Hadron Production via the Coalescence model

Applications to deuteron, ^{3}He, Ξ_c and X(3872)

HyeongOck Yun

Coalescence model

2-dimension coalescence model

- Hadron transverse momentum distribution

$$\frac{d^2N_h}{d^2P_T} = g_h \int \prod_{i=1}^{N} d^2x_i d^2p_i \ f_i(x_i, p_i) W(r_1, \cdots r_{N-1}, k_1, \cdots, k_{N-1}) \delta^{(2)} \left(P_T - \sum_{j=1}^{N} p_j \right)$$

g_h : statistical factor, $f_i(x_i, p_i)$: p_{T} distribution of constituent , $W(r, k)$: Wigner function

- Normalization condition

$$\int d^2x_i d^2p_i \ f_i(\vec{x}_i, \vec{p}_i) = N_i, \quad \int \prod_{i=1}^{N-1} d^2r_i d^2k_i W_H(\vec{r}_1, \cdots, \vec{r}_{N-1}, \vec{k}_1, \cdots, \vec{k}_{N-1}) = (2\pi)^{2(N-1)}$$

$$f(x_i, p_i) = \frac{d^2N_i}{Ad^2p_{iT}}$$

Coalescence model

2-body coalescence

- Wigner function :

Gaussian type Wigner function - $W(\vec{r},\vec{k}) = 4\exp\left[-\frac{r^2}{\sigma^2} - \sigma^2 k^2\right] \quad \rightarrow \quad 4\exp\left[-\frac{(r')^2}{\sigma^2} - \sigma^2 (k')^2\right],$ Primed frame : CM frame

Sudden hadronization approximation

$\rightarrow \quad 4\exp\left[-\frac{\gamma r^2}{\sigma^2} - \frac{\sigma^2 k^2}{\gamma}\right]$

For simplicity, Shown in 1D

Coordinate : $R^\mu = \frac{m_1 x_1^\mu + m_2 x_2^\mu}{m_1 + m_2}, \quad r^\mu = x_1^\mu - x_2^\mu,$

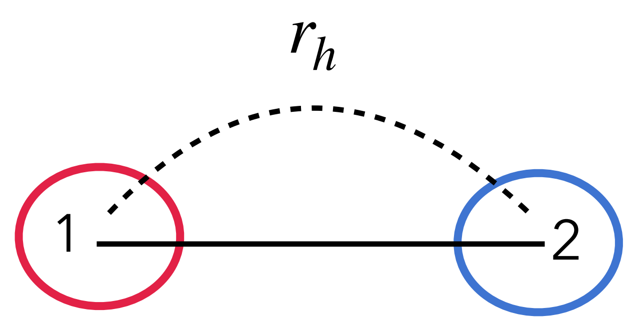
$P^\mu = p_1^\mu + p_2^\mu, \quad k^\mu = \frac{m_2 p_1^\mu - m_1 p_2^\mu}{m_1 + m_2}$

Lorentz transformation : $\Delta t' = \gamma(\Delta t - \beta r_x), \quad r'_x = \gamma(r_x - \beta \Delta t)$

$\Delta E' = \gamma(\Delta E - \beta k_x), \quad k'_x = \gamma(k_x - \beta \Delta E)'$

- 2 body coalescence formula

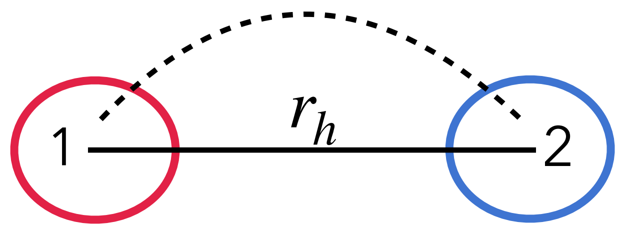
$$\frac{d^2 N_h}{d^2 P_T} = \frac{g_h}{g_1 g_2} (2\sqrt{\pi}\sigma)^2 \frac{1}{A} \int d^2 p_1 d^2 p_2 \, \frac{d^2 N_1}{d^2 p_{1T}} \frac{d^2 N_2}{d^2 p_{2T}} \exp\left[-\sigma^2 (k')^2\right] \delta^{(2)}(P_T - p_{1T} - p_{2T}), \quad \langle r_h^2 \rangle = \frac{3}{2}\sigma^2$$



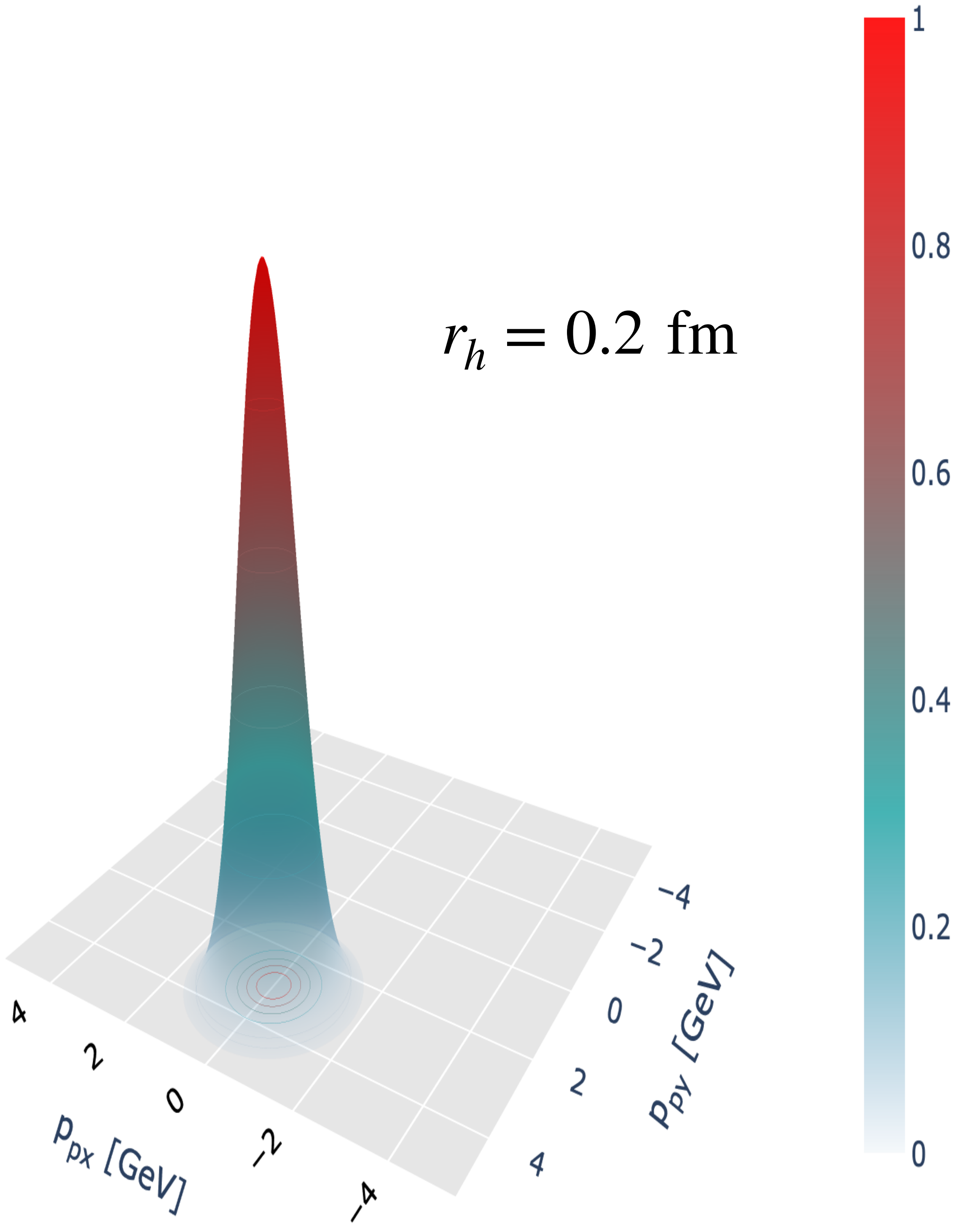
Coalescence model

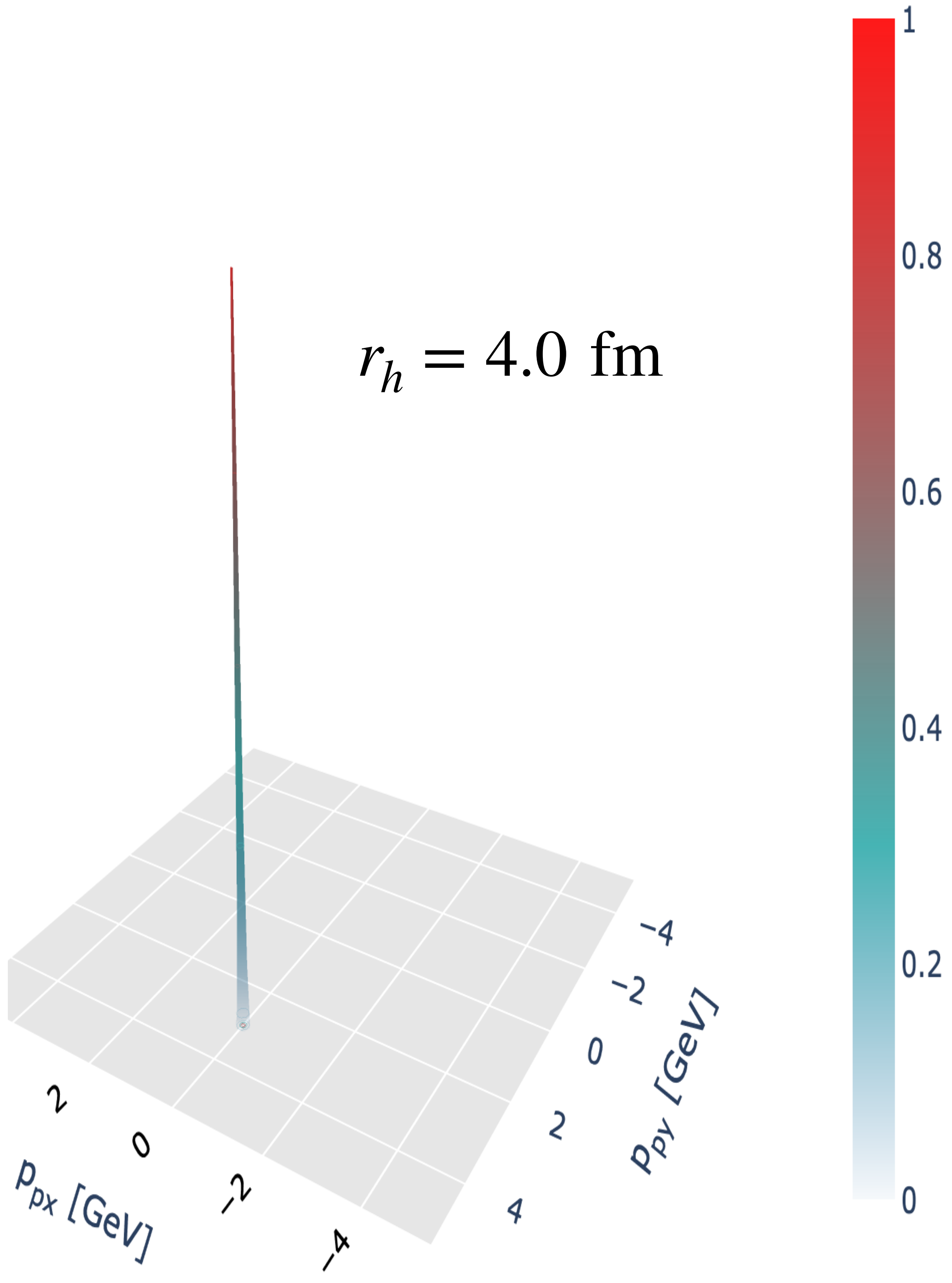
Wigner function

$\exp\left(-\sigma^2 k^2\right)$



$\langle r_h^2 \rangle = \frac{3}{2}\sigma^2$





- In $\sigma \to \infty$ limit, $W(\vec{r}, \vec{k}) = 4\exp\left[-\frac{r'^2}{\sigma^2}\right] \times \left(\frac{\pi}{\sigma^2}\right) \delta^{(2)}(\vec{k}')$

- Coalescence formula :

$$\frac{d^2N_h}{d^2P_T} = \frac{g_h}{g_1 g_2}(2\pi)^2 \left(\frac{\gamma}{A}\right) \frac{d^2N_1}{d^2p_{1T}}\Big|_{\vec{p}_{1T}=\frac{\vec{P}_{hT}}{2}} \frac{d^2N_2}{d^2p_{2T}}\Big|_{\vec{p}_{2T}=\frac{\vec{P}_{hT}}{2}}$$

- Coalescence formula for nuclei i (in experiment paper) :

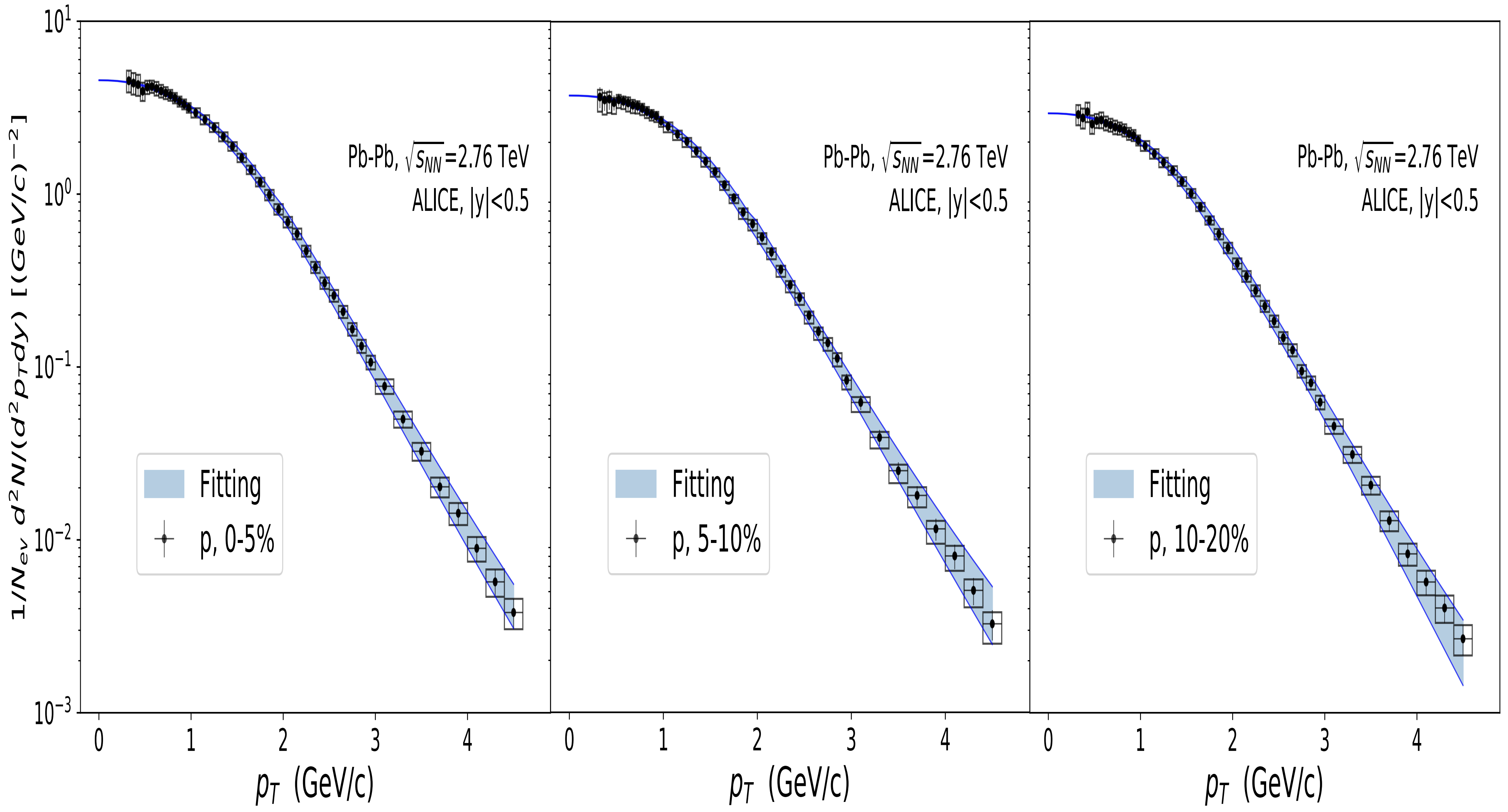
$$E_i \frac{d^3N_i}{dp_i^3} = B_A \left(E_p \frac{d^3N_p}{dp_p^3}\right)^A, \quad A = \text{mass number}, \quad p_i = Ap_p$$

The increase of B_A can be explained by the Lorentz boost effect (γ)

Deuteron and ^3He

Proton distribution and coalescence area A

- Fitting (ALICE Collaboration, Phys. Rev. C 88, 044910)



$$\frac{d^2N_{p,n}}{d^2p_T}\Big|_{t=t_f} = r_f \frac{d^2N_{p,n}}{d^2p_T}\Big|_{Exp}$$

$$r_f = \frac{\textit{the number of proton at formation point}}{\textit{the number of final proton}}$$

$$\frac{d^2N_d}{d^2P_T} = g_d(2\pi)^2\gamma\left(\frac{1}{A}\right)\frac{d^2N_p}{d^2p_{pT}}\Big|_{\vec{p}_{pT}=\frac{\vec{P}_T}{2}}\frac{d^2N_n}{d^2p_{nT}}\Big|_{\vec{p}_{nT}=\frac{\vec{P}_T}{2}}$$

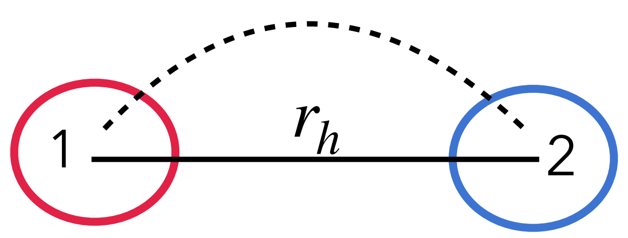
$$\frac{d^2N_{^3He}}{d^2P_T} = g_{^3He}(2\pi)^4\gamma^2\left(\frac{1}{A^2}\right)\frac{d^2N_p}{d^2p_{pT}}\Big|_{\vec{p}_{pT}=\frac{\vec{P}_T}{3}}\frac{d^2N_p}{d^2p_{pT}}\Big|_{\vec{p}_{pT}=\frac{\vec{P}_T}{3}}\frac{d^2N_n}{d^2p_{nT}}\Big|_{\vec{p}_{nT}=\frac{\vec{P}_T}{3}}$$

$$\frac{d^2N_d}{d^2P_T} = g_d(2\pi)^2\gamma\left(\frac{r_f^2}{A}\right)\frac{d^2N_p}{d^2p_{pT}}\Big|^{Exp}_{\vec{p}_{pT}=\frac{\vec{P}_T}{2}}\frac{d^2N_n}{d^2p_{nT}}\Big|^{Exp}_{\vec{p}_{nT}=\frac{\vec{P}_T}{2}}$$

$$\frac{d^2N_{^3He}}{d^2P_T} = g_{^3He}(2\pi)^4\gamma^2\left(\frac{r_f^3}{A^2}\right)\frac{d^2N_p}{d^2p_{pT}}\Big|^{Exp}_{\vec{p}_{pT}=\frac{\vec{P}_T}{3}}\frac{d^2N_p}{d^2p_{pT}}\Big|^{Exp}_{\vec{p}_{pT}=\frac{\vec{P}_T}{3}}\frac{d^2N_n}{d^2p_{nT}}\Big|^{Exp}_{\vec{p}_{nT}=\frac{\vec{P}_T}{3}}$$

Determine A and r_f correctly → d and ^3He can be explained simultaneously

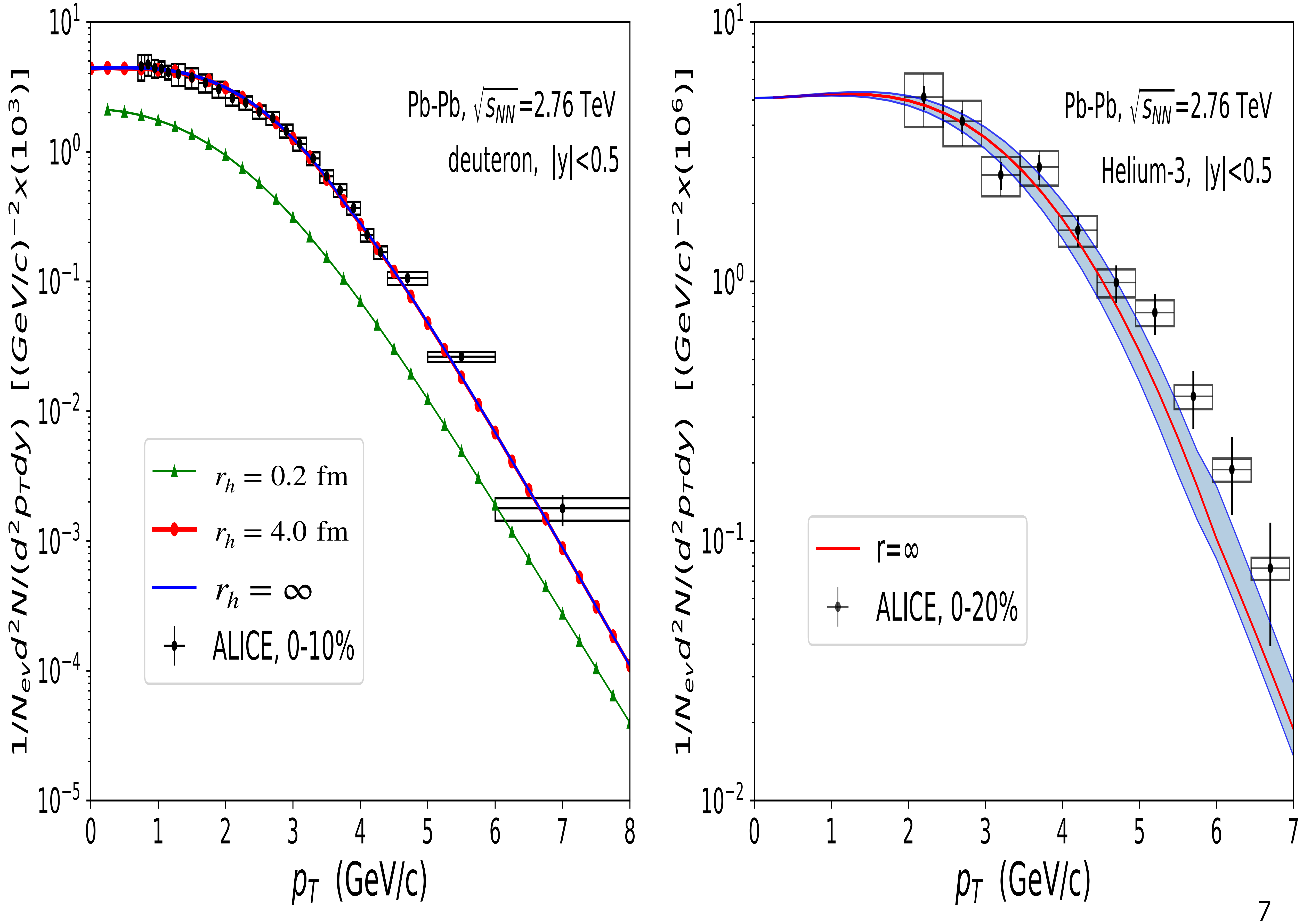
Deuteron and Helium-3



$$\langle r_h^2 \rangle = \frac{3}{2}\sigma^2$$

Pb-Pb collision, $\sqrt{s_{NN}} = 2.76$ TeV

- HyeongOck Yun, at al. Phys. Rev. C 107, 014906 (2023)



1. $$\frac{d^2N_d}{d^2P_T} = g_d(2\sqrt{\pi})^2\sigma^2\left(\frac{r_f^2}{A}\right)\int d^2p_p d^2p_n \frac{d^2N_p^{Exp}}{d^2p_{pT}}\frac{d^2N_n^{Exp}}{d^2p_{nT}} \times \exp\left[-\sigma^2(k')^2\right]\delta^{(2)}(P_T - p_{pT} - p_{nT})$$

2. $$\frac{d^2N_d}{d^2P_T}(\sigma \to \infty) = g_d(2\pi)^2\gamma\left(\frac{r_f^2}{A}\right)\frac{d^2N_p}{d^2p_{pT}}\Big|^{Exp}_{\vec{p}_{pT}=\frac{\vec{P}_T}{2}}\frac{d^2N_n}{d^2p_{nT}}\Big|^{Exp}_{\vec{p}_{nT}=\frac{\vec{P}_T}{2}}$$

G. Röpke, Phys. Rev. C 79 (2009) 014002

- Parameter

  $r_f = 0.368$, $A_{0-10\%} = 608fm^2$, $r_h^d \sim 4$ fm

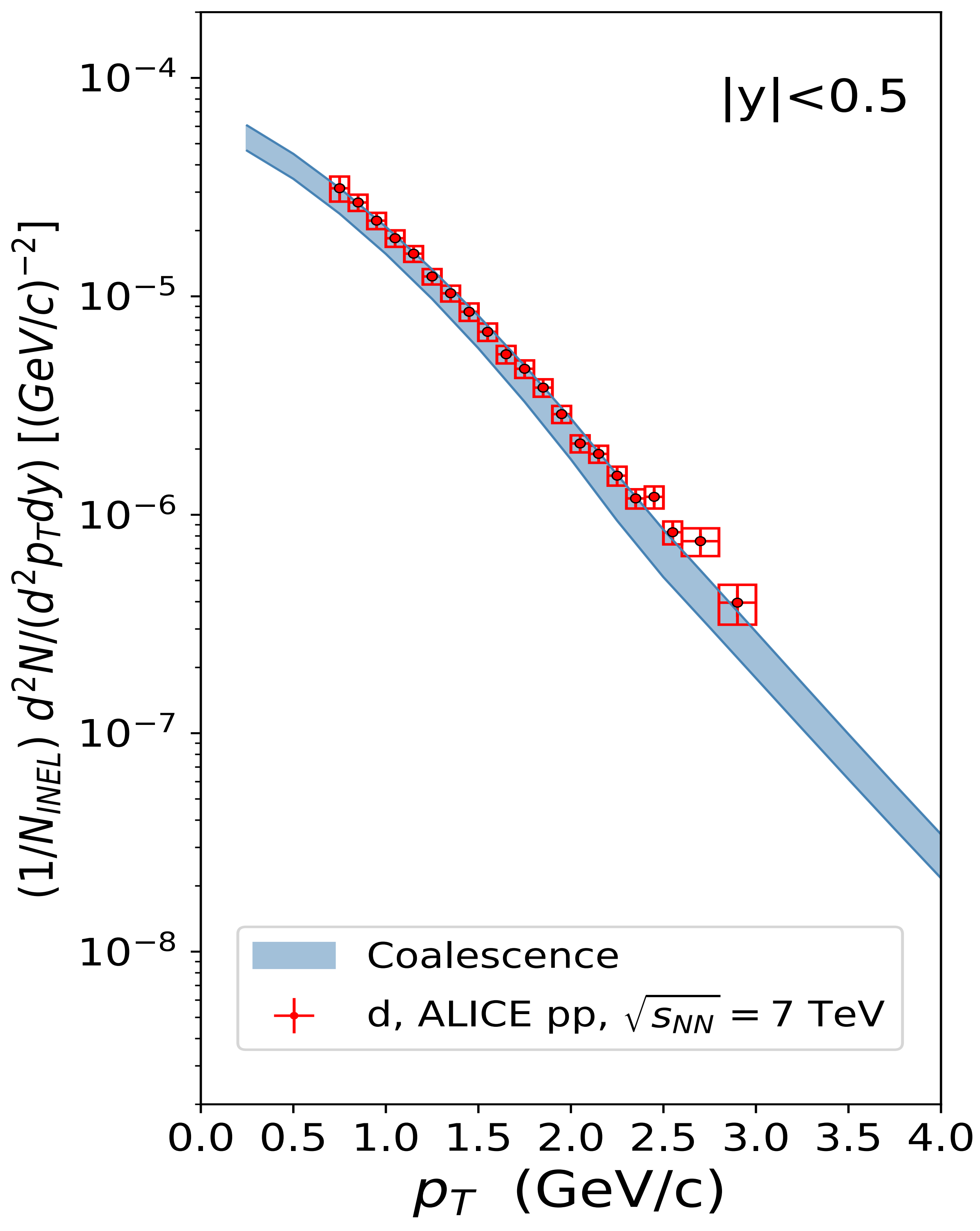
  ($r_f = 0.368$ is consistent with the statistical hadronization model)

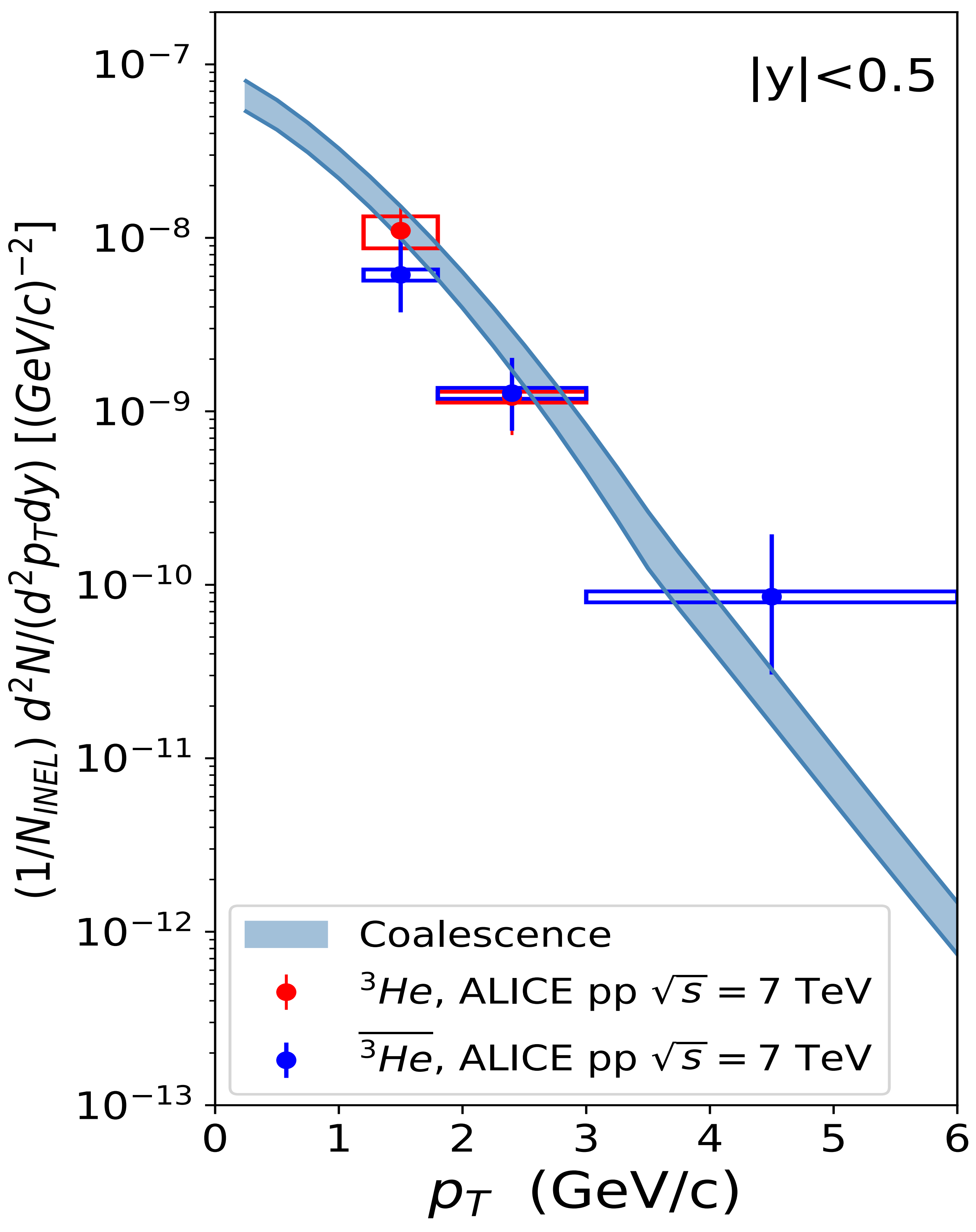
$\sigma \to \infty$ limit coalescence model well describe large molecule state

Deuteron and Helium-3

pp collision, $\sqrt{s} = 7$ TeV

HyeongOck Yun, at al. Phys.Lett.B 851 (2024) 138569





- Coalescence area

$$A_{\rm pp}^{7\ \rm TeV} = A_{\rm PbPb}^{2.76\ \rm TeV} \times \frac{6.01}{1447.5} = 2.52\ \rm fm^2$$

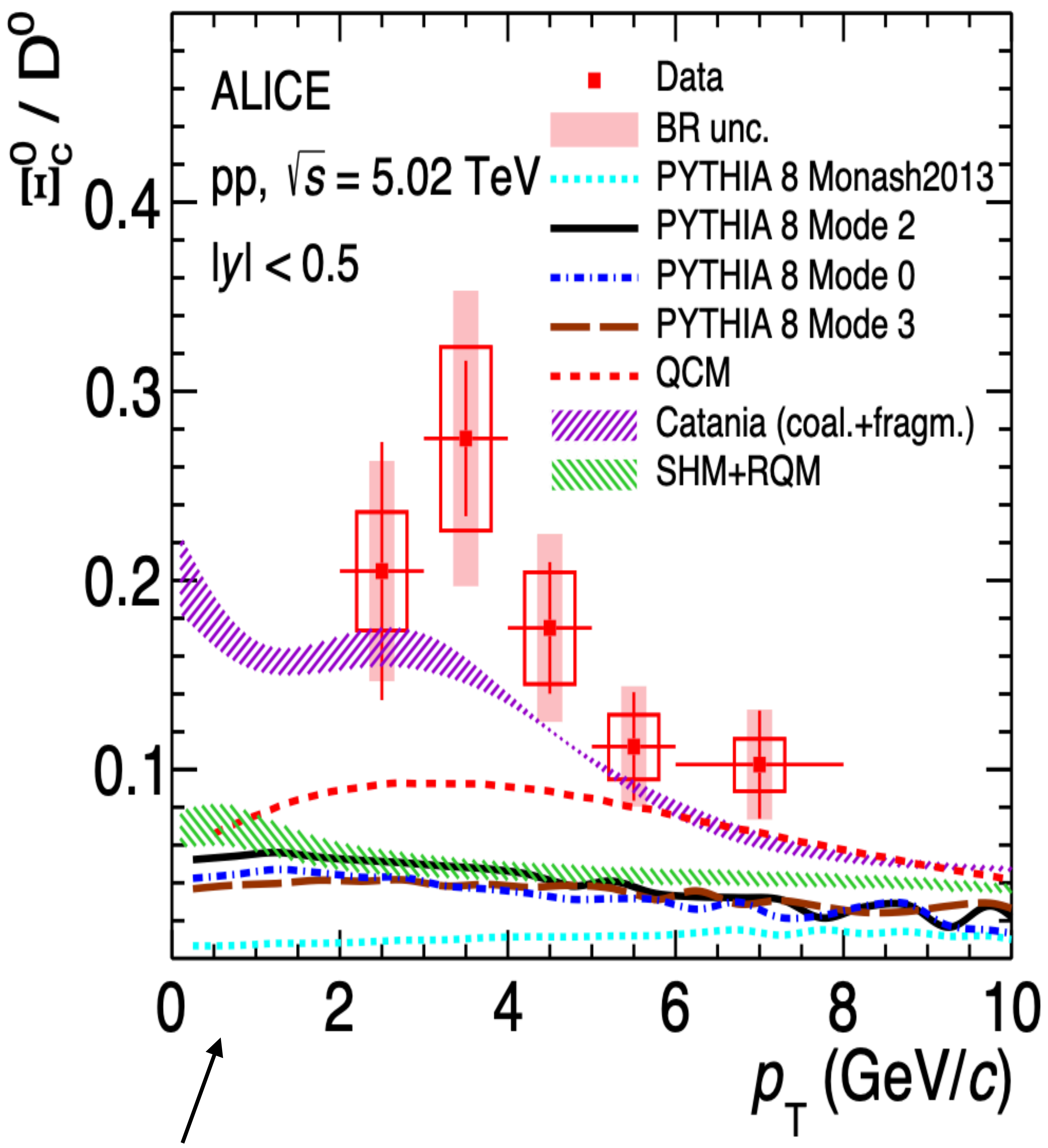
$$\left(\left.\frac{dN}{d\eta}\right|_{\rm pp}^{7\ \rm TeV} = 6.01, \quad \left.\frac{dN}{d\eta}\right|_{\rm PbPb}^{2.76\ \rm TeV} = 1447.5 \right)$$

Coalescence model explains hadron production well even in small system

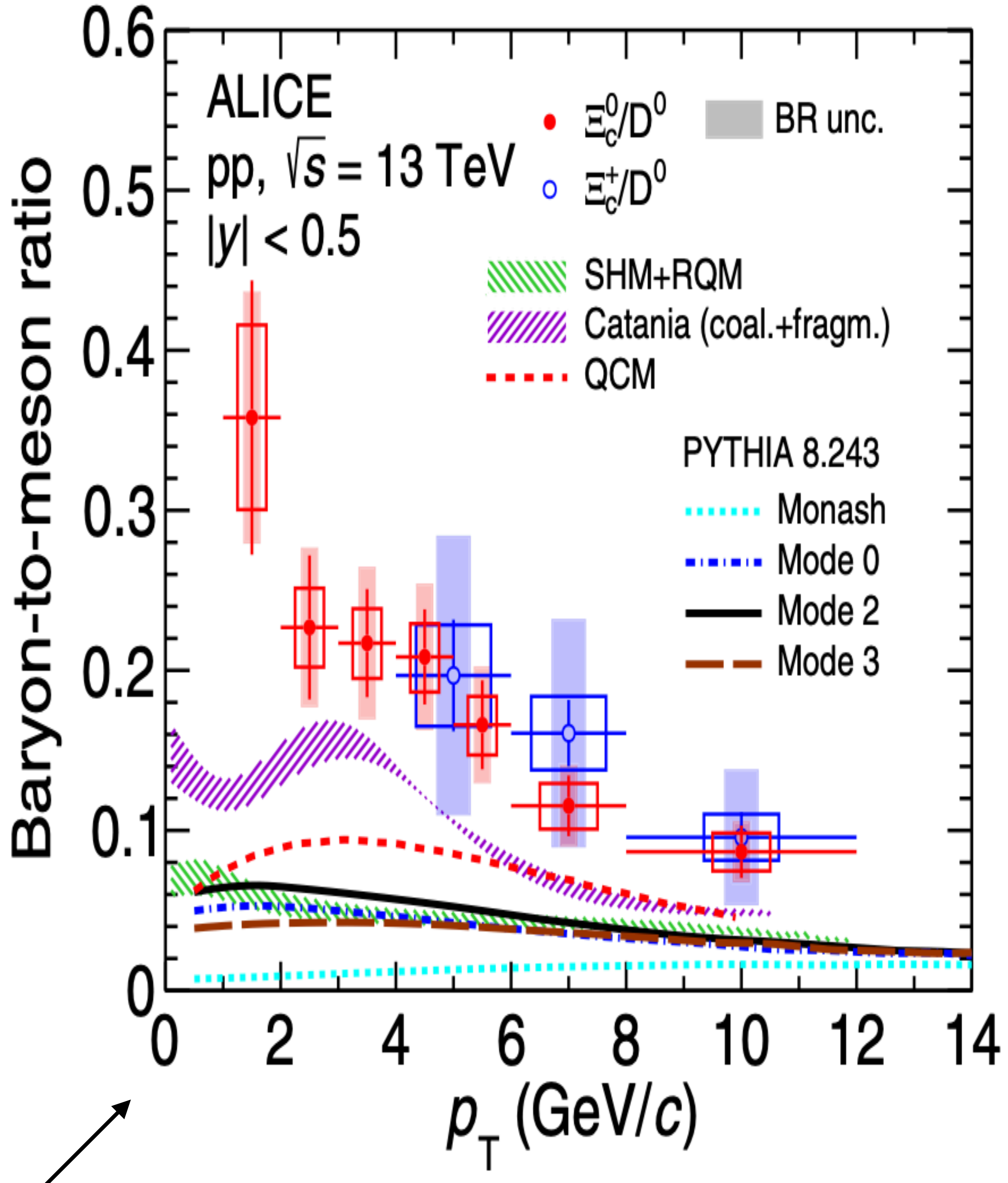
Application to Ξ_c in pp collisoin

Experiment data and Diquark

pp collision

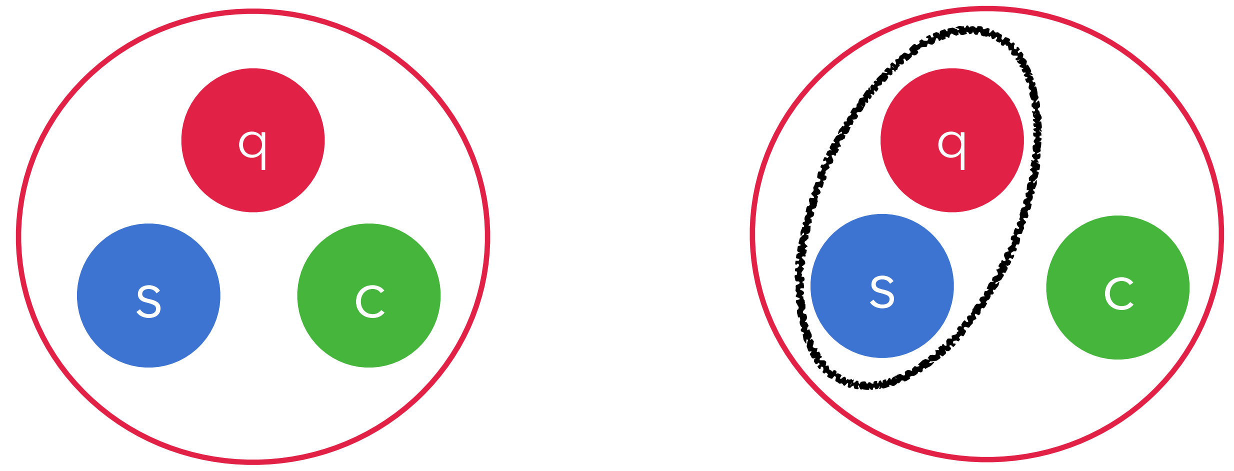


ALICE Collaboration,
JHEP 10, 159 (2021)



ALICE Collaboration,
Phys. ReV. Lett 127, 272001, (2021)

Ξ_c



3-body configuration

[qs] diquark + c quark configuration

According to quark model, [qs] diquark (color $\bar{3}$, spin 0) state is bound state

$$\Xi_{c,\ \text{total}} = \Xi_{c,\ 3-\text{body}} + \Xi_{c,\ [qs]+c}$$

Constituent transverse momentum distribution

Diquark and Charm quark

- [qs] diquark distribution

: Blast-wave model + thermal model

- Blast-wave model - p_T shape of diquark

$$\frac{1}{p_T}\frac{dN}{dp_T} \sim m_T \int_0^R I_0\left(\frac{p_T \sinh\rho}{T_{kin}}\right) K_1\left(\frac{m_T \cosh\rho}{T_{kin}}\right) rdr,$$

$I_0(x),\ K_1(x)$: Bessel function

$\rho = \tanh^{-1}\beta_r,\quad \beta_r = \left(\frac{r}{R}\right)^n \beta_s$

Parameter of blast-wave model

: ALICE Collaboration, Eur. Phys. J. C 80 (2020) 693

Collision system	T_k	$\langle\beta\rangle$	n
pp, 5.02 TeV, MB	181	0.198	6.248

- Thermal model - Diquark yield determination

$$N^{[qs],\ m\,=\,580\ \mathrm{MeV}}_{T_c\,=\,165\ \mathrm{MeV}} = 0.565,\quad N^{[qs],\ m\,=\,580\ \mathrm{MeV}}_{T_c\,=\,181\ \mathrm{MeV}} = 0.889$$

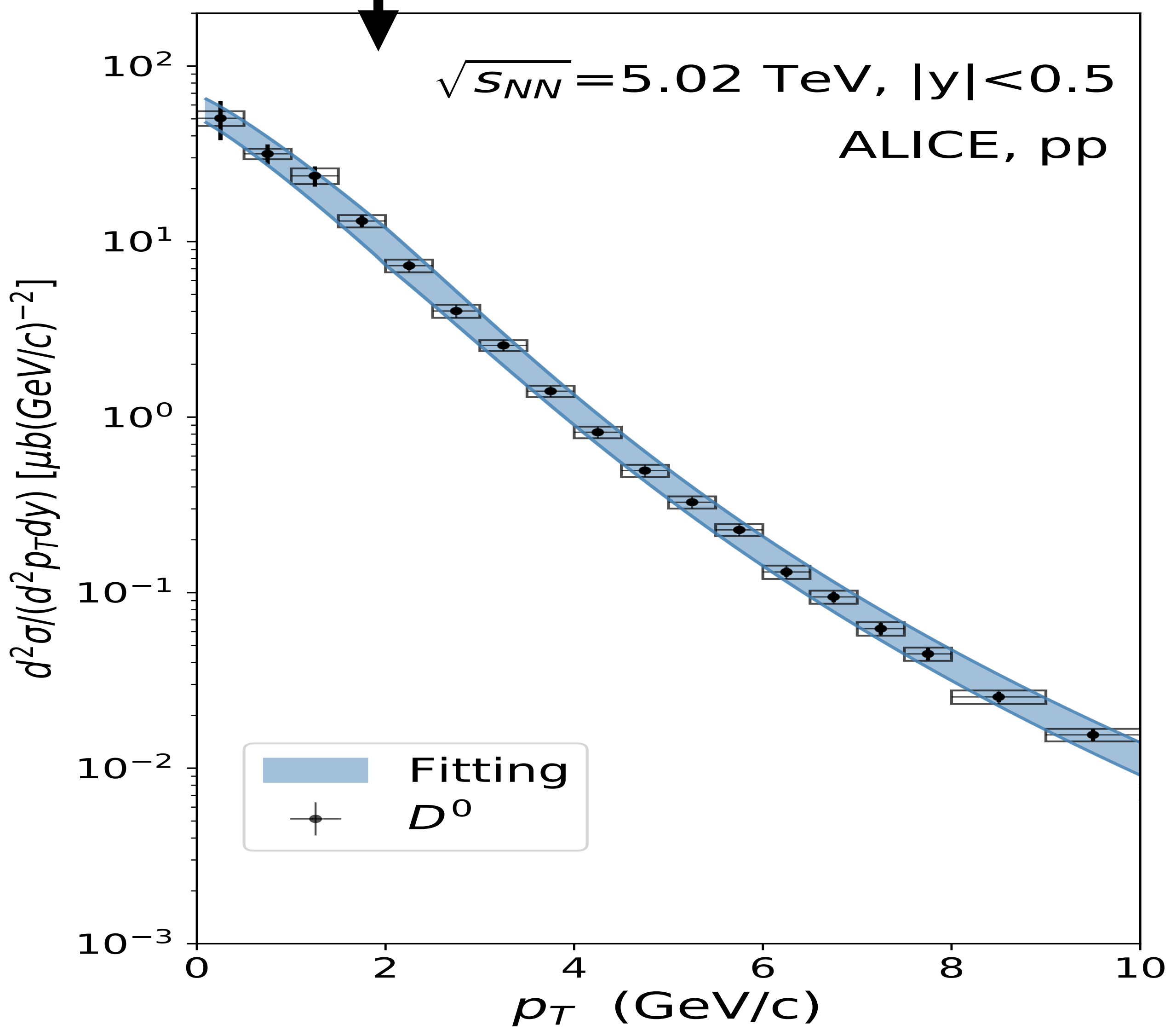
$$N^{[qs],\ m\,=\,970\ \mathrm{MeV}}_{T_c\,=\,165\ \mathrm{MeV}} = 0.120,\quad N^{[qs],\ m\,=\,970\ \mathrm{MeV}}_{T_c\,=\,181\ \mathrm{MeV}} = 0.229$$

- Charm quark distribution

: D meson distribution + cross section ratio scaling

$$\frac{dN_c}{d^2p_T dy} = \frac{\sigma_{c\bar{c}}}{\sigma_{D^0}} \times \frac{dN_{D^0}}{d^2p_T dy}$$

Eur. Phys. J. C 79, 388 (2019)



- Cross section

$$\frac{d\sigma_{c\bar{c}}}{dy}\Big|_{|y|<0.5} = 1165\ \mu b,$$

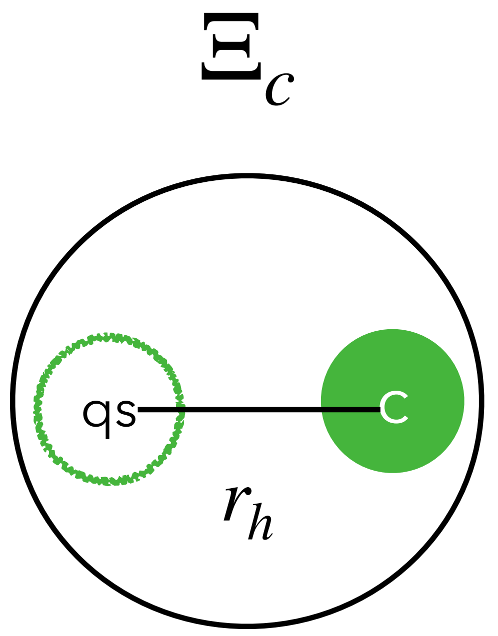
$$\frac{d\sigma_{D^0}}{dy}\Big|_{|y|<0.5} = 447\ \mu b$$

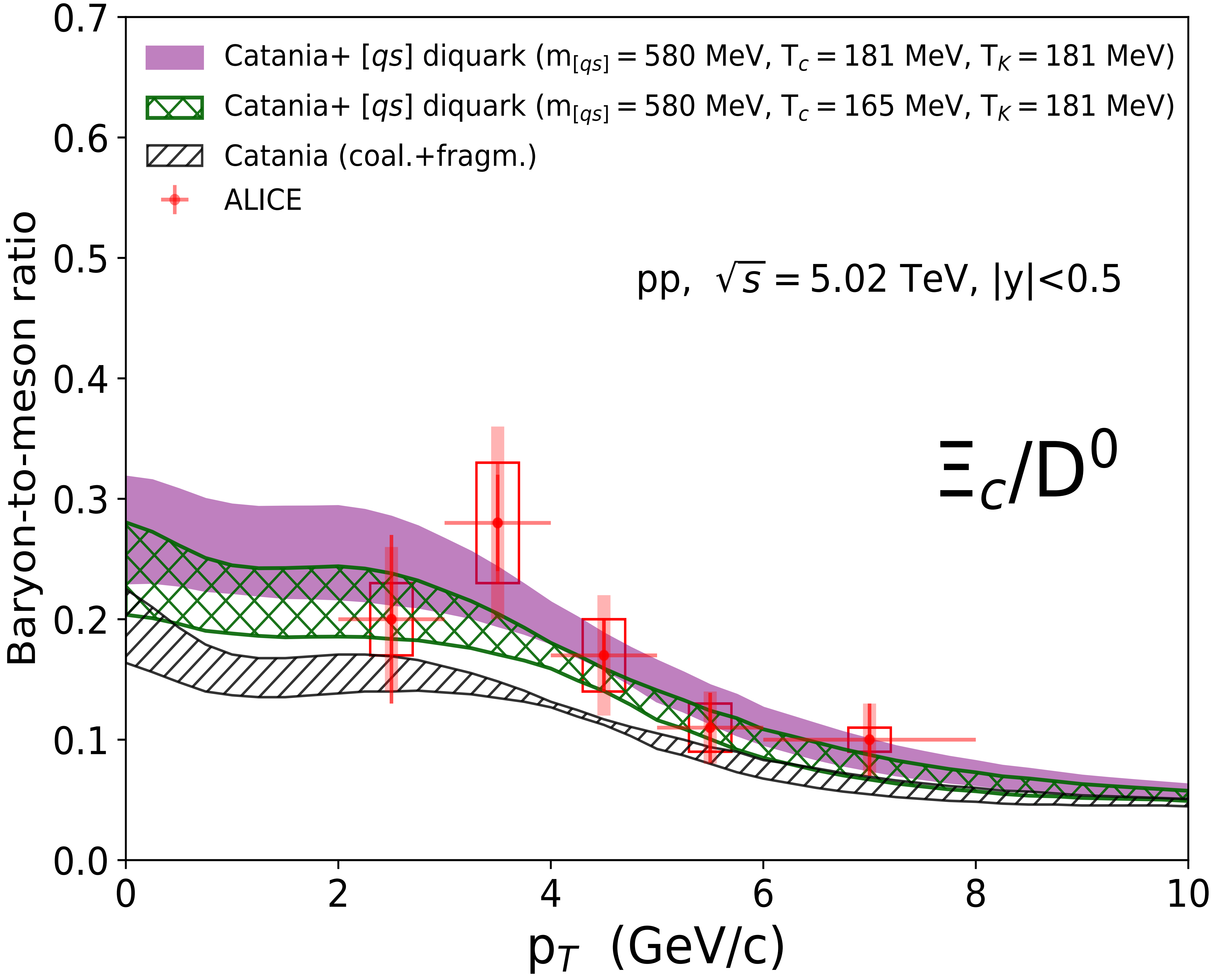
Phys. Rev. D 105, L011103 (2022)

Ξ_c/D ratio

[qs] + c coalescence

HyeongOck Yun, at al. Phys.Lett.B 851 (2024) 138569





- 2 body Coalescence

$$\frac{d^2N_{\Xi_c}}{d^2P_T} = \frac{g_{\Xi_c}}{g_{[qs]}g_c}(2\sqrt{\pi})^2\sigma^2\frac{1}{A}\int d^2p_{[qs]T}d^2p_{cT}\ \frac{d^2N_{[qs]}}{d^2p_{[qs]T}}\frac{d^2N_c}{d^2p_{cT}}$$
$$\times \exp\left[-\sigma^2(k')^2\right]\delta^{(2)}(P_T - p_{[qs]T} - p_{cT})$$

- Parameter

$$r_h = 0.444\ \text{fm},\ A_{pp}^{5.02\ \text{TeV}} = 1.80\ \text{fm}^2, m_{qs} = 580\ \text{MeV}$$

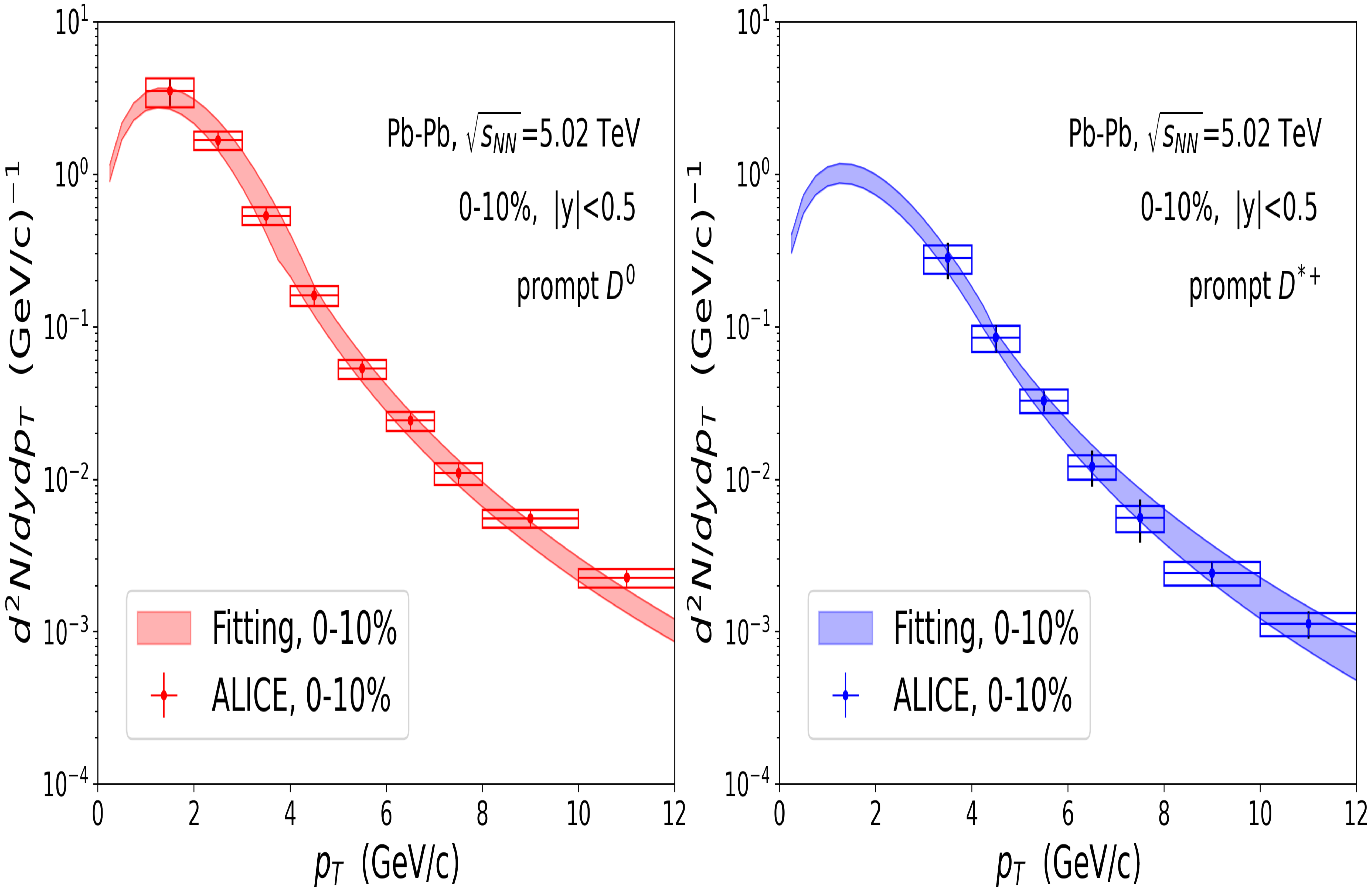
$[qs] + c$ coalescence can provide additional enhancement.

Molecular structure X(3872) in PbPb Collision

D meson distribution

Pb-Pb collisions at 5.02TeV

○ Fitting (ALICE Collaboration, JHEP 01 (2022) 174)



○ Feed-down (Pb-Pb collisions, 5.02TeV)

- Experimental data :
  $dN^{D^0}_{0-10\%}/dy = 6.819 \pm 0.457(stat.)^{+0.912}_{-0.936}(syst.)$

- Decay channel

  $Br(D*(2007)^0 \rightarrow D^0\pi^0) = (64.7 \pm 0.9)\ \%$

  $Br(D*(2007)^0 \rightarrow D^0\gamma) = (35.3 \pm 0.9)\ \%$

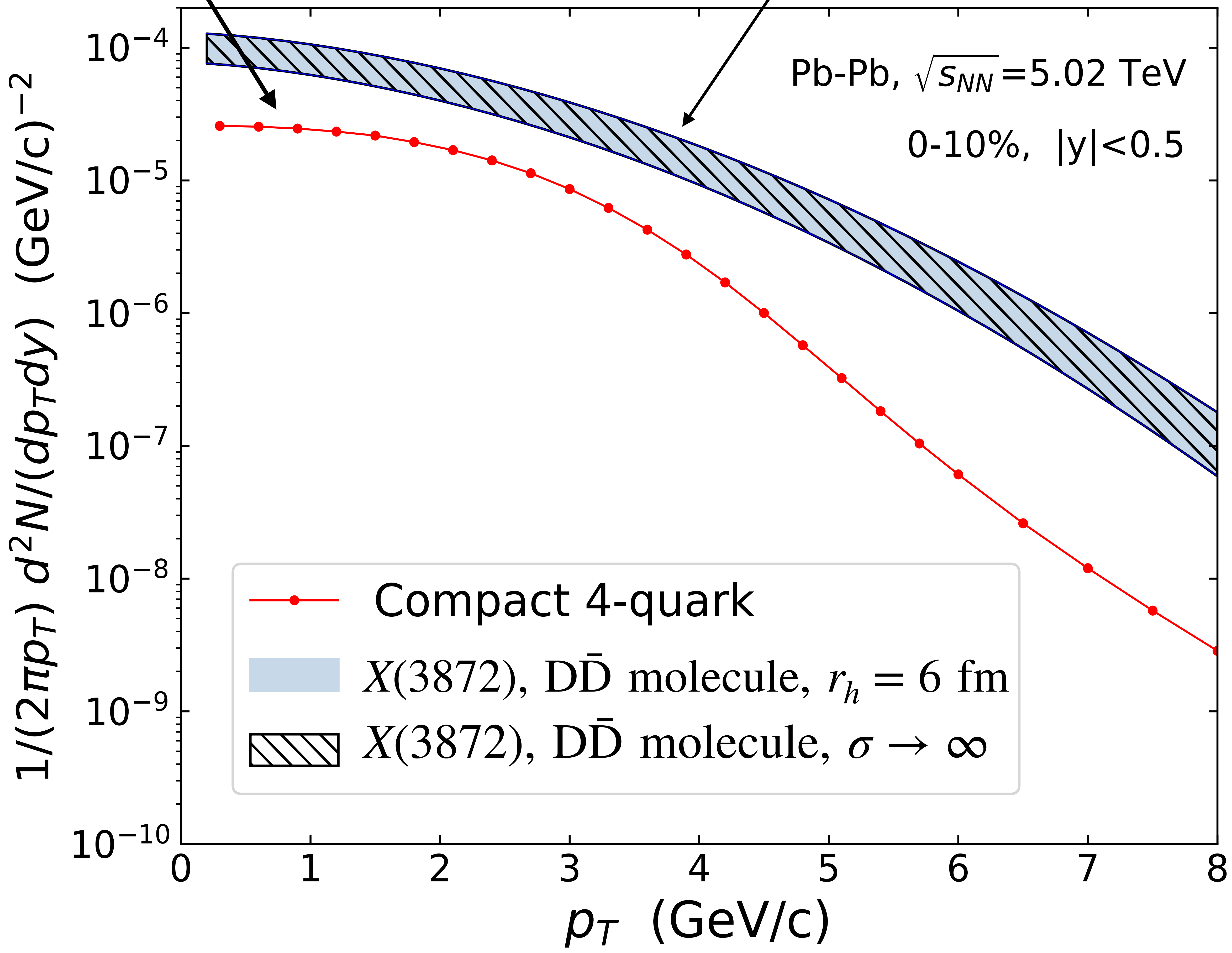
  $Br(D*(2010)^+ \rightarrow D^0\pi^+) = (67.7 \pm 0.5)\ \%$

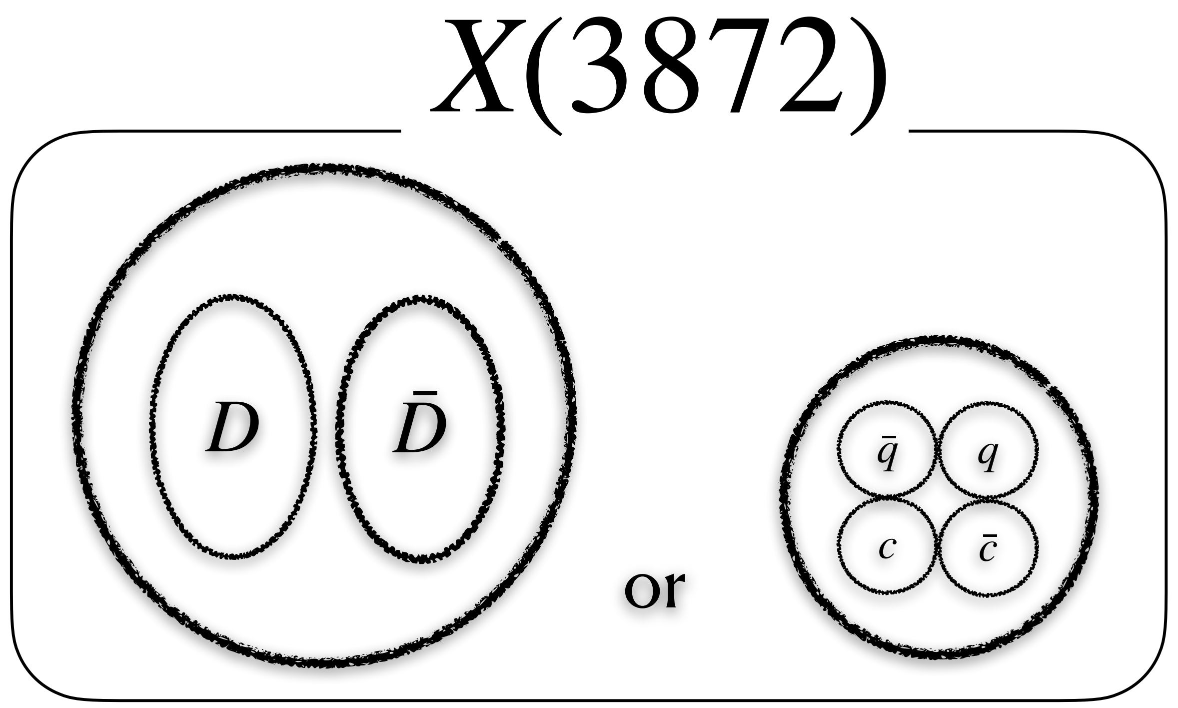
- From Statistical hadronization model, 31% of measured D^0 participate in coalescence

X(3872) p_T distribution

S. H. Lee and S. Cho, Phys. Rev. C 101, 024902
+ Scaling (× 1.63, 2.76TeV → 5.02TeV)

H. Yun at al, Phys.Rev.C 107 (2023) 1, 014906





- Yields

$$dN^{DD^*}_{coal}/dy = (2.47 \pm 0.71) \times 10^{-3},\ \ dN^{4q}_{coal}/dy = 6.2 \times 10^{-4}$$

Two possible configurations are markedly different

Measurement of the p_T distribution in heavy-ion collisions → Confirmation of the structure

Summary

- We confirmed that the coalescence model works well in both small and large systems by comparing the calculations of deuteron and helium-3
- We calculated the Ξ_c/D ratio in pp collisions, and found that the additional diquark contribution can help explain the experimental data
- $D\bar{D}$ molecular X(3872) : By measuring p_T distribution in heavy ion collisions, the structure of X(3872) can be inferred

Back up

Ongoing study
: X(3872) at high p_T

X(3872) structure

$D - \bar{D}^*$ molecular structure

- In $(q\bar{c})$, $(c\bar{q})$ basis

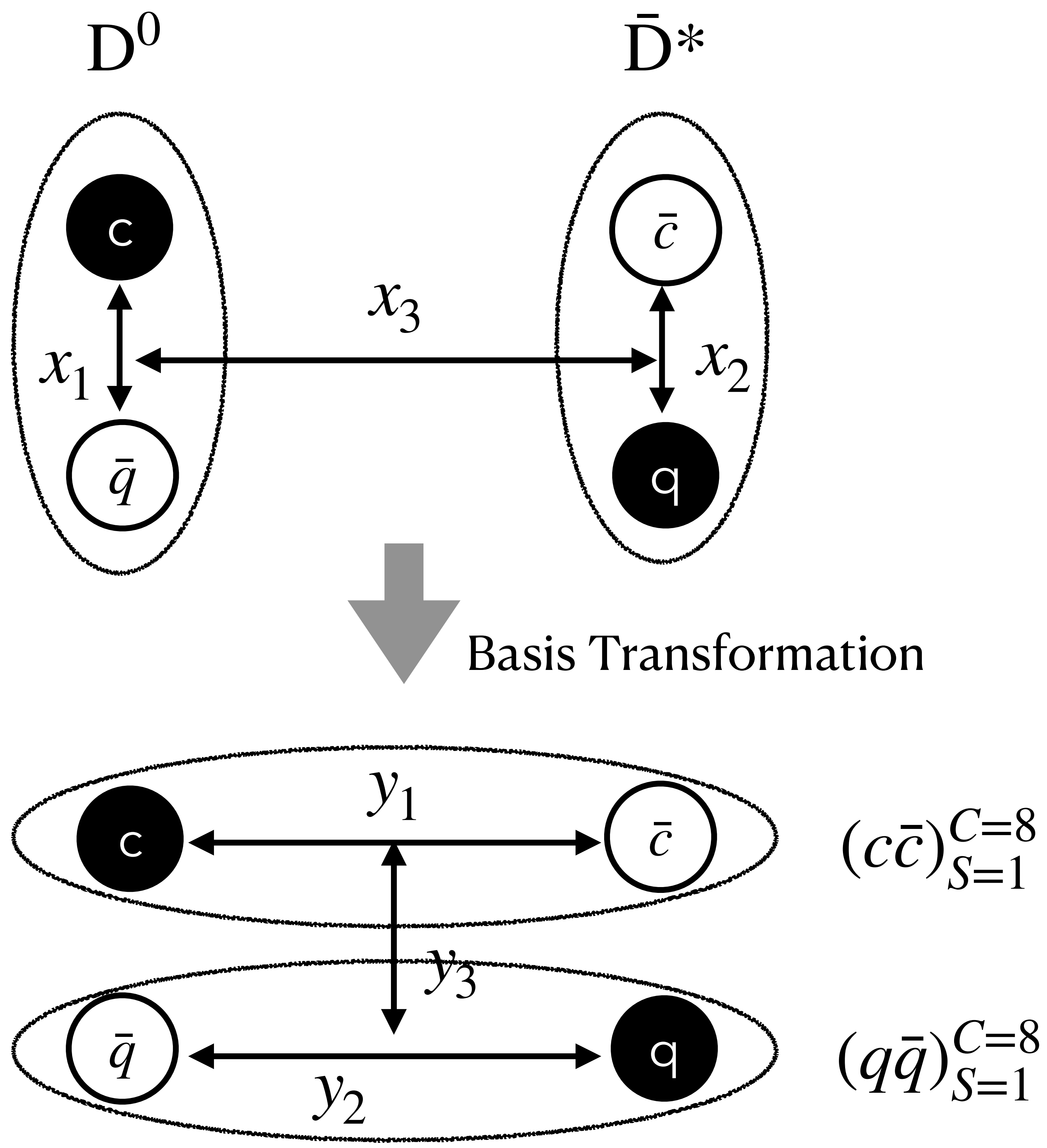
$$|1'\rangle = (q\bar{c})^{C=1}_{S=0} \otimes (c\bar{q})^{C=1}_{S=1} \quad \longleftarrow \quad \mathbf{D - \bar{D}^*}$$

$$|2'\rangle = (q\bar{c})^{C=8}_{S=0} \otimes (c\bar{q})^{C=8}_{S=1}$$

- Transformation into $(c\bar{c})$, $(q\bar{q})$ basis

$$|1\rangle = (c\bar{c})^{C=8}_{S=1} \otimes (q\bar{q})^{C=8}_{S=1}$$

$$|2\rangle = (c\bar{c})^{C=1}_{S=1} \otimes (c\bar{q})^{C=1}_{S=1}$$



$$|1'\rangle = \frac{2\sqrt{2}}{3}|1\rangle + \frac{1}{3}|2\rangle,$$ **$D^0\ \bar{D}^*$ is mostly composed of $(c\bar{c})^{C=8}_{S=1} \otimes (q\bar{q})^{C=8}_{S=1}$ (~90%)**

$$|2'\rangle = -\frac{1}{3}|1\rangle + \frac{2\sqrt{2}}{3}|2\rangle$$

Freeze out condition

Formation time

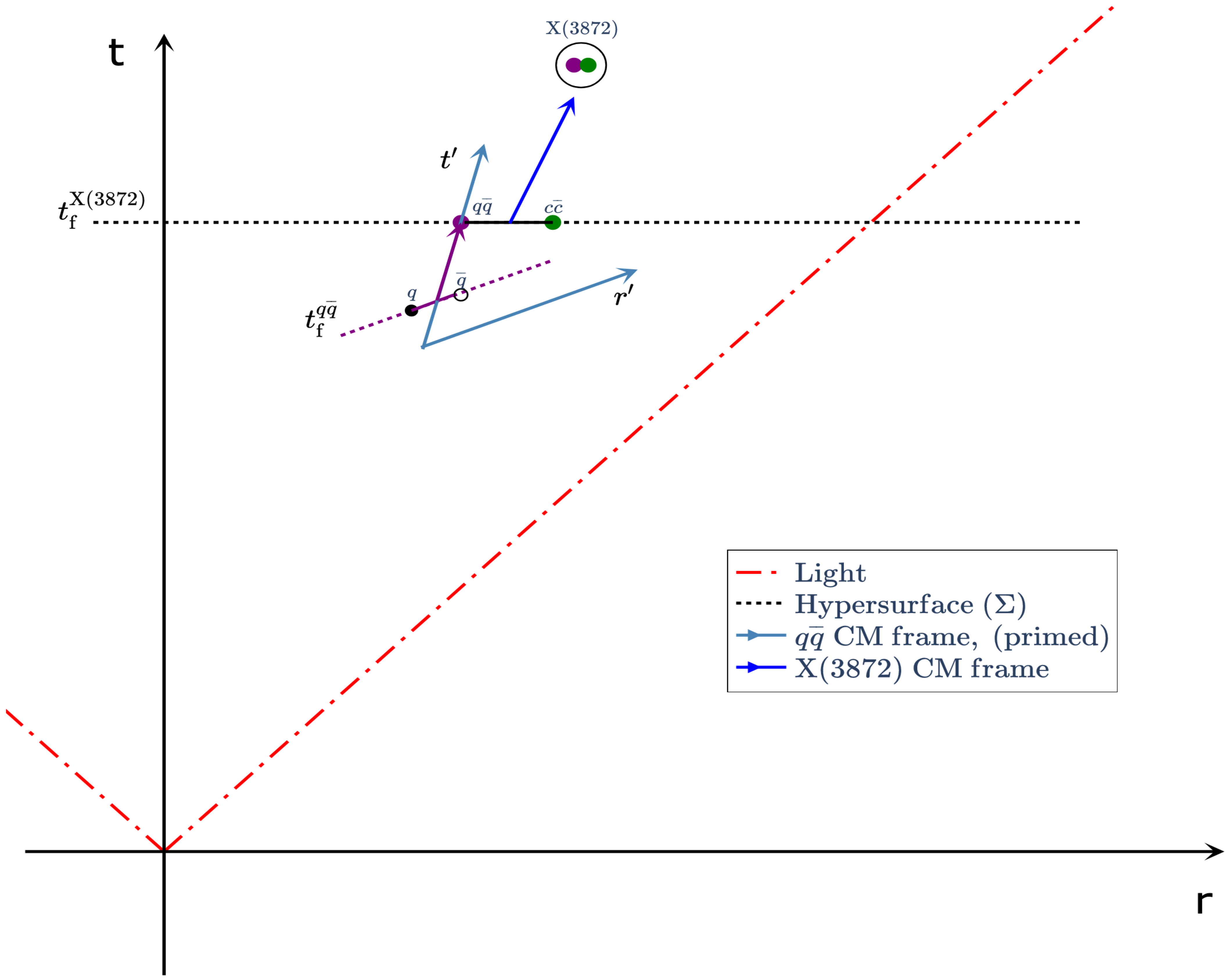
- Due to the large formation time, quark and anti-quark can coexist at $[q\bar{q}]$ CM frame

Color octet state (large formation time)

$$: W(\vec{r}, \vec{k})_{[q\bar{q}]} = 4\exp\left[-\frac{r^2}{\gamma\sigma^2} - \frac{\sigma^2 k^2}{\gamma}\right]$$

Color singlet hadron (short formation time)

$$:W(\vec{r}, \vec{k})_{X(3872)} = 4\exp\left[-\frac{\gamma r^2}{\sigma^2} - \frac{\sigma^2 k^2}{\gamma}\right]$$



$q\bar{q}$ distribution

- Parton distribution at low p_{T} ($p_{\mathrm{T}} < 2.0$)

  : Blast-wave model + Thermal model

$$\frac{1}{p_T}\frac{dN}{dp_T} \sim m_T \int_0^R I_0\left(\frac{p_T \sinh\rho}{T_{kin}}\right) K_1\left(\frac{m_T \cosh\rho}{T_{kin}}\right) rdr$$

Yield	$u,\ d$	s	$g_{S=1}^{C=8}$
N_{pp}	2.58	1.48	4.21
N_{PbPb}	616	335	924

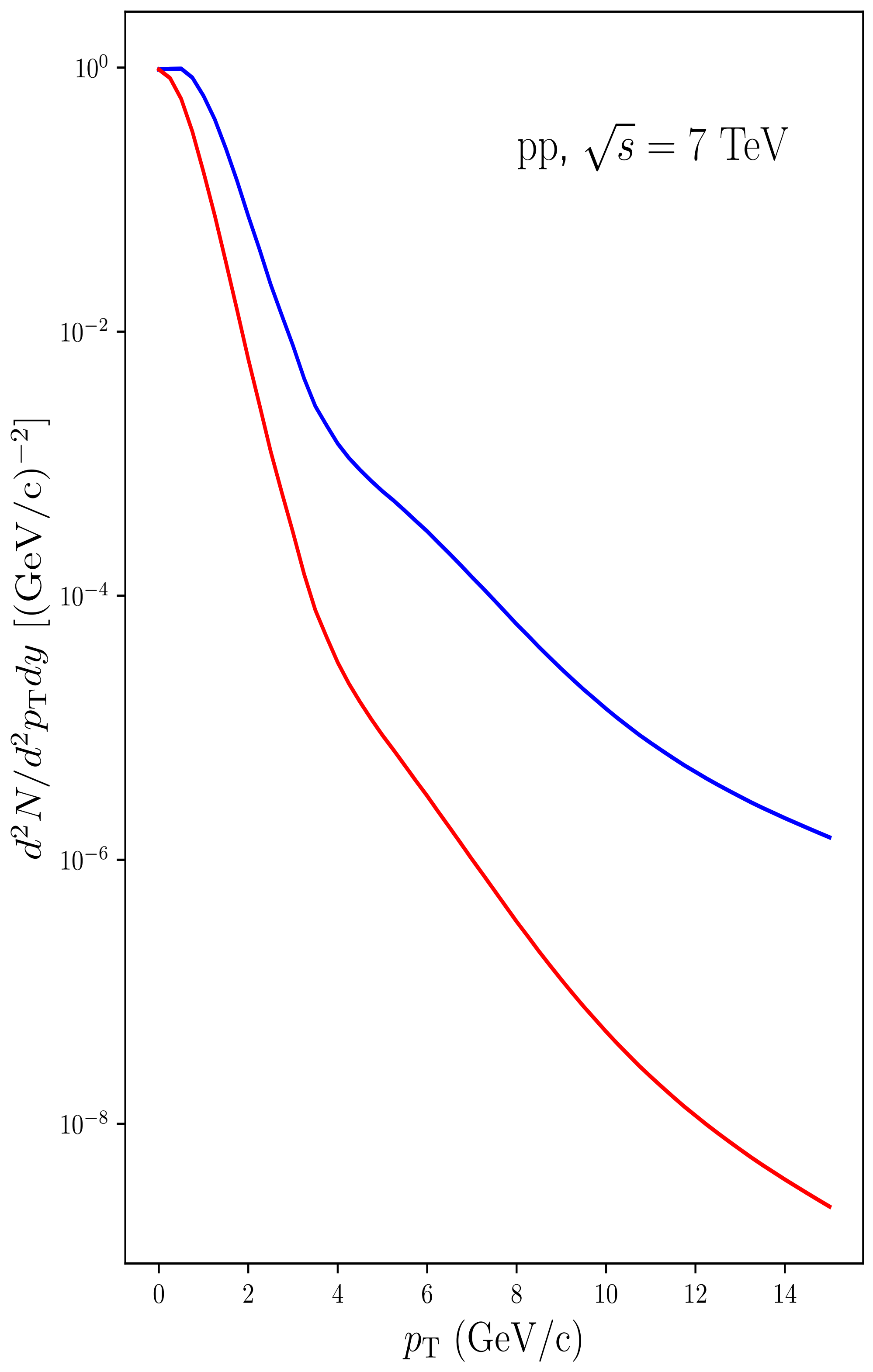
- Parton distribution at high p_{T} ($p_{\mathrm{T}} < 2.0$)

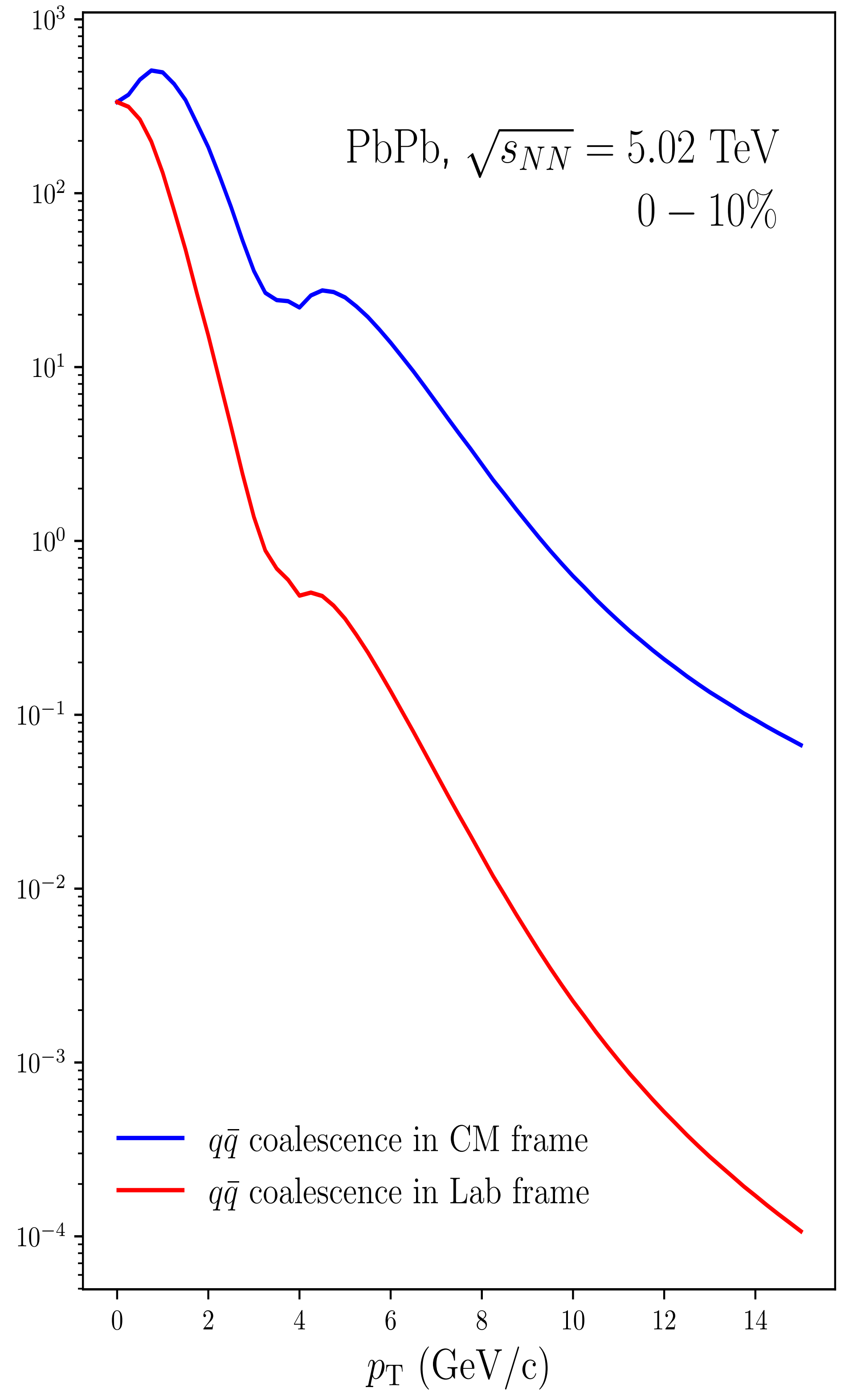
  : Minijet distribution

$$\frac{dN_{jet}^{u,d}}{d^2p_{\mathrm{T}}} = 24.68\left[1+\left(\frac{p_{\mathrm{T}}}{5.11}\right)^2\right]^{-8.01} + 0.55\left[1+\left(\frac{p_{\mathrm{T}}}{5.65}\right)^2\right]^{-2.56}$$

$$\frac{dN^{\mathrm{CM}}_{[q\bar{q}]^{S=1}_{C=8}}}{d^2P_{\mathrm{T}}} = g_{[q\bar{q}]^{S=1}_{C=8}}(2\sqrt{\pi}\sigma)^2\frac{\gamma^2}{A}\int d^2p_{q\mathrm{T}}d^2p_{\bar{q}\mathrm{T}}\ \frac{dN_q}{d^2p_{q\mathrm{T}}}\frac{dN_{\bar{q}}}{d^2p_{\bar{q}\mathrm{T}}}\exp\left(-\sigma^2|\mathbf{k}'|^2\right)\delta^{(2)}(\mathbf{P}_{\mathrm{T}}-\mathbf{p}_{q\mathrm{T}}-\mathbf{p}_{\bar{q}\mathrm{T}})$$

$$\frac{dN^{\mathrm{Lab}}_{[q\bar{q}]^{S=1}_{C=8}}}{d^2P_{\mathrm{T}}} = g_{[q\bar{q}]^{S=1}_{C=8}}(2\sqrt{\pi}\sigma)^2\frac{1}{A}\int d^2p_{q\mathrm{T}}d^2p_{\bar{q}\mathrm{T}}\ \frac{dN_q}{d^2p_{q\mathrm{T}}}\frac{dN_{\bar{q}}}{d^2p_{\bar{q}\mathrm{T}}}\exp\left(-\sigma^2|\mathbf{k}'|^2\right)\delta^{(2)}(\mathbf{P}_{\mathrm{T}}-\mathbf{p}_{q\mathrm{T}}-\mathbf{p}_{\bar{q}\mathrm{T}})$$





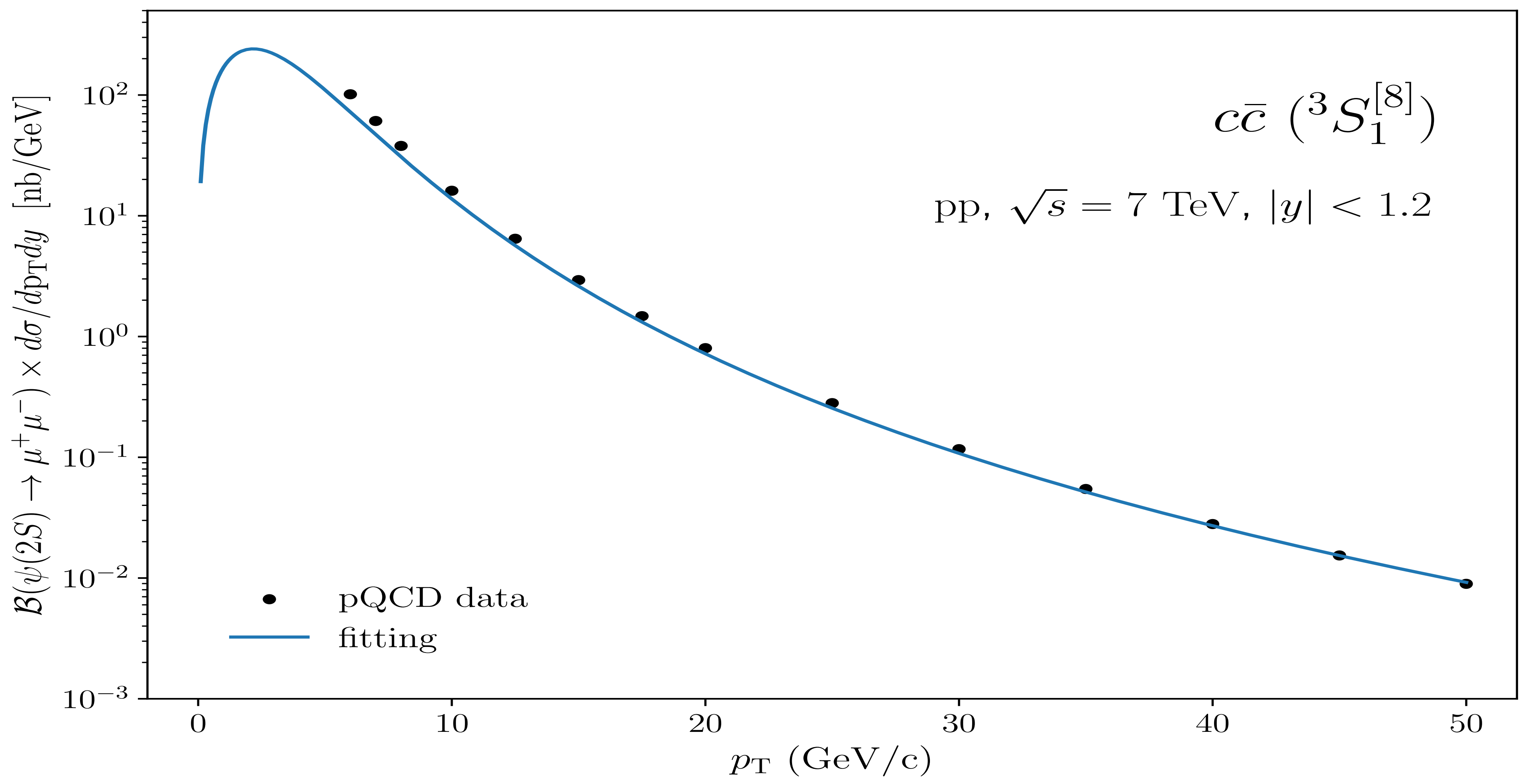
$c\bar{c}$ octet p_T distribution

perturbative QCD

- $c\bar{c}$ octet cross section

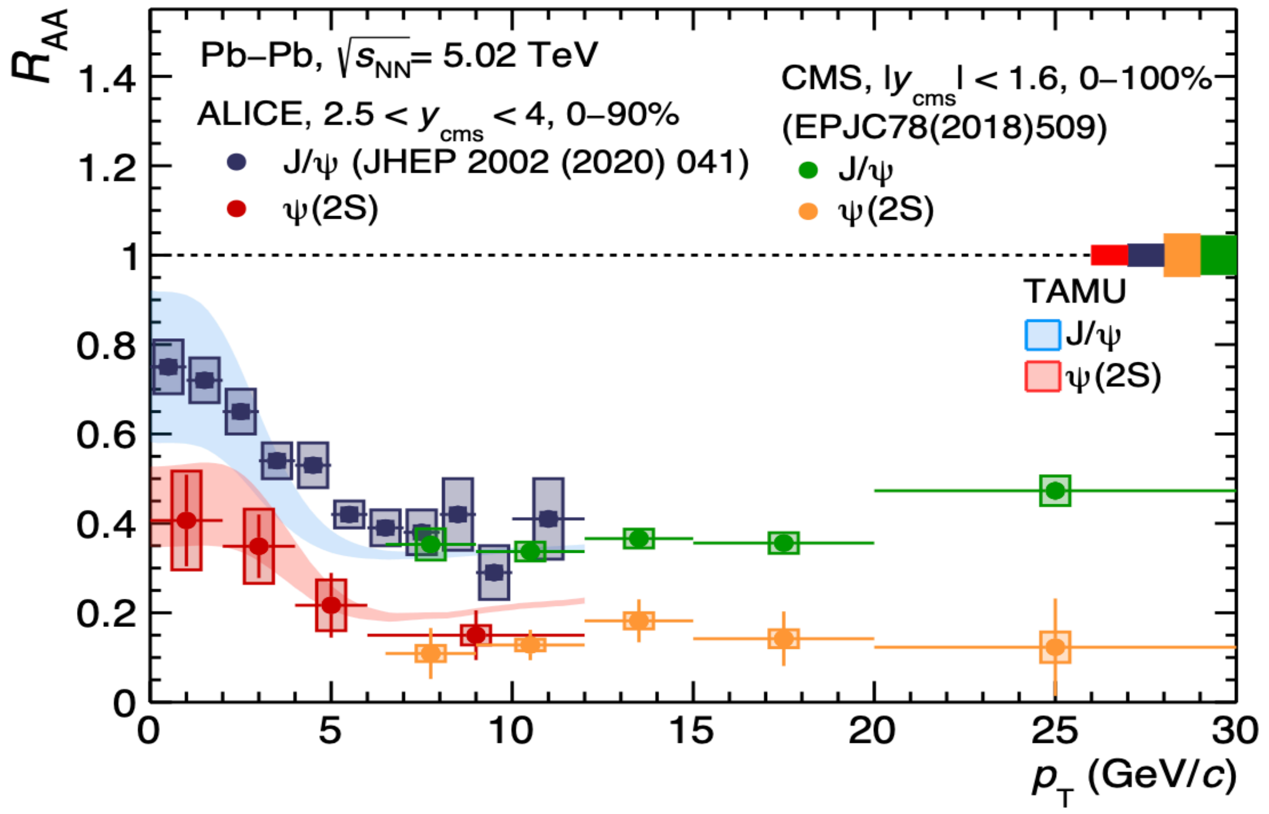
$$\frac{d\sigma^{[c\bar{c}(^3S_1^{[8]})]}}{dp_T dy} = \frac{1}{\langle \mathcal{O}^{\psi(2S)}_{[c\bar{c}(^3S_1^{[8]})]} \rangle} \frac{d\sigma^{[c\bar{c}(^3S_1^{[8]})] \to \psi(2S)}}{dp_T dy}$$

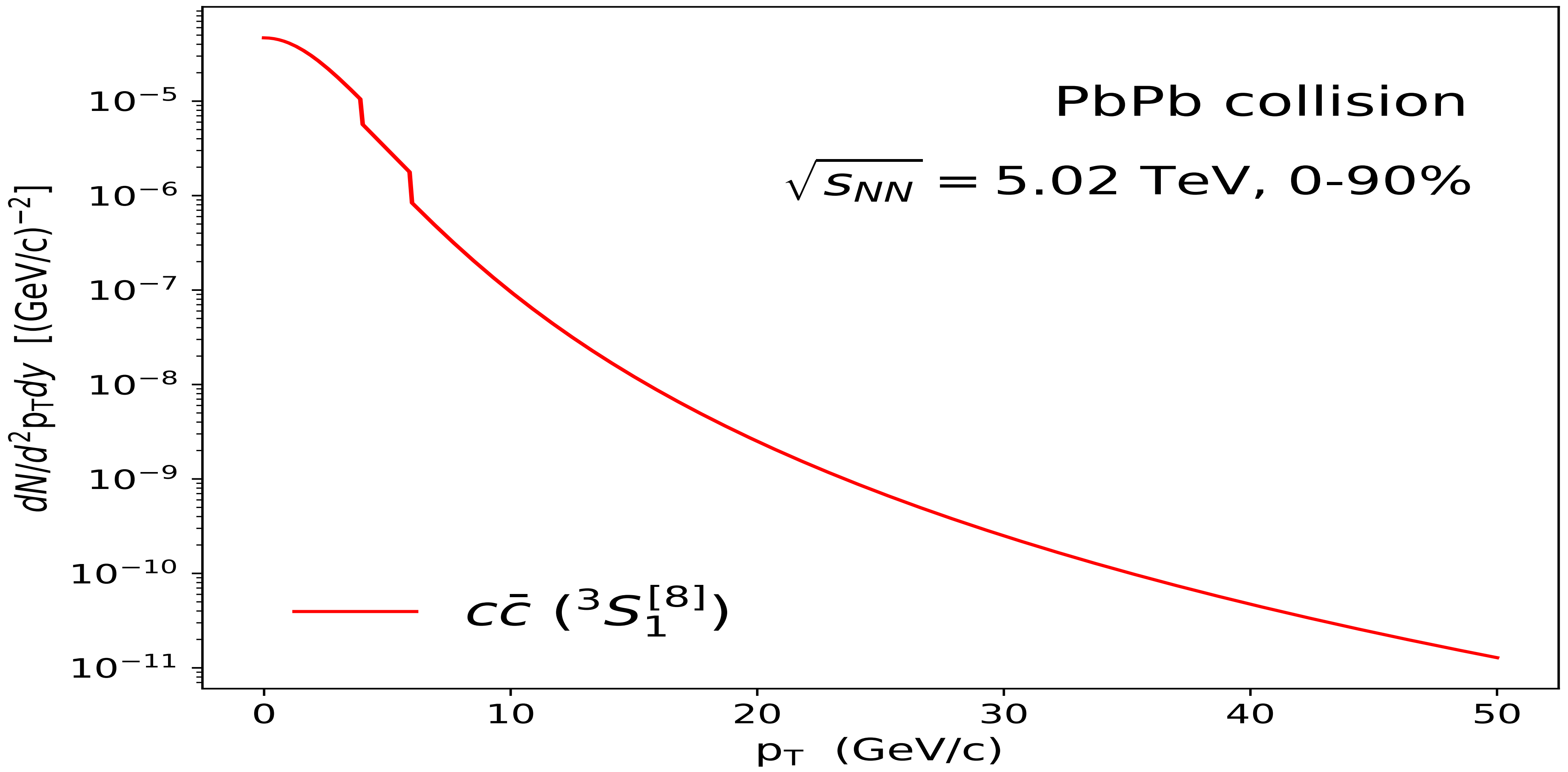
$$= 2.40 \times 10^{-5} p_T \left[1 + \left(\frac{p_T}{4.812} \right)^2 \right]^{-2.954}$$



- Nuclear modification factor

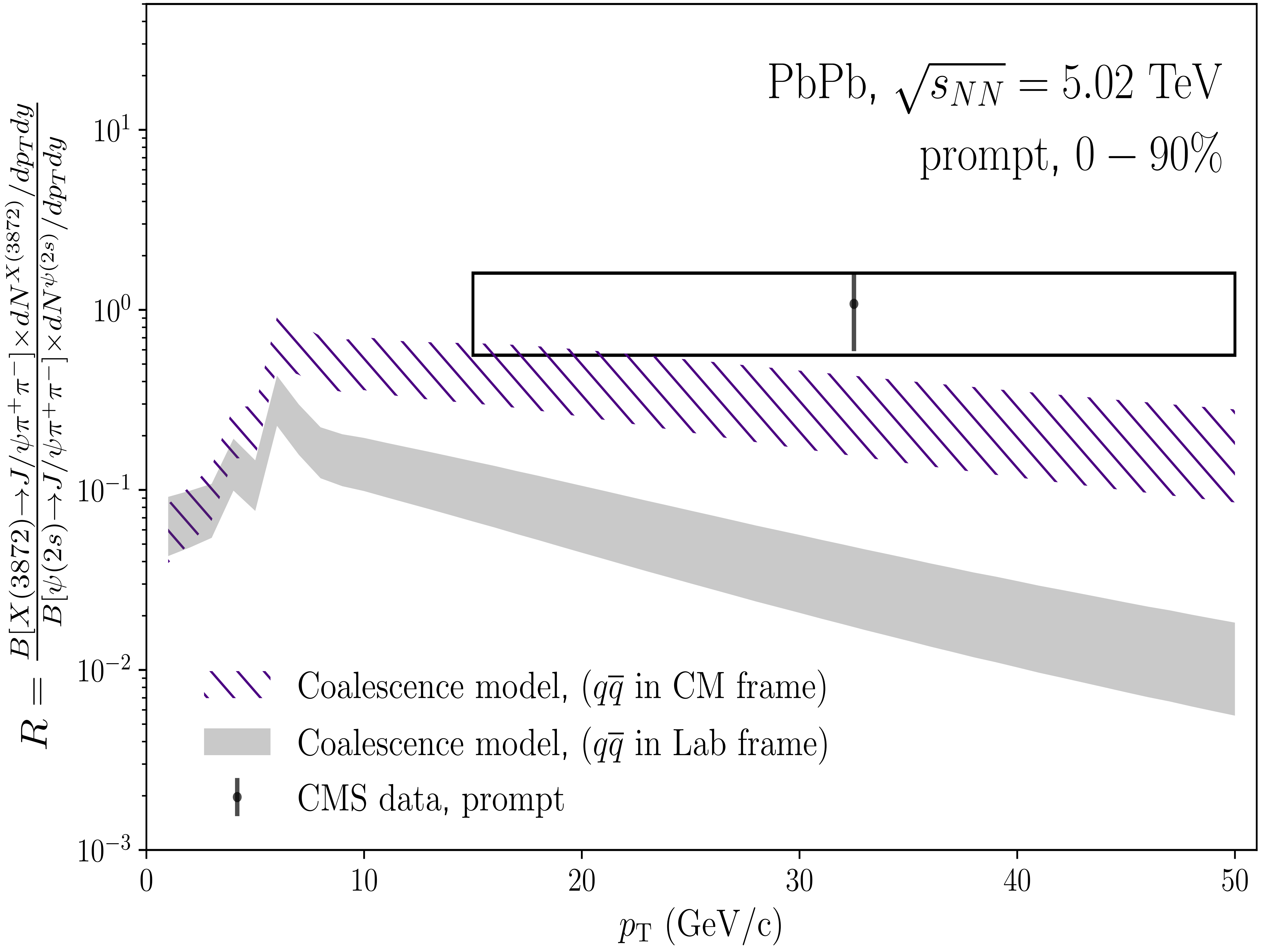
$$R_{AA} = \begin{cases} 0.35 (p_T < 4) \\ 0.2 \ \ (4 \leq p_T < 6) \\ 0.1 \ \ (6 \leq p_T) \end{cases}$$





X(3872) at high p_T

$[q\bar{q}] + [c\bar{c}]$, PbPb collision



$$m_{[q\bar{q}]} = 600 \text{ MeV}, \quad m_{[c\bar{c}]} = 3272 \text{ MeV},$$

$$r_h^{[q\bar{q}]} = 6 \text{ fm}, \quad r_h^{[q\bar{q}]-[c\bar{c}]} = 0.6 \text{ fm},$$

Relative momentum : $k^\mu = \dfrac{m_2 p_1^\mu - m_1 p_2^\mu}{m_1 + m_2}$

$\rightarrow [q\bar{q}]$ (at low p_T) + $[c\bar{c}]$ (at high p_T) coalescence is possible

Ξ_c/D

From diquark