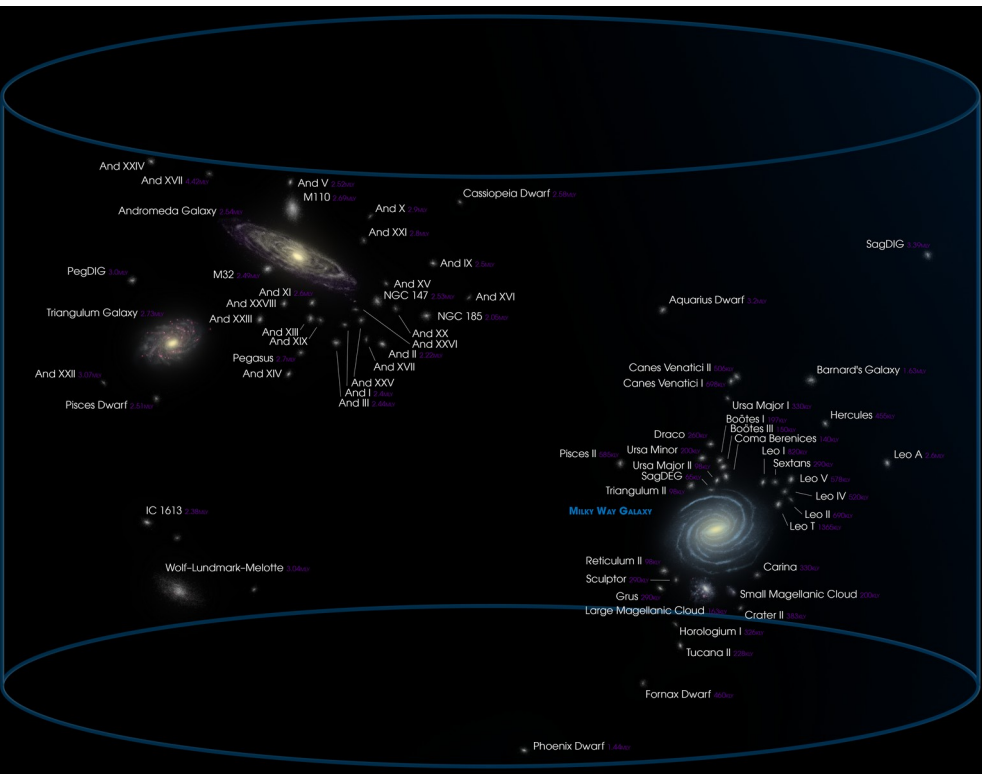


May the Force be Accurate:

Towards better orbital models

Till Sawala¹, Jenni Häkkinen¹, Gabor Racz¹

¹University of Helsinki



Orbits in the Local Group: initial-value problem

- What is the future of the MW – LMC?
- What is the future of the MW – M31?
- What are the past and future orbits of satellites?

To solve the problem numerically, we generally have to integrate the mutual gravitational forces*:

$$\mathbf{F}_{j \rightarrow i}(\mathbf{R}) = -G \int d^3\mathbf{r} \int d^3\mathbf{r}' \rho_i(\mathbf{r}) \rho_j(\mathbf{r}') \frac{\mathbf{R} + \mathbf{r} - \mathbf{r}'}{|\mathbf{R} + \mathbf{r} - \mathbf{r}'|^3}.$$

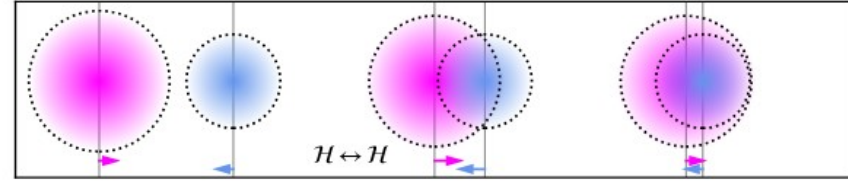
Force on halo **i** due to halo **j** is the force acting on every element of halo **i** due to every element of halo **j** (*and vice versa*).

*and account for dynamical friction (where a related issue arises, ask me later).

In practice, we use approximations:

Full Halo - Halo interaction ($H - H$):

$$\mathbf{F}_{j \rightarrow i}(\mathbf{R}) = -G \int d^3\mathbf{r} \int d^3\mathbf{r}' \rho_i(\mathbf{r}) \rho_j(\mathbf{r}') \frac{\mathbf{R} + \mathbf{r} - \mathbf{r}'}{|\mathbf{R} + \mathbf{r} - \mathbf{r}'|^3}.$$



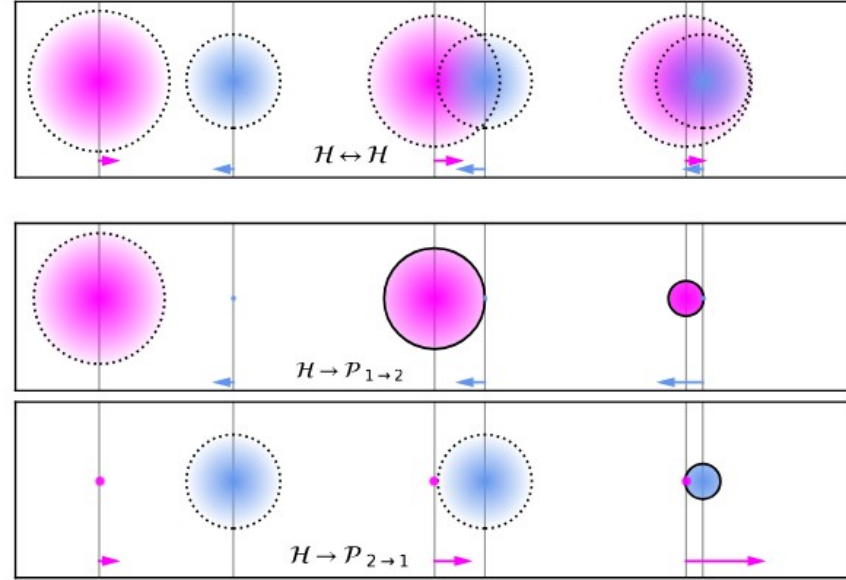
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Assume one of the two is a point mass: ($H - P$):

$$\begin{aligned} \mathbf{F}_{j \rightarrow i}(\mathbf{R}) &= \int d^3\mathbf{r} \left[M_i \delta^3(\mathbf{r}) \right] \mathbf{g}_j(\mathbf{R} + \mathbf{r}) \\ &= M_i \mathbf{g}_j(\mathbf{R}). \end{aligned}$$



In practice, we use approximations:

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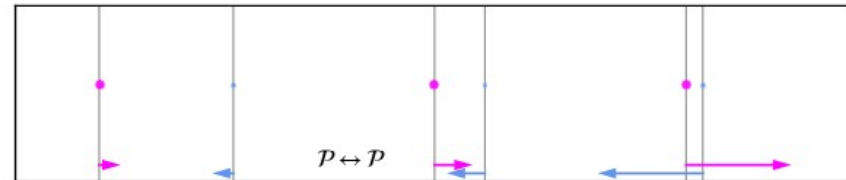
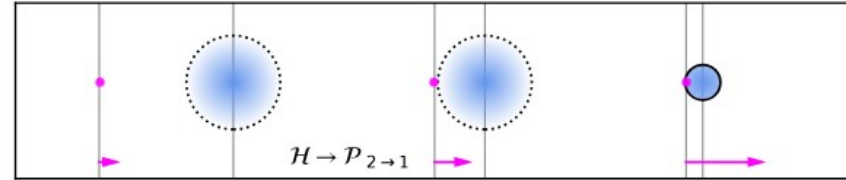
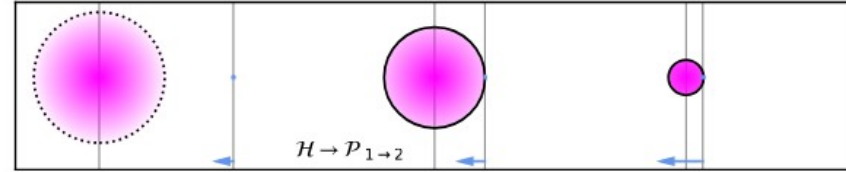
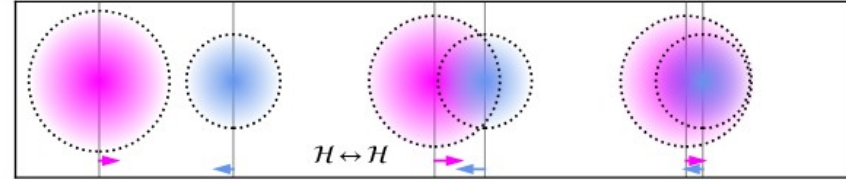
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Assume both halos are point masses ($P - P$):

$$\mathbf{F}_{j \rightarrow i}(\mathbf{R}) = M_i \mathbf{g}_j(\mathbf{R}) = -G M_i M_j \frac{\mathbf{R}}{R^3}$$



In $P - P$, if at least one halo is extended, force errors can be very large (even with softening).

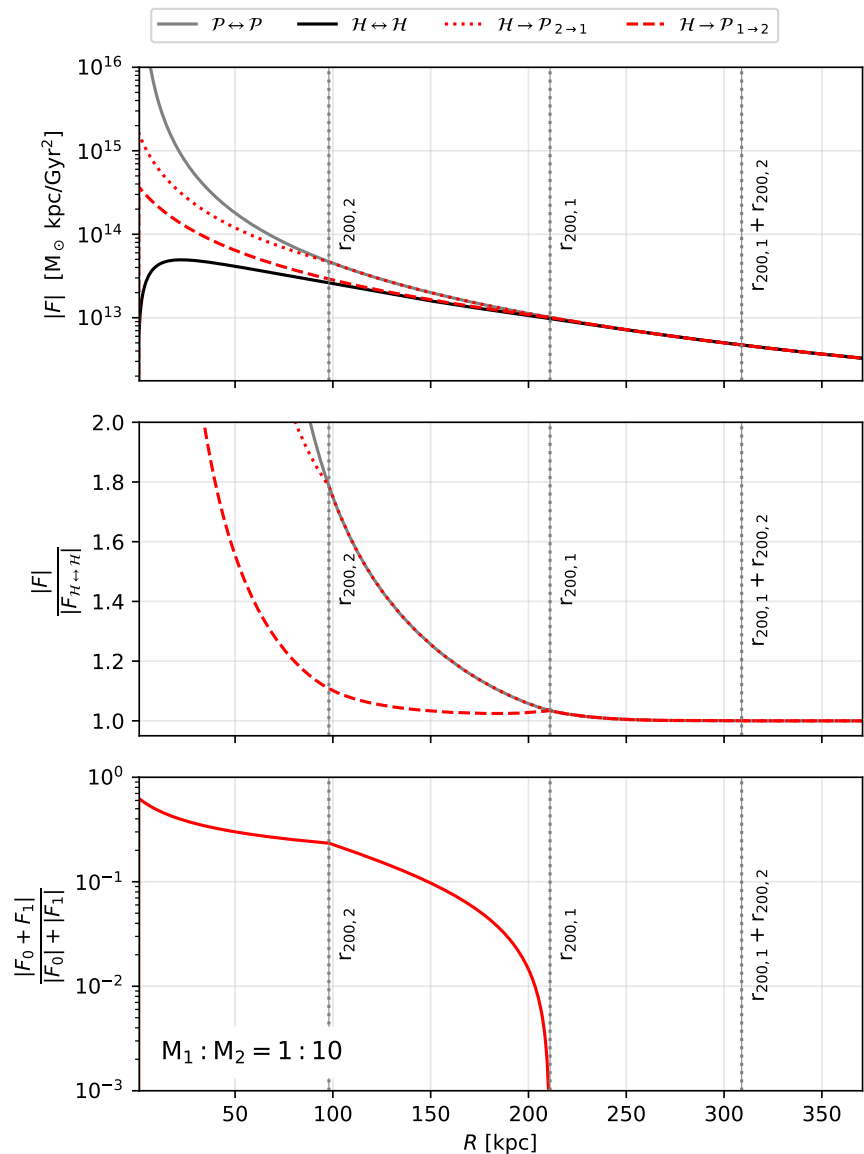
In $H - P$, force errors are reduced, but momentum is not conserved. This is particularly consequential for $N > 2$.

Cautun+ (2018), MW - M31 - LMC,
 $H - P$

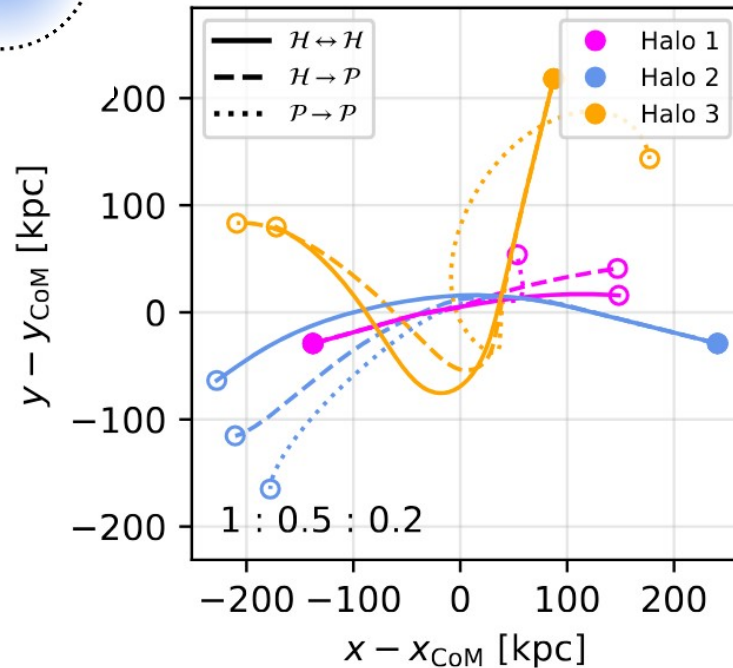
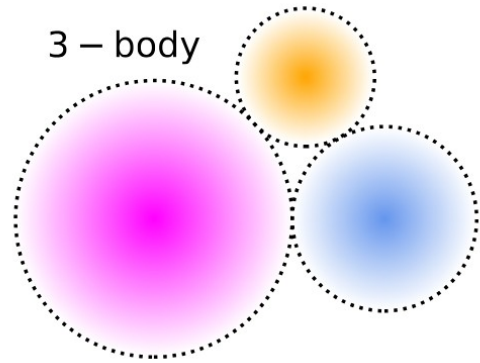
Large force error, asymmetric

T.S.+ (2025), MW-M31-LMC-M33
 $P - P$ (softened, $\text{eps} = 20$ kpc)

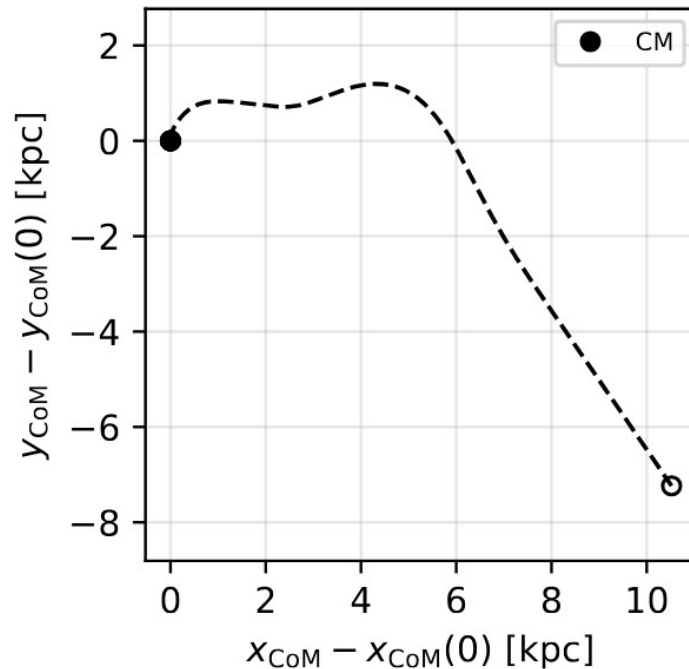
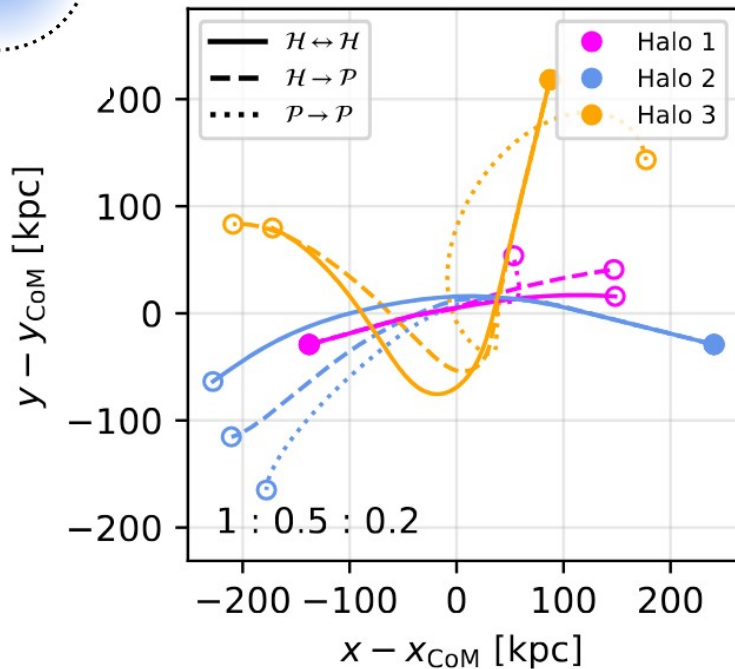
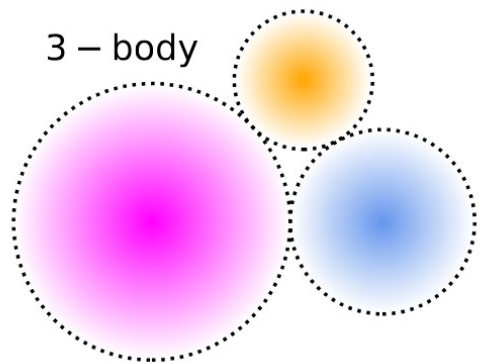
Even larger force error, symmetric



Simple test problem: three NFW halos starting from rest.



Simple test problem: three NFW halos starting from rest.

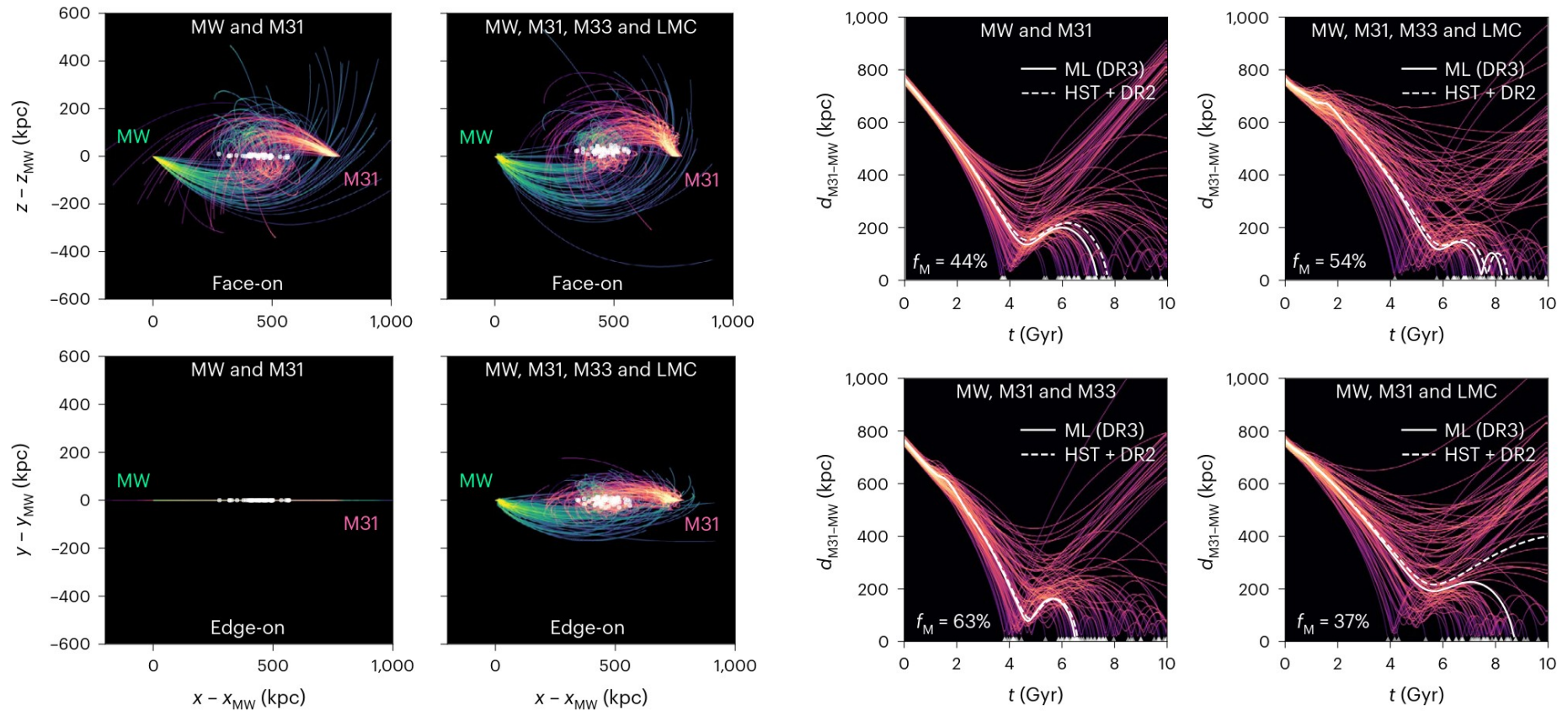


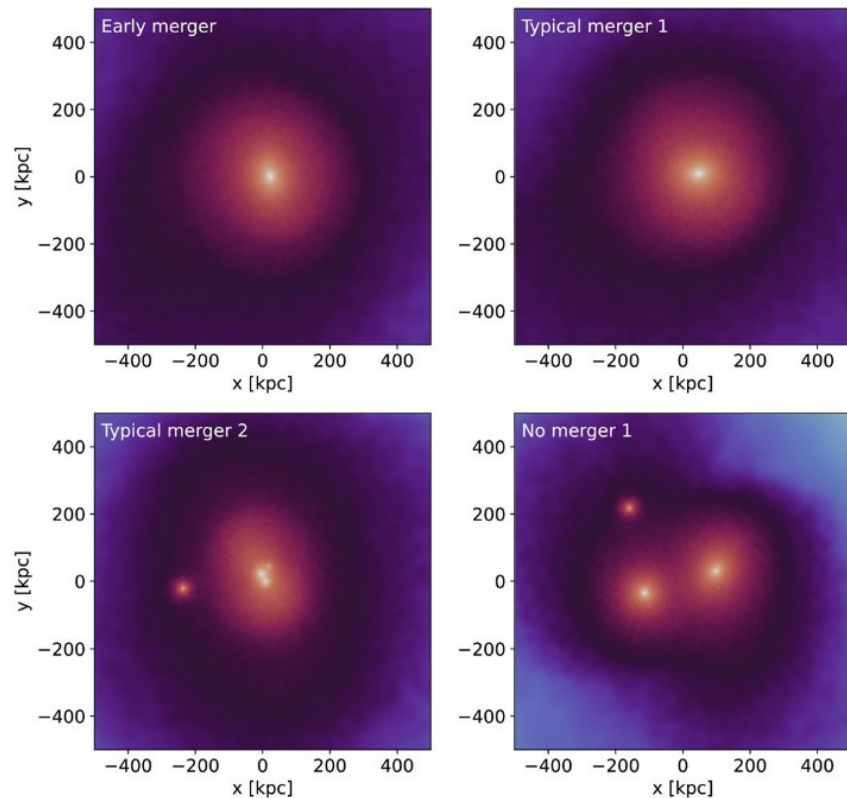
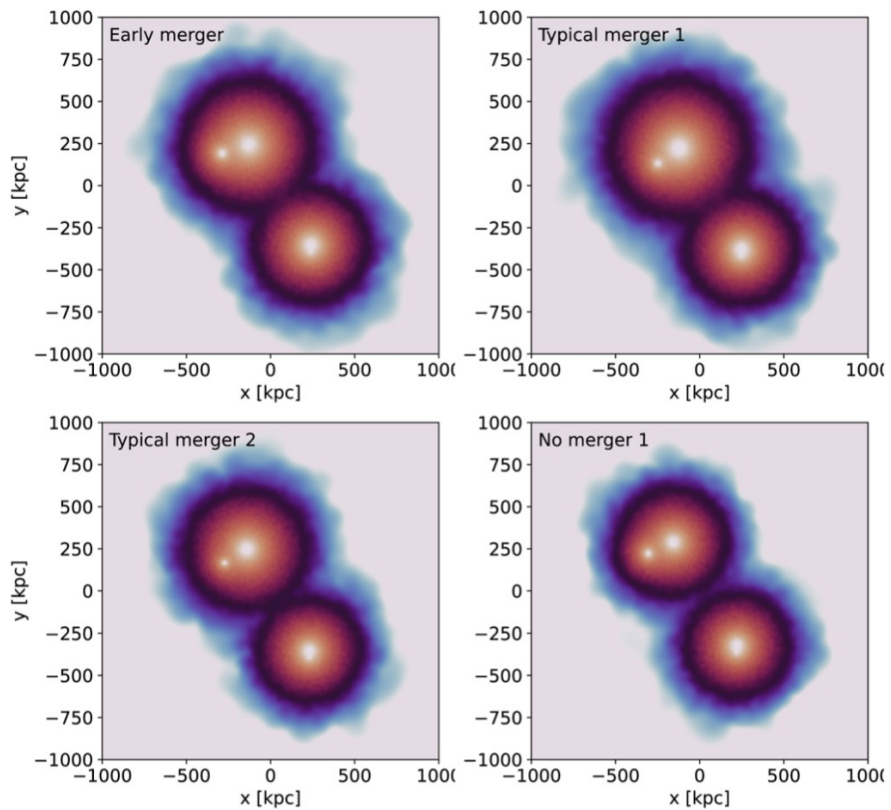
No certainty of a Milky Way–Andromeda collision

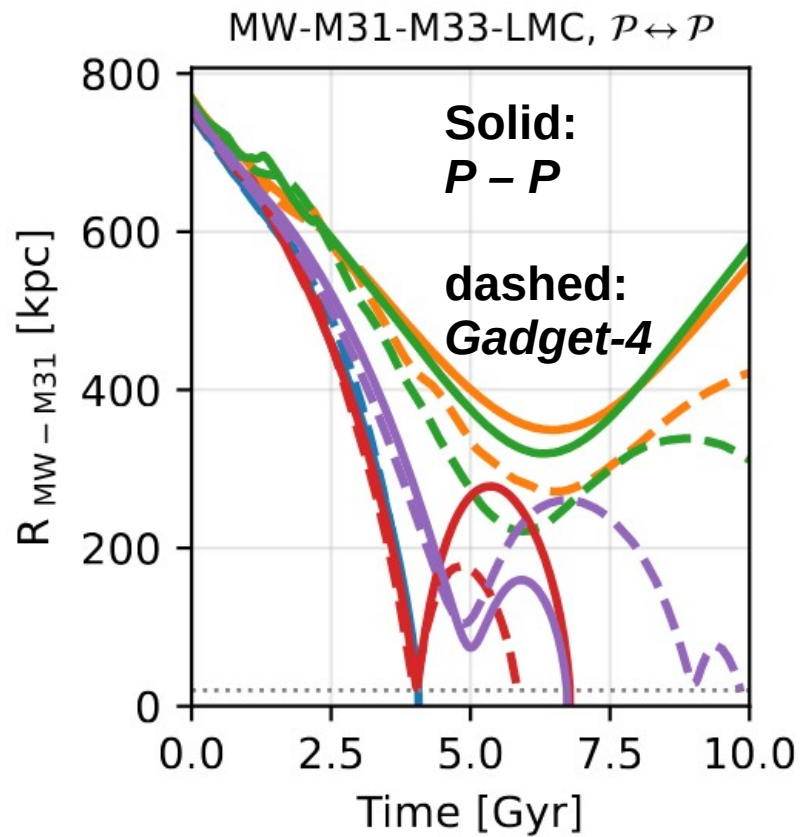
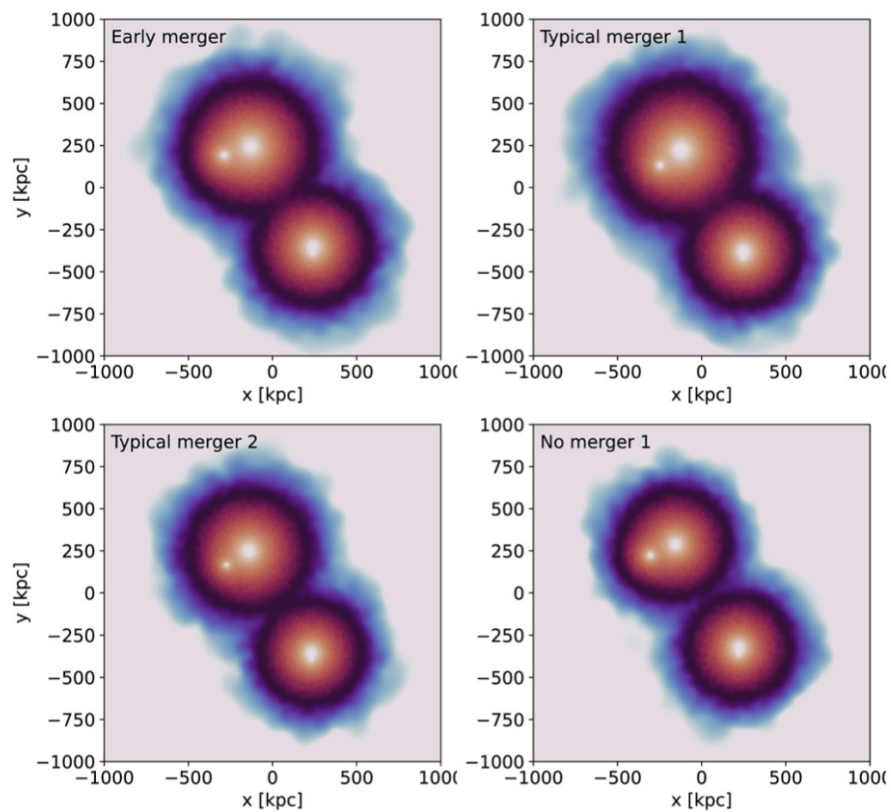
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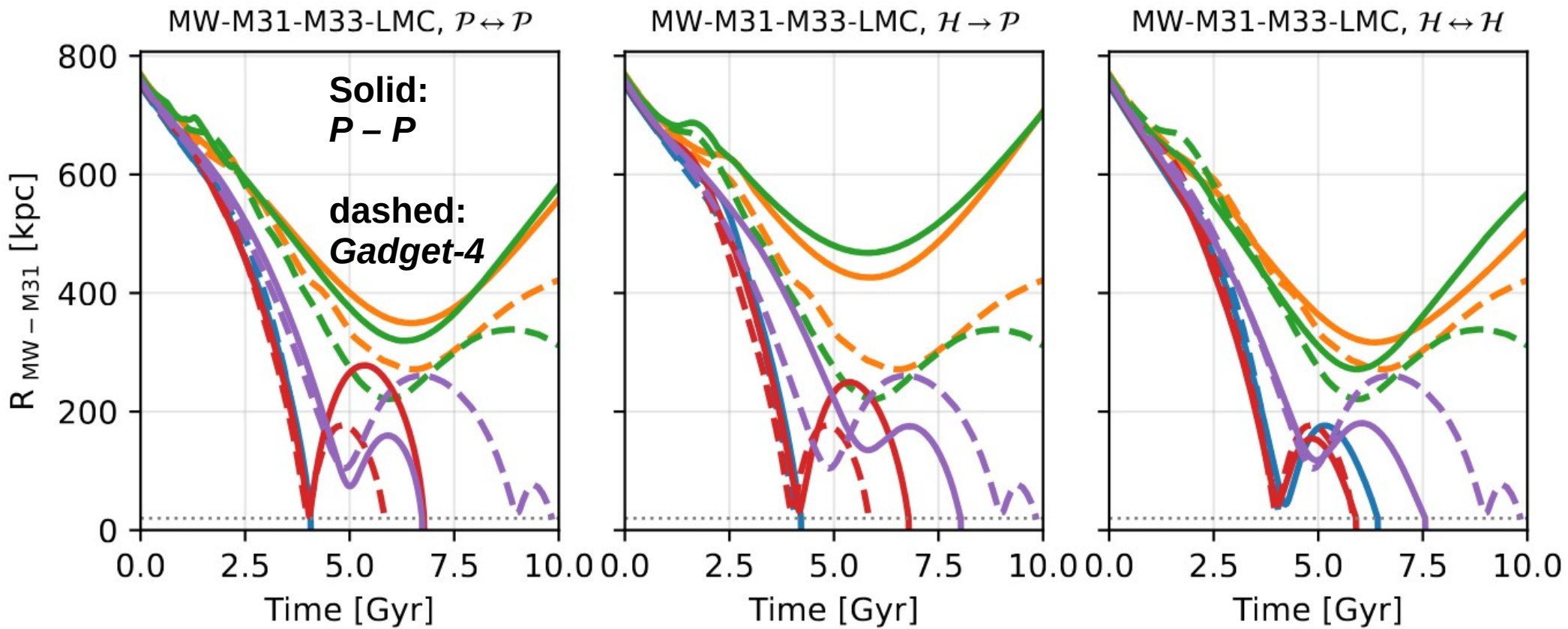
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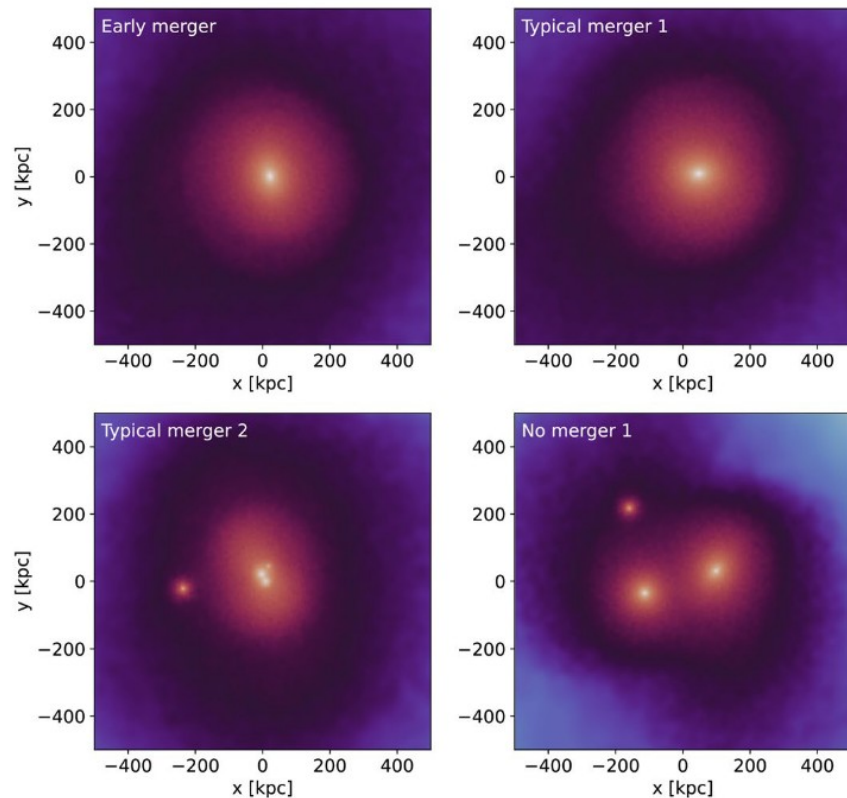
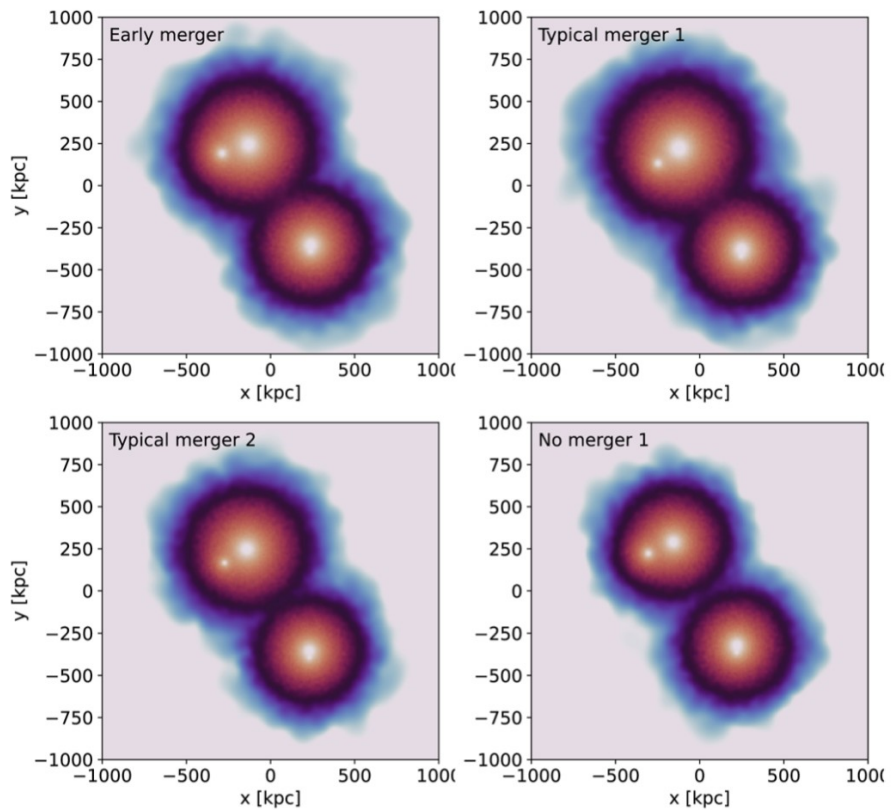
Till Sawala ^{1,2}✉, Jehanne Delhomelle^{1,3}, Alis J. Deason ², Carlos S. Frenk ²,
Jenni Häkkinen ¹, Peter H. Johansson ¹, Atte Keitaanranta ¹,
Alexander Rawlings¹ & Ruby Wright ^{1,4}



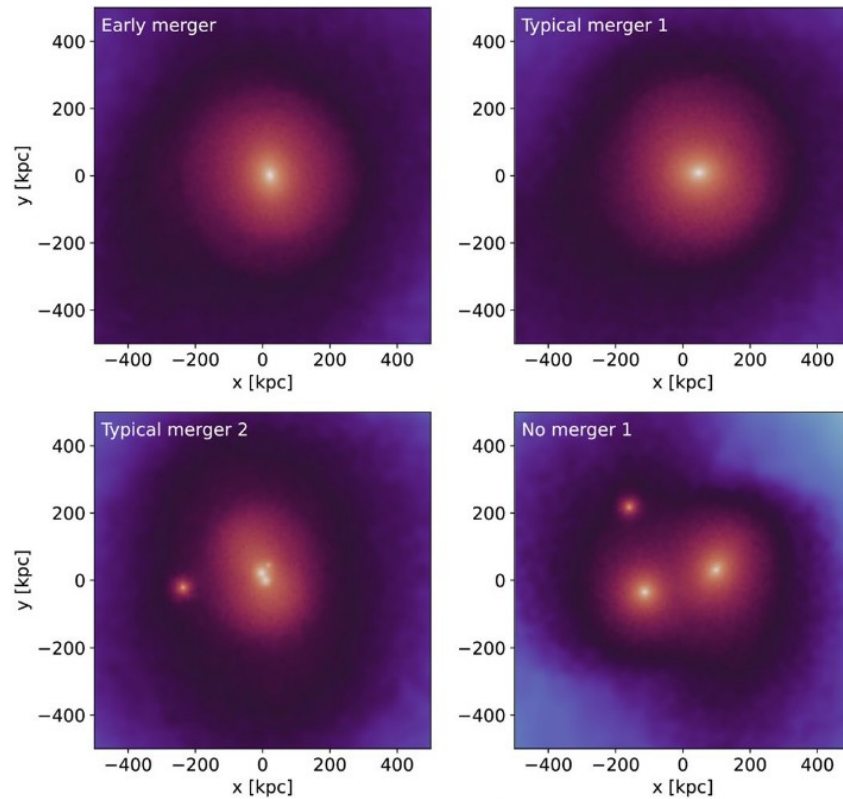
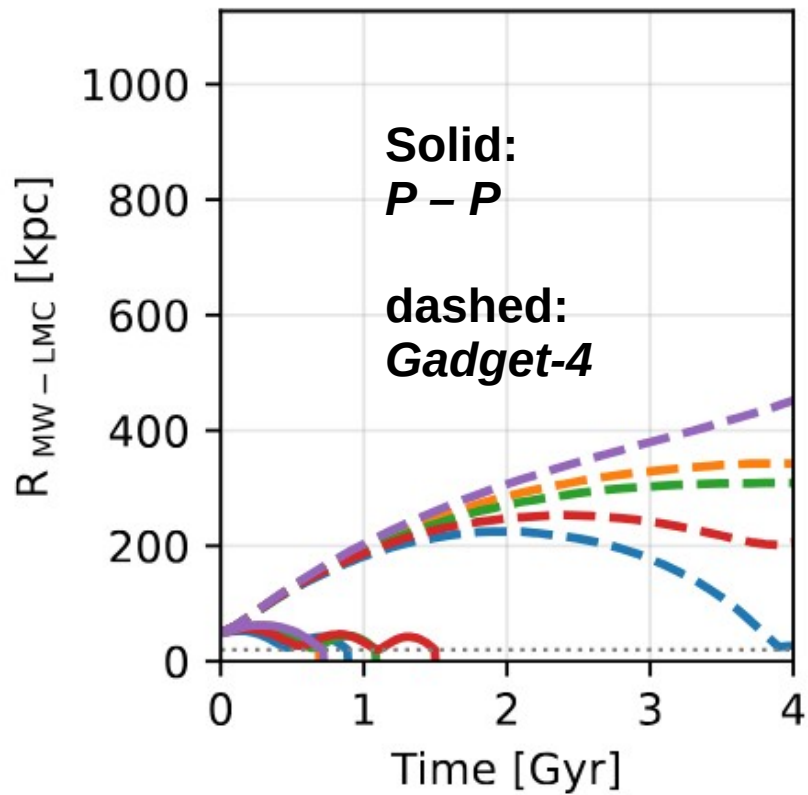


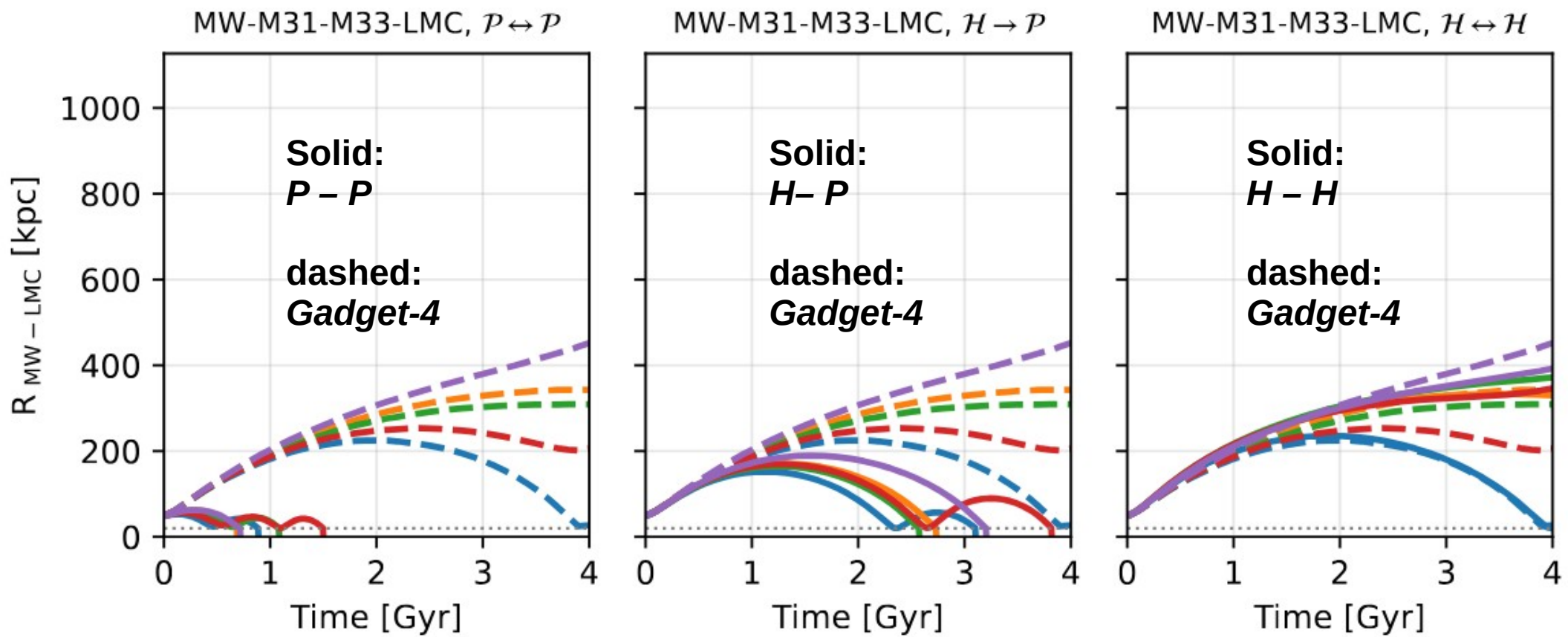






MW-M31-M33-LMC, $\mathcal{P} \leftrightarrow \mathcal{P}$



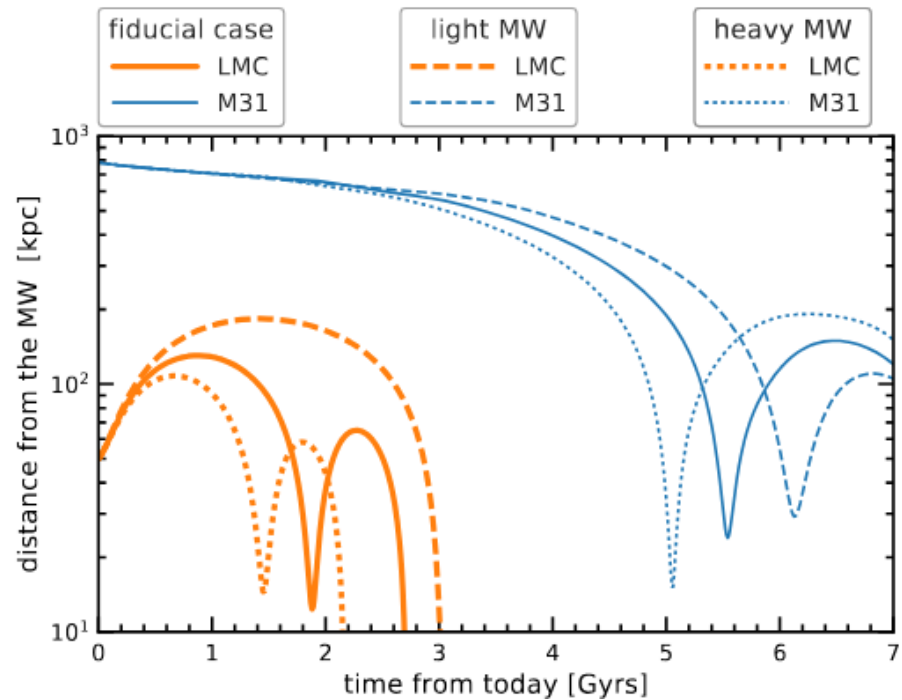


The aftermath of the Great Collision between our Galaxy and the Large Magellanic Cloud

Marius Cautun ¹★, Alis J. Deason ¹, Carlos S. Frenk¹ and Stuart McAlpine ^{1,2}

¹Department of Physics, Institute of Computational Cosmology, Durham University, South Road, Durham DH1 3LE, UK

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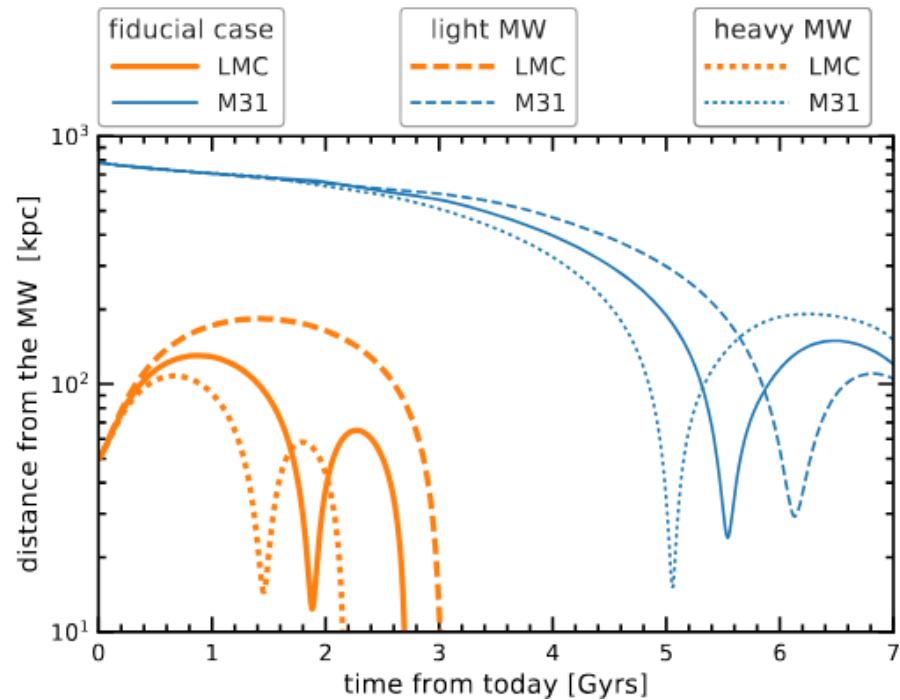
H - P integration

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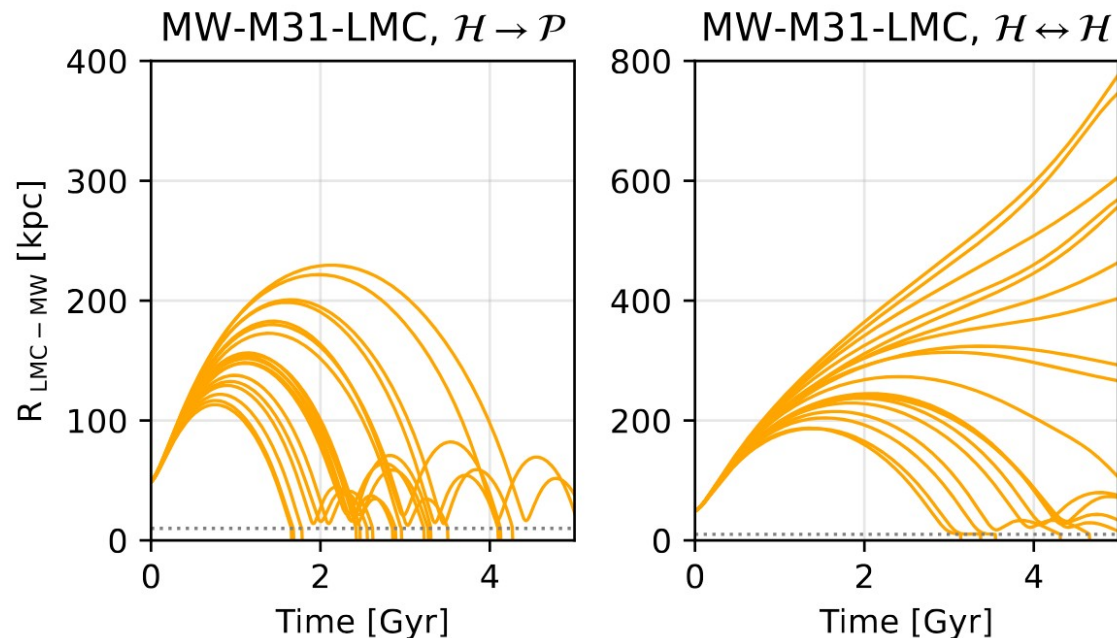
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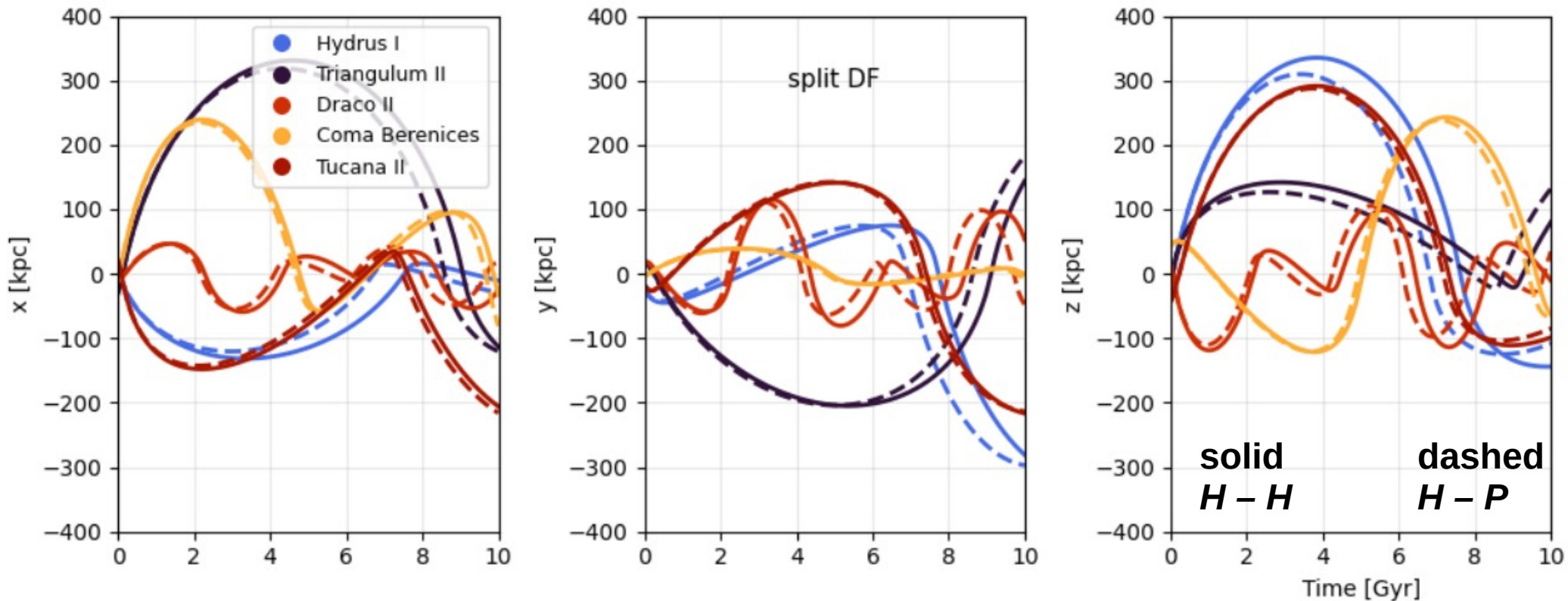
H - P integration



MC samples, different force models.

Lower mass satellites

(Gaia DR2 data from Simon et al. 2018)



Conclusions (so far)

- Integrating orbits seems simple, but it can be surprisingly wrong!
→ see (maybe) upcoming paper.
- As data gets (even) better (Gaia DR4👁👁), it may be worth investing some effort into the force calculations.
- Even **$H - H$** is making a lot of simplifying assumptions (e.g. rigid halos, simple DF). → see upcoming paper by J. Häkkinen.