MODELLING SCATTERING MATRICES FOR ASTEROID REGOLITH USING MACHINE LEARNING

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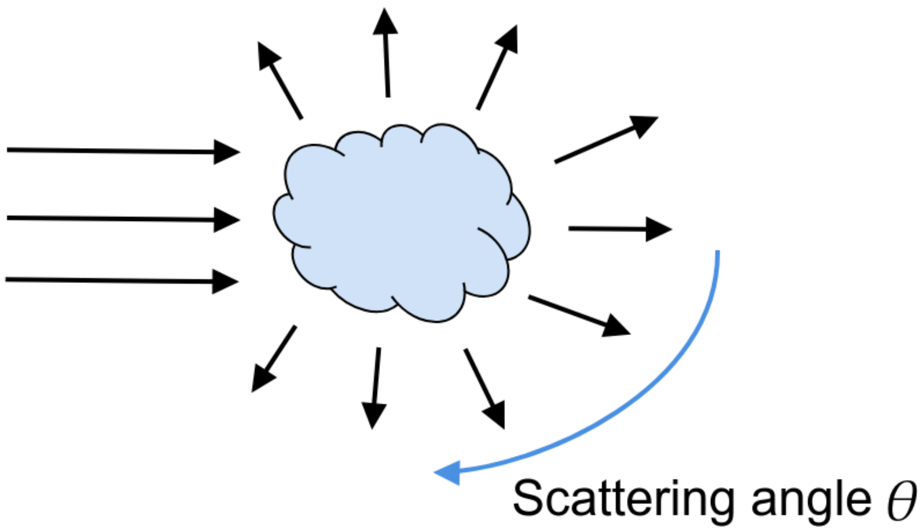
Background

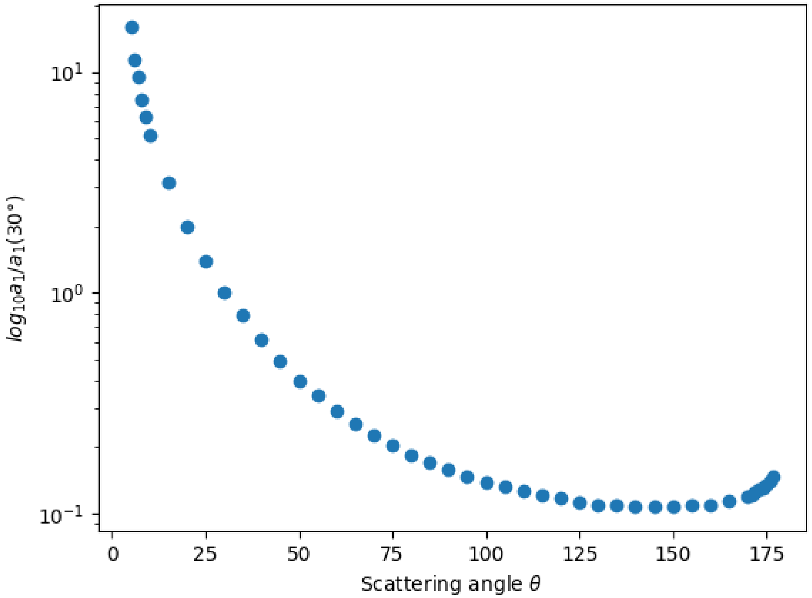
- When modelling scattering processes in asteroid regolith, single scatterers can be described with a 4x4 Mueller matrix, the scattering matrix
  - Contains information about size, shape and composition of particles
  - Block-diagonal form when averaged over a particle ensemble

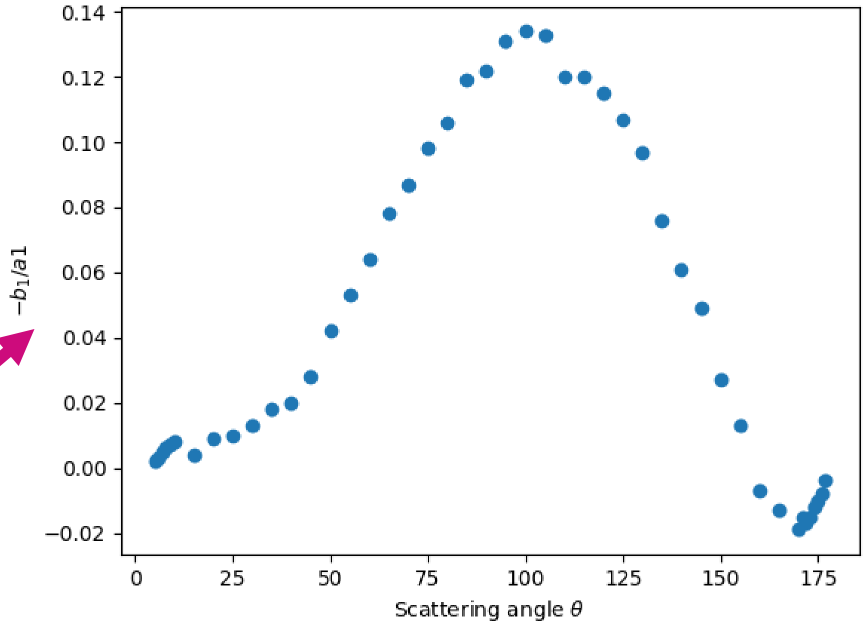
$$\mathbf{F}(\theta) = \begin{pmatrix} a_1(\theta) & b_1(\theta) & 0 & 0 \\ b_1(\theta) & a_2(\theta) & 0 & 0 \\ 0 & 0 & a_3(\theta) & b_2(\theta) \\ 0 & 0 & -b_2(\theta) & a_4(\theta) \end{pmatrix}$$

  - Decomposition into so-called pure Mueller matrices allows incorporation into geometric optics code (Muinonen, K., & Penttilä, A.,2024)



Example: Measurement of volcanic ash at 647 nm





$$\mathbf{F}(\theta) = \begin{pmatrix} a_1(\theta) & b_1(\theta) & 0 & 0 \\ b_1(\theta) & a_2(\theta) & 0 & 0 \\ 0 & 0 & a_3(\theta) & b_2(\theta) \\ 0 & 0 & -b_2(\theta) & a_4(\theta) \end{pmatrix}$$

Measurements from Granada-Amsterdam database by Merikallio et al. 2015, Munoz et al. 2025

Parameterized scattering matrix

- Scattering matrix elements can be modelled with analytical functions and certain symmetry relations as a function of scattering angle θ
  - For detailed model explanation see, Muinonen, K., &Leppälä, A. S. (2025)
- 48 parameters in 6 functions:

$$\mathrm{a}_1 = \frac{D_1}{1+d_1 q^{\delta_1}} + \frac{R_1}{1+r_1 p^{\rho_1}} + S_1 e^{-s_1 p^{\sigma_1}},$$

$$\frac{a_j}{a_1} = D_j p^{\delta_j} + R_j q^{\rho_j} + S_j e^{-s_j p^{\sigma_j}}, \qquad j = 2, 3, 4,$$

$$\frac{b_j}{a_1} = A_j p^{\beta_j} q^{\gamma_j} + H_j p^{\beta_j}\left(1 - e^{-h_j q^{\eta_j}}\right) + K_j q^{\gamma_j}\left(1 - e^{-k_j p^{\kappa_j}}\right), \qquad j = 1, 2,$$

$$where \qquad p = \cos\frac{\theta}{2} = \sqrt{1-q^2}, \qquad q = \sin\frac{\theta}{2}.$$

- Finding parameters for optimal fit is not straightforward

Physics-informed neural networks (PINNs)

- Embed physical laws into neural network training process
- PINNs are efficient
- Physical laws are incorporated through:
  - Soft constraints with a custom loss function $Loss = Loss_{data} + Loss_{physics}$
  - Hard constraints with custom network architectures

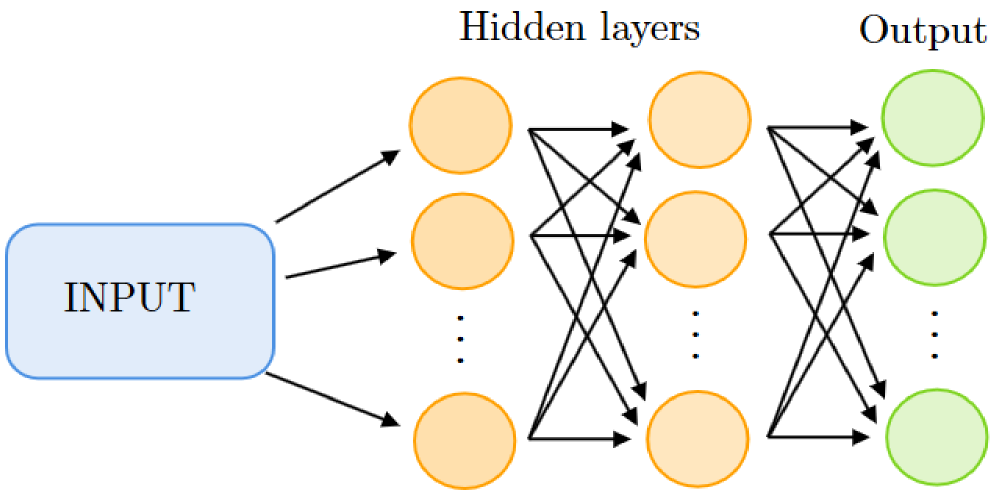


Fig 1: Simplified data-driven neural network architecture

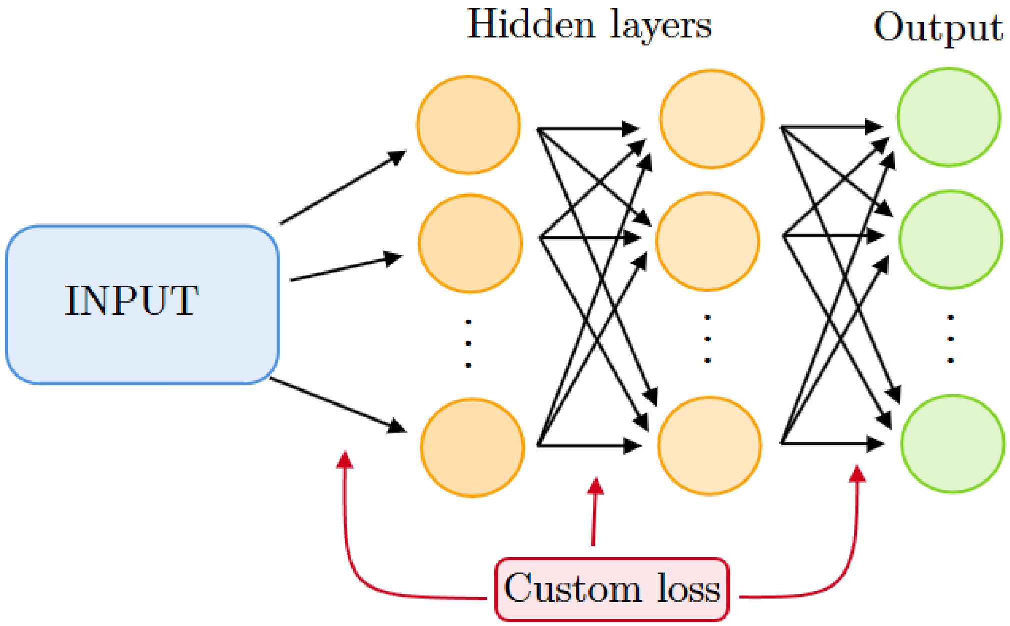


Fig 2: Simplified physics-informed neural network architecture

Neural network architecture

- Two combined deep neural networks: a_1 and all other scattering matrix elements with interrelations
- NN learning constrained with custom loss functions

$$Loss_1 = Loss_{fit}$$
$$Loss_2 = Loss_{fit} + Loss_{symmetry}$$

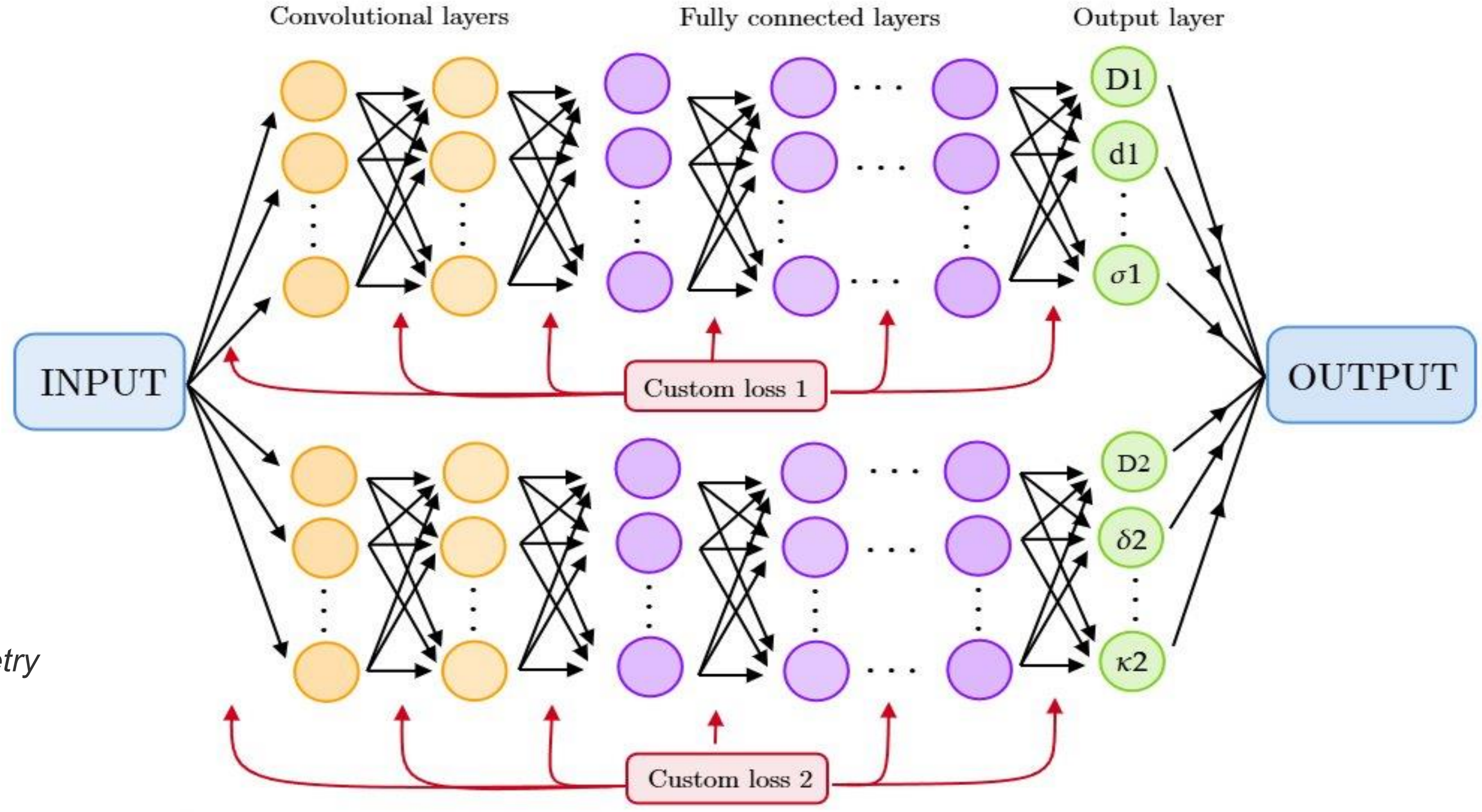
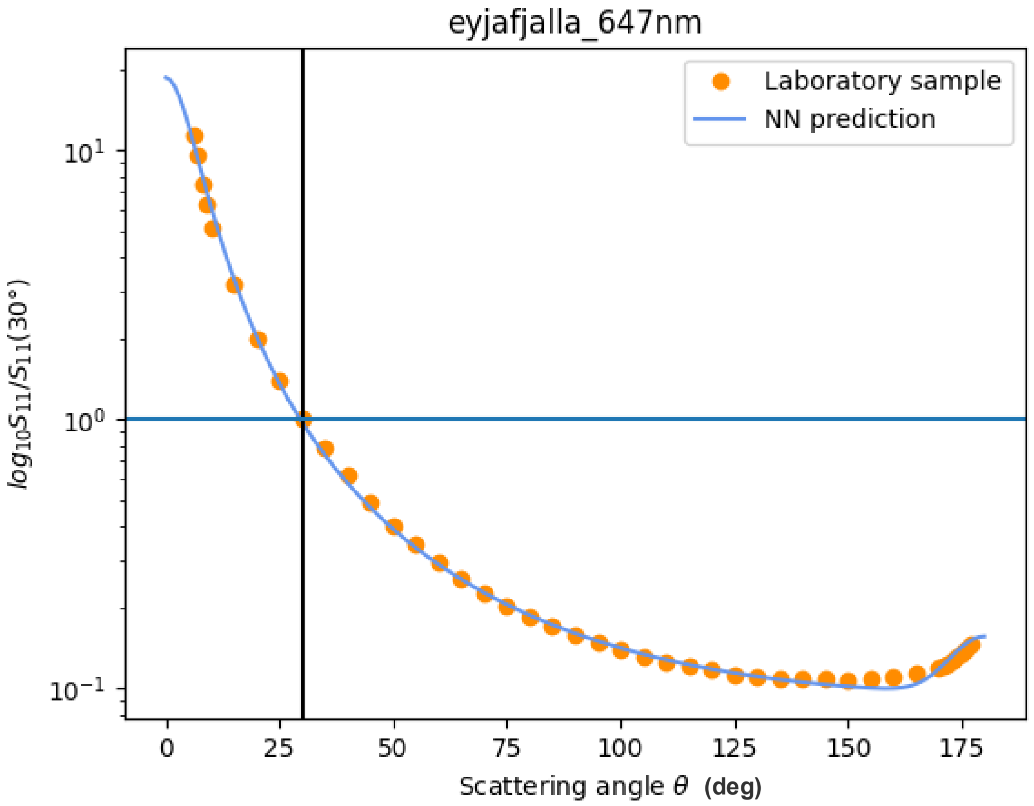


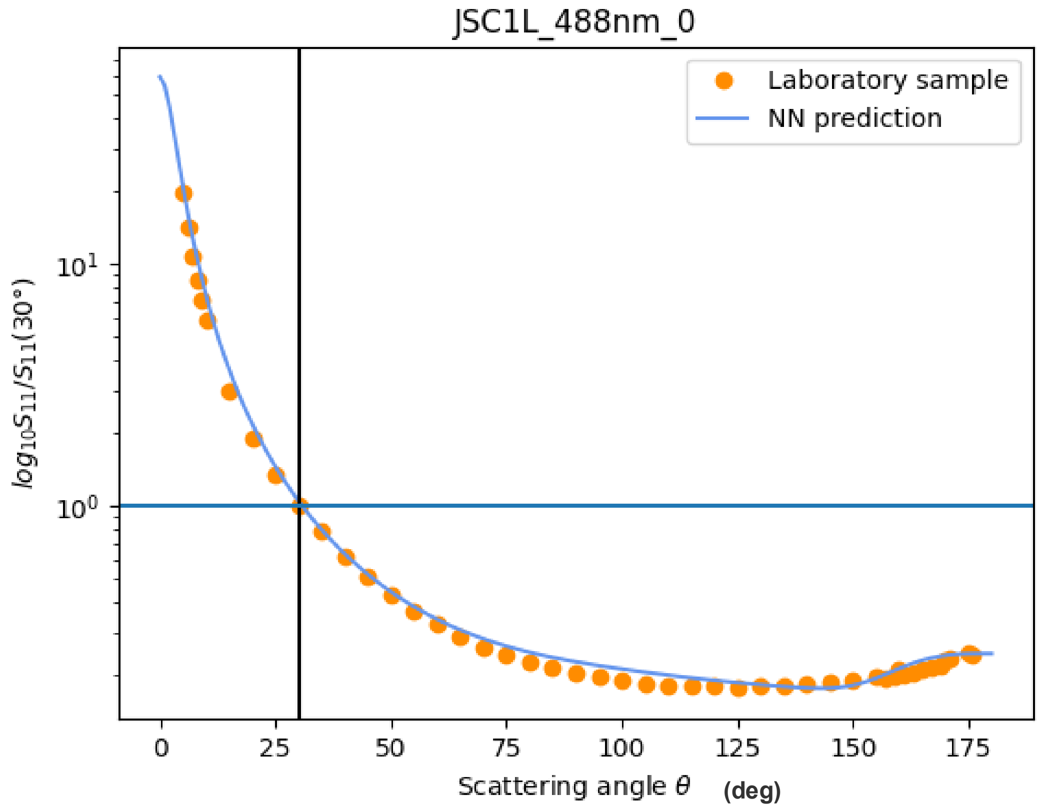
Fig 3: Employed neural network architecture

Preliminary results: a_1

- Granada-Amsterdam laboratory measurements are used as a benchmark for neural network (NN) performance



Volcanic ash



Lunar analogue

Conclusions and future work

- PINNs are efficient in parameter estimation
- Parameters are estimated with two combined deep neural networks
- The preliminary results show that reasonable fits of a_1 laboratory measurements can be computed
- Next steps:
  - Implementation of the second neural network is in progress
  - a_1 NN can be tweaked to further increase accuracy

THANK YOU FOR LISTENING

Questions or comments?

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