Waves in ultra-magnetized plasmas

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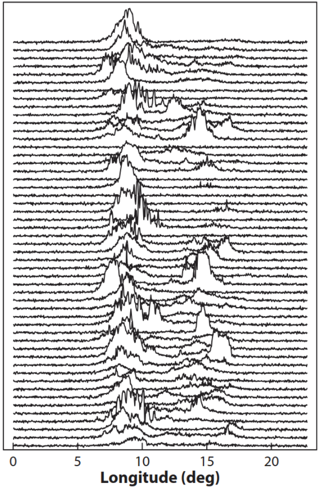
Nordic-Baltic Astronomy days

Centre of Excellence
neutronstars.fi

JENNY AND ANTTI WIHURI FOUNDATION

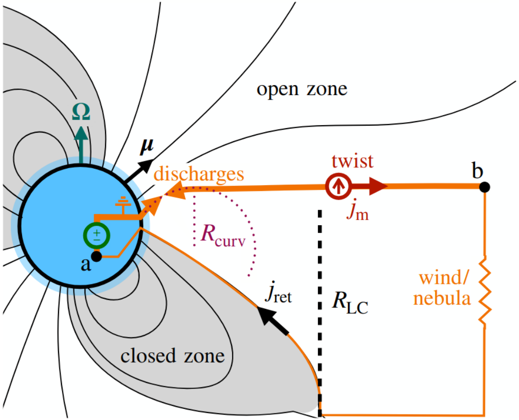
Motivation

- Pulsar radio-wave emissions
- Frequency ranges: 100 MHz – 1 GHz
- Many open questions
  - Origin of radio emission
  - Generation of polarization modes
- This talk: Generation of the X-mode

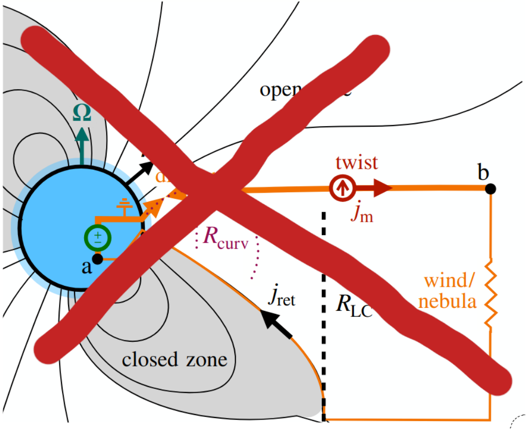


Picture from: Philippov and Kramer, 2022, Data: von Hoensbroech et al., 1998

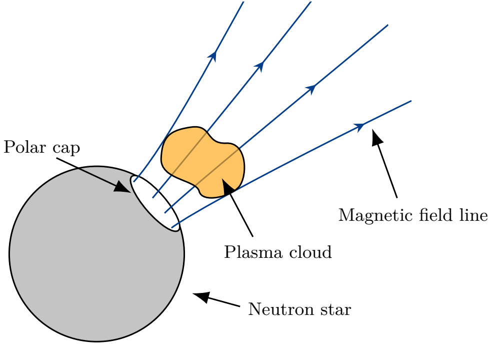
Motivation



Nättilä and Salmi 2026 in prep.

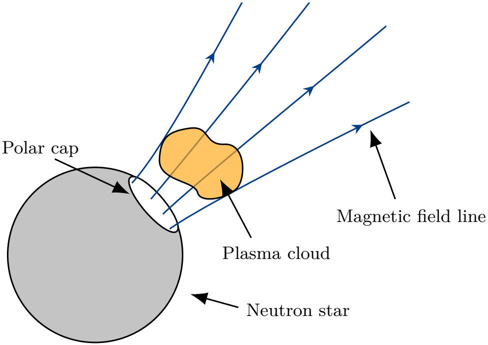


Nättilä and Salmi 2026 in prep. (adapted)



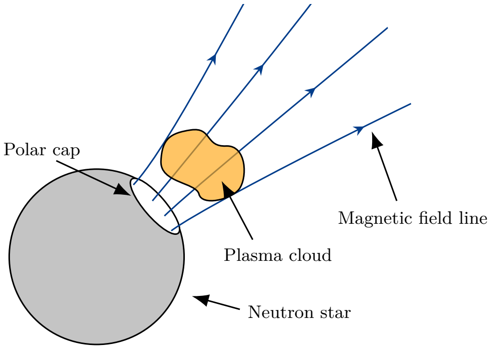
Pair plasmas in polar caps

- Positron-electron plasma described by distribution functions f_{e} and f_{p}
- Pair production process



Pair plasmas in polar caps

- Positron-electron plasma described by distribution functions f_e and f_p
- Pair production process
- Current: $\boldsymbol{J} = e(n_p \boldsymbol{v}_p - n_e \boldsymbol{v}_e)$
  - $n_{p/e}$: electron / positron density
  - $\boldsymbol{v}_{p/e}$: electron / positron velocity

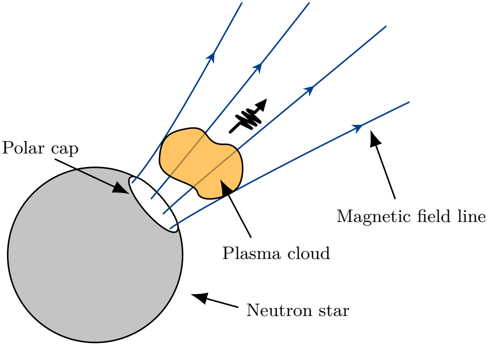


- Currents acts as a source

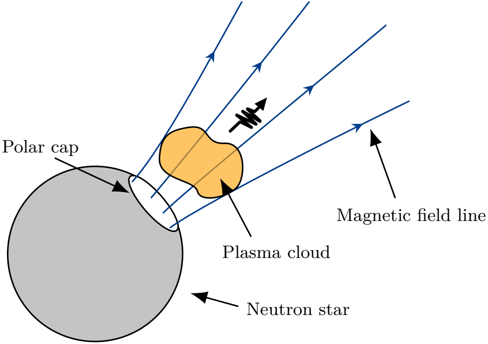
$$-\partial_t^2 \boldsymbol{E} + c^2 \boldsymbol{k} \times (\boldsymbol{k} \times \boldsymbol{E}) = 4\pi \partial_t \boldsymbol{J}$$

- $\boldsymbol{E}$: electric field, $\boldsymbol{J}$: current, $\boldsymbol{k}$ is the wave vector, c: speed of light

- The current defines which waves are emitted



- Charged particles in a magnetic field move in a helical path.
- Magnetic field strength is very high, $B_0 \sim 10^{12}\,\mathrm{G}$
  - Comparison: refrigerator magnet $\sim 10\,\mathrm{G}$



Parallel current

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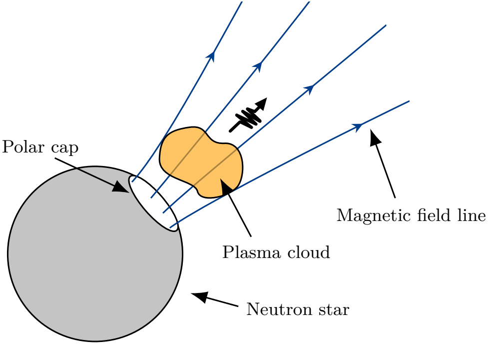
- High $B_0 \implies$ extremely fast synchrotron radiation time scale $\sim 10^{-18}\,\mathrm{s}$ (rough estimate)



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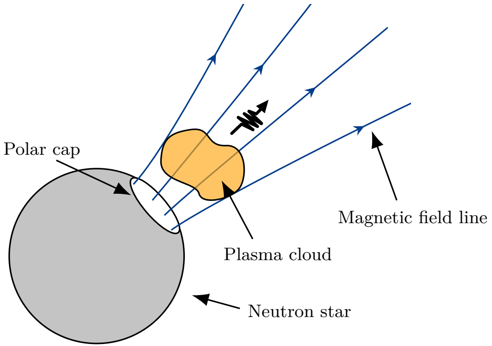
- High $B_0 \implies$ extremely fast synchrotron radiation time scale $\sim 10^{-18}\,\mathrm{s}$ (rough estimate)
- Only parallel currents w.r.t. the magnetic field



Parallel currents cannot excite the X-mode

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$$-\partial_t^2 \boldsymbol{E} + c^2 \boldsymbol{k} \times (\boldsymbol{k} \times \boldsymbol{E}) = 4\pi \partial_t \boldsymbol{J} \implies \partial_t^2 E^X + c^2 k^2 E^X = -4\pi \partial_t \boldsymbol{J} \cdot \left(\hat{\boldsymbol{k}} \times \hat{\boldsymbol{B}}_0\right)$$

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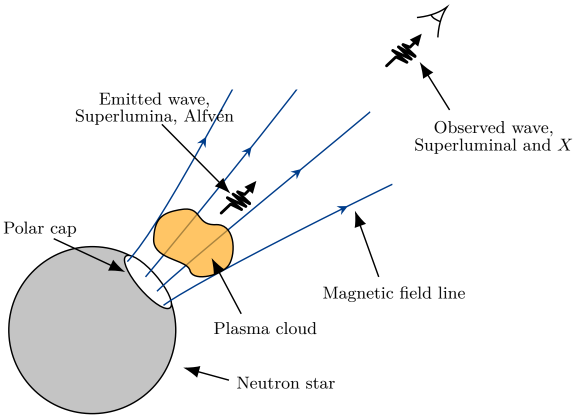
After the synchrotron radiation, there is no perpendicular motion. Hence, $\partial_t \boldsymbol{J} \cdot \left(\hat{\boldsymbol{k}} \times \hat{\boldsymbol{B}}_0\right) = 0$, and $E^X = 0$.

The problem

- Parallel currents can excite two modes: Alfvén and superluminal
  - Alfvén waves are damped, and hence not observed
  - The superluminal waves are connected to observations



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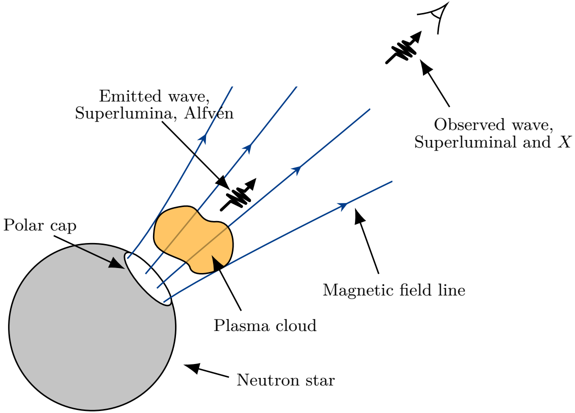

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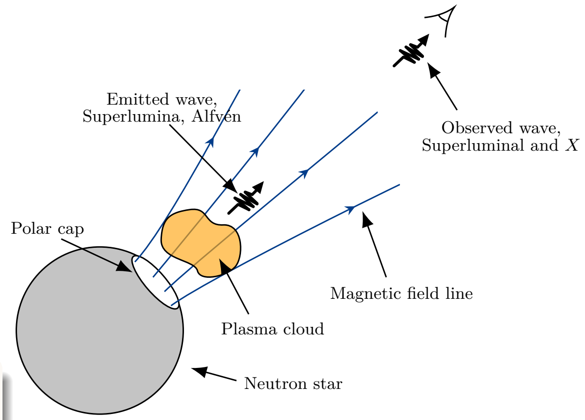

Since the parallel currents cannot excite the X-mode, we need to look for a perpendicular current

Pre-radiation time scales

- Gyrotime scale: $t_\Omega \sim \Omega^{-1} = B_0^{-1} e^{-1} m_\mathrm{e}$
- Radiation time scale: $t_\mathrm{rad} \sim B_0^{-2} \gg t_\Omega$
- Radiation falls of rapidly as moving away from the star
- Investigate time scales $t_\Omega \ll t \ll t_\mathrm{rad}$
- Goal: Find a current that can excite the X-mode before the radiation happens

- To find the current, we need to solve the Vlasov-equation

$$\underbrace{\partial_t f_{\mathrm{p/e}}}_{\text{Time evolution}} + \underbrace{i\boldsymbol{k}\cdot\boldsymbol{v} f_{\mathrm{p/e}}}_{\text{Phase mixing}} + \underbrace{\Omega_{\mathrm{p/e}}\gamma^{-1}\partial_\phi f_{\mathrm{p/e}}}_{\text{Gyro-motion}} + \underbrace{q_{\mathrm{p/e}} m^{-1}(\boldsymbol{E}+\boldsymbol{\beta}\times\boldsymbol{B})\cdot\nabla_{c\boldsymbol{u}} f_{\mathrm{p/e}}}_{\text{Wave-Particle interaction}} = \underbrace{-\boldsymbol{\nabla}\cdot\left(\boldsymbol{F}_{\mathrm{RR}} f_{\mathrm{p/e}}\right)}_{\text{Radiation reaction}}$$

Reduced Vlasov-equation

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- Pre-radiation scale ($t \ll t_{\mathrm{rad}}$): Drop the radiation reaction force, and allow the plasma to have a perpendicular temperature.

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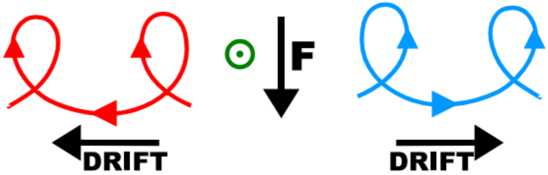
- Drop the wave-particle interaction; we are interested in the bulk-motion

- Current due to the diamagnetic drift

Diamagnetic drift

- Pressure gradients causes the particles to drift

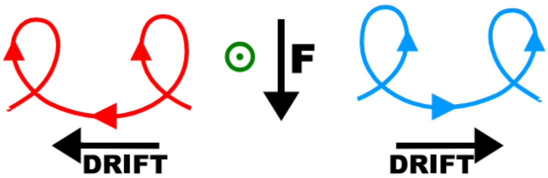


Picture from: https://en.wikipedia.org/wiki/Guiding_center

Diamagnetic drift

- Pressure gradients causes the particles to drift
- With the current, solve the wave equation

$$\partial_t^2 E^X + c^2 k^2 E^X = -4\pi \partial_t \boldsymbol{J} \cdot \left(\hat{\boldsymbol{k}} \times \hat{\boldsymbol{B}}_0\right)$$



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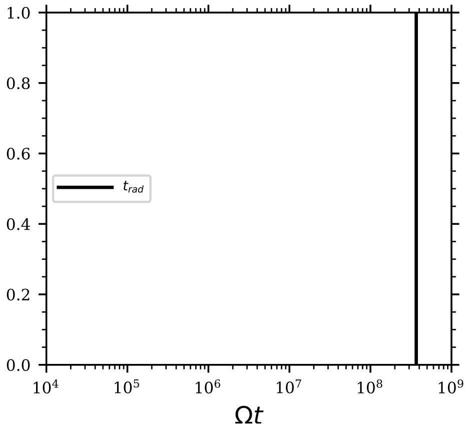
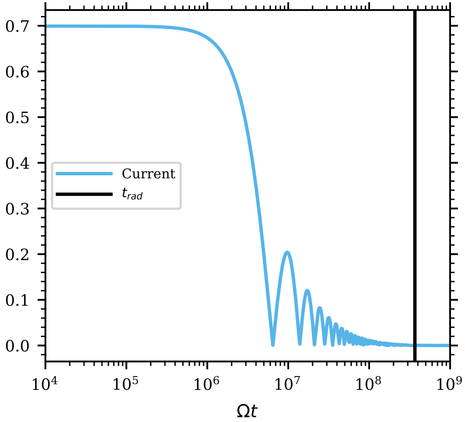
- Time scales $t_\Omega \ll t \ll t_{\mathrm{rad}}$
- $\Omega = eB_0/mc$ is the gyrofrequency



Linho+ 2026 (in. prep)

Results

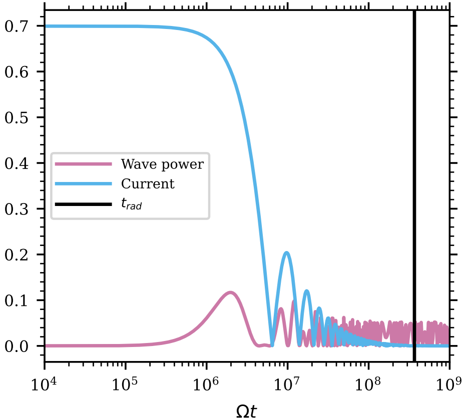
- Time scales $t_\Omega \ll t \ll t_{\rm rad}$
- $\Omega = eB_0/mc$ is the gyrofrequency
- Current caused by the diamagnetic drift



Linho+ 2026 (in. prep)

Results

- Time scales $t_\Omega \ll t \ll t_{\mathrm{rad}}$
- $\Omega = eB_0/mc$ is the gyrofrequency
- Current caused by the diamagnetic drift
- Diamagnetic current excites the electric field



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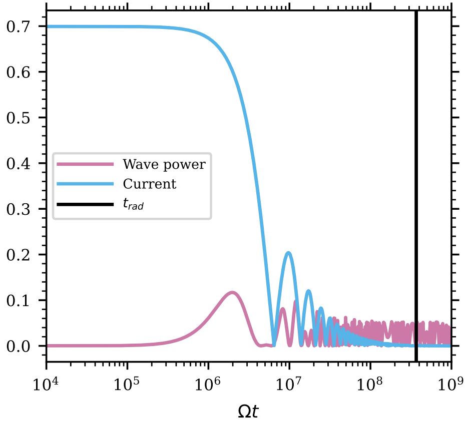
Future

- Results depend heavily on the perpendicular temperature
- We ignore many important phenomena
  - The wave-particle interaction
  - Gradients in the background magnetic field
  - Charge separation
- t_{rad} is highly dependent of the distance from the star



Thank you! Questions?