QuanTIC
Quantum Technologies
Information & Complexity

CIBioFi
Centre for Bioinformatics
and Photonics

80 años
Construyendo Futuro

Universidad del Valle

OPTIMIZATION OF THE SCHRÖDINGER-POISSON MODEL USING B-SPLINES

COSMOLOGY in COLOMBIA
CoCo 2025

JEISON S. LENIS
CÉSAR A. VALENZUELA-TOLEDO
J. BAYRON ORJUELA-QUINTANA
JAVIER MADROÑERO P.

CONTENTS

MOTIVATION

Is possible to explain the formation of structures in a way other than the N-body model?

Picture generates with AI (ChatGPT)

MOTIVATION

N-body system

$$E_N = \sum_{i=1}^{N} \left(\frac{\mathbf{u}_i^2}{2a^2} - \frac{mG}{2a} \sum_{j=1, j\neq i}^{N} \frac{1}{|\mathbf{x}_i - \mathbf{x}_j|} \right),$$

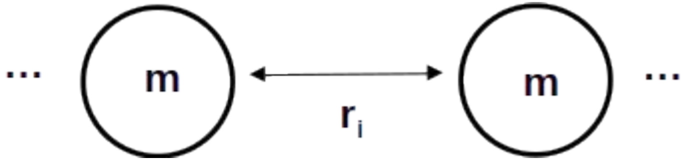
$$\dot{\mathbf{x}}_i = \frac{\partial E_N}{\partial \mathbf{u}_i}, \quad \dot{\mathbf{u}}_i = \frac{\partial E_N}{\partial \mathbf{x}_i}. \quad \Rightarrow \quad \{\mathbf{x}_i(t), \mathbf{u}_i(t)\}.$$

MOTIVATION

N-body system

$$E_N = \sum_{i=1}^{N} \left(\frac{\mathbf{u}_i^2}{2a^2} - \frac{mG}{2a} \sum_{j=1, j\neq i}^{N} \frac{1}{|\mathbf{x}_i - \mathbf{x}_j|} \right),$$

$$\dot{\mathbf{x}}_i = \frac{\partial E_N}{\partial \mathbf{u}_i}, \quad \dot{\mathbf{u}}_i = \frac{\partial E_N}{\partial \mathbf{x}_i}. \quad \Rightarrow \quad \{\mathbf{x}_i(t), \mathbf{u}_i(t)\}.$$

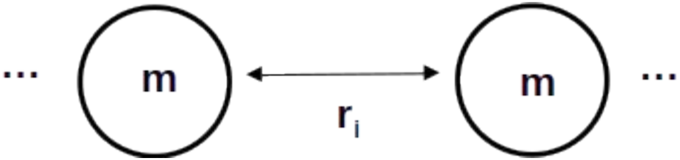


MOTIVATION

N-body system

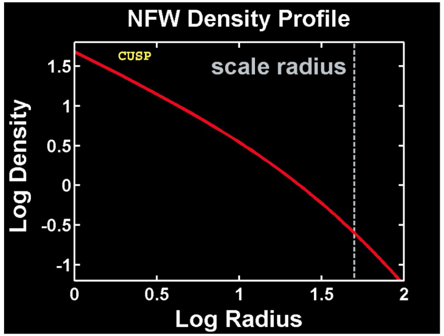
$$E_N = \sum_{i=1}^{N} \left(\frac{\mathbf{u}_i^2}{2a^2} - \frac{mG}{2a} \sum_{j=1, j\neq i}^{N} \frac{1}{|\mathbf{x}_i - \mathbf{x}_j|} \right),$$

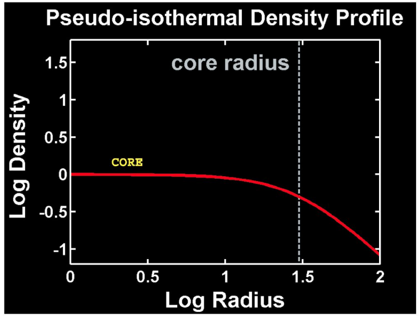
$$\dot{\mathbf{x}}_i = \frac{\partial E_N}{\partial \mathbf{u}_i}, \quad \dot{\mathbf{u}}_i = \frac{\partial E_N}{\partial \mathbf{x}_i}. \Rightarrow \quad \{\mathbf{x}_i(t), \mathbf{u}_i(t)\}.$$



CUSP

CORE





A. Del Popolo & M. Le Delliou, 2021. [Review of Solutions to the Cusp-Core Problem of the ΛCDM Model](#).

MOTIVATION

Dust model

$$\nabla \times \mathbf{u}_d = 0,$$

$$\partial_t \mathbf{u}_d = -\frac{1}{a^2}\left(\mathbf{u}_d \cdot \nabla\right)\mathbf{u}_d - \nabla \phi_d,$$

$$\partial_t n_d = -\frac{1}{a^2}\nabla \cdot \left(n_d \mathbf{u}_d\right),$$

$$\Delta \phi_d = \frac{4\pi G \rho_0}{a}\left(n_d - 1\right).$$

MOTIVATION

Dust model

$$\nabla \times \mathbf{u}_d = 0,$$

$$\partial_t \mathbf{u}_d = -\frac{1}{a^2}\left(\mathbf{u}_d \cdot \nabla\right)\mathbf{u}_d - \nabla\phi_d,$$

$$\partial_t n_d = -\frac{1}{a^2}\nabla \cdot \left(n_d \mathbf{u}_d\right),$$

$$\Delta\phi_d = \frac{4\pi G\rho_0}{a}\left(n_d - 1\right).$$

$$f_d(t, \mathbf{x}, \mathbf{u}) = n_d(t, \mathbf{x})\delta_D\left[\mathbf{u} - \nabla\phi_d(t, \mathbf{x})\right],$$

MOTIVATION

Dust model

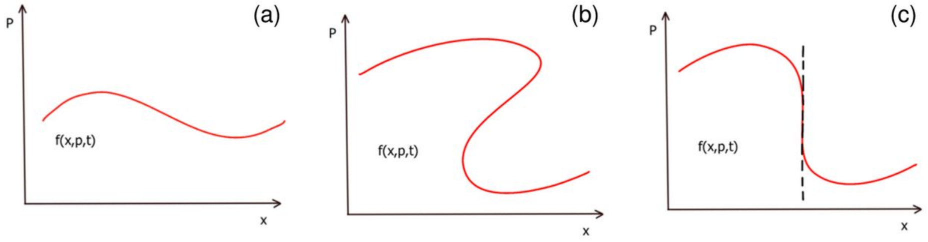
$$\nabla \times \mathbf{u}_d = 0,$$

$$\partial_t \mathbf{u}_d = -\frac{1}{a^2}\left(\mathbf{u}_d \cdot \nabla\right)\mathbf{u}_d - \nabla\phi_d,$$

$$\partial_t n_d = -\frac{1}{a^2}\nabla \cdot \left(n_d \mathbf{u}_d\right),$$

$$\Delta\phi_d = \frac{4\pi G\rho_0}{a}\left(n_d - 1\right).$$

$$f_d(t,\mathbf{x},\mathbf{u}) = n_d(t,\mathbf{x})\delta_D\left[\mathbf{u} - \nabla\phi_d(t,\mathbf{x})\right],$$



V. Loaíza. 2023. Numerical study of the Schrödinger-Poisson model for the formation of cosmic structures.

SCHRÖDINGER–POISSON MODEL

$$i\partial_\tau \psi(\tau,\xi) = \left[-\frac{1}{2}\Delta_\xi + a(\tau)U(\tau,\xi)\right]\psi(\tau,\xi),$$

$$\Delta_\xi U(\tau,\xi) = \left|\psi(\tau,\xi)\right|^2 - 1.$$

SCHRÖDINGER–POISSON MODEL

scalar complex field

scalefactor

$$i\partial_\tau \psi(\tau,\xi) = \left[-\frac{1}{2}\Delta_\xi + a(\tau)\, U(\tau,\xi)\right]\psi(\tau,\xi),$$

$$\Delta_\xi U(\tau,\xi) = \left|\psi(\tau,\xi)\right|^2 - 1. \Longrightarrow U\left(\left|\psi^2\right|\right)$$

self-consistent potential

SCHRÖDINGER–POISSON MODEL

$$i\partial_\tau \psi(\tau,\xi) = \Big[\underbrace{-\frac{1}{2}\Delta_\xi + \overbrace{a(\tau)U(\tau,\xi)}^{\mathbf{V}}\Big]}_{\mathbf{H}}\psi(\tau,\xi),$$

$$\Delta_\xi U(\tau,\xi) = \left|\psi(\tau,\xi)\right|^2 - 1.$$

Conformal Transformation

$$d\tau \equiv \frac{1}{a^2}\left[\frac{3}{2}H_0^2\Omega_{m,0}\right]^{\frac{1}{2}} dt, \quad U(\tau,\xi) \equiv \frac{a}{\mu}\left[\frac{3}{2}H_0^2\Omega_{m,0}\right]^{-\frac{1}{2}} V(\tau,\xi),$$

$$\xi \equiv \frac{1}{\mu^{\frac{1}{2}}}\left[\frac{3}{2}H_0^2\Omega_{m,0}\right]^{\frac{1}{4}} \mathbf{x}, \quad \psi(\tau,\xi) \equiv \frac{\Psi(\tau,\xi)}{\sqrt{\left|\Psi(\tau,\xi)\right|^2}}.$$

Kopp, Vattis, and Skordis. Solving the Vlasov equation in two spatial dimensions with the Schrödinger method. arXiv:1711.00140
T. Zimermann. 2018. A Simple Model for the Temporal Evolution of Cold Dark Matter.

SCHRÖDINGER-POISSON MODEL

$$i\partial_\tau \psi(\tau,\xi) = \underbrace{\left[-\frac{1}{2}\Delta_\xi + \overbrace{a(\tau)U(\tau,\xi)}^{V}\right]}_{H} \psi(\tau,\xi),$$

$$\Delta_\xi U(\tau,\xi) = \left|\psi(\tau,\xi)\right|^2 - 1.$$

Madelung representation

$$\psi(t,\mathbf{x}) := \sqrt{n(t,\mathbf{x})}\exp\left(i\frac{\phi(t,\mathbf{x})}{\mu}\right),$$

$$\nabla\times\mathbf{u} = 0, \qquad \partial_t n = -\frac{1}{a^2}\nabla\cdot(n\mathbf{u}),$$

$$\partial_t\mathbf{u} = -\frac{1}{a^2}(\mathbf{u}\cdot\nabla)\mathbf{u} - \nabla V + \underbrace{\frac{\mu^2}{2a^2}\nabla\left(\frac{\Delta\sqrt{n}}{\sqrt{n}}\right)}_{Q},$$

$$\Delta V = \frac{4\pi G\rho_0}{a}(n-1).$$

$$n(t,\mathbf{x}) \neq 0, \qquad \mathbf{u}(t,\mathbf{x}) \equiv \nabla\phi(t,\mathbf{x}).$$

SCHRÖDINGER-POISSON MODEL

$$i\partial_\tau \psi(\tau,\xi) = \underbrace{\left[-\frac{1}{2}\Delta_\xi + \overbrace{a(\tau)U(\tau,\xi)}^{V}\right]}_{H}\psi(\tau,\xi),$$

$$\Delta_\xi U(\tau,\xi) = \left|\psi(\tau,\xi)\right|^2 - 1.$$

Madelung representation

$$\psi(t,\mathbf{x}) := \sqrt{n(t,\mathbf{x})}\exp\left(i\frac{\phi(t,\mathbf{x})}{\mu}\right),$$

$$\nabla\times\mathbf{u} = 0, \qquad \partial_t n = -\frac{1}{a^2}\nabla\cdot(n\mathbf{u}),$$

$$\partial_t \mathbf{u} = -\frac{1}{a^2}(\mathbf{u}\cdot\nabla)\mathbf{u} - \nabla V + \underbrace{\frac{\mu^2}{2a^2}\nabla\left(\frac{\Delta\sqrt{n}}{\sqrt{n}}\right)}_{Q},$$

$$\Delta V = \frac{4\pi G\rho_0}{a}(n-1). \qquad \mu \equiv \frac{\hbar}{m}$$

$$n(t,\mathbf{x}) \neq 0, \qquad \mathbf{u}(t,\mathbf{x}) \equiv \nabla\phi(t,\mathbf{x}).$$

SCHRÖDINGER-POISSON MODEL

$$i\partial_\tau \psi(\tau,\xi) = \underbrace{\left[-\frac{1}{2}\Delta_\xi + \overbrace{a(\tau)U(\tau,\xi)}^{\mathbf{V}}\right]}_{\mathbf{H}} \psi(\tau,\xi),$$

$$\Delta_\xi U(\tau,\xi) = \left|\psi(\tau,\xi)\right|^2 - 1.$$

FUZZY DARK MATTER

Madelung representation

$$\psi(t,\mathbf{x}) := \sqrt{n(t,\mathbf{x})}\exp\left(i\frac{\phi(t,\mathbf{x})}{\mu}\right),$$

$$\nabla\times\mathbf{u} = 0, \qquad \partial_t n = -\frac{1}{a^2}\nabla\cdot(n\mathbf{u}),$$

$$\partial_t\mathbf{u} = -\frac{1}{a^2}(\mathbf{u}\cdot\nabla)\mathbf{u} - \nabla V + \underbrace{\frac{\mu^2}{2a^2}\nabla\left(\frac{\Delta\sqrt{n}}{\sqrt{n}}\right)}_{Q},$$

$$\Delta V = \frac{4\pi G\rho_0}{a}(n-1). \qquad \mu \equiv \frac{\hbar}{m}$$

$$n(t,\mathbf{x}) \neq 0, \qquad \mathbf{u}(t,\mathbf{x}) \equiv \nabla\phi(t,\mathbf{x}).$$

SCHRÖDINGER–POISSON MODEL

$$i\partial_\tau \psi(\tau,\xi) = \underbrace{\Big[-\frac{1}{2}\Delta_\xi + \overbrace{a(\tau)U(\tau,\xi)}^{V}\Big]}_{H}\psi(\tau,\xi),$$

$$\Delta_\xi U(\tau,\xi) = \big|\psi(\tau,\xi)\big|^2 - 1.$$

Defines time propagation

Madelung representation

$$\psi(t,\mathbf{x}) := \sqrt{n(t,\mathbf{x})}\exp\left(i\frac{\phi(t,\mathbf{x})}{\mu}\right),$$

$$\nabla\times\mathbf{u} = 0, \qquad \partial_t n = -\frac{1}{a^2}\nabla\cdot(n\mathbf{u}),$$

$$\partial_t \mathbf{u} = -\frac{1}{a^2}(\mathbf{u}\cdot\nabla)\mathbf{u} - \nabla V + \underbrace{\frac{\mu^2}{2a^2}\nabla\left(\frac{\Delta\sqrt{n}}{\sqrt{n}}\right)}_{Q},$$

$$\Delta V = \frac{4\pi G\rho_0}{a}(n-1).$$

$$n(t,\mathbf{x}) \neq 0, \qquad \mathbf{u}(t,\mathbf{x}) \equiv \nabla\phi(t,\mathbf{x}).$$

SCHRÖDINGER-POISSON MODEL

$$i\partial_\tau \psi(\tau,\xi) = \underbrace{\left[-\frac{1}{2}\Delta_\xi + \overbrace{a(\tau)U(\tau,\xi)}^{V}\right]}_{H}\psi(\tau,\xi),$$

$$\Delta_\xi U(\tau,\xi) = \left|\psi(\tau,\xi)\right|^2 - 1.$$

Dynamical systems

ΛCDM
Einstein - De Sitter
Casimir

Y. Choquet-Bruhat,2008. General Relativity and the Einstein Equations.
S. Watson, 2000. An exposition on inflationary cosmology.
F. Pace, et. al., 2010. Spherical collapse model in dark-energy cosmologies.

SCHRÖDINGER–POISSON MODEL

V

$$i\partial_\tau \psi\left(\tau,\xi\right) = \left[-\frac{1}{2}\Delta_\xi + a(\tau)U\left(\tau,\xi\right)\right]\psi\left(\tau,\xi\right),$$

$$\Delta_\xi U\left(\tau,\xi\right) = \left|\psi\left(\tau,\xi\right)\right|^2 - 1.$$

H

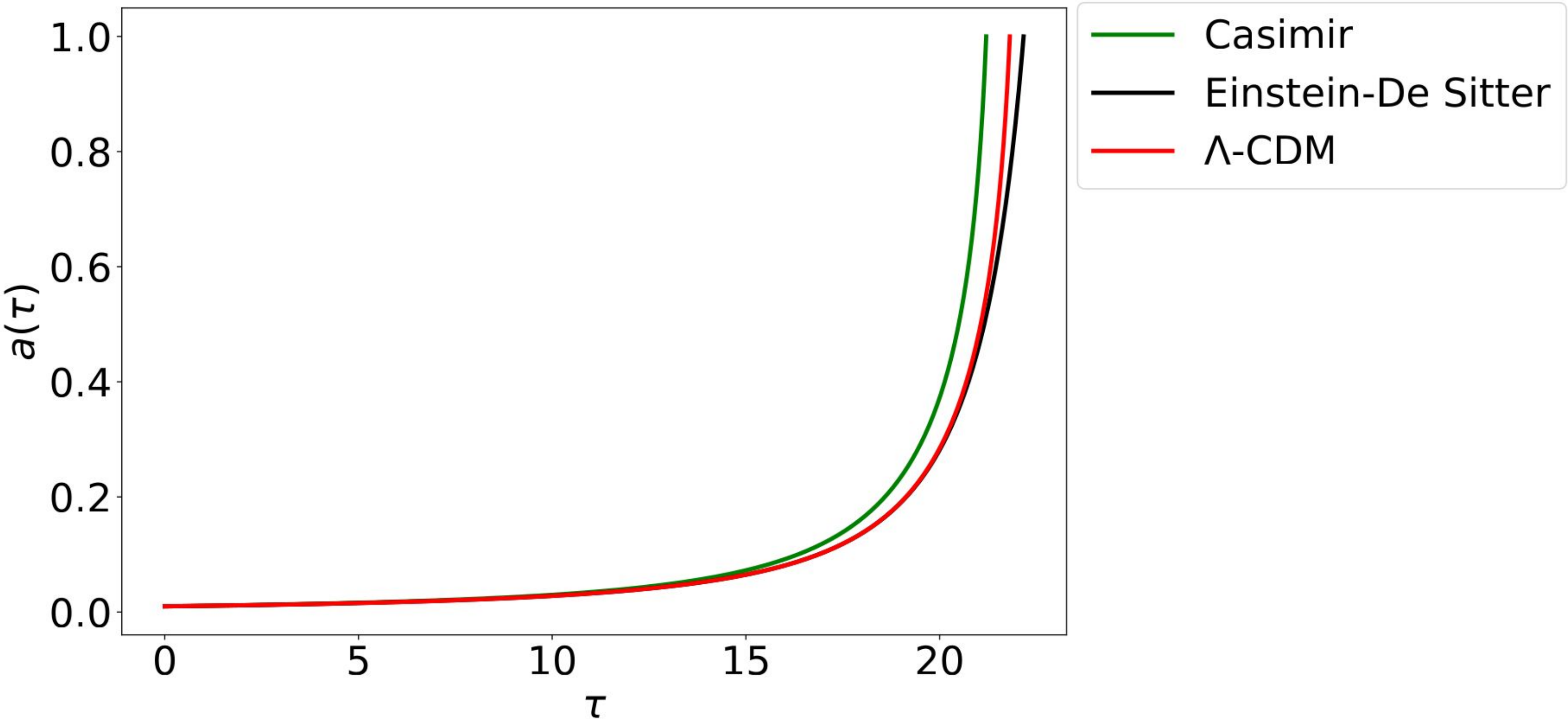
Dynamical systems

ΛCDM
Einstein - De Sitter
Casimir

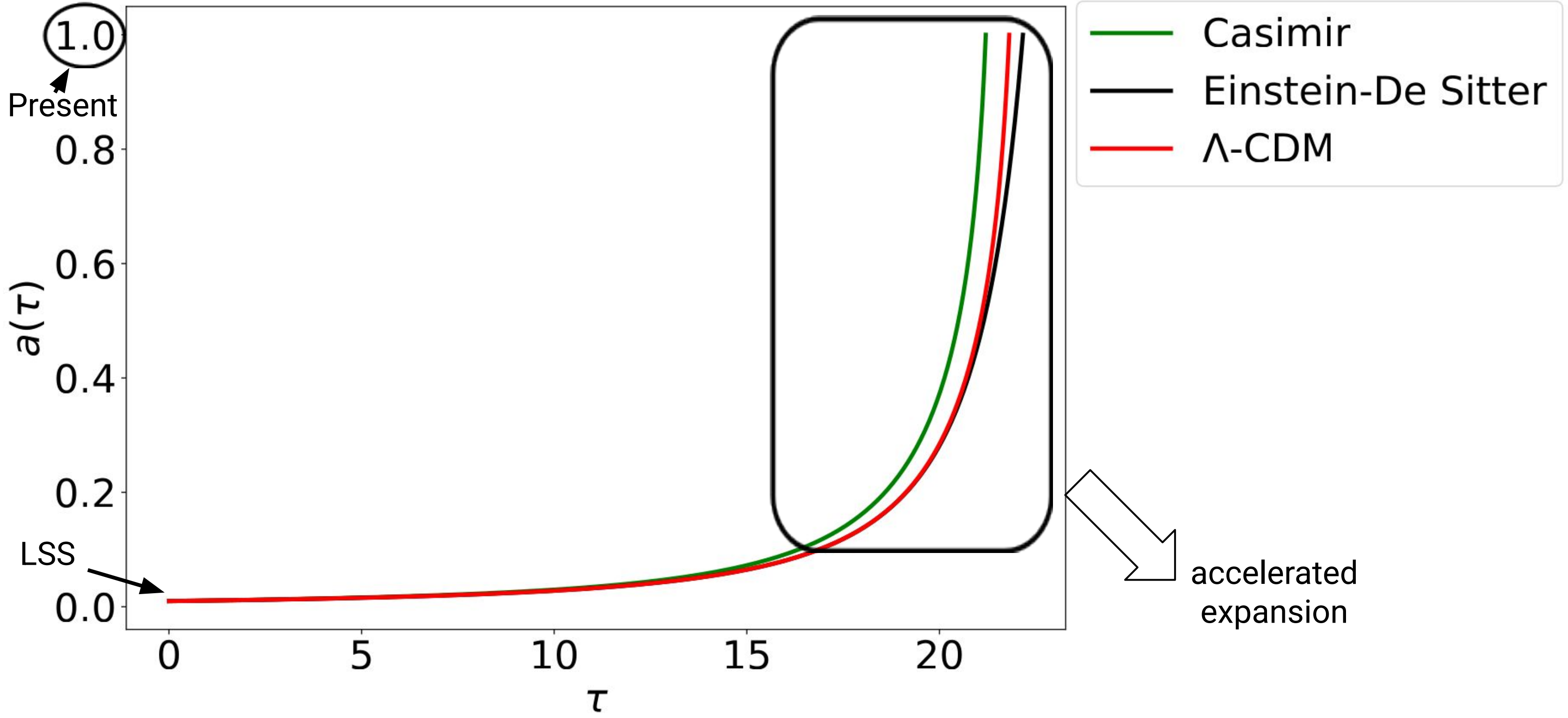
geometric considerations

Y. Choquet-Bruhat,2008. General Relativity and the Einstein Equations.
S. Watson, 2000. An exposition on inflationary cosmology.
F. Pace, et. al., 2010. Spherical collapse model in dark-energy cosmologies.

SCHRÖDINGER– POISSON MODEL



SCHRÖDINGER– POISSON MODEL



NUMERICAL IMPLEMENTATION

Crank-Nicolson propagator

$$\left(\hat{\mathrm{I}} + \frac{\mathrm{i}}{2}\hat{\hat{\mathrm{H}}}_1\right)\psi(\tau + \Delta\tau) = \left(\hat{\mathrm{I}} - \frac{\mathrm{i}}{2}\hat{\hat{\mathrm{H}}}_1\right)\psi(\tau),$$

NUMERICAL IMPLEMENTATION

Crank-Nicolson propagator

$$\left(\hat{\mathrm{I}} + \frac{\mathrm{i}}{2}\hat{\hat{\mathrm{H}}}_1\right)\psi(\tau+\Delta\tau) = \left(\hat{\mathrm{I}} - \frac{\mathrm{i}}{2}\hat{\hat{\mathrm{H}}}_1\right)\psi(\tau),$$

Identity matrix

$$\hat{\hat{\mathrm{H}}}_1 = \hat{\mathrm{K}} + \hat{\hat{\mathrm{V}}}_1$$

$$\hat{\hat{\mathrm{V}}}_1 = \frac{1}{\Delta\tau}\int_{\tau}^{\tau+\Delta\tau} d\tau' \hat{\mathrm{V}}(\tau')$$

$$\approx \frac{1}{2}\left(a(\tau)U(\tau) + a(\tau+\Delta\tau)U(\tau+\Delta\tau) + O(\Delta\tau^2)\right)$$

NUMERICAL IMPLEMENTATION

Crank-Nicolson propagator

$$\left(\hat{\mathbf{I}} + \frac{\mathrm{i}}{2}\hat{\hat{\mathbf{H}}}_1\right)\psi(\tau + \Delta\tau) = \left(\hat{\mathbf{I}} - \frac{\mathrm{i}}{2}\hat{\hat{\mathbf{H}}}_1\right)\psi(\tau),$$

$$\hat{\hat{\mathbf{H}}}_1 = \hat{\mathbf{K}} + \hat{\hat{\mathbf{V}}}_1$$

How?

$$\hat{\hat{\mathbf{V}}}_1 = \frac{1}{\Delta\tau}\int_{\tau}^{\tau+\Delta\tau} d\tau' \hat{\mathbf{V}}(\tau')$$

$$\approx \frac{1}{2}\left(a(\tau)U(\tau) + a(\tau+\Delta\tau)U(\tau+\Delta\tau) + O(\Delta\tau^2)\right)$$

NUMERICAL IMPLEMENTATION

Crank-Nicolson propagator

$$\left(\hat{\mathrm{I}} + \frac{\mathrm{i}}{2}\hat{\bar{\mathrm{H}}}_1\right)\psi(\tau+\Delta\tau) = \left(\hat{\mathrm{I}} - \frac{\mathrm{i}}{2}\hat{\bar{\mathrm{H}}}_1\right)\psi(\tau),$$

$$\hat{\bar{\mathrm{H}}}_1 = \hat{\mathrm{K}} + \hat{\bar{\mathrm{V}}}_1$$

$$\hat{\bar{\mathrm{V}}}_1 = \frac{1}{\Delta\tau}\int_{\tau}^{\tau+\Delta\tau} d\tau' \hat{\mathbf{V}}(\tau')$$

$$\approx \frac{1}{2}\left(a(\tau)U(\tau) + a(\tau+\Delta\tau)U(\tau+\Delta\tau) + O(\Delta\tau^2)\right)$$

Predictor-corrector scheme

$$\psi(\tau) \xrightarrow{\hat{\bar{\mathbf{V}}}_1' = a(\tau)U(\tau)} \tilde{\psi}(\tau+\Delta\tau),$$

$$\psi(\tau) \xrightarrow{\hat{\bar{\mathbf{V}}}_1 = \left(a(\tau)U(\tau) + a(\tau+\Delta\tau)\tilde{U}(\tau+\Delta\tau)\right)} \psi(\tau+\Delta\tau).$$

NUMERICAL IMPLEMENTATION

Crank-Nicolson propagator

$$\left(\hat{\mathrm{I}} + \frac{\mathrm{i}}{2}\hat{\bar{\mathrm{H}}}_1\right)\psi(\tau+\Delta\tau) = \left(\hat{\mathrm{I}} - \frac{\mathrm{i}}{2}\hat{\bar{\mathrm{H}}}_1\right)\psi(\tau),$$

$$\hat{\bar{\mathrm{H}}}_1 = \hat{\mathrm{K}} + \hat{\bar{\mathrm{V}}}_1$$

$$\hat{\bar{\mathrm{V}}}_1 = \frac{1}{\Delta\tau}\int_{\tau}^{\tau+\Delta\tau} d\tau' \hat{\mathrm{V}}(\tau')$$

$$\approx \frac{1}{2}\left(a(\tau)U(\tau) + a(\tau+\Delta\tau)U(\tau+\Delta\tau) + O(\Delta\tau^2)\right)$$

Predictor-corrector scheme

$$\psi(\tau) \xrightarrow{\hat{\bar{\mathrm{V}}}'_1 = a(\tau)U(\tau)} \tilde{\psi}(\tau+\Delta\tau),$$

$$\psi(\tau) \xrightarrow{\hat{\bar{\mathrm{V}}}_1 = \left(a(\tau)U(\tau) + a(\tau+\Delta\tau)\tilde{U}(\tau+\Delta\tau)\right)} \psi(\tau+\Delta\tau).$$

Predictor

Corrector

NUMERICAL IMPLEMENTATION

Crank-Nicolson propagator

$$\left(\hat{\mathbf{I}} + \frac{i}{2}\hat{\hat{\mathbf{H}}}_1\right)\psi(\tau+\Delta\tau) = \left(\hat{\mathbf{I}} - \frac{i}{2}\hat{\hat{\mathbf{H}}}_1\right)\psi(\tau),$$

$$\hat{\hat{\mathbf{H}}}_1 = \hat{\mathbf{K}} + \hat{\hat{\mathbf{V}}}_1$$

$$\hat{\hat{\mathbf{V}}}_1 = \frac{1}{\Delta\tau}\int_{\tau}^{\tau+\Delta\tau} d\tau' \hat{\mathbf{V}}(\tau')$$

$$\approx \frac{1}{2}\left(a(\tau)U(\tau) + a(\tau+\Delta\tau)U(\tau+\Delta\tau) + O(\Delta\tau^2)\right)$$

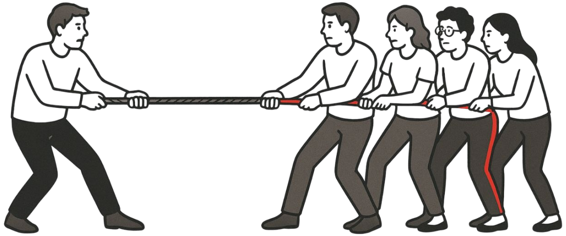
Predictor-corrector scheme

$$\psi(\tau) \xrightarrow{\hat{\hat{\mathbf{V}}}_1' = a(\tau)U(\tau)} \tilde{\psi}(\tau+\Delta\tau),$$

$$\psi(\tau) \xrightarrow{\hat{\hat{\mathbf{V}}}_1 = \left(a(\tau)U(\tau) + a(\tau+\Delta\tau)\tilde{U}(\tau+\Delta\tau)\right)} \psi(\tau+\Delta\tau).$$

Predictor

Corrector



NUMERICAL IMPLEMENTATION

Crank-Nicolson propagator

$$\left(\hat{\mathbf{I}} + \frac{\mathrm{i}}{2}\hat{\hat{\mathbf{H}}}_1\right)\psi(\tau+\Delta\tau) = \left(\hat{\mathbf{I}} - \frac{\mathrm{i}}{2}\hat{\hat{\mathbf{H}}}_1\right)\psi(\tau),$$

$$\hat{\hat{\mathbf{H}}}_1 = \hat{\mathbf{K}} + \hat{\hat{\mathbf{V}}}_1$$

$$\hat{\hat{\mathbf{V}}}_1 = \frac{1}{\Delta\tau}\int_{\tau}^{\tau+\Delta\tau} d\tau' \hat{\mathbf{V}}(\tau')$$

$$\approx \frac{1}{2}\left(a(\tau)U(\tau) + a(\tau+\Delta\tau)U(\tau+\Delta\tau) + O(\Delta\tau^2)\right)$$

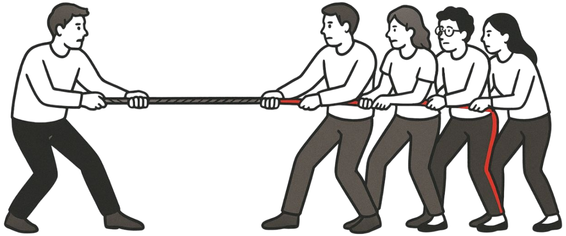
Predictor-corrector scheme

$$\psi(\tau) \xrightarrow{\hat{\hat{\mathbf{V}}}_1' = a(\tau)U(\tau)} \tilde{\psi}(\tau+\Delta\tau),$$

$$\psi(\tau) \xrightarrow{\hat{\hat{\mathbf{V}}}_1 = \left(a(\tau)U(\tau) + a(\tau+\Delta\tau)\tilde{U}(\tau+\Delta\tau)\right)} \psi(\tau+\Delta\tau).$$

Predictor

Corrector



One spatial dimension

NUMERICAL IMPLEMENTATION

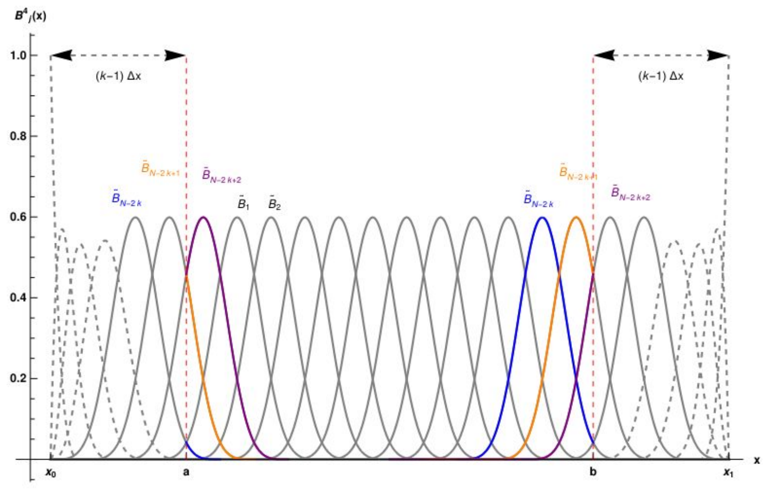
B-spline representation

$$\left(\mathbf{B} + \frac{\mathrm{i}}{2}\bar{\mathbf{H}}_1\right)\psi(\tau + \Delta\tau) = \left(\mathbf{B} - \frac{\mathrm{i}}{2}\bar{\mathbf{H}}_1\right)\psi(\tau).$$

NUMERICAL IMPLEMENTATION

B-spline representation

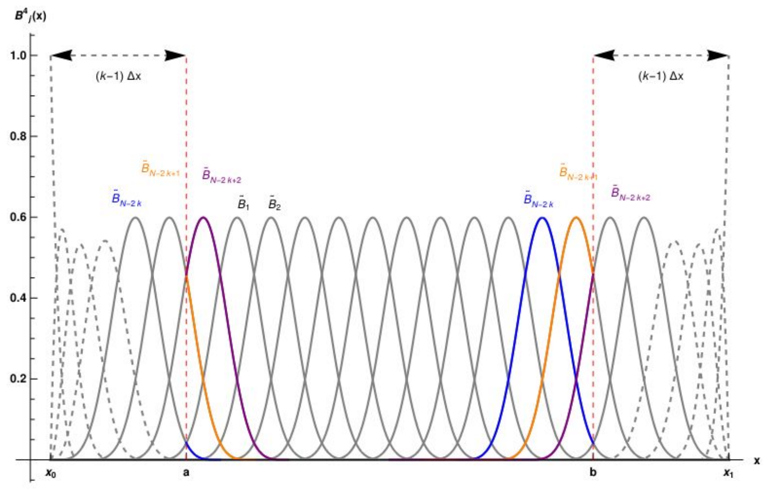
$$\left(\mathbf{B}+\frac{\mathrm{i}}{2}\bar{\mathbf{H}}_1\right)\psi(\tau+\Delta\tau)=\left(\mathbf{B}-\frac{\mathrm{i}}{2}\bar{\mathbf{H}}_1\right)\psi(\tau).$$

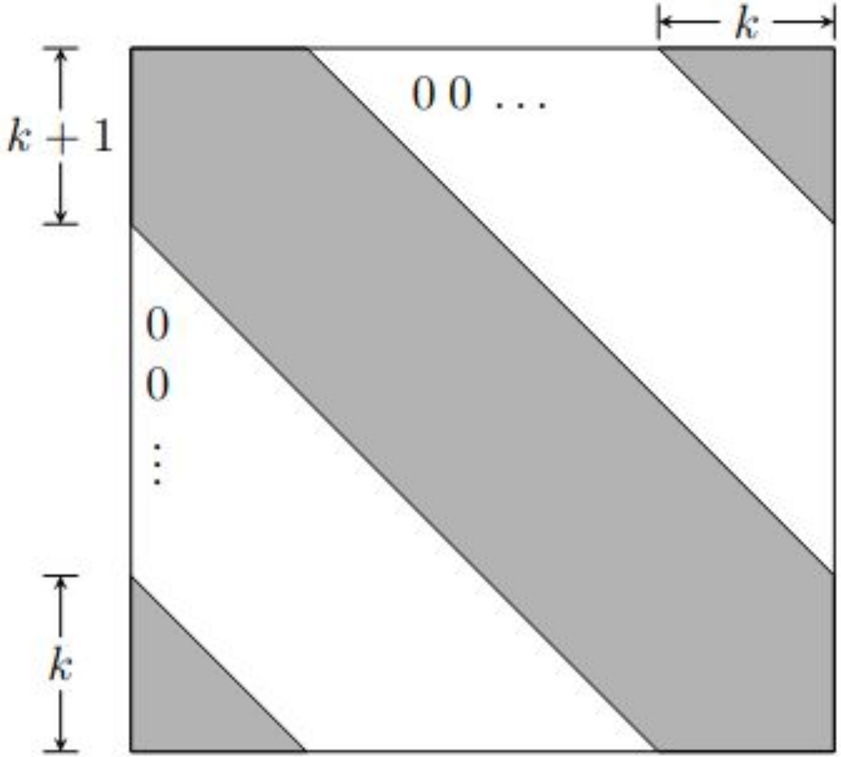


NUMERICAL IMPLEMENTATION

B-spline representation

$$\left(\mathbf{B}+\frac{\mathrm{i}}{2}\bar{\mathbf{H}}_1\right)\psi(\tau+\Delta\tau)=\left(\mathbf{B}-\frac{\mathrm{i}}{2}\bar{\mathbf{H}}_1\right)\psi(\tau).$$

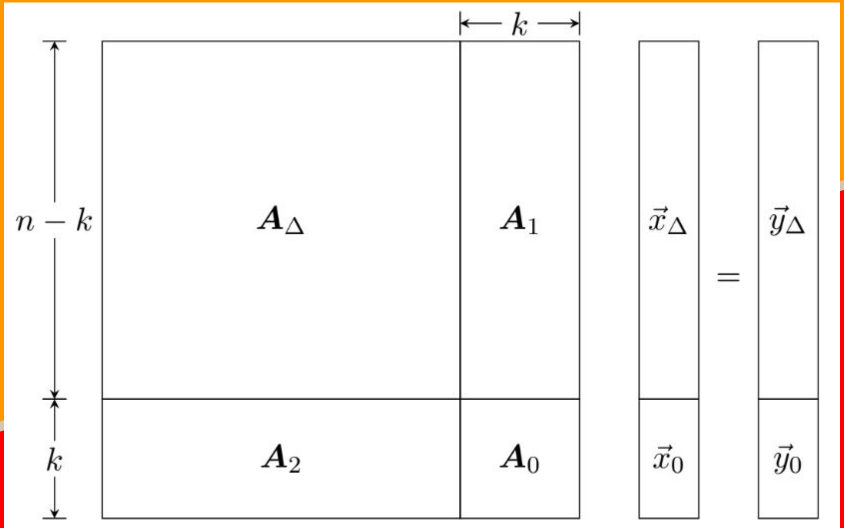


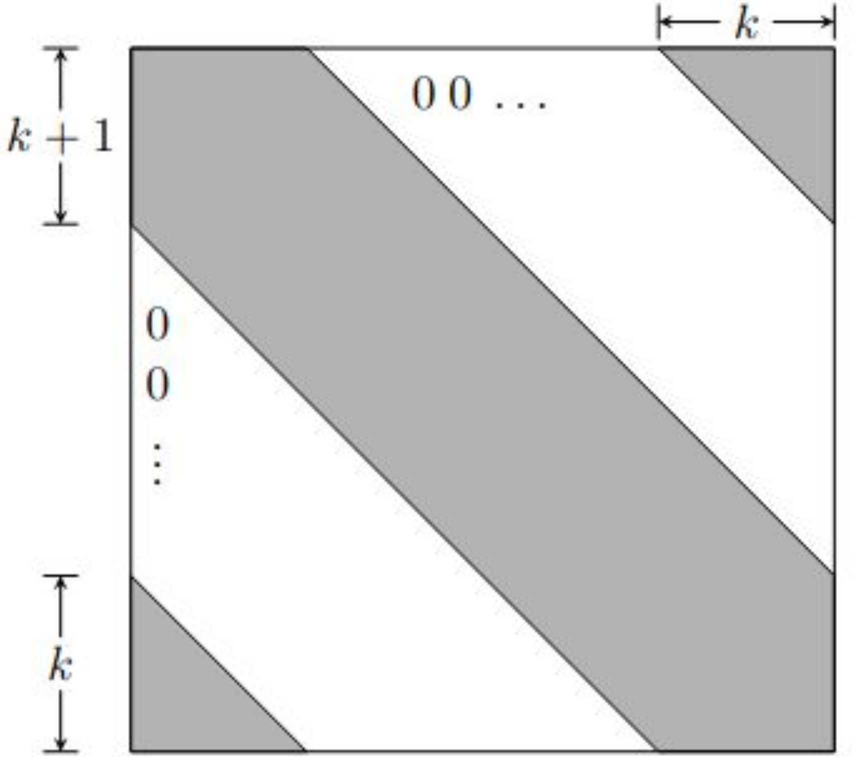


V. Loaíza. 2023. Numerical study of the Schrödinger-Poisson model for the formation of cosmic structures.

NUMERICAL IMPLEMENTATION

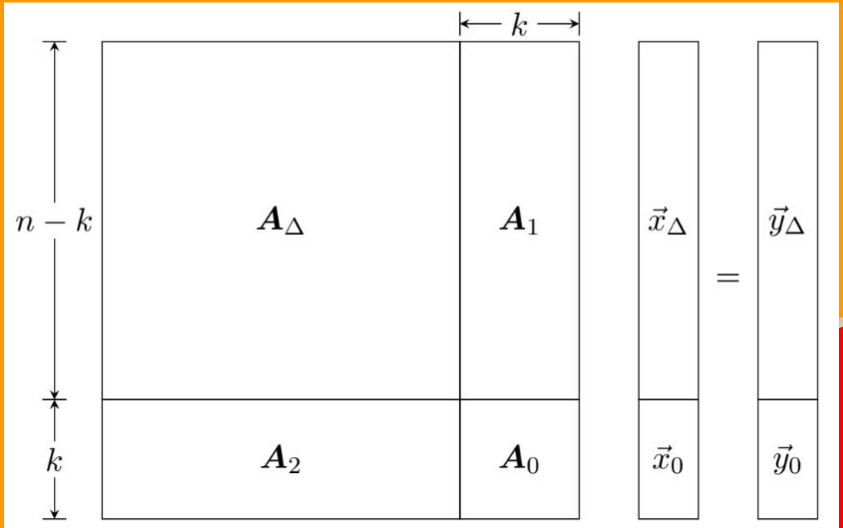
B-spline representation

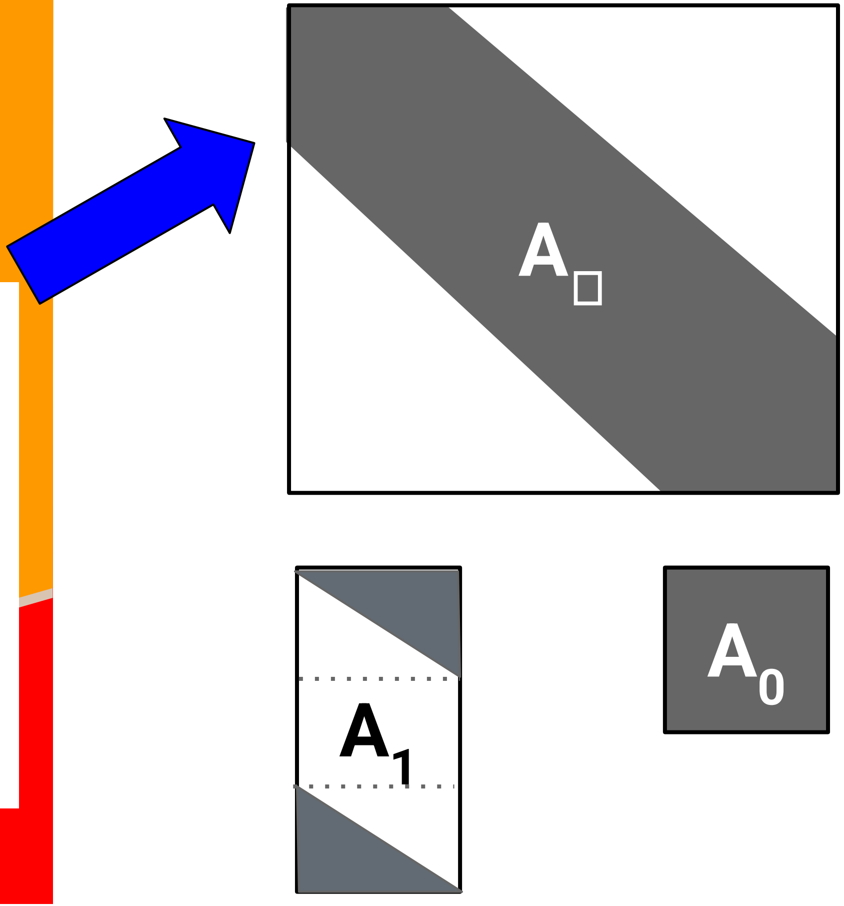




V. Loaíza. 2023. Numerical study of the Schrödinger-Poisson model for the formation of cosmic structures.

NUMERICAL IMPLEMENTATION





V. Loaíza. 2023. Numerical study of the Schrödinger-Poisson model for the formation of cosmic structures.

PRELIMINARY RESULTS

Matter Power Spectrum (1D)

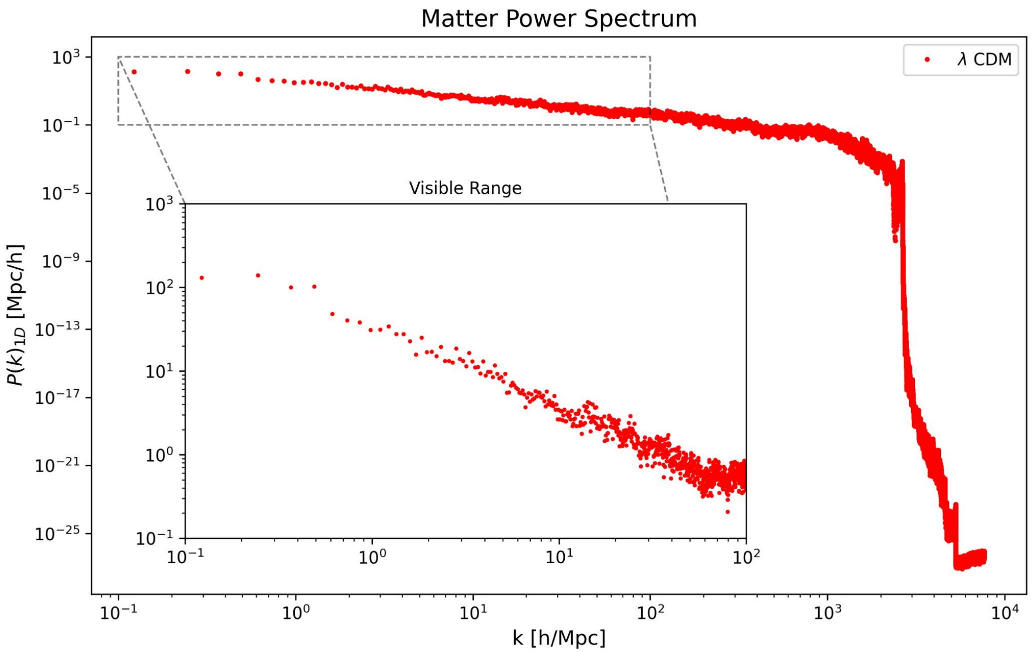
$$\delta(\tau, x) = \frac{\rho(\tau, x) - \rho_0(\tau)}{\rho_0(\tau)},$$

$$\delta(x) = |\psi(x)|^2 - 1, \;\; \delta(k) = \mathcal{F}(\delta(x)).$$

$$\langle \delta_0(k)\delta_0(k')\rangle = (2\pi)^3 \delta_D^{(3)}(k + k')P_{lin}(k).$$

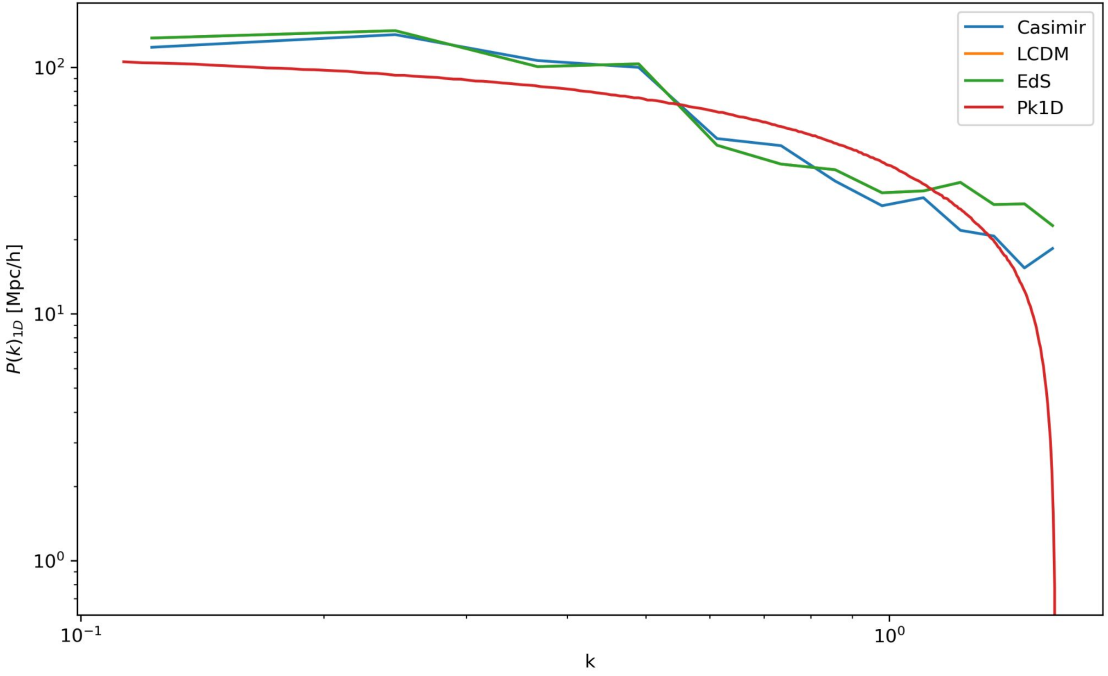
$$\Rightarrow P(k)_{1D} = \frac{1}{L}\langle \delta(k)\delta(k)\rangle.$$

~100 h^{-1} Mpc



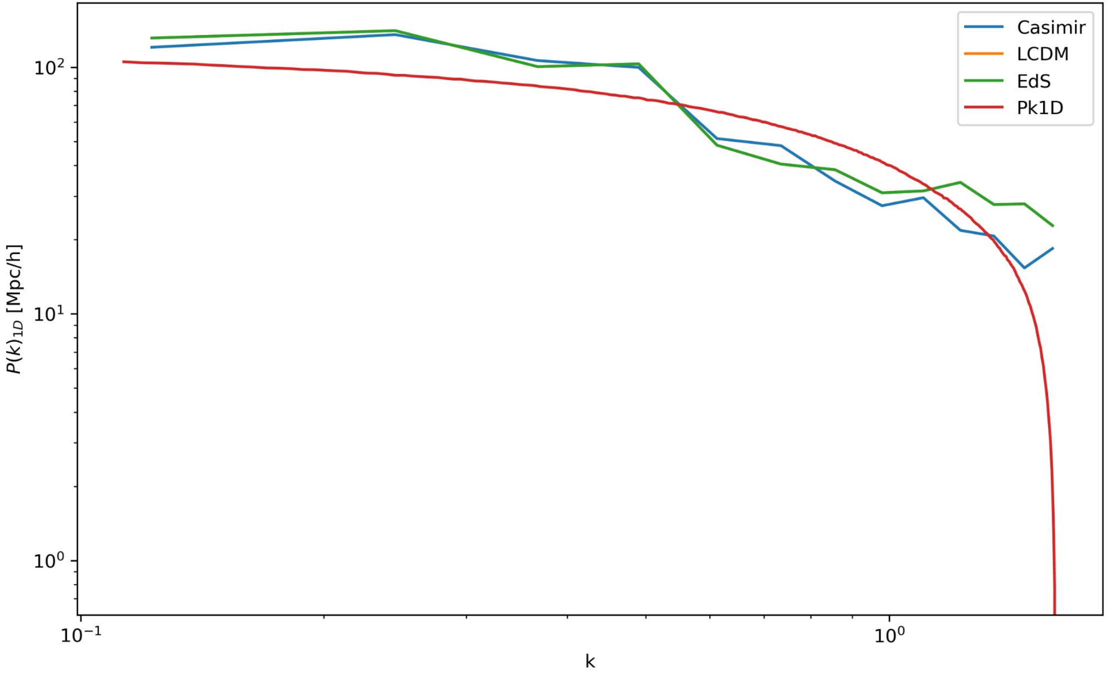
PRELIMINARY RESULTS

Observational regime



PRELIMINARY RESULTS

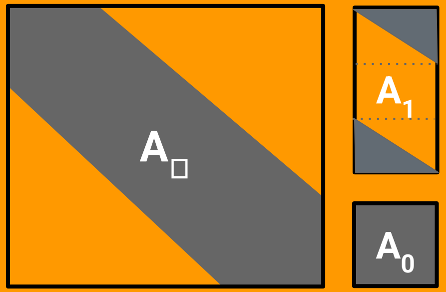
Observational regime



computational runtime of 2–6 months

OPTIMIZATION

To optimize computing time, let's keep in mind some of the things mentioned previously.

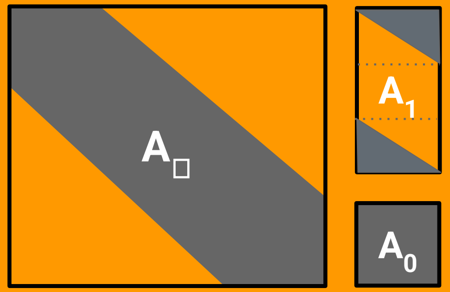


Overlapping matrix - B-spline

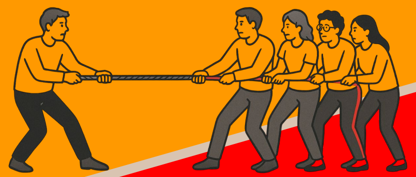
Predictor-corrector scheme

OPTIMIZATION

To optimize computing time, let's keep in mind some of the things mentioned previously.

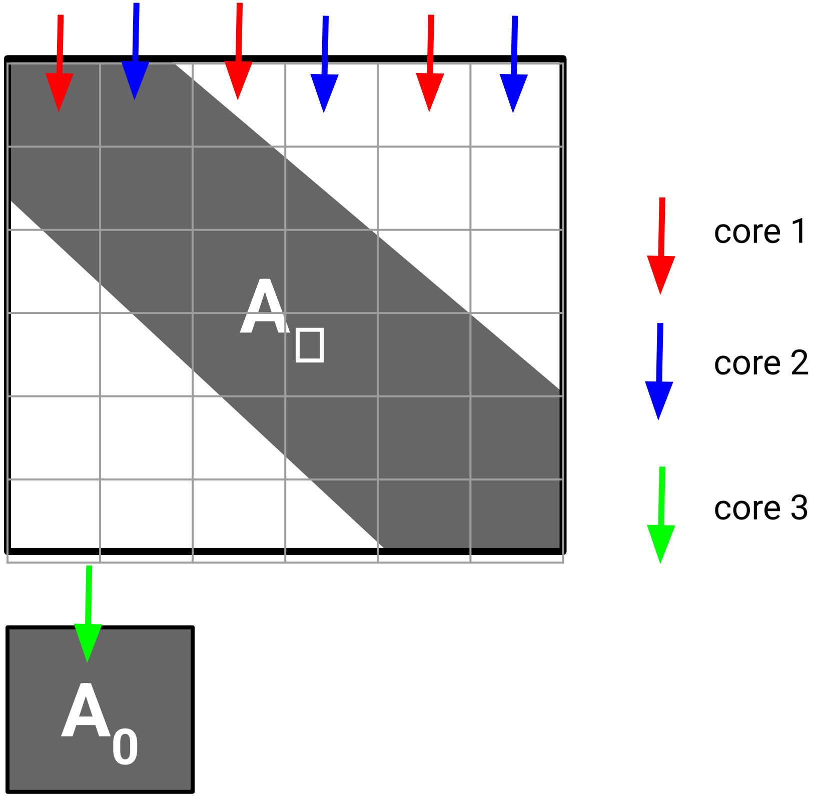


Overlapping matrix - B-spline

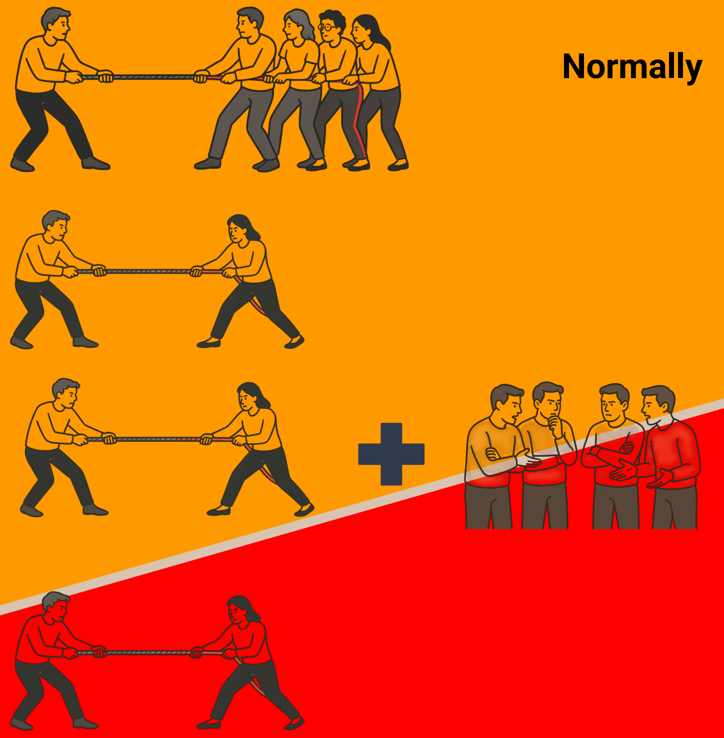


Predictor-corrector scheme

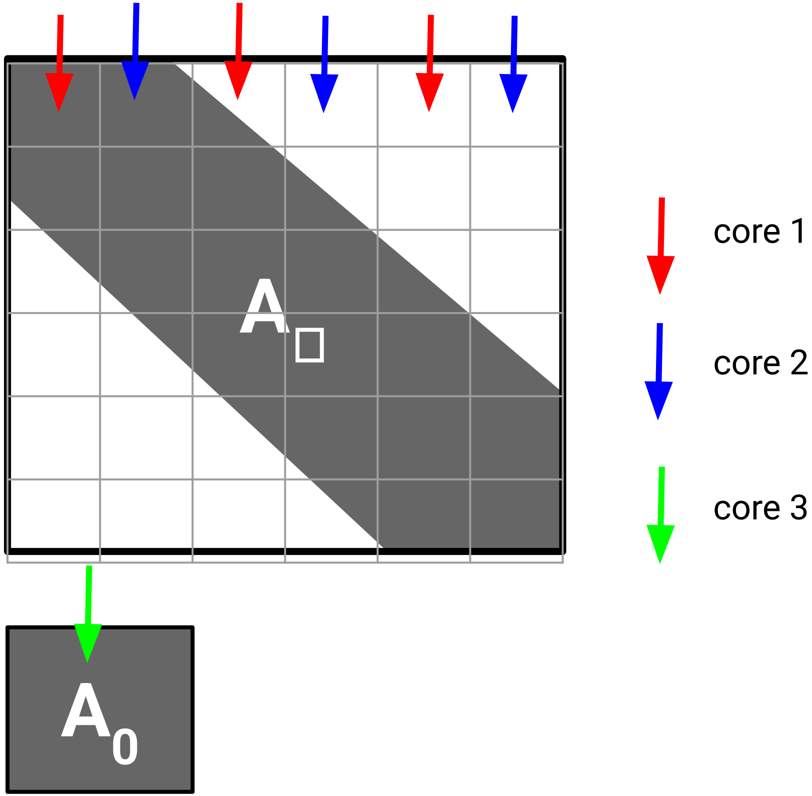
Message Passing Interface (MPI)



OPTIMIZATION

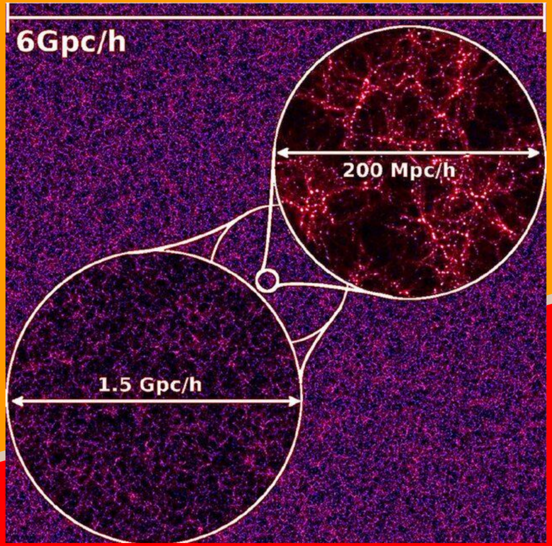


Message Passing Interface (MPI)



PERSPECTIVES

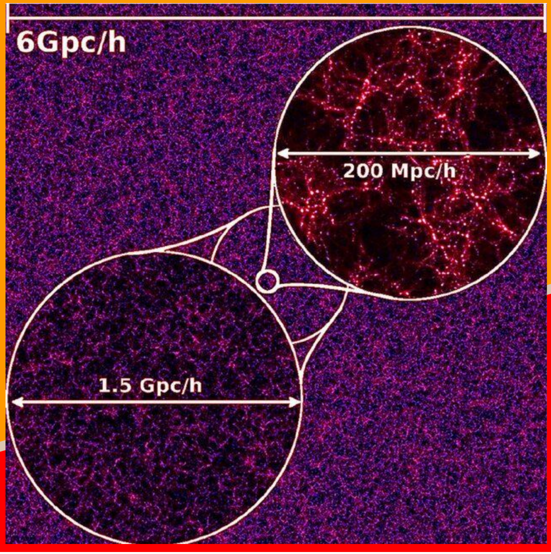
In order to obtain more realistic values, the scale must be expanded, as is the case with baryon acoustic oscillations (BAO).

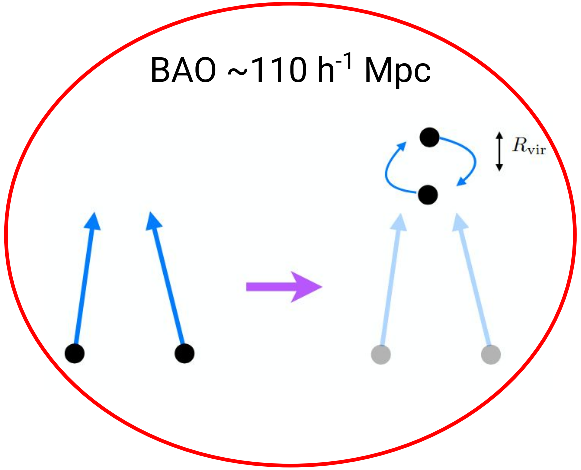


W. Watson, et. al, 2013. Statistics of extreme objects in the Juropa Hubble Volume simulation.

PERSPECTIVES

In order to obtain more realistic values, the scale must be expanded, as is the case with baryon acoustic oscillations (BAO).





W. Watson, et. al, 2013. Statistics of extreme objects in the Juropa Hubble Volume simulation.

M. Ivanov, 2022. Effective Field Theory for Large Scale Structure.

PERSPECTIVES

- Enlarge the box dimension (if possible to 1Gpc) to compare with N-body simulations.
- By using the self-consistent potential implement the numerical method to Bose-Einstein condensates (Gross-Pitaevskii), plasma physics, among others.
- Increase the number of random seeds to improve system statistics.

THANK YOU

Dark energy models

ΛCDM: Cosmological standard model.

$$\frac{da}{d\tau} = a^3 \left[\frac{2}{3\Omega_{m,0}} \left(\frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} \right) \right]^{\frac{1}{2}}$$

Einstein-De Sitter: Only contains matter.

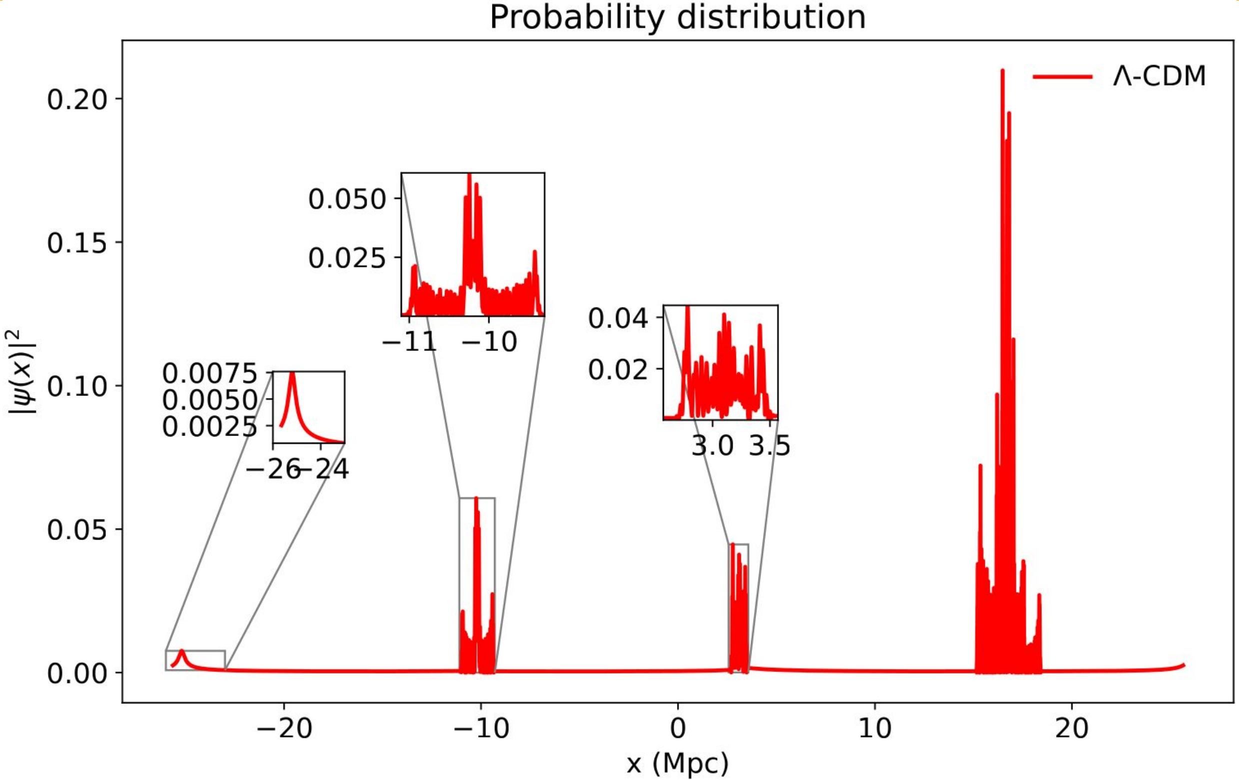
$$a(t) = \left(t + \frac{2}{3H_0} \right)^{\frac{2}{3}},$$

Casimir: Related to vacuum energy.

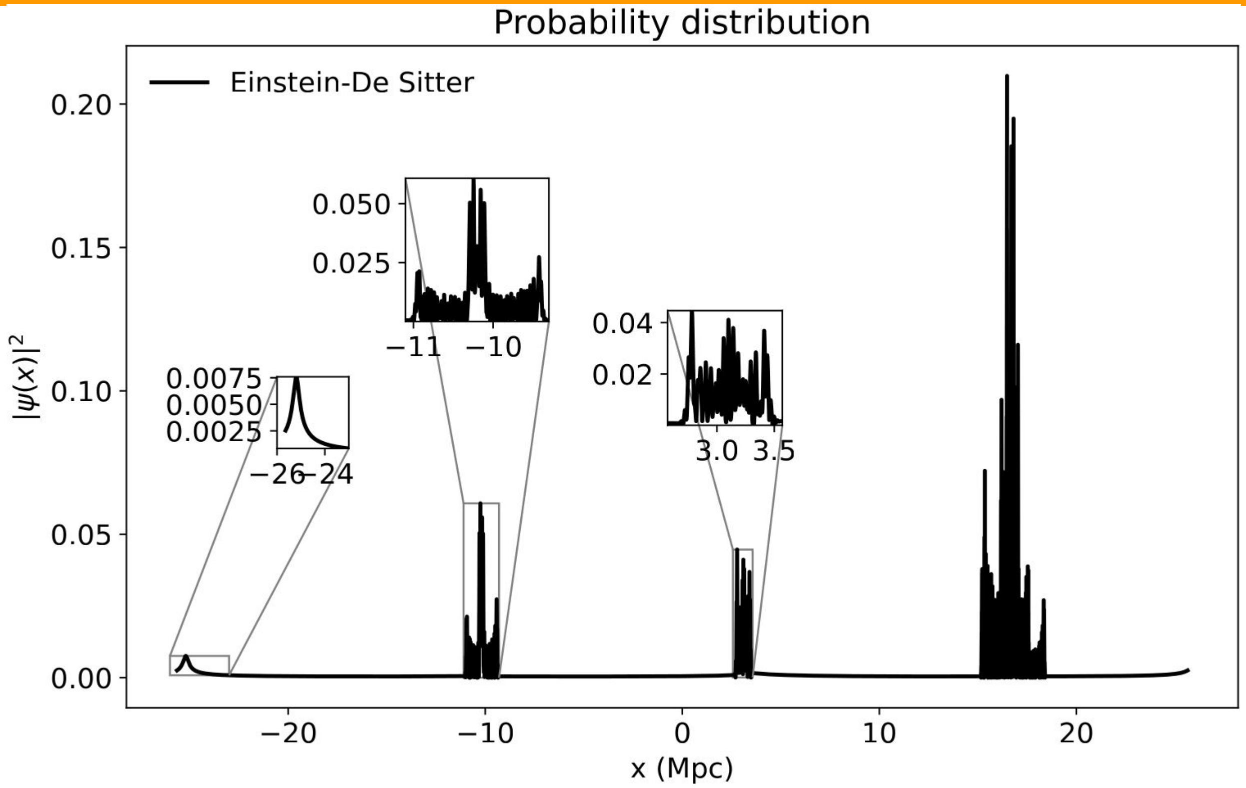
$$\omega(a) = -\frac{1}{3} \frac{3\Omega_{DE,0}a^4 + \Omega_{Cas,0}}{\Omega_{DE,0}a^4 - \Omega_{Cas,0}}$$

Y. Choquet-Bruhat,2008. General Relativity and the Einstein Equations.
S. Watson, 2000. An exposition on inflationary cosmology.
F. Pace, et. al., 2010. Spherical collapse model in dark-energy cosmologies.

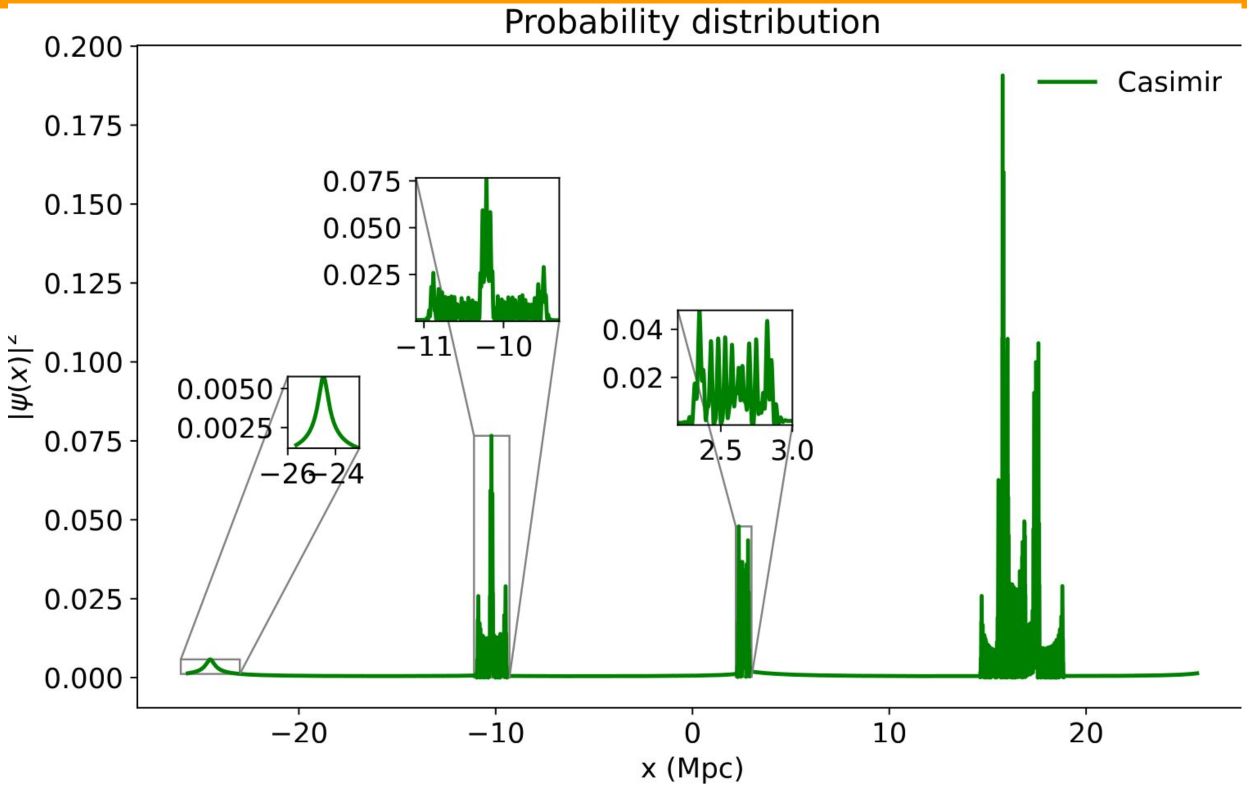
Probability distributions



Probability distributions



Probability distributions



SCHRÖDINGER–POISSON MODEL

V

$$i\partial_\tau \psi(\tau,\xi) = \underbrace{\left[-\frac{1}{2}\Delta_\xi + a(\tau) U(\tau,\xi)\right]}_{H} \psi(\tau,\xi),$$

$$\Delta_\xi U(\tau,\xi) = \left|\psi(\tau,\xi)\right|^2 - 1.$$

H

Dynamical systems

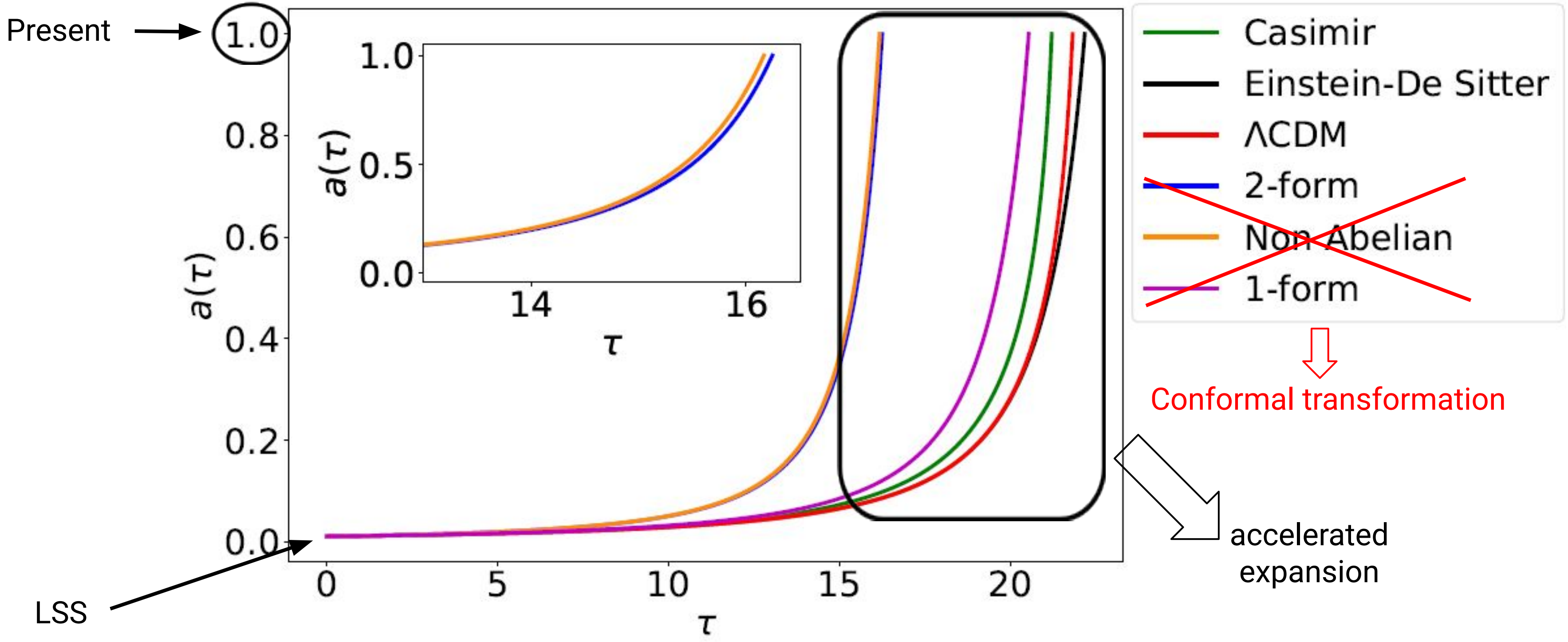
ΛCDM
Einstein - De Sitter
Casimir

1-form
2-form
Non Abelian

couplings with external fields

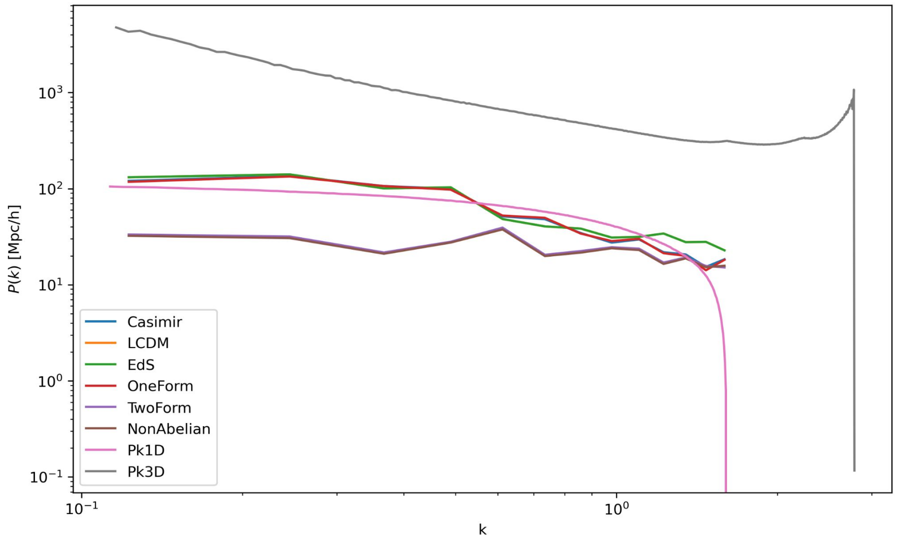
J. P. B. Almeida, et. al, 2020. Arbitrarily coupled p-forms in cosmological backgrounds.
J. P. B. Almeida, et. al., 2019. Anisotropic 2-form dark energy.
A. Guarnizo, et. al., 2020. Dynamical analysis of cosmological models with non-abelian gauge vector fields.

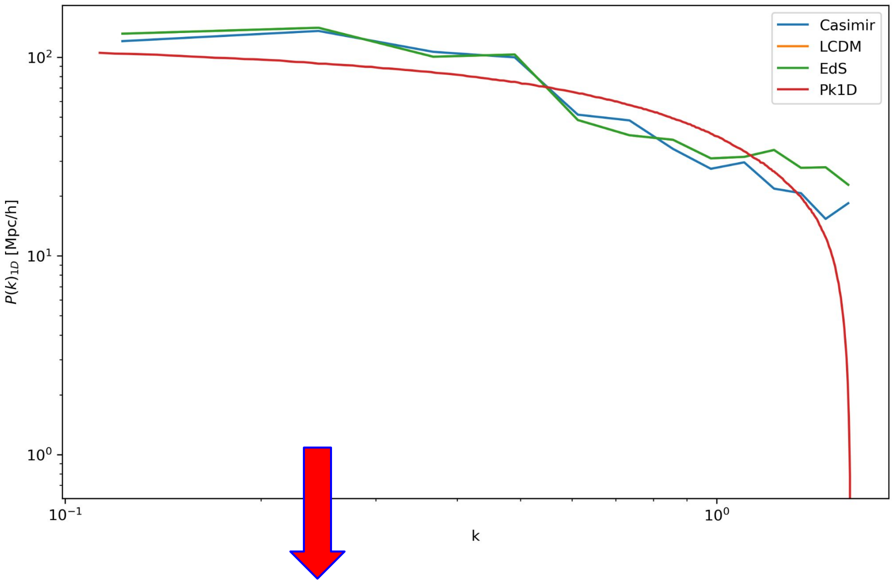
SCHRÖDINGER– POISSON MODEL



PRELIMINARY RESULTS

Observational regime





computational runtime of 2–6 months

Dark energy models

1-Form

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R + \mathcal{L}_{(1)} + \mathcal{L}_\phi + \mathcal{L}_x\right),$$

where,

$$\mathcal{L}_{(1)} = -\frac{1}{4}f_1(\phi)F_{(1)\mu\nu}F_{(1)}^{\mu\nu},$$

$$\mathcal{L}_\phi = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi),$$

$\mathcal{L}_X$ = Perfect fluid of matter or radiation.

$$X \equiv \frac{1}{\sqrt{6}M_P}\frac{\dot\phi}{H}, \qquad Y \equiv \frac{1}{\sqrt{3}M_P}\frac{\sqrt{V}}{H},$$

$$\Sigma = \frac{\dot\sigma}{H}, \qquad \Omega_i = \frac{\rho_i}{3H^2M_P^2}, \quad i = (1), r, m.$$

$$\Rightarrow \Omega_{DE} = X^2 + Y^2 + \Sigma^2 + \Omega_{(1)}.$$

then, the autonomous system is,

$$X' = \frac{3}{2}X\left(X^2 - Y^2 + \Sigma^2 - 1 - \frac{1}{3}\Omega_{(1)} + \frac{1}{3}\Omega_r\right) + \frac{\sqrt{6}}{2}\left(\lambda Y^2 - 2\mu\Omega_{(1)}\right),$$

$$Y' = \frac{1}{2}Y\left(3X^2 - 3Y^2 + 3\Sigma^2 + 3 - \sqrt{6}\lambda X + \Omega_{(1)} + \Omega_r\right),$$

$$\Sigma' = \frac{1}{2}\Sigma\left(3X^2 - 3Y^2 + 3\Sigma^2 - 3 + \Omega_{(1)} + \Omega_r\right) + 2\Omega_{(1)},$$

$$\Omega'_{(1)} = \Omega_{(1)}\left(3X^2 - 3Y^2 + 3\Sigma^2 + 4\Sigma - 1 + 2\sqrt{6}\lambda X + \Omega_{(1)} + \Omega_r\right),$$

$$\Omega'_r = \Omega_r\left(3X^2 - 3Y^2 + 3\Sigma^2 - 1 + \Omega_{(1)} + \Omega_r\right).$$

With the parameters

$$\lambda = 2$$

$$\mu = 5$$

$$z_0 = 5.5 \cdot 10^7.$$

J. P. B. Almeida, et. al, 2020. Arbitrarily coupled p-forms in cosmological backgrounds.

Dark energy models

2-Form

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{12}f(\phi)H_{\alpha\beta\gamma}H^{\alpha\beta\gamma} + P_f(Z)\right],$$

by the rotational symmetry in the system:

$$ds^2 = -N(t)^2dt^2 + e^{2\alpha(t)}\left[e^{-4\sigma(t)}dx^2 + e^{2\sigma(t)}\left(dy^2 + dz^2\right)\right],$$

with N(t) the lapse function, and $a \equiv e^{\alpha(t)}$ the geometric mean of the 3 scalefactors.

$$H \equiv \dot\alpha, \qquad \Sigma = \frac{\dot\sigma}{H},$$

$$\rho_B = \frac{f(\phi)}{2}e^{-4(\alpha+\sigma)}\dot v_B^2, \qquad \rho_f = \dot\chi^2\partial_Z P_f - P_f.$$

$$x_1 = \frac{\dot\phi}{\sqrt{6}HM_P}, \quad x_2 = \frac{\sqrt{V}}{\sqrt{3}HM_P}, \quad \Omega_i = \frac{\rho_i}{3H^2M_P^2}, i = B, r, m.$$

$$\Rightarrow \Omega_m = 1 - x_1^2 - x_2^2 - \Sigma^2 - \Omega_r^2 - \Omega_B^2.$$

then, the autonomous system is,

$$x_1' = \frac{3}{2}x_1\left(x_1^2 - x_2^2 + \Sigma^2 - 1 - \frac{1}{3}\Omega_B + \frac{1}{3}\Omega_r\right) + \frac{\sqrt{6}}{2}\left(\lambda x_2^2 - \mu\Omega_B\right),$$

$$x_2' = \frac{1}{2}x_2\left(3x_1^2 - 3x_2^2 + 3\Sigma^2 + 3 - \sqrt{6}\lambda x_1 - \Omega_B + \Omega_r\right),$$

$$\Sigma' = \frac{1}{2}\Sigma\left(3x_1^2 - 3x_2^2 + 3\Sigma^2 - 3 - \Omega_B + \Omega_r\right) - 2\Omega_B,$$

$$\Omega_B' = \Omega_B\left(3x_1^2 - 3x_2^2 + 3\Sigma^2 + 4\Sigma + 1 + \sqrt{6}\mu x_1 - \Omega_B + \Omega_r\right),$$

$$\Omega_r' = \Omega_r\left(3x_1^2 - 3x_2^2 + 3\Sigma^2 - 1 - \Omega_B + \Omega_r\right),$$

With the parameters

$$\lambda = 2, \quad x_2 = 1\cdot 10^{-14}, \quad \Omega_r = 0.999961,$$
$$\mu = 5.5, \quad \Omega_B = 1\cdot 10^{-10}, \quad z_0 = 4.5\cdot 10^7,$$
$$x_1 = 1\cdot 10^{-13}.$$

J. P. B. Almeida, et. al., 2019. Anisotropic 2-form dark energy.

Dark energy models

Non Abelian $$S = \int d^4x\sqrt{-\tilde{g}}\left[\frac{M_P^2}{2}R - \frac{1}{4}F^{\alpha}_{\mu\nu}F^{\mu\nu}_{\alpha} + \frac{\kappa}{96}\left(\tilde{F}^{\mu\nu}_{\alpha}F^{\alpha}_{\mu\nu}\right)^2 - \frac{1}{2}m^2_{\alpha}A^{\alpha}_{\mu}A^{\alpha}_{\mu}\right],$$

since,

$$F^{\alpha}_{\mu\nu} = \partial_\mu A^{\alpha}_{\nu} - \partial_\nu A^{\alpha}_{\mu} + g\epsilon^{\alpha}_{bc}A^{b}_{\mu}A^{c}_{\nu},$$
$$\tilde{F}^{\mu\nu}_{\alpha} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F^{\alpha}_{\rho\sigma}.$$

$$x \equiv \frac{1}{\sqrt{2}M_P}\frac{\dot{\phi}}{aH}, \qquad y \equiv \frac{1}{\sqrt{2}M_P}\frac{g\phi^2}{a^2H},$$
$$w \equiv \frac{1}{\sqrt{2}M_P}\frac{m\phi}{aH}, \qquad z \equiv \frac{1}{\sqrt{2}M_P}\frac{\phi}{a}.$$
$$\Rightarrow 1 = x^2 + y^2 + w^2 + 4\alpha x^2 z^4, \qquad \alpha \equiv M_P^4\kappa g^2.$$

then, the autonomous system is,

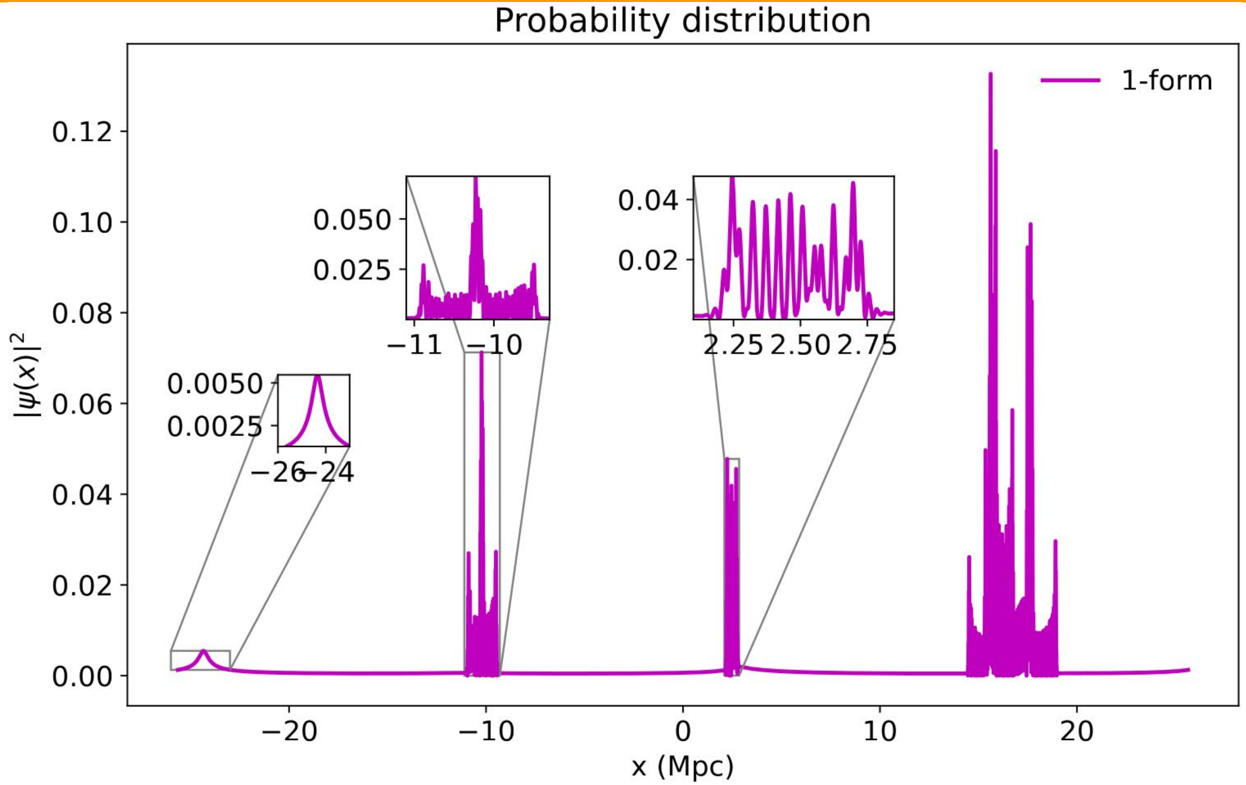
$$q = \frac{1 + y^2 - w^2 + x^2\left(1 - 12\alpha z^3\right) + 2r}{2};$$
$$x' = qx - \frac{\frac{2y^2}{z} + 8\alpha x^2 z^3 + x\left(1 - 12\alpha z^4\right)}{1 + 4\alpha z^4},$$
$$y' = y\left(-1 + 1 + \frac{2x}{z}\right),$$
$$w' = w\left(q + \frac{x}{z}\right), \qquad w' = 0 \to massless,$$
$$z' = x - z,$$
$$\Omega_r' = 2\left(-1 + q\right)\Omega_r.$$

With the initial conditions

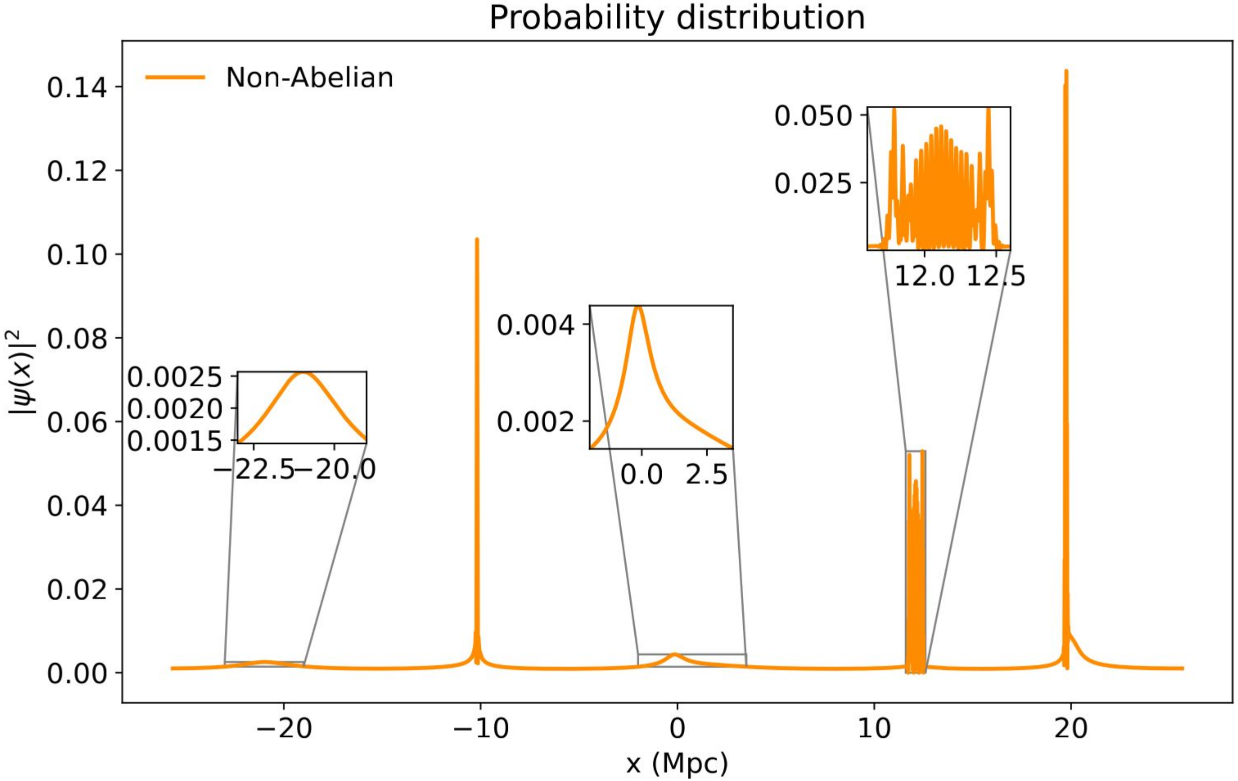
$$\alpha = 10^9, \qquad w_i = 3\cdot 10^{-27}, \qquad x_i = 3\cdot 10^{-30},$$
$$y_i = 10^{-3}, \qquad z_i = 1.1\cdot 10^5, \qquad \Omega_{r_i} = 0.99998,$$
$$z_0 = 1.74\cdot 10^8.$$

A. Guarnizo, et. al., 2020. Dynamical analysis of cosmological models with non-abelian gauge vector fields.

Probability distributions



Probability distributions



Probability distributions