathbf{x}} \phi(\tau, \mathbf{x}) = 0$$

$$\frac{\partial}{\partial \tau} \mathbf{u}_B(\tau, \mathbf{x}) + \mathcal{H}(\tau) \mathbf{u}_B(\tau, \mathbf{x}) + \frac{\nabla_{\mathbf{x}} [P_B(\rho_B)]}{\rho_B(\tau, \mathbf{x})} = -\nabla_{\mathbf{x}} \phi(\tau, \mathbf{x})$$



$$\frac{\partial}{\partial \tau} \tilde{\theta}_C(\mathbf{k}, \tau) + \frac{2}{\tau} \tilde{\theta}_C(\mathbf{k}, \tau) + \frac{6}{\tau^2} \tilde{\delta}(\mathbf{k}, \tau) = 0$$

$$\frac{\partial}{\partial \tau} \tilde{\theta}_B(\mathbf{k}, \tau) + \frac{2}{\tau} \tilde{\theta}_B(\mathbf{k}, \tau) + \frac{6}{\tau^2} \left[\tilde{\delta}(\mathbf{k}, \tau) - \frac{k^2}{k_J^2} \tilde{\delta}_B(\mathbf{k}, \tau) \right] = 0$$



$$\frac{\partial^2}{\partial \tau^2} \tilde{\delta}_B(\mathbf{k}, \tau) + \frac{2}{\tau} \frac{\partial}{\partial \tau} \tilde{\delta}_B(\mathbf{k}, \tau) - \frac{6}{\tau} \left[\tilde{\delta}(\mathbf{k}, \tau) - \frac{k^2}{k_J^2} \tilde{\delta}_B(\mathbf{k}, \tau) \right] = 0$$

$$g(\mathbf{k}, \tau) \equiv \frac{\tilde{\delta}_B(\mathbf{k}, \tau)}{\tilde{\delta}_C(\mathbf{k}, \tau)}$$

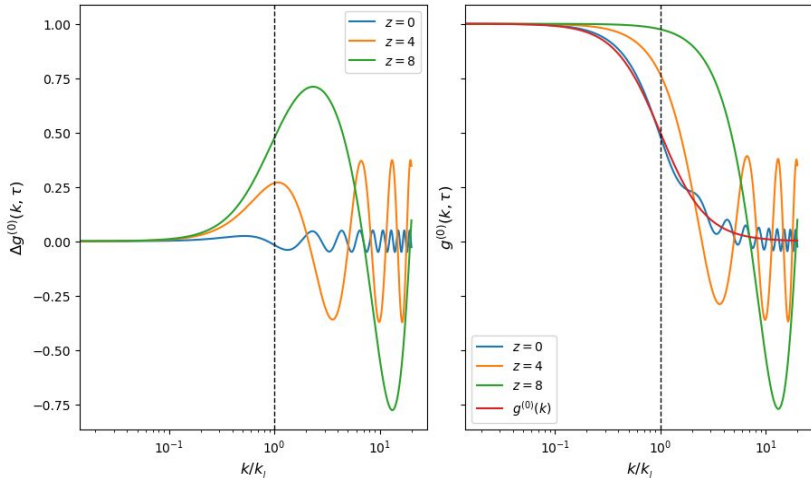
M. shoji and E. Komatsu 2009
G. Somogyi and R. Smith 2010

Número de onda de Jeans

$$k_J(\tau) = \frac{\sqrt{6}}{C_s(\tau)} \tau$$

$$g_1^{(0)}(k, \tau) = c_1 \tau^{-\frac{5}{2}} \left(1 + \sqrt{1 - \frac{24}{25} \left(1 + \frac{k^2}{k_J^2} \right)} \right) + c_2 \tau^{-\frac{5}{2}} \left(1 - \sqrt{1 - \frac{24}{25} \left(1 + \frac{k^2}{k_J^2} \right)} \right) + \frac{1}{1 + \frac{k^2}{k_J^2}}$$

Modos de decaimiento Modos de crecimiento



Los resultados de Fonseca y Castañeda (2023) son consistentes con los obtenidos por Shoji and Komatsu (2009).

Aproximación analítica a segundo orden

Las ecuaciones de movimiento pueden resolverse a través de las siguientes expansiones perturbativas.

$$\tilde{\delta}_C(\mathbf{k}, \tau) = \sum_{n=1}^{\infty} a^n(\tau) \tilde{\delta}_{n,C}(\mathbf{k})$$

$$\tilde{\delta}_B(\mathbf{k}, \tau) = \sum_{n=1}^{\infty} a^n(\tau) \tilde{\delta}_{n,B}(\mathbf{k}) = \sum_{n=1}^{\infty} a^n(\tau) \tilde{g}_n(\mathbf{k}, \tau) \tilde{\delta}_{n,C}(\mathbf{k})$$

$$\tilde{\theta}_C(\mathbf{k}, \tau) = \sum_{n=1}^{\infty} \dot{a}(\tau) a^{n-1}(\tau) \tilde{\theta}_{n,C}(\mathbf{k})$$

$$\tilde{\theta}_B(\mathbf{k}, \tau) = \sum_{n=1}^{\infty} \dot{a}(\tau) a^{n-1}(\tau) \tilde{\theta}_{n,B}(\mathbf{k}) = \sum_{n=1}^{\infty} \dot{a}(\tau) a^{n-1}(\tau) \tilde{h}_n(\mathbf{k}, \tau) \tilde{\theta}_{n,C}(\mathbf{k})$$

$$g_n(\mathbf{k}, \tau) \equiv \frac{\tilde{\delta}_{n,B}(\mathbf{k}, \tau)}{\tilde{\delta}_{n,C}(\mathbf{k}, \tau)}$$

JFF para densidad de fluctuaciones en materia.

$$h_n(\mathbf{k}, \tau) \equiv \frac{\tilde{\theta}_{n,B}(\mathbf{k}, \tau)}{\tilde{\theta}_{n,C}(\mathbf{k}, \tau)}$$

JFF para la divergencia de la velocidad.

Encontramos las expresiones

$$\sum_{n=1}^{\infty} \left\{ \left[n a^{n-1}(\tau) \dot{a}(\tau) g_n(\mathbf{k}, \tau) + a^n(\tau) \dot{g}_n(\mathbf{k}, \tau) \right] \tilde{\delta}_{n,C}(\mathbf{k}) + \dot{a}(\tau) a^{n-1}(\tau) \tilde{\theta}_{n,C}(\mathbf{k}) h_n(\mathbf{k}, \tau) \right\}$$

$$= -\frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^3 \mathbf{k}_1 d^3 \mathbf{k}_2 \frac{\mathbf{k} \cdot \mathbf{k}_2}{k_2^2} \delta^D(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}) \sum_{\ell=1}^{\infty} \sum_{m=1}^{\infty} \left\{ a^{\ell+m-1}(\tau) \dot{a}(\tau) g_{\ell}(\mathbf{k}_1, \tau) h_m(\mathbf{k}_2, \tau) \tilde{\delta}_{\ell,C}(\mathbf{k}_1) \tilde{\theta}_{m,C}(\mathbf{k}_2) \right\}$$

Ecuación de continuidad

$$\begin{aligned}
& \sum_{n=1}^{\infty} \left\{ \left[\ddot{a}(\tau) a^{n-1}(\tau) + \dot{a}^2(\tau) a^{n-2}(\tau) (n-1) \right] \tilde{\theta}_{n,C}(\mathbf{k}) h_n(\mathbf{k}, \tau) + \dot{a}(\tau) a^{n-1}(\tau) \tilde{\theta}_{n,C}(\mathbf{k}) \dot{h}_n(\mathbf{k}, \tau) \right. \\
& \quad \left. + \frac{2}{\tau} \dot{a}(\tau) a^{n-1}(\tau) \tilde{\theta}_{n,C}(\mathbf{k}) h_n(\mathbf{k}, \tau) + \frac{6}{\tau^2} a^n(\tau) \left[f_C + f_B g_n(\mathbf{k}, \tau) \right] \delta_{n,C}(\mathbf{k}) \right\} \\
& = -\frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^3 \mathbf{k}_1 d^3 \mathbf{k}_2 \delta^D(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}) k^2 \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{2k_1^2 k_2^2} \sum_{\ell=1}^{\infty} \sum_{m=1}^{\infty} \left\{ \dot{a}^2(\tau) a^{\ell+m-2}(\tau) \right. \\
& \quad \times h_{\ell}(\mathbf{k}_1, \tau) h_m(\mathbf{k}_2, \tau) \tilde{\theta}_{\ell,C}(\mathbf{k}_1) \tilde{\theta}_{m,C}(\mathbf{k}_2) \left. \right\} + k^2 c_s^2(\tau) \sum_{n=1}^{\infty} a^n(\tau) \delta_{n,C}(\mathbf{k}) g_n(\mathbf{k}, \tau) - \frac{k^2 c_s^2(\tau)}{2(2\pi)^3} \\
& \quad \times \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^3 \mathbf{k}_1 d^3 \mathbf{k}_2 \delta^D(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}) \sum_{\ell=1}^{\infty} \sum_{m=1}^{\infty} \left\{ a^{\ell+m}(\tau) \delta_{\ell,C}(\mathbf{k}_1) \delta_{m,C}(\mathbf{k}_2) \right. \\
& \quad \times g_{\ell}(\mathbf{k}_1, \tau) g_m(\mathbf{k}_2, \tau) \left. \right\} + \frac{k^2 c_s^2(\tau)}{3(2\pi)^6} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^3 \mathbf{k}_1 d^3 \mathbf{k}_2 d^3 \mathbf{k}_3 \delta^D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 - \mathbf{k}) \\
& \quad \times \sum_{\ell=1}^{\infty} \sum_{m=1}^{\infty} \sum_{p=1}^{\infty} \left\{ a^{\ell+m+p}(\tau) \delta_{\ell,C}(\mathbf{k}_1) \delta_{m,C}(\mathbf{k}_2) \delta_{p,C}(\mathbf{k}_3) g_{\ell}(\mathbf{k}_1, \tau) g_m(\mathbf{k}_2, \tau) g_p(\mathbf{k}_3, \tau) \right\}
\end{aligned}$$

Función del filtrado.

Interacción entre los modos.

Fluctuaciones en la densidad de materia para un fluido mixto de materia bariónica y CDM

Efectos de presión bariónica

Función del filtrado.

Ecuación de Euler

Para una teoría a segundo orden

$$\ddot{g}_2(\mathbf{k}, \tau) + \frac{10}{\tau} \dot{g}_2(\mathbf{k}, \tau) + \frac{1}{\tau^2} \left[20 + 6 \frac{k^2}{k_J^2} \right] g_2(\mathbf{k}, \tau) - \frac{1}{\tau^2} \left[6 + 10 \frac{A_2(\mathbf{k})}{\tilde{\delta}_{2,C}(\mathbf{k})} - \frac{4B_2(\mathbf{k})}{\tilde{\delta}_{2,C}(\mathbf{k})} \right] = 0$$

$$\tilde{\delta}_{2,C}(\mathbf{k}) = \frac{1}{(2\pi)^3} \int d\mathbf{q} F_2^{(s)}(\mathbf{q}, \mathbf{k} - \mathbf{q}) \tilde{\delta}_{1,C}(\mathbf{q}) \tilde{\delta}_{1,C}(\mathbf{k} - \mathbf{q})$$

$$A_2(\mathbf{k}) \equiv \frac{2}{\tau} \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^3\mathbf{k}_1 d^3\mathbf{k}_2 \frac{\mathbf{k} \cdot \mathbf{k}_2}{k_2^2} \delta^D(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}) g_1(\mathbf{k}_1) g_1(\mathbf{k}_2) \tilde{\delta}_{1,C}(\mathbf{k}_1) \tilde{\delta}_{1,C}(\mathbf{k}_2)$$

Función simétrica!

Función de filtrado a primer orden

$$B_2(\mathbf{k}) \equiv \frac{4}{\tau^2} \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^3\mathbf{k}_1 d^3\mathbf{k}_2 \delta^D(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}) \left[-\frac{3k^2}{4k_J^2} - k^2 \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{2k_1^2 k_2^2} \right] g_1(\mathbf{k}_1) g_1(\mathbf{k}_2) \tilde{\delta}_{1,C}(\mathbf{k}_1) \tilde{\delta}_{1,C}(\mathbf{k}_2)$$

Modos de acople con materia bariónica

$$g_2^{(0)}(\mathbf{k}, \tau) = c_1 \tau^{-\frac{9}{2}} \left(1 + \sqrt{1 - \frac{4}{81} \left(20 + 6 \frac{k^2}{k_J^2} \right)} \right) + c_2 \tau^{-\frac{9}{2}} \left(1 - \sqrt{1 - \frac{4}{81} \left(20 + 6 \frac{k^2}{k_J^2} \right)} \right) + \frac{\frac{10}{3} - \frac{7}{3} \left[1 - \frac{\tilde{\delta}'_{2,C}(\mathbf{k})}{\tilde{\delta}_{2,C}(\mathbf{k})} \right]}{\frac{10}{3} + \frac{k^2}{k_J^2}}$$

Modos de decaimiento

Modos de crecimiento

$$\tilde{\delta}'_{2,C}(\mathbf{k}) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^3\mathbf{k}_1 d^3\mathbf{k}_2 \delta^D(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}) F_2^{(s)}(\mathbf{k}_1, \mathbf{k}_2) \tilde{\delta}_{1,C}(\mathbf{k}_1) \tilde{\delta}_{1,C}(\mathbf{k}_2)$$

Nuestro aporte...

Aplicando el método desarrollado por Makino (1992)

$$\frac{\tilde{\delta}_{2,B}^{(0)}(\mathbf{k})}{\tilde{\delta}_{2,C}^{(0)}(\mathbf{k})} = g_2^{(0)}(\mathbf{k}) = \frac{\frac{10}{3} - \frac{7}{3} \left(1 - \frac{\tilde{\delta}'_{2,C}^{(0)}(\mathbf{k})}{\tilde{\delta}_{2,C}^{(0)}(\mathbf{k})} \right)}{\frac{10}{3} + \frac{k^2}{k_J^2}}$$

$$\tilde{\delta}'_{2,C}^{(0)}(\mathbf{k}) = \frac{\tau^4}{(2\pi)^3} \int d\mathbf{q} F_2^{(s)}(\mathbf{q}, \mathbf{k} - \mathbf{q})$$

$$F_2^{(s)}(\mathbf{q}, \mathbf{k} - \mathbf{q}) + \frac{3}{14} \frac{k^2}{k_J^2}$$

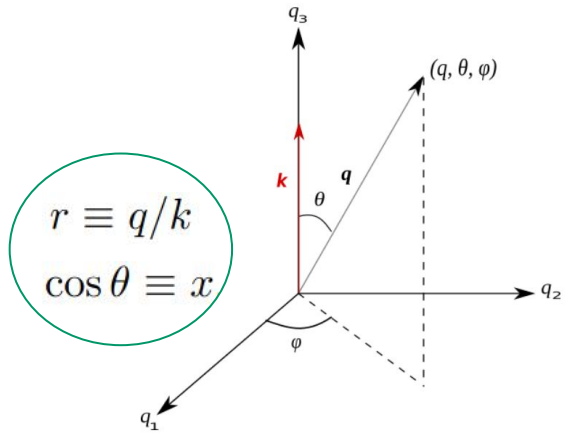
$$\tilde{\delta}_{2,C}^{(0)}(\mathbf{k}) = \frac{\tau^4}{(2\pi)^3} \int d\mathbf{q} F_2^{(s)}(\mathbf{q}, \mathbf{k} - \mathbf{q})$$

$$F_2^{(s)}(\mathbf{q}, \mathbf{k} - \mathbf{q}) = \frac{5}{7} + \frac{1}{2} \frac{\mathbf{q} \cdot (\mathbf{k} - \mathbf{q})}{q^2 |\mathbf{k} - \mathbf{q}|^2} [q^2 + |\mathbf{k} - \mathbf{q}|^2] + \frac{2}{7} \frac{[\mathbf{q} \cdot (\mathbf{k} - \mathbf{q})]^2}{q^2 |\mathbf{k} - \mathbf{q}|^2}$$

Colapso esférico.

Desplazamiento.

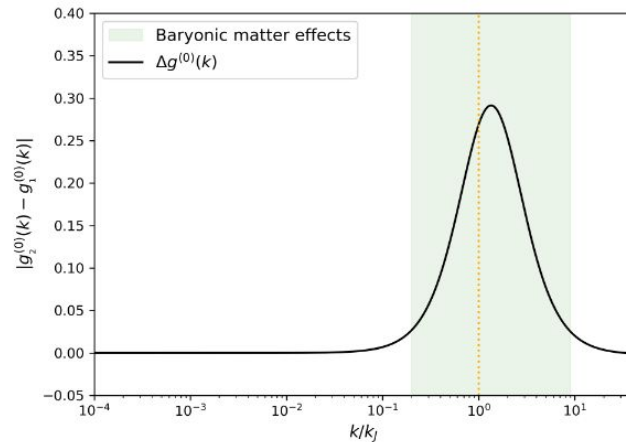
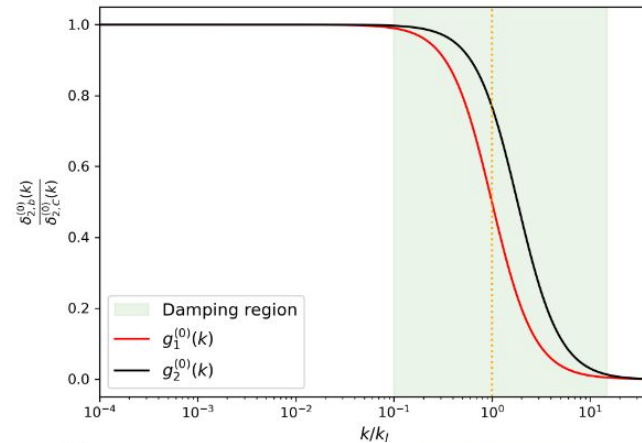
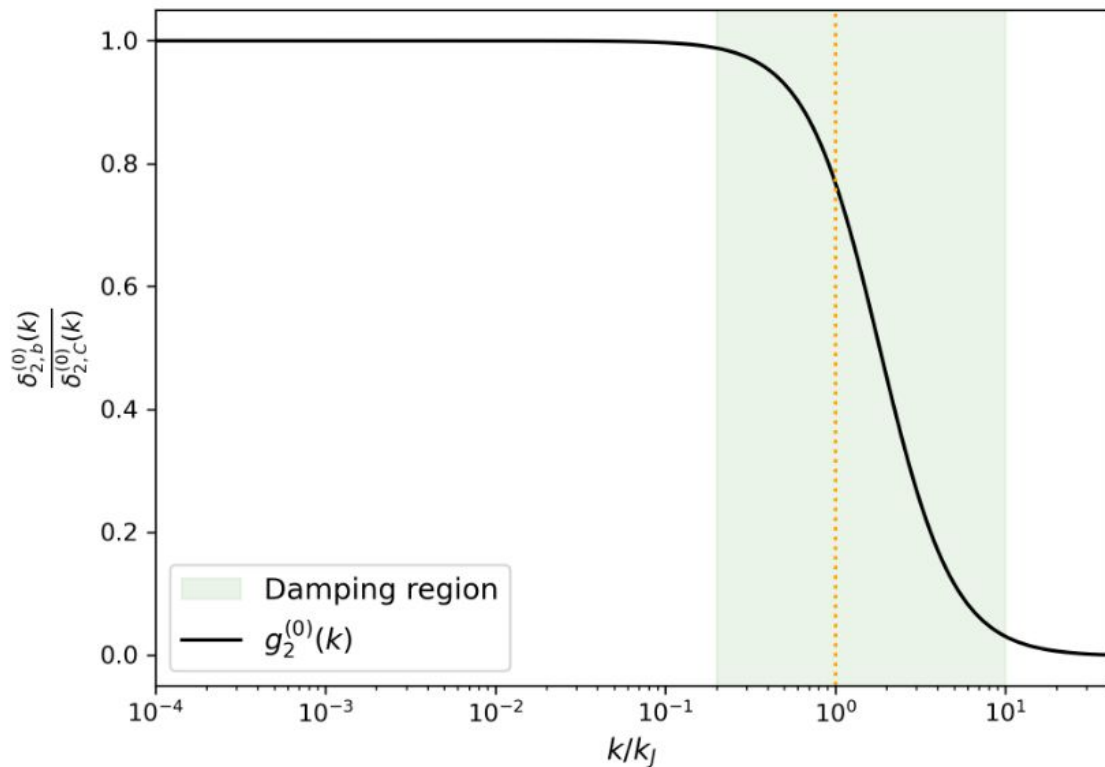
Efectos de marea.



$$\int d\mathbf{q} = \frac{1}{(2\pi)^3} \int \int \int dq_1 dq_2 dq_3 q^2 \sin \theta dq d\theta d\varphi = \frac{1}{(2\pi)^2} \int \int q^2 dq d(\cos \theta)$$

$$\frac{\tilde{\delta}_{2,B}^{(0)}(\mathbf{k})}{\tilde{\delta}_{2,C}^{(0)}(\mathbf{k})} = \left[\frac{10}{3} + \frac{k^2}{k_J^2} \right]^{-1} \left[1 + \frac{7}{3} \frac{\int_0^\infty \int_{-1}^1 dx dr \frac{r \frac{3r + 7x - 10rx^2}{1 + r^2 - 2rx} + 3 \frac{k^2}{k_J^2} r^2}{\left(1 + r^2 \frac{k^2}{k_J^2} \right) \left(1 + (1 + r^2 - 2rx) \frac{k^2}{k_J^2} \right)}}{\int_0^\infty \int_{-1}^1 dx dr r \frac{3r + 7x - 10rx^2}{1 + r^2 - 2rx}} \right]$$

Descripción de fluctuaciones bariónicas (materia)



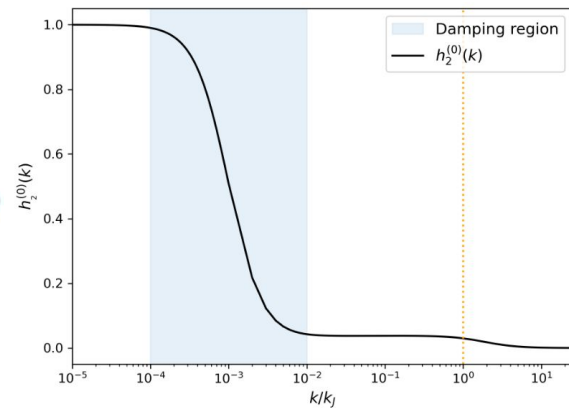
Fonseca, Castañeda and García (2025)

Descripción de fluctuaciones bariónicas (divergencia de velocidad)

$$h_2^{(0)}(\mathbf{k}) = \frac{1}{\tilde{\theta}_{2,C}^{(0)}(\mathbf{k})} \left[\frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d^3\mathbf{k}_1 d^3\mathbf{k}_2 \delta^D(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}) \right.$$

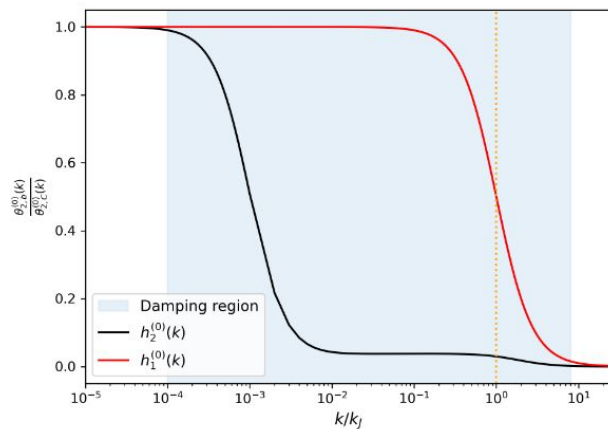
Kernel

$$\left[2F_2^{(s)}(\mathbf{k}_1, \mathbf{k}_2) - G_2^{(s)}(\mathbf{k}_1, \mathbf{k}_2) \right] g_1^{(0)}(\mathbf{k}_1) g_1^{(0)}(\mathbf{k}_2) \tilde{\delta}_{1,C}^{(0)}(\mathbf{k}_1) \tilde{\delta}_{1,C}^{(0)}(\mathbf{k}_2) \left. \right] - 2 \frac{\tilde{\delta}_{2,C}^{(0)}(\mathbf{k})}{\tilde{\theta}_{2,C}^{(0)}(\mathbf{k})} g_2^{(0)}(\mathbf{k})$$



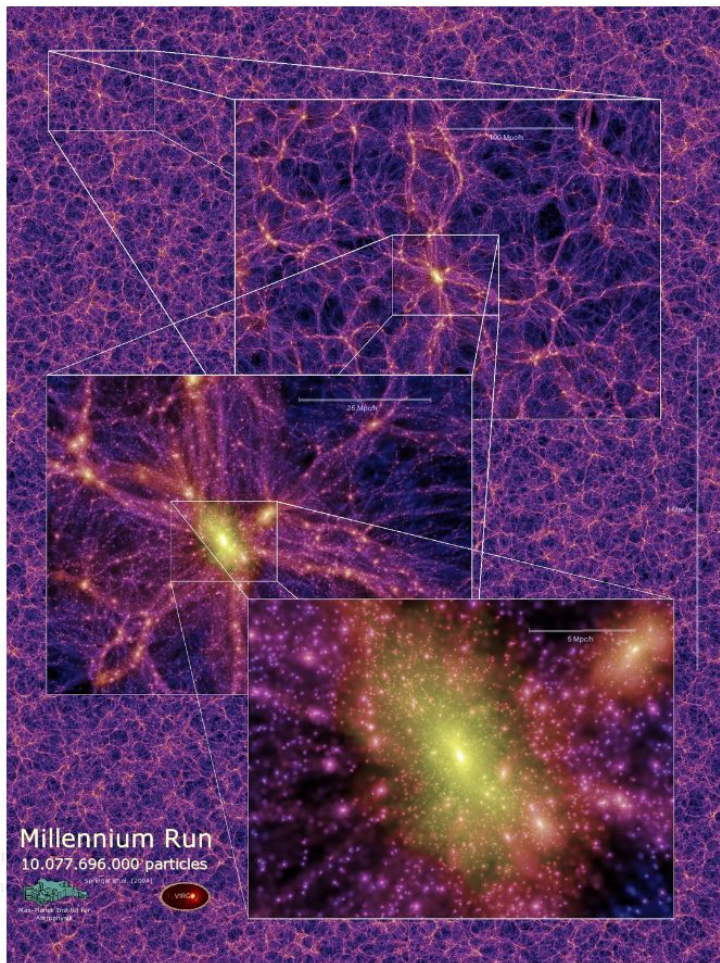
$$h_2^{(0)}(\mathbf{k}) = -7 \frac{\int_0^{\infty} \int_{-1}^1 dx dr \frac{r(r+x-2rx^2)}{(1+r^2-2rx) \left(1+r^2 \frac{k^2}{k_J^2}\right) \left(1+\frac{k^2}{k_J^2}(1+r^2-2rx)\right)}}{\int_0^{\infty} \int_{-1}^1 dx dr r \frac{7x-r-6rx^2}{1+r^2-2rx}} + \frac{2}{\frac{10}{3} + \frac{k^2}{k_J^2}}$$

$$\times \frac{\int_0^{\infty} \int_{-1}^1 dx dr r \frac{3r+7x-10rx^2}{1+r^2-2rx}}{\int_0^{\infty} \int_{-1}^1 dx dr r \frac{7x-r-6rx^2}{1+r^2-2rx}} \left[1 + \frac{7}{3} \frac{\int_0^{\infty} \int_{-1}^1 dx dr \frac{r \frac{3r+7x-10rx^2}{1+r^2-2rx} + 3 \frac{k^2}{k_J^2} r^2}{\left(1+r^2 \frac{k^2}{k_J^2}\right) \left(1+(1+r^2-2rx) \frac{k^2}{k_J^2}\right)} \int_0^{\infty} \int_{-1}^1 dx dr r \frac{3r+7x-10rx^2}{1+r^2-2rx} \right]$$



Consideraciones finales

1. Demostramos que las funciones de filtrado de Jeans exhiben una variación considerable en la escala de filtrado, como consecuencia de los efectos no lineales inducidos por la presión bariónica a segundo orden.
2. Estas variaciones presentan relevantes implicaciones para corregir la masa de filtrado de Jeans, lo que indica que la temperatura del gas bariónico que se infiere del flujo del espectro de potencias de materia, estaría subestimada.
3. La función de filtrado de Jeans, no es simplemente una herramienta para modelar el gas cósmico, sino que establece un paradigma físico para el sesgo dependiente de la escala. Las futuras mediciones de alta precisión del sesgo galáctico, particularmente a altos corrimientos al rojo y para poblaciones de galaxias ricas en gas, podrían buscar la huella impresa por este filtrado físico fundamental, revelando así la conexión entre la cosmología lineal y la formación no lineal de galaxias.
4. Seguimos trabajando en esto. La idea es conectar el campo de velocidades peculiares con el campo de densidad de materia, para luego conectar con las observaciones.



¡Muchas gracias!

<https://arxiv.org/pdf/2511.08820>