

Testing the Quasi-Static and Sub-Horizon Approximations in Horndesky Theories.

PhD. Santiago García-Serna

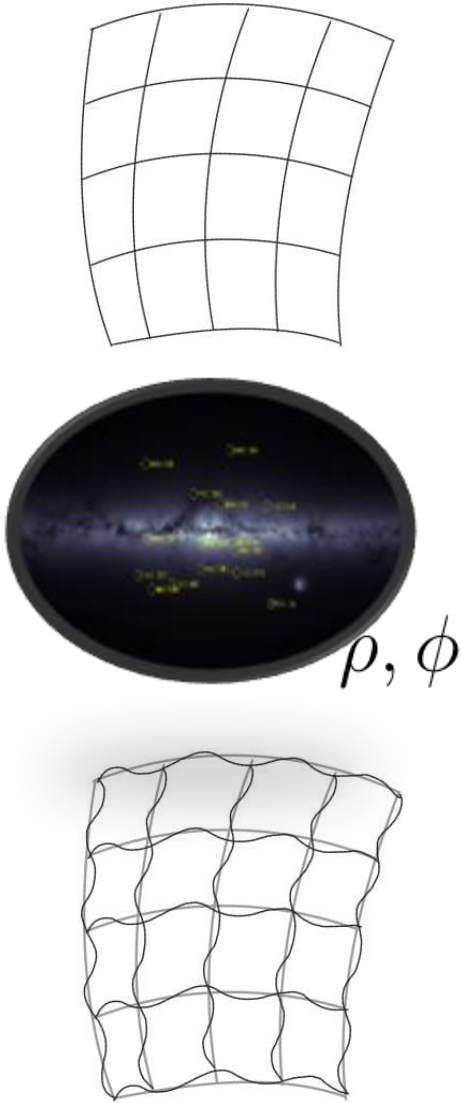
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Greco

Facultad de Ciencias Naturales y Exactas
Universidad del Valle - Colombia



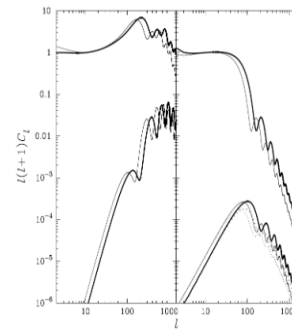
Input



SOLVERS

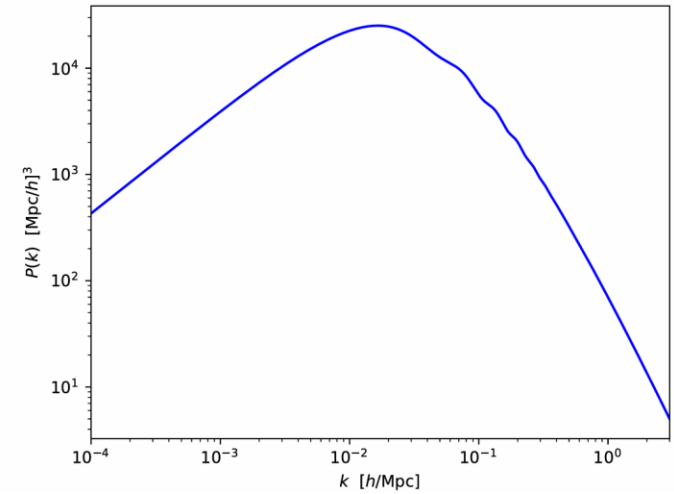
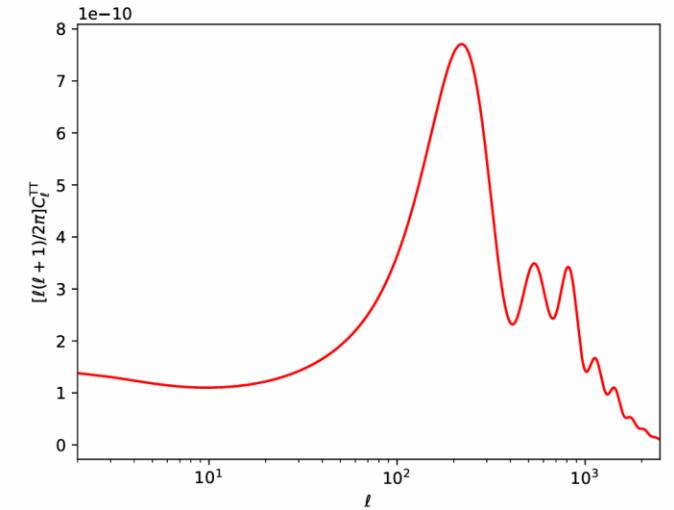


CMBFAST



Code for Anisotropies in the Microwave Background

Output





Horndeski

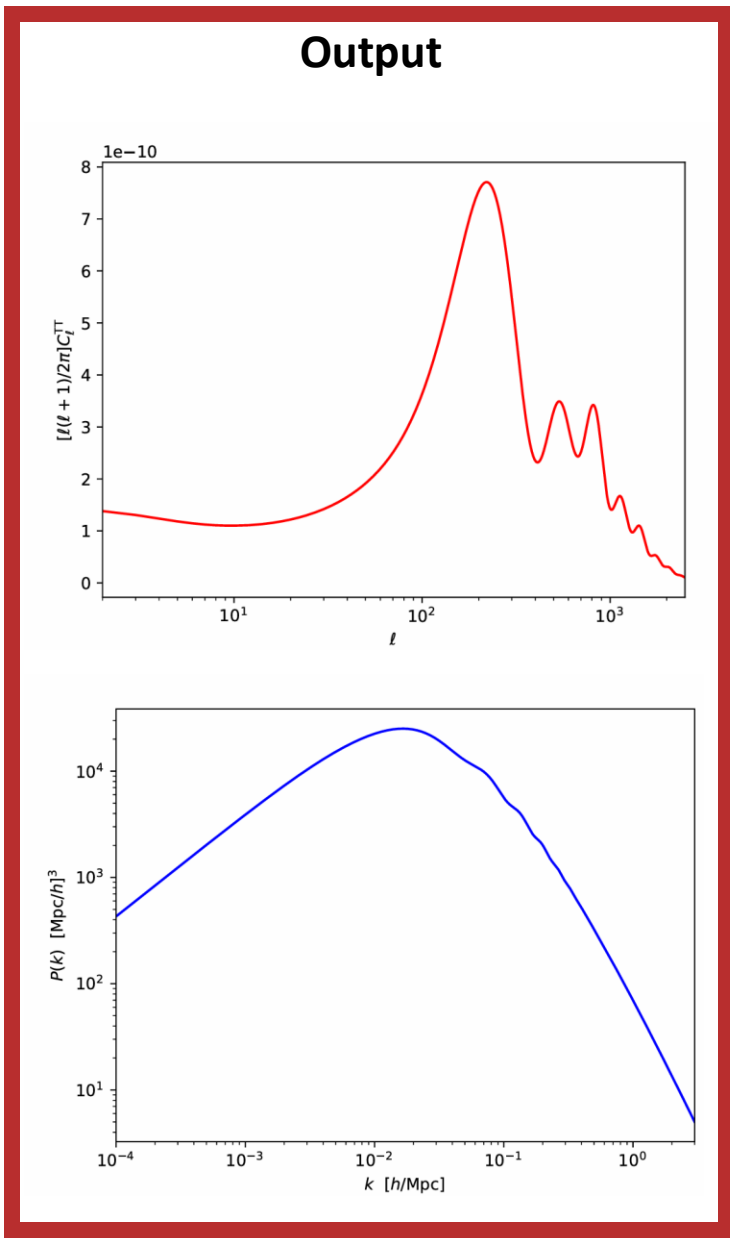
FULL

Horndeski

FULL →



= →



Horndeski

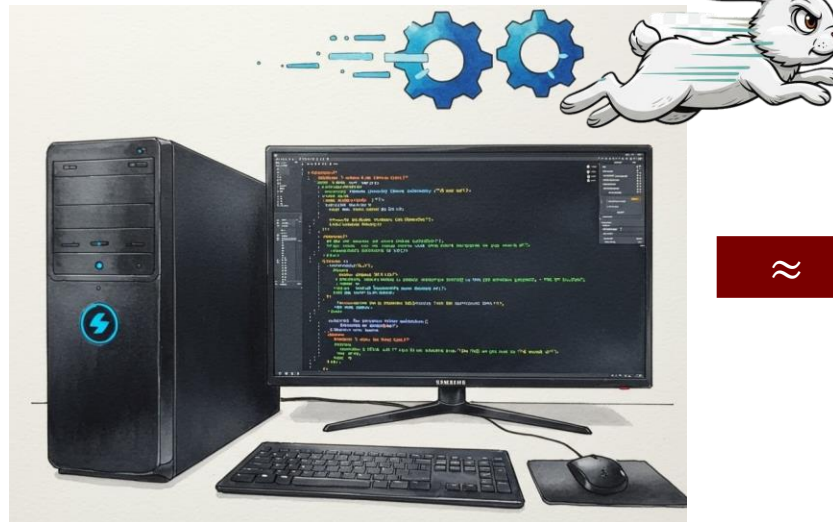
QSA – SHA



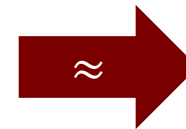
Horndeski

QSA – SHA

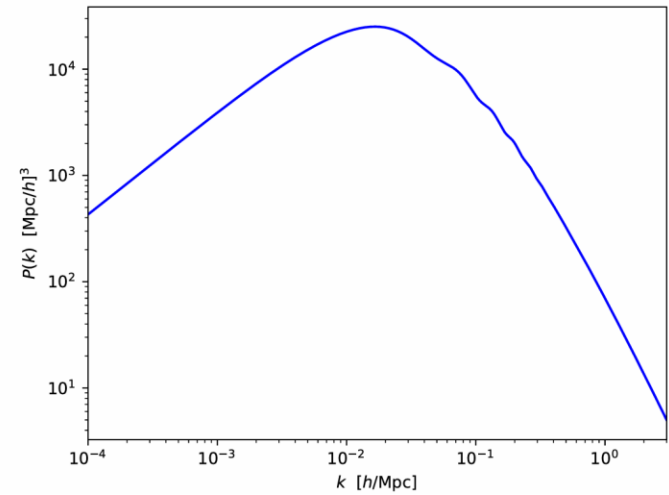
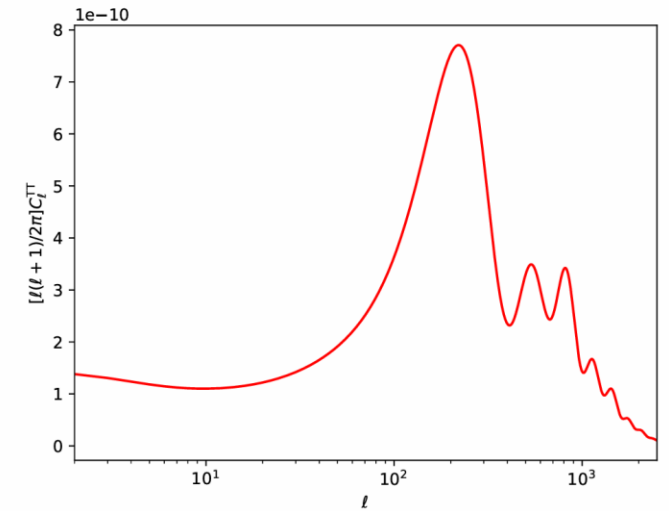
hi_class



Einstein Boltzmann solver



Output



Horndeski

FULL →



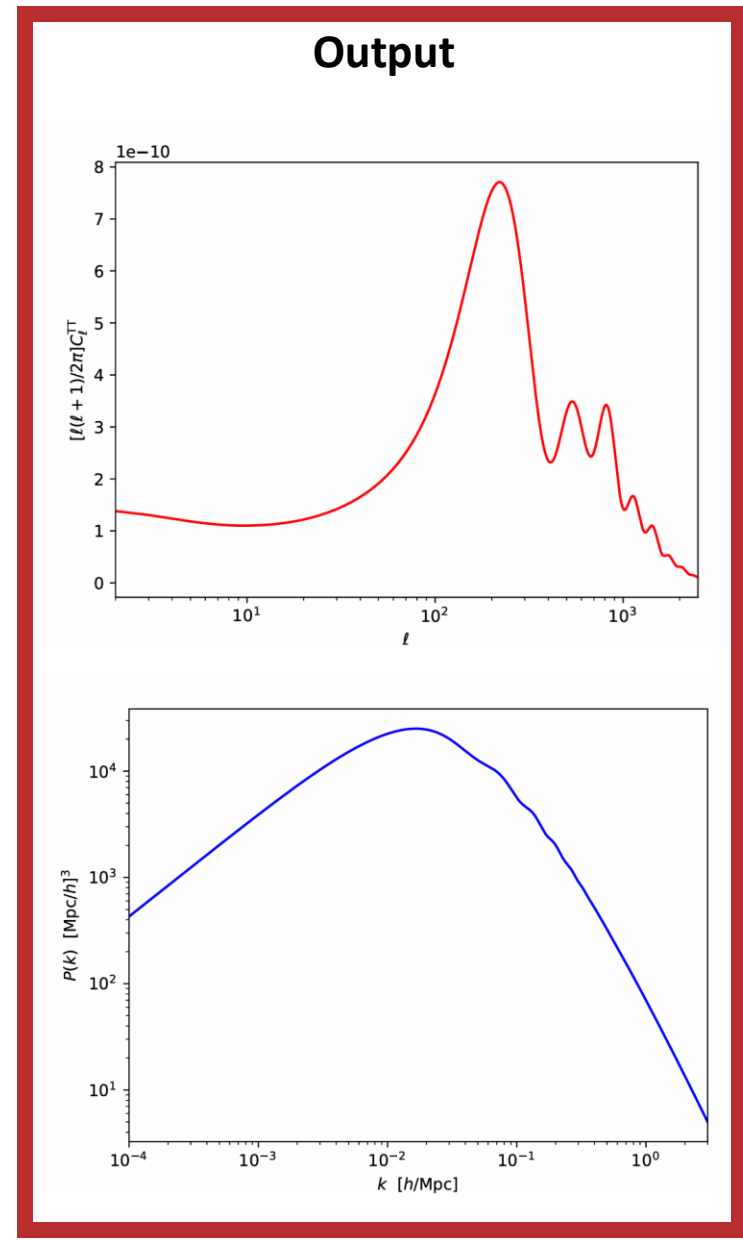
= →

QSA – SHA →



≈ →

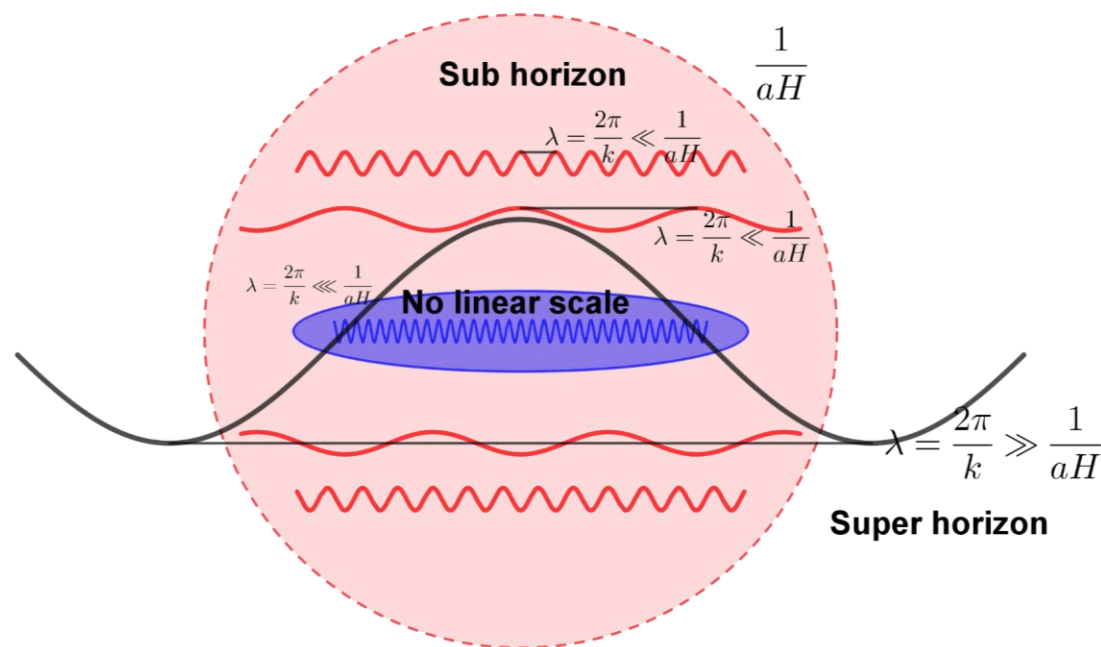
Einstein Boltzmann solver



What is the QSA and SHA?

$$ds^2 = -\{1 + 2\Psi(\mathbf{x}, t)\}dt^2 + a(t)^2\{1 + 2\Phi(\mathbf{x}, t)\}\delta_{ij} dx^i dx^j$$

$$k \gg aH, \quad \Phi \gg \frac{\dot{\Phi}}{H}, \quad \Psi \gg \frac{\dot{\Psi}}{H}$$



What is the QSA and SHA?

$$ds^2 = -\{1 + 2\Psi(\mathbf{x}, t)\}dt^2 + a(t)^2\{1 + 2\Phi(\mathbf{x}, t)\}\delta_{ij} dx^i dx^j$$

$$k \gg aH, \quad \Phi \gg \frac{\dot{\Phi}}{H}, \quad \Psi \gg \frac{\dot{\Psi}}{H}$$

$$\delta R = 6\ddot{\Phi} + 24H\dot{\Phi} - 6H\dot{\Psi} + 4\frac{k^2}{a^2}\Phi + \left(2\frac{k^2}{a^2} - 12\dot{H} + 24H^2\right)\Psi$$

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$$\stackrel{\text{QSA}}{\approx} 4\frac{k^2}{a^2}\Phi + \left(2\frac{k^2}{a^2} - 12\dot{H} + 24H^2\right)\Psi$$

What is the QSA and SHA?

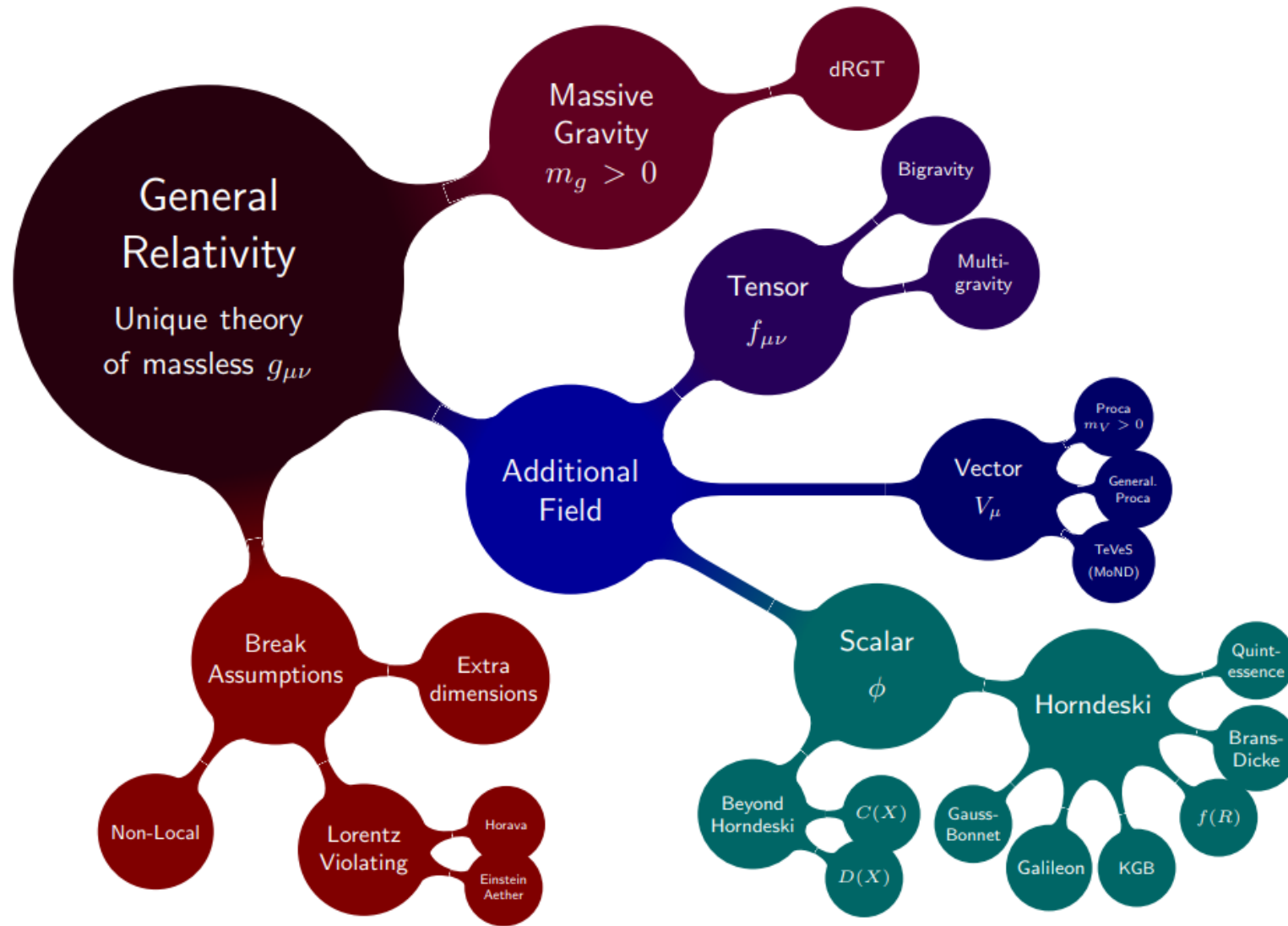
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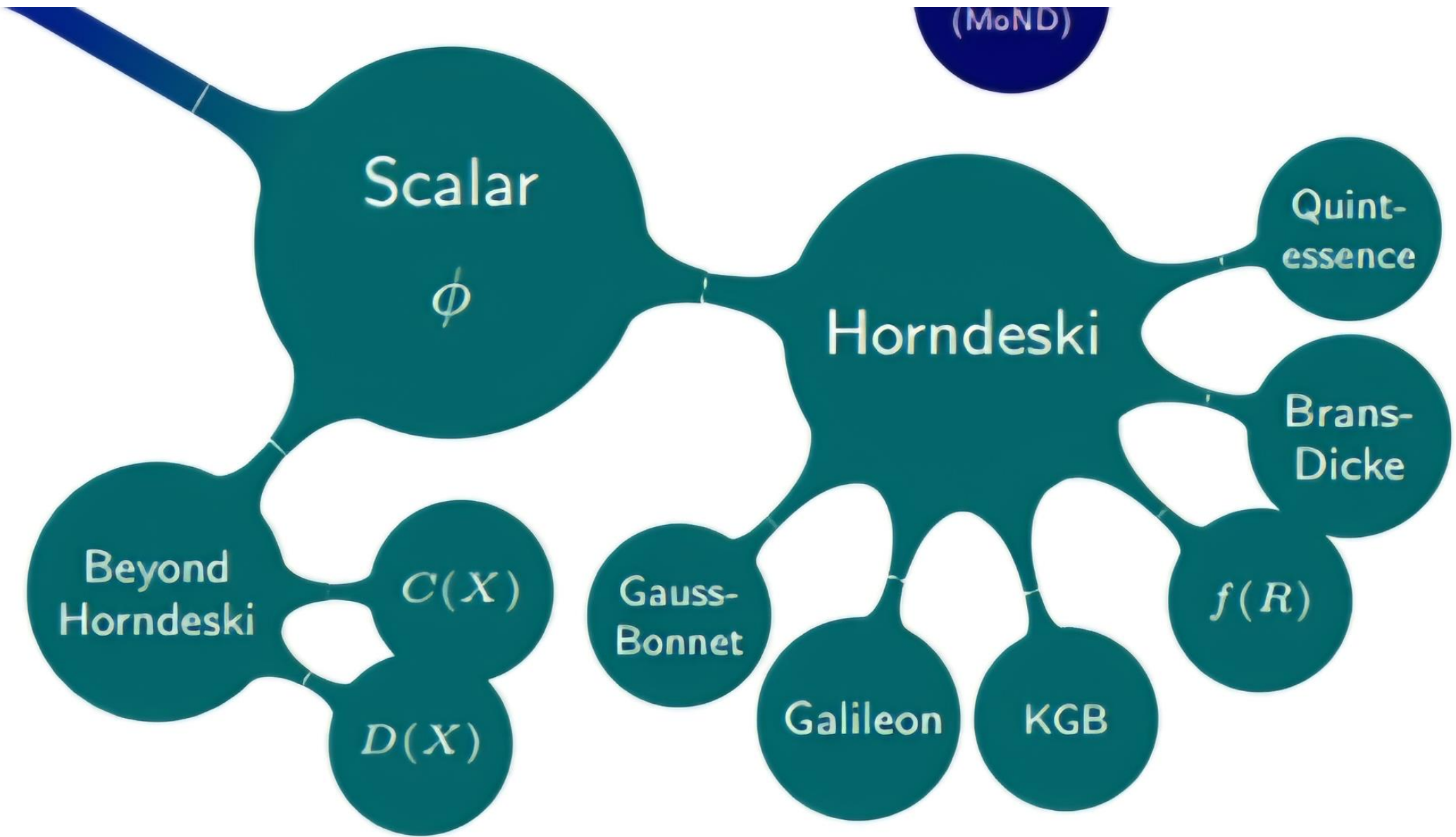
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$$\stackrel{\text{QSA}}{\approx} 4\frac{k^2}{a^2}\Phi + \left(2\frac{k^2}{a^2} - \cancel{12\dot{H}} + \cancel{24H^2}\right)\Psi$$

$$\stackrel{\text{SHA}}{\approx} 4\frac{k^2}{a^2}\Phi + 2\frac{k^2}{a^2}\Psi.$$



J. M. Ezquiaga and M. Zumalacárregui (2018,



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$$g_{\mu\nu} \rightarrow \bar{g}_{\mu\nu} + \delta g_{\mu\nu} \quad G_{\mu\nu} \rightarrow \bar{G}_{\mu\nu} + \delta G_{\mu\nu} \quad T_{\mu\nu} \rightarrow \bar{T}_{\mu\nu} + \delta T_{\mu\nu}$$

$$0 = -\kappa \bar{\rho}_m \delta_m + A_1 \dot{\Phi} + A_2 \dot{\Psi} + \left(A_3 + A_4 \frac{k^2}{a^2} \right) \Phi + \left(A_5 + A_6 \frac{k^2}{a^2} \right) \Psi,$$

$$0 = -\frac{a\kappa \bar{\rho}_m V_m}{k^2} + C_1 \dot{\Phi} + C_2 \dot{\Psi} + C_3 \Phi + C_4 \Psi,$$

$$0 = B_1 \ddot{\Phi} + B_2 \ddot{\Psi} + B_3 \dot{\Phi} + B_4 \dot{\Psi} + B_5 \Phi + B_6 \Psi,$$

$$0 = D_1 \ddot{\Phi} + D_2 \dot{\Phi} + D_3 \dot{\Psi} + \left(D_4 + D_5 \frac{k^2}{a^2} \right) \Phi + \left(D_6 + D_7 \frac{k^2}{a^2} \right) \Psi.$$

Designing Horndeski and the effective fluid approach

Rubén Arjona,^{*} Wilmar Cardona,[†] and Savvas Nesseris[‡] [arXiv:1904.06294v2](https://arxiv.org/abs/1904.06294v2)

$$g_{\mu\nu} \rightarrow \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$$

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$$A_3 \frac{k^2}{a^2} \Psi + A_6 \frac{k^2}{a^2} \delta\phi - \kappa \rho_m \delta_m \simeq 0,$$

$$B_6 \frac{k^2}{a^2} \Psi + B_8 \frac{k^2}{a^2} \Phi + B_7 \frac{k^2}{a^2} \delta\phi \simeq 0,$$

$$D_7 \frac{k^2}{a^2} \Psi + \left(D_9 \frac{k^2}{a^2} - M^2 \right) \delta\phi + D_{10} \frac{k^2}{a^2} \Phi \simeq 0,$$

$$G_4(\Psi + \Phi) + G_{4\phi} \delta\phi = 0$$

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$$G_4(\Psi + \Phi) + G_{4\phi} \delta\phi = 0$$

$$D_9 = -K_X + 2G_{3\phi} - 4H\dot{\phi}G_{3X} \\ - \ddot{\phi} (2G_{3X} + \dot{\phi}^2 G_{3XX}) - \dot{\phi}^2 G_{3\phi X}$$

Designing Horndeski and the effective fluid approach

Rubén Arjona,* Wilmar Cardona,† and Savvas Nesseris‡

arXiv:1904.06294v2

QSA-SHA coefficients

$$\begin{aligned} \varepsilon &\equiv \frac{aH}{k}, & \delta &\equiv \frac{\dot{\varepsilon}}{\varepsilon H}, & \xi &\equiv \frac{\ddot{\varepsilon}}{\varepsilon H^2}, & \chi &\equiv \frac{\ddot{\varepsilon}'}{\varepsilon H^3}, \\ \varepsilon_\Phi &\equiv \frac{\dot{\Phi}}{\Phi H}, & \varepsilon_\Psi &\equiv \frac{\dot{\Psi}}{\Psi H}, & \chi_\Phi &\equiv \frac{\dot{\varepsilon}_\Phi}{\varepsilon_\Phi H}, & \chi_\Psi &\equiv \frac{\dot{\varepsilon}_\Psi}{\varepsilon_\Psi H}. \end{aligned}$$

Tracking the validity of the quasi-static and sub-horizon approximations in modified gravity

J. Bayron Orjuela-Quintana, Savvas Nesseris

arXiv:2303.14251 [gr-qc]

QSA-SHA coefficients

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$$\kappa\delta\rho_{\text{DE}} = W_1\dot{\Phi} + W_2\dot{\Psi} + \left(W_3 + W_4\frac{k^2}{a^2}\right)\Phi + \left(W_5 + W_6\frac{k^2}{a^2}\right)\Psi,$$

$$\kappa\delta P_{\text{DE}} = Y_1\ddot{\Phi} + Y_2\ddot{\Psi} + Y_3\dot{\Phi} + Y_4\dot{\Psi} + \left(Y_5 + Y_6\frac{k^2}{a^2}\right)\Phi + \left(Y_7 + Y_8\frac{k^2}{a^2}\right)\Psi,$$

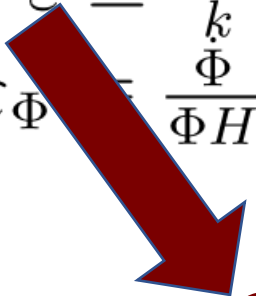
$$\frac{a\kappa\bar{\rho}_{\text{DE}}}{k^2}V_{\text{DE}} = Z_1\dot{\Phi} + Z_2\dot{\Psi} + Z_3\Phi + Z_4\Psi,$$

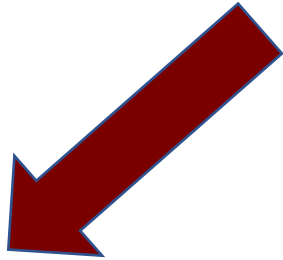
$$\kappa\bar{\rho}_{\text{DE}}\pi_{\text{DE}} = -\frac{k^2}{a^2}(\Phi + \Psi),$$

Tracking the validity of the quasi-static and sub-horizon approximations in modified gravity

QSA-SHA coefficients

$$\begin{aligned} \varepsilon &\equiv \frac{aH}{k}, & \delta &\equiv \frac{\dot{\varepsilon}}{\varepsilon H}, & \xi &\equiv \frac{\ddot{\varepsilon}}{\varepsilon H^2}, & \chi &\equiv \frac{\ddot{\varepsilon}}{\varepsilon H^3}, \\ \varepsilon_\Phi &\equiv \frac{\dot{\Phi}}{\Phi H}, & \varepsilon_\Psi &\equiv \frac{\dot{\Psi}}{\Psi H}, & \chi_\Phi &\equiv \frac{\dot{\varepsilon}_\Phi}{\varepsilon_\Phi H}, & \chi_\Psi &\equiv \frac{\dot{\varepsilon}_\Psi}{\varepsilon_\Psi H}. \end{aligned}$$


$$V_{\text{DE}} = \frac{a\dot{F}F + 6\frac{k^2}{a^2}F_R \frac{\bar{\rho}_m}{\bar{\rho}_{\text{DE}}}\delta_m}{2F \left(3\frac{k^2}{a^2}F_R + F\right)} \approx 0$$


$$\dot{F}\delta_m \approx 0$$

$$\dot{F} \propto H \times \text{Perturbation} \sim 0$$

Tracking the validity of the quasi-static and sub-horizon approximations in modified gravity

QSA-SHA coefficients

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QSA-SHA “0-order”

$$V_{\text{DE}} = \frac{a\dot{F} F + 6\frac{k^2}{a^2} F_R \bar{\rho}_m}{2F \left(3\frac{k^2}{a^2} F_R + F \bar{\rho}_{\text{DE}} \right)} \delta_m \quad \dot{F} = 6\frac{k^3}{a^3} (\delta + \xi - 2) F_R \varepsilon^3$$

$$V_{\text{DE}} = \frac{a}{F \left(F + 3\frac{k^2}{a^2} F_R \right)} \left\{ F(F - 1) + \frac{k^2}{a^2} (3F - 4) F_R \right\} \frac{\bar{\rho}_m}{\bar{\rho}_{\text{DE}}} \delta_m \frac{k}{a} \varepsilon$$

Tracking the validity of the quasi-static and sub-horizon approximations in modified gravity

QSA-SHA coefficients

$$\begin{aligned} \epsilon &\equiv \frac{aH}{k}, & \delta &\equiv \frac{\dot{\epsilon}}{\epsilon H}, & \xi &\equiv \frac{\ddot{\epsilon}}{\epsilon H^2}, & \chi &\equiv \frac{\ddot{\dot{\epsilon}}}{\epsilon H^3}, \\ \epsilon_\Phi &\equiv \frac{\dot{\Phi}}{\Phi H}, & \epsilon_\Psi &\equiv \frac{\dot{\Psi}}{\Psi H}, & \chi_\Phi &\equiv \frac{\dot{\epsilon}_\Phi}{\epsilon_\Phi H}, & \chi_\Psi &\equiv \frac{\dot{\epsilon}_\Psi}{\epsilon_\Psi H}. \end{aligned}$$

- Standar approach $V_{\text{DE}} \sim \mathcal{O}(\epsilon^3)$
- QSA-SHA coefficients $V_{\text{DE}} \sim \mathcal{O}(\epsilon)$

Tracking the validity of the quasi-static and sub-horizon approximations in modified gravity

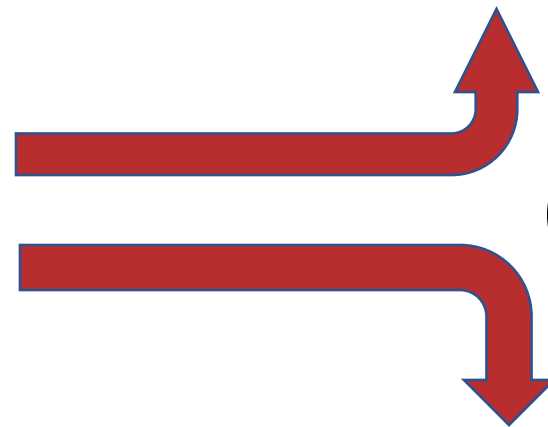
Horndeski

QSA
- SHA



$$\begin{aligned} \varepsilon &\equiv \frac{aH}{k}, & \delta &\equiv \frac{\dot{\varepsilon}}{\varepsilon H}, \\ \xi &\equiv \frac{\dot{\varepsilon}}{\varepsilon H^2}, & \chi &\equiv \frac{\varepsilon}{\varepsilon H^3}, \\ \varepsilon_\Phi &\equiv \frac{\Phi}{\Phi H}, & \varepsilon_\varphi &\equiv \frac{\dot{\varphi}}{\varphi H}, \\ \chi_\Phi &\equiv \frac{\dot{\varepsilon}_\Phi}{\varepsilon_\Phi H}, & \chi_\varphi &\equiv \frac{\dot{\varepsilon}_\varphi}{\varepsilon_\varphi H}. \end{aligned}$$

• $\pi_{DE} = 0$



Coefficients
0 - order

$$\begin{aligned} \varepsilon &\equiv \frac{aH}{k}, & \delta &\equiv \frac{\dot{\varepsilon}}{\varepsilon H}, \\ \xi &\equiv \frac{\dot{\varepsilon}}{\varepsilon H^2}, & \chi &\equiv \frac{\varepsilon}{\varepsilon H^3}, \\ \varepsilon_\Phi &\equiv \frac{\Phi}{\Phi H}, & \varepsilon_\Psi &\equiv \frac{\Psi}{\Psi H}, \\ \chi_\Phi &\equiv \frac{\dot{\varepsilon}_\Phi}{\varepsilon_\Phi H}, & \chi_\Psi &\equiv \frac{\dot{\varepsilon}_\Psi}{\varepsilon_\Psi H}. \end{aligned}$$

• $\pi_{DE} \neq 0$

Coefficients for Horndesky

- $\pi_{DE} \neq 0$
$$\begin{aligned} \varepsilon &\equiv \frac{aH}{k}, & \delta &\equiv \frac{\dot{\varepsilon}}{\varepsilon H}, & \xi &\equiv \frac{\ddot{\varepsilon}}{\varepsilon H^2}, & \chi &\equiv \frac{\ddot{\varepsilon}}{\varepsilon H^3}, \\ \varepsilon_\Phi &\equiv \frac{\dot{\Phi}}{\Phi H}, & \varepsilon_\Psi &\equiv \frac{\dot{\Psi}}{\Psi H}, & \chi_\Phi &\equiv \frac{\dot{\varepsilon}_\Phi}{\varepsilon_\Phi H}, & \chi_\Psi &\equiv \frac{\dot{\varepsilon}_\Psi}{\varepsilon_\Psi H}. \end{aligned}$$

$$A = A_0 + \frac{k^2}{a^2} A_1 + \left(A_1 + \frac{k^2}{a^2} A_2 + \frac{k^4}{a^4} A_3 \right) \varepsilon + \mathcal{O}(\varepsilon^2, \varepsilon_\Psi, \varepsilon_\Psi)$$

- $\pi_{DE} = 0$
$$\begin{aligned} \varepsilon &\equiv \frac{aH}{k}, & \delta &\equiv \frac{\dot{\varepsilon}}{\varepsilon H}, & \xi &\equiv \frac{\ddot{\varepsilon}}{\varepsilon H^2}, & \chi &\equiv \frac{\ddot{\varepsilon}}{\varepsilon H^3}, \\ \varepsilon_\Phi &\equiv \frac{\dot{\Phi}}{\Phi H}, & \varepsilon_\varphi &\equiv \frac{\dot{\varphi}}{\varphi H}, & \chi_\Phi &\equiv \frac{\dot{\varepsilon}_\Phi}{\varepsilon_\Phi H}, & \chi_\varphi &\equiv \frac{\dot{\varepsilon}_\varphi}{\varepsilon_\varphi H}. \end{aligned}$$

$$A = A_0 + \frac{k^2}{a^2} A_1 + \left(A_1 + \frac{k^2}{a^2} A_2 + \frac{k^4}{a^4} A_3 \right) \varepsilon + \mathcal{O}(\varepsilon^2, \varepsilon_\varphi, \varepsilon_\Phi)$$

Coefficients for Horndesky

$$\begin{aligned}
 \frac{G}{G_N} \propto & \mathcal{D}5^{(0)} + \frac{k^2 \mathcal{D}6^{(0)}}{a^2} + \left(\frac{k^2 \mathcal{D}8^{(0)}}{a^2} + \mathcal{D}7^{(0)} \right) \frac{k\varepsilon}{a} + \frac{k^2 \mathcal{D}9^{(0)} \varepsilon^2}{a^2} + \frac{k^3 \mathcal{D}10^{(0)} \varepsilon^3}{a^3} \\
 & / \left(\mathcal{F}1^{(\theta)} + \frac{k^4 \mathcal{F}3^{(\theta)}}{a^4} + \frac{k^2 \mathcal{F}2^{(\theta)}}{a^2} + \left(\frac{k^5 \mathcal{F}6^{(\theta)}}{a^5} + \frac{k^3 \mathcal{F}5^{(\theta)}}{a^3} + \frac{k \mathcal{F}4^{(\theta)}}{a} \right) \varepsilon \right. \\
 & + \left. \left(\frac{k^4 \mathcal{F}8^{(\theta)}}{a^4} + \frac{k^2 \mathcal{F}7^{(\theta)}}{a^2} \right) \varepsilon^2 + \left(\frac{k^5 \mathcal{F}10^{(\theta)}}{a^5} + \frac{k^3 \mathcal{F}9^{(\theta)}}{a^3} \right) \varepsilon^3 \right. \\
 & \left. + \frac{k^4 \mathcal{F}11^{(\theta)} \varepsilon^4}{a^4} + \frac{k^5 \mathcal{F}12^{(\theta)} \varepsilon^5}{a^5} \right)
 \end{aligned}$$

Coefficients for Horndesky

$$\begin{aligned} \frac{G}{G_N} \propto & \mathcal{D}5^{(0)} + \frac{k^2 \mathcal{D}6^{(0)}}{a^2} + \left(\frac{k^2 \mathcal{D}8^{(0)}}{a^2} + \mathcal{D}7^{(0)} \right) \frac{k\varepsilon}{a} + \frac{k^2 \mathcal{D}9^{(0)} \varepsilon^2}{a^2} + \cancel{\frac{k^3 \mathcal{D}10^{(0)} \varepsilon^3}{a^3}} \\ & / \left(\mathcal{F}1^{(\theta)} + \frac{k^4 \mathcal{F}3^{(\theta)}}{a^4} + \frac{k^2 \mathcal{F}2^{(\theta)}}{a^2} + \left(\frac{k^5 \mathcal{F}6^{(\theta)}}{a^5} + \frac{k^3 \mathcal{F}5^{(\theta)}}{a^3} + \frac{k \mathcal{F}4^{(\theta)}}{a} \right) \varepsilon \right. \\ & + \left. \left(\frac{k^4 \mathcal{F}8^{(\theta)}}{a^4} + \frac{k^2 \mathcal{F}7^{(\theta)}}{a^2} \right) \varepsilon^2 + \cancel{\left(\frac{k^5 \mathcal{F}10^{(\theta)}}{a^5} + \frac{k^3 \mathcal{F}9^{(\theta)}}{a^3} \right) \varepsilon^3} \right. \\ & \left. + \cancel{\left(\frac{k^4 \mathcal{F}11^{(\theta)} \varepsilon^4}{a^4} + \frac{k^5 \mathcal{F}12^{(\theta)} \varepsilon^5}{a^5} \right)} \right) \end{aligned}$$

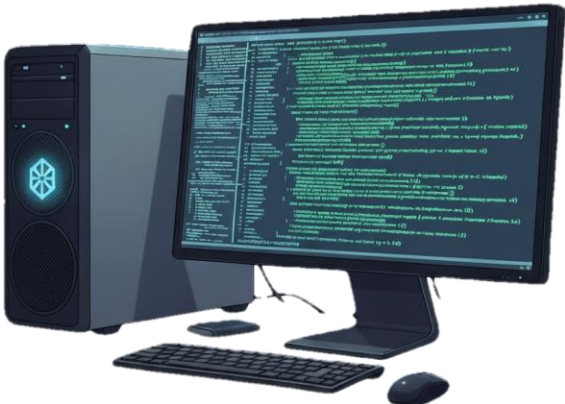
Horndeski

FULL



$\Phi, \Psi, P(\mathbf{k}), C(I), \dots$

QSA – SHA



Einstein Boltzmann solver

\approx

