Sundman and the final parsec problem

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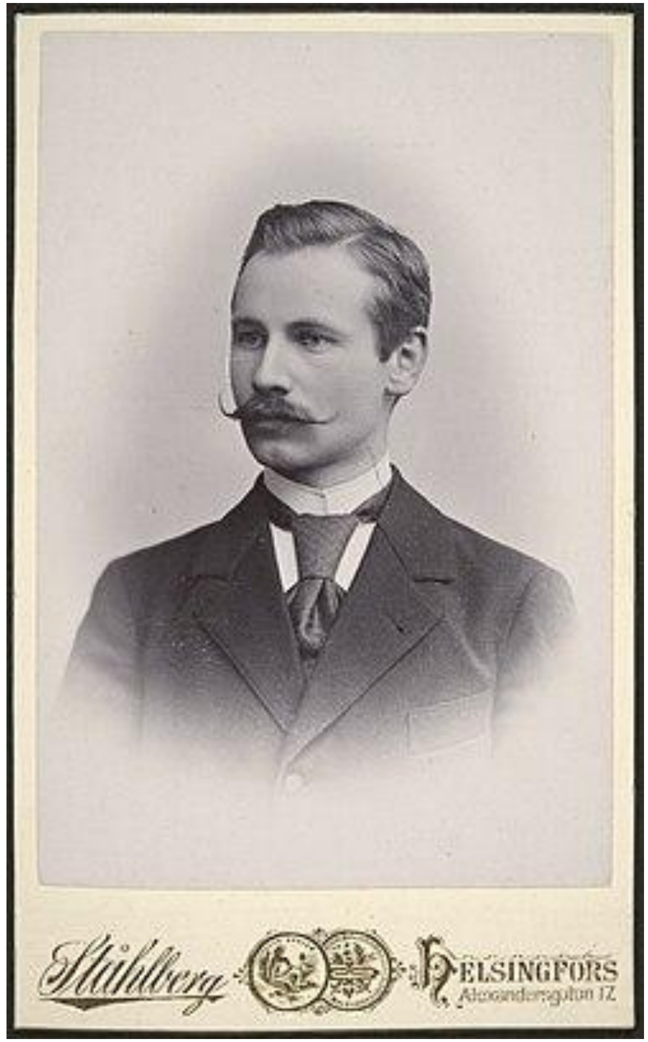
Ståhlberg
HELSINGFORS

Massive black hole binaries in active galactic nuclei

M. C. Begelman, R. D. Blandford & M. J. Rees

Nature **287**, 307–309 (1980) | Cite this article

Abstract

Most theoretical discussions of active galactic nuclei (including quasars) attribute their energy production either to an accreting black hole or to a precursor stage—for instance a dense star cluster or a supermassive star—whose inevitable end point is a massive black hole[1]. We explore here the possibility that some active nuclei may contain two massive black holes in orbit about each other. This hypothesis suggests a new interpretation for the observed bending[2] and apparent precession[3] of radio jets emerging from these objects and may indeed be verified through detection of the direct consequences of orbital motion.

Few-Body Problem: Theory and Computer Simulations
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The Slingshot Revisited

Sverre Aarseth

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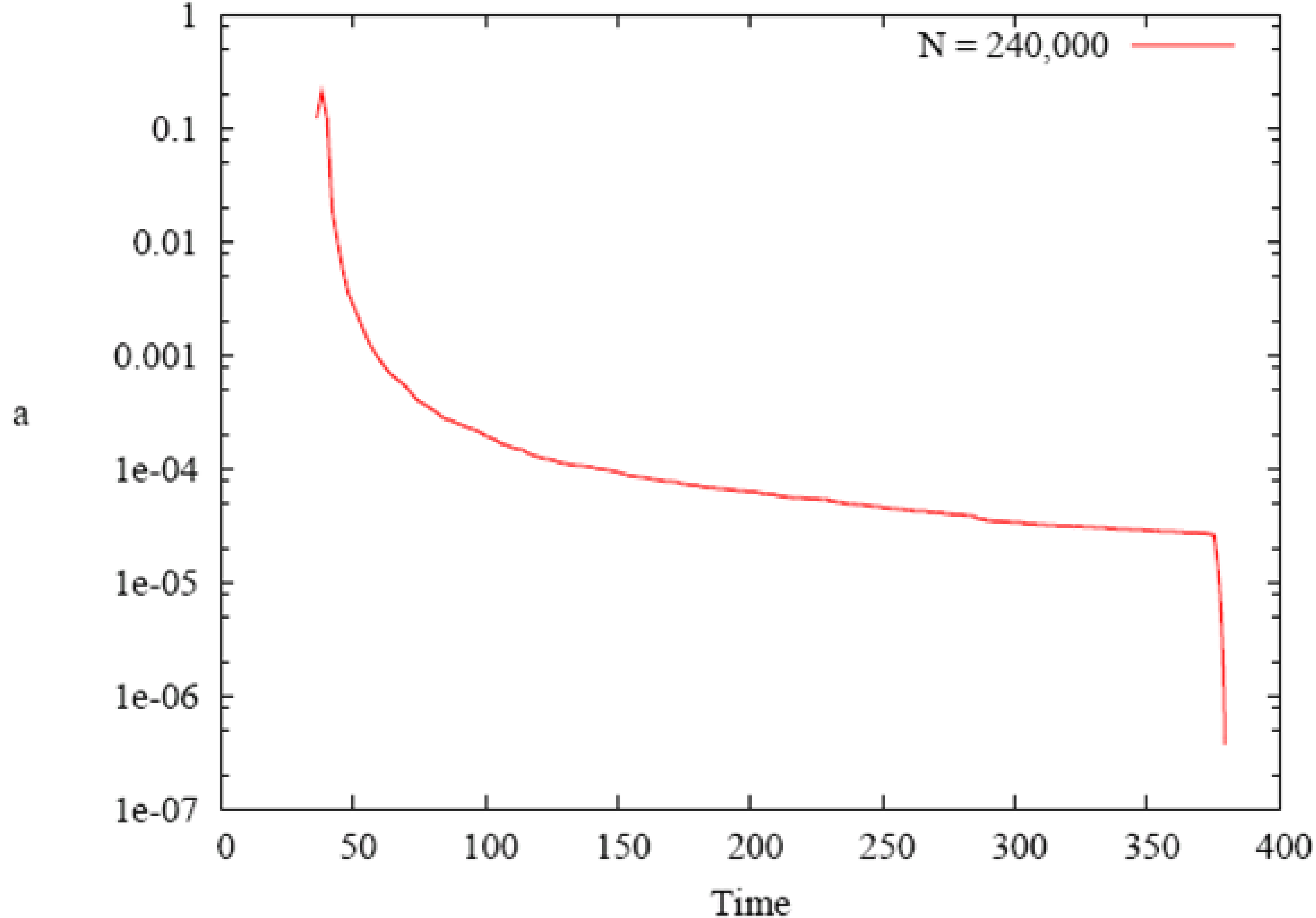
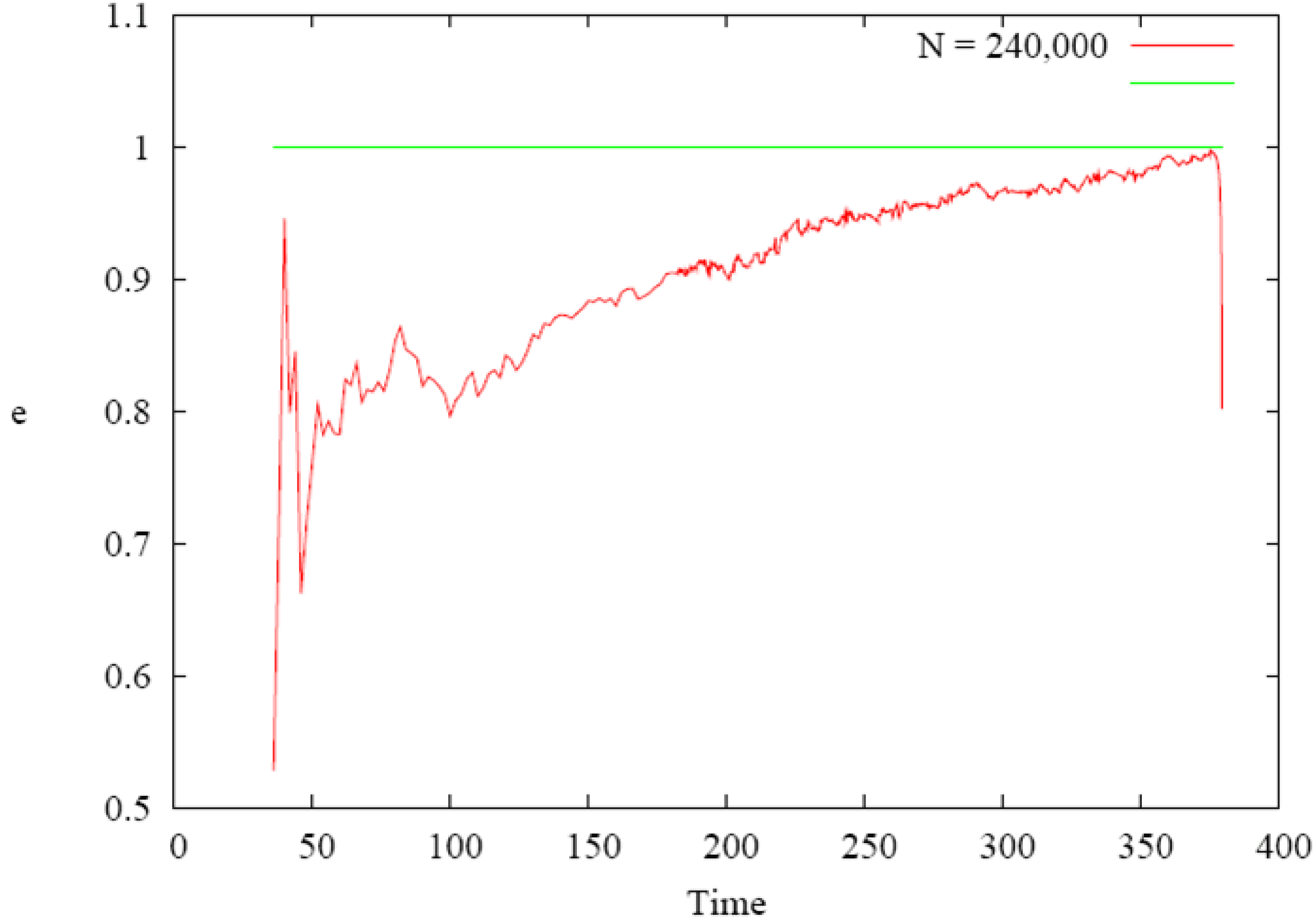


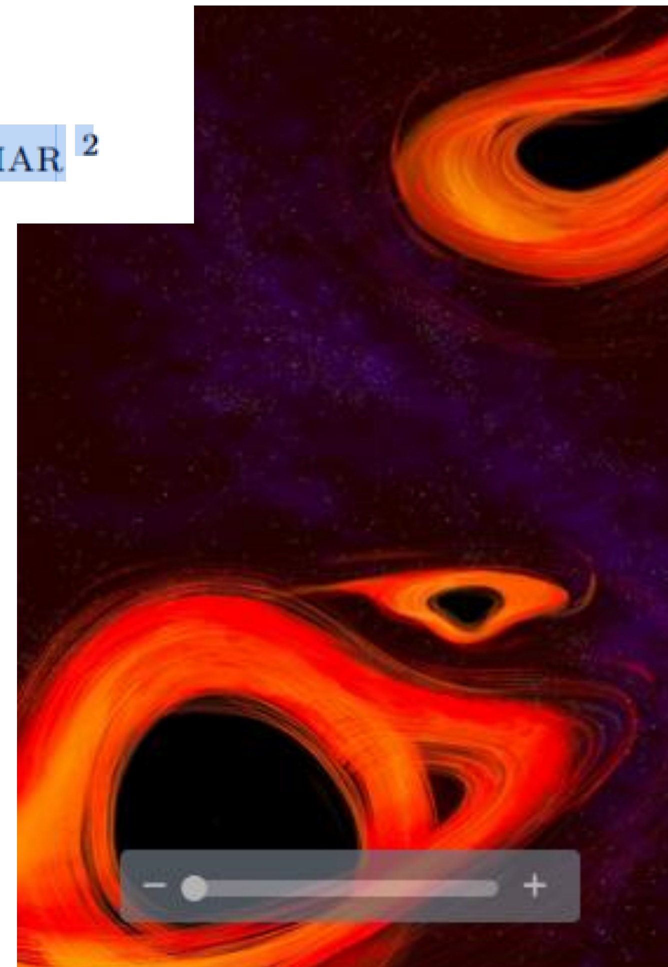
N = 240,000
a
1
0.1
0.01
0.001
1e-04
1e-05
1e-06
1e-07
0
50
100
150
200
250
300
350
400
Time



N = 240,000
e
Time
1.1
1
0.9
0.8
0.7
0.6
0.5
0
50
100
150
200
250
300
350
400

Unveiling Supermassive Black Holes And Their Binaries

Mauri Valtonen [1] & Achamveedu Gopakumar [2]



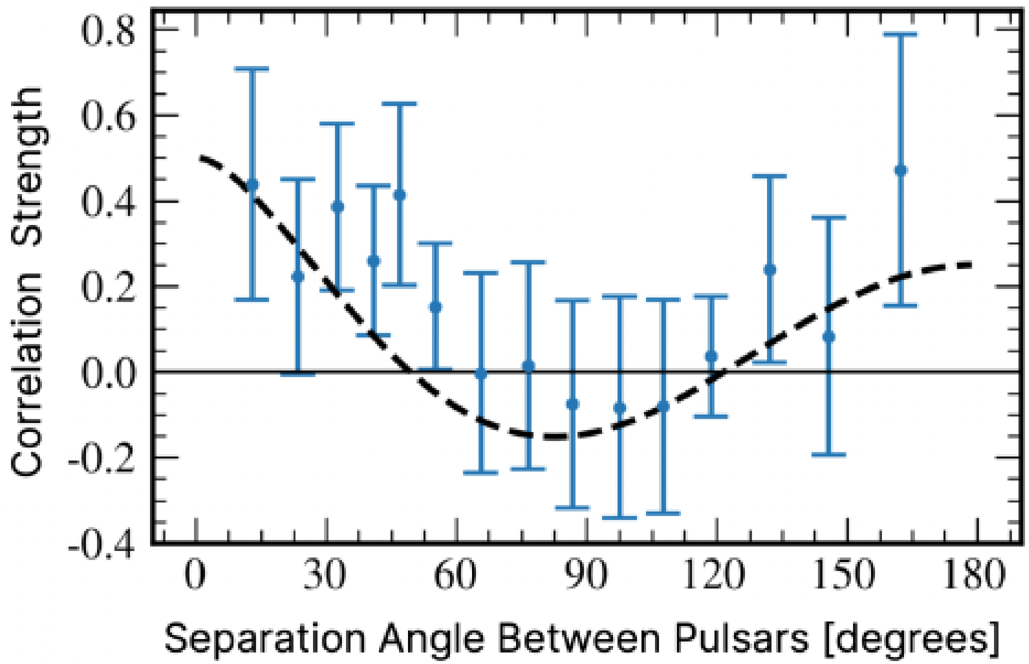


Figure 10.2: Evidence for a Gravitational Wave Background in Pulsar Timing Data. Einstein's general relativity predicts that a stochastic background of low-frequency gravitational waves should produce a specific spatial correlation (Hellings-Downs curve, black dashed) in the timing residuals of an array of millisecond pulsars. By averaging correlations across pulsar pairs, the NANOGrav Collaboration detects a signal (blue points) consistent with this predicted pattern, providing strong evidence for a universe filled with gravitational waves, most likely from merging supermassive black hole binaries. (Credit: NANOGrav Collaboration)

V. *Illustrations of the Dynamical Theory of Gases.*—Part I. *On the Motions and Collisions of Perfectly Elastic Spheres.* *By* J. C. MAXWELL, *M.A., Professor of Natural Philosophy in Marischal College and University of Aberdeen**.

SO many of the properties of matter, especially when in the gaseous form, can be deduced from the hypothesis that their minute parts are in rapid motion, the velocity increasing with the temperature, that the precise nature of this motion becomes a subject of rational curiosity. Daniel Bernouilli, Herapath, Joule, Krönig, Clausius, &c. have shown that the relations between pressure, temperature, and density in a perfect gas can be explained by supposing the particles to move with uniform velocity in straight lines, striking against the sides of the containing vessel and thus producing pressure. It is not necessary to suppose each particle to travel to any great distance in the same straight line; for the effect in producing pressure will be the same if the particles strike against each other; so that the straight line described may be very short. M. Clausius has determined the mean length of path in terms of the average distance

* Communicated by the Author, having been read at the Meeting of the British Association at Aberdeen, September 21, 1859.

- Henri Poincare:

Sur le probleme des
trois corps
Bull.Astron. 8, 12, 1891

King Oscar Prize 1889

2 SUNDMAN'S INEQUALITY

Szebehely gives Sundman's inequality as

$$\mathbf{c}^2 \leqslant I\,(\ddot{I} - 2h) - \dot{I}^2/4, \qquad (1)$$

where $\mathbf{c}$ is the angular momentum, h the total energy and I the moment of inertia of the three-body system. For the case of $\dot{I} = 0$ Szebehely (1973) derives

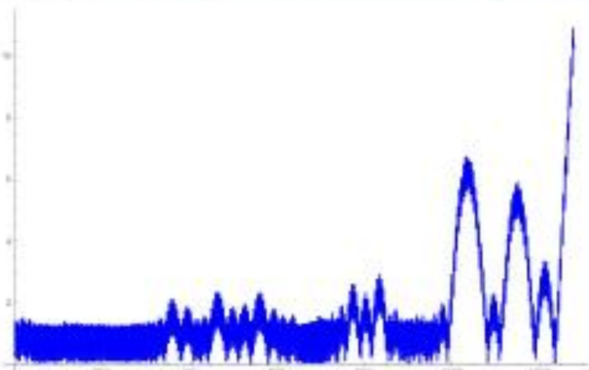
$$\mathbf{c}^4 \leqslant (2\,|h|)^2 I_{min} I_{max}, \qquad (2)$$

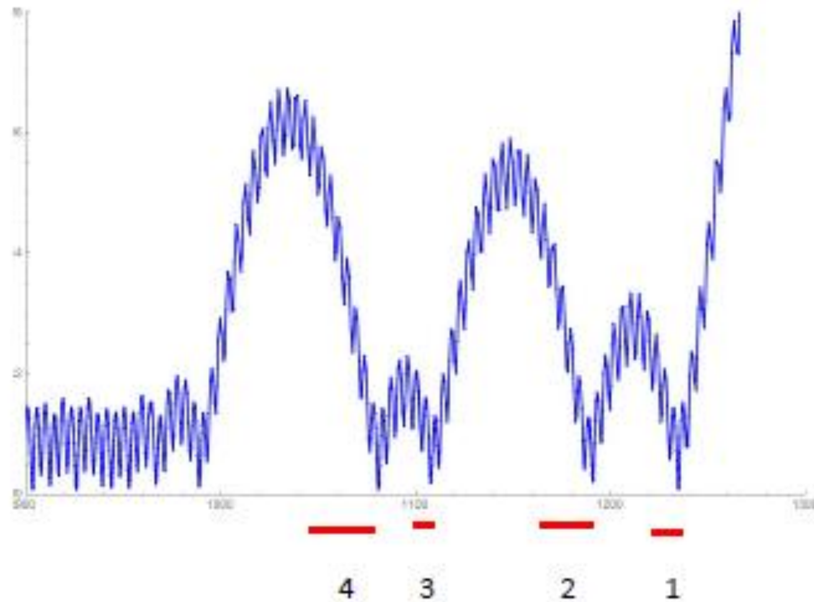
where I_{min} is the minimum value of the moment of inertia and I_{max} is its previous maximum value. Szebehely (1973) inferred

M2	M3	ein0	eex0	sincl	Omega0	bomegaex0	aex
0.9	0.01	0.6	0.9	1.6015684190895385	0.150107149412273	6.270699226920502	10

h=-0.4979 c^2=0.3133

n	Imin	Imax	4h^2 Imin Imax/c^4
1	0.0926	3.0326	2.8
2	0.2165	5.9026	12.9
3	0.1898	2.2887	4.4
4	0.0982	6.7170	6.7





- Sundman’s inequality limits the phase space of all possible orbits: the probability for decrease of angular momentum is greater than its increase -> binary goes to zero angular momentum **c** = 0.

- See Memoirs RAS 80, 77 (1975)

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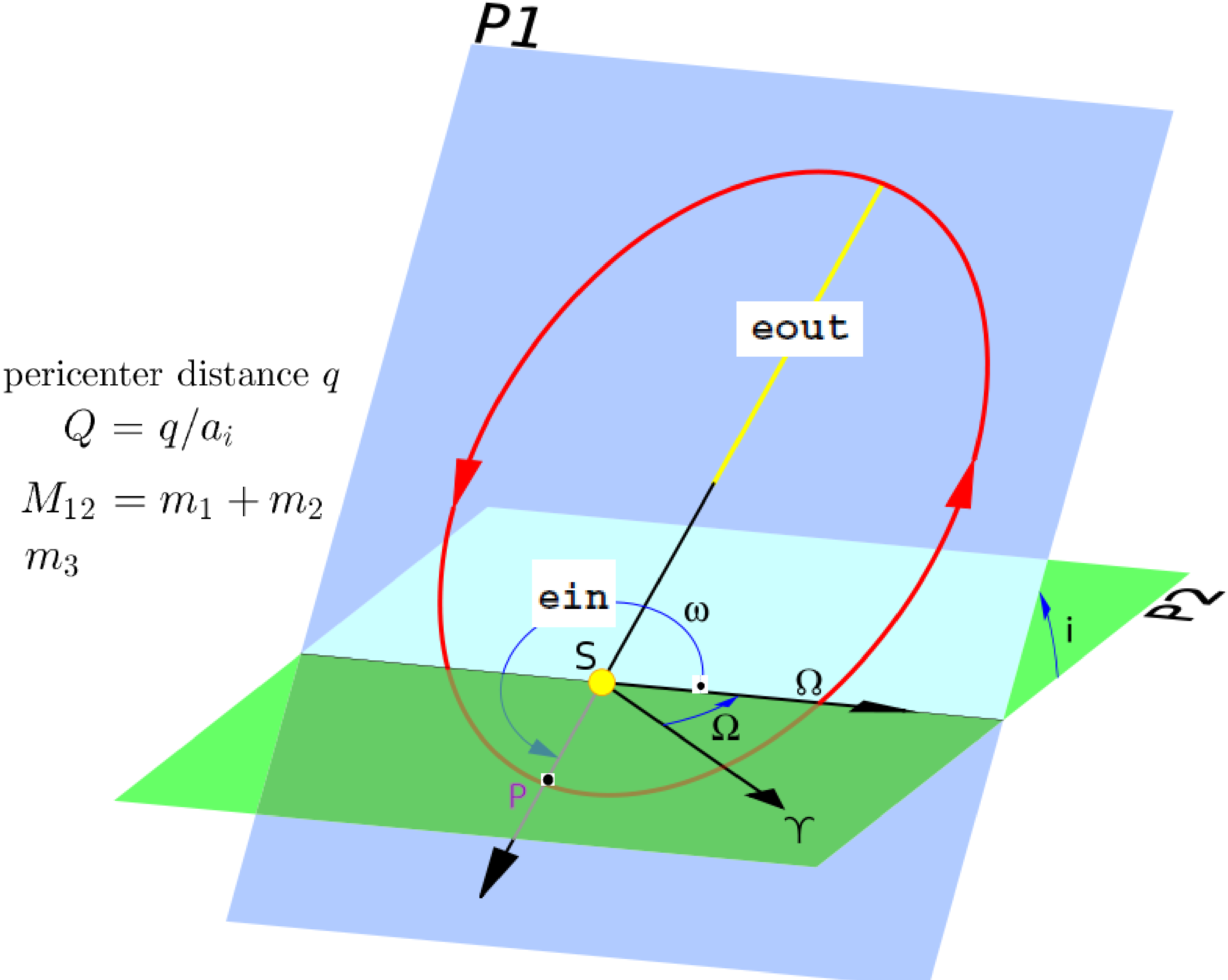
Gravitational Scattering

Douglas C. Heggie

Univ. of Edinburgh, Scotland


P1
eout
pericenter distance q
$Q = q/a_i$
$M_{12} = m_1 + m_2$
m_3
ein
ω
S
Ω
Ω
i
P2
P
ϒ

Heggie, Roy & Haddow

2. Energy change in a single encounter

$$\begin{aligned}\frac{\delta\varepsilon}{\varepsilon} \simeq -\frac{\sqrt{\pi}}{4}\frac{m_3}{M_{12}}Q^{-3}K^{5/2}e^{-(2/3)K}\,\{e_1\,[\sin(2\omega+nt_0)(\cos 2i-1)\\ -\sin(2\omega+nt_0)\cos(2i)\cos(2\Omega)-3\sin(nt_0+2\omega)\cos(2\Omega)\\ -4\sin(2\Omega)\cos(2\omega+nt_0)\cos i]+e_2(1-e_i^2)\,[\sin(2\omega+nt_0)(1-\cos 2i)\\ -\sin(2\omega+nt_0)\cos(2i)\cos(2\Omega)-3\sin(nt_0+2\omega)\cos(2\Omega)\\ -4\cos(nt_0+2\omega)\sin(2\Omega)\cos i]+e_4\sqrt{1-e_i^2}\,[-2\cos(2i)\cos(2\omega+nt_0)\sin(2\Omega)\\ -6\cos(2\omega+nt_0)\sin(2\Omega)-8\cos(2\Omega)\sin(2\omega+nt_0)\cos i]\}\,.\end{aligned} \tag{2.1}$$

Here n is the mean motion of the binary and t_0 is a reference time. The true anomaly of the binary $M=n(t-t_0)$.

K is defined as

$$K=\sqrt{2}\sqrt{\frac{M_{12}}{M_{12}+m_3}}\,Q^{3/2}.$$

The functions e_1, e_2 and e_4 are

$$\begin{aligned}e_1 &= J_{-1}(e_i)-2e_iJ_0(e_i)+2e_iJ_2(e_i)-J_3(e_i),\\ e_2 &= J_{-1}(e_i)-J_3(e_i),\\ e_4 &= J_{-1}(e_i)-e_iJ_0(e_i)-e_1J_2(e_i)+J_3(e_i).\end{aligned}$$

$$\frac{\delta\varepsilon}{\varepsilon} \sim 10^{-3}\frac{m_3}{M_{12}}(Q/Q_1)^{-7}(9 + 7cos\mathbf{i} - 2cos^2\mathbf{i})$$

- Direct orbit: cos i = 1

( ) = 14

Carries away angular momentum

Retrograde orbit: cos i = -1

( ) = 0

Does nothing

See also Memoirs of RAS 80, 61 (1975)

Conclusion

- By 1975 it was already possible to infer that in supermassive binaries in a star field e -> 0 (two Memoirs of RAS papers).
- By 2005 it was confirmed by N-body simulations (Aarseth) and by analytic solution the three-body problem (Heggie).

Citations (ADS)

- Poincare 1891, 5
- Sundman 1907, (31)
- Szebehely 1973, 3
- Aarseth 2006, 6
- Heggie 2006, 8