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Transport coefficients of compressed baryonic matter at finite magnetic field

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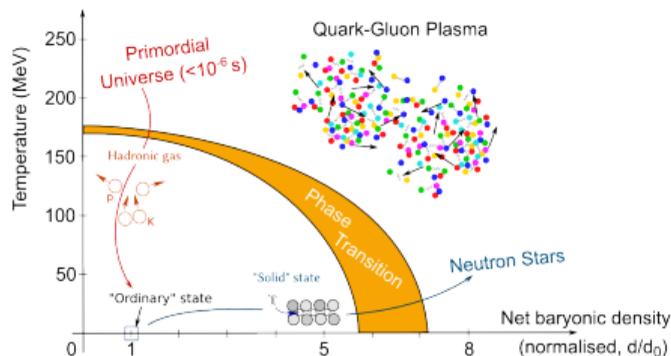
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Introduction

In the context of the QCD phase diagram there are two extreme scenarios,

- **Hot and dense phases** – the quark-gluon plasma that existed in the early universe just after the Big Bang – at high temperatures and nearly vanishing net baryon densities. Probed at ultra-relativistic heavy-ion collision experiments at the LHC and RHIC.
- **Cold nuclear matter** – systems like the atomic nuclei and neutron stars – that correspond to conditions of low temperatures and high density. Upcoming facilities of Compressed Baryonic Matter (CBM) experiment in FAIR and Nuclotron-based Ion Collider fAcility (NICA) at JINR.



QCD phase diagram. ¹

CBM will probe the highest baryonic densities ever obtained in laboratory environments

In this work, we employ the chiral effective model to obtain the constituent quark mass in this non-pQCD regime and determine the transport coefficients in the kinetic theory framework.

¹A. Maire, PhD thesis, Strasbourg U., 2011.

Constituent quark mass from chiral effective model

We use the chiral effective model to obtain the expressions for the constituent quark masses. We start with a Lagrangian density,²

$$L = L_{kin} + L_{BX} + L_{SSB} + L_{scale-break} + L_{ESB}, \quad (1)$$

where, $L_{BX} = - \sum_{i=n,p} \bar{\psi}_i m_i^* \psi_i$, $m_i^* = -g_{\sigma i} \sigma - g_{\zeta i} \zeta - g_{\delta i} \delta$,

$$L_{SSB} = -\frac{1}{2} k_0 \chi^2 (\sigma^2 + \zeta^2 + \delta^2) + k_1 (\sigma^2 + \zeta^2 + \delta^2)^2 + k_2 \left(\frac{\sigma^4}{2} + \frac{\delta^4}{2} + 3\sigma^2 \delta^2 + \zeta^4 \right) + k_3 \chi (\sigma^2 - \delta^2) \zeta - k_4 \chi^4,$$

$$L_{scale-break} = -\frac{1}{4} \chi^4 \ln \frac{\chi^4}{\chi_0^4} + \frac{d}{3} \chi^4 \ln \left(\frac{(\sigma^2 - \delta^2) \zeta}{\sigma_0^2 \zeta_0} \left(\frac{\chi}{\chi_0} \right)^3 \right),$$

$$L_{ESB} = -\left(\frac{\chi}{\chi_0} \right)^2 \text{Tr} \left[\text{diag} \left(\frac{1}{2} m_\pi^2 f_\pi (\sigma + \delta), \frac{1}{2} m_\pi^2 f_\pi (\sigma - \delta), \left(\sqrt{2} m_K^2 f_K - \frac{1}{\sqrt{2}} m_\pi^2 f_\pi \right) \zeta \right) \right].$$

Here, m_i^* is the effective mass of the baryon field ψ_i . Since we consider nuclear matter, $i = p, n$ stand for proton and neutron, respectively. σ, δ and ζ are the scalar fields and χ is the dilaton field.

²A. Mishra *et al.*, *Phys. Rev. C* **69**, 024903, arXiv: nucl-th/0308064 (2004).

Constituent quark mass from chiral effective model

The equations of motion for the fields σ , ζ , δ and χ , obtained from the Lagrangian density in Eq. (1), are ³

$$k_0 \chi^2 \sigma - 4k_1 \sigma (\sigma^2 + \zeta^2 + \delta^2) - 2k_2 (\sigma^3 + 3\sigma \delta^2) - 2k_3 \chi \sigma \zeta - \frac{d}{3} \chi^4 \left(\frac{2\sigma}{\sigma^2 - \delta^2} \right) + \left(\frac{\chi}{\chi_0} \right)^2 m_\pi^2 f_\pi - \sum_i g_{\sigma i} \rho_i^s = 0,$$

$$k_0 \chi^2 \zeta - 4k_1 \zeta (\sigma^2 + \zeta^2 + \delta^2) - 4k_2 \zeta^3 - k_3 \chi (\sigma^2 - \delta^2) - \frac{d}{3} \frac{\chi^4}{\zeta} + \left(\frac{\chi}{\chi_0} \right)^2 \left[\sqrt{2} m_K^2 f_K - \frac{1}{\sqrt{2}} m_\pi^2 f_\pi \right] - \sum_i g_{\zeta i} \rho_i^s = 0,$$

$$k_0 \chi^2 \delta - 4k_1 \delta (\sigma^2 + \zeta^2 + \delta^2) - 2k_2 \delta (\delta^2 + 3\sigma^2) + 2k_3 \chi \delta \zeta + \frac{2}{3} d \chi^4 \left(\frac{\delta}{\sigma^2 - \delta^2} \right) - \sum_i g_{\delta i} \rho_i^s = 0,$$

$$k_0 \chi (\sigma^2 + \zeta^2 + \delta^2) - k_3 \zeta (\sigma^2 - \delta^2) + \chi^3 \left[1 + 4 \ln \frac{\chi}{\chi_0} \right] \\ + (4k_4 - d) \chi^3 - \frac{4}{3} d \chi^3 \ln \left[\left(\frac{(\sigma^2 - \delta^2) \zeta}{\sigma_0^2 \zeta_0} \right) \left(\frac{\chi}{\chi_0} \right)^3 \right] + \left(\frac{2\chi}{\chi_0^2} \right) \left[m_\pi^2 f_\pi \sigma + \left(\sqrt{2} m_K^2 f_K - \frac{1}{\sqrt{2}} m_\pi^2 f_\pi \right) \zeta \right] = 0,$$

where $\rho_i^s \equiv \langle \bar{\psi}_i \psi_i \rangle$ is the scalar density of the i^{th} baryon. Solving the coupled equations, one can obtain the values of σ , ζ , δ and χ as a function of baryon density ρ_i .

³P. Papazoglou et al., *Phys. Rev. C* **59**, 411–427, arXiv: nucl-th/9806087 (1999).

Constituent quark mass from chiral effective model

Comparing with the explicit symmetry breaking term in the QCD Lagrangian and using $\delta=0$ for simplicity, we get

$$m_q \langle \bar{q}q \rangle = \frac{1}{2} m_\pi^2 f_\pi \sigma, \quad (2)$$

where $m_q = (m_u + m_d)/2$ is the current quark mass and $\langle \bar{q}q \rangle$ is the quark condensate.

By normalizing the density-dependent quark condensate, we express the constituent u -quark mass at finite T and μ_B as ⁴,

$$M_q(\rho_B, T) = \frac{\langle \bar{q}q \rangle(\rho_B, T)}{\langle \bar{q}q \rangle(\rho_B = 0, T = 0)} M_q(\rho_B = 0, T = 0) \quad (3)$$

where $\langle \bar{q}q \rangle^{1/3}(\rho_B = 0) = (263.5 \text{ MeV})$ and $M_q(\rho_B = 0) = 313 \text{ MeV}$ are the vacuum expectation values of the quark condensate and the constituent quark mass respectively.

⁴D. R. J. Marattukalam et al., arXiv: 2410.22890 (nucl-th) (Oct. 2024).

Constituent quark mass from chiral effective model

The number densities have the form,

$$\rho_i = 2 \frac{4\pi}{(2\pi)^3} \int_0^\infty p^2 (f_i^* - \bar{f}_i^*) dp, \quad (4)$$

$$\rho_i^s = 2 \frac{4\pi}{(2\pi)^3} \int_0^\infty \frac{m_i^*}{E_i^*} (f_i^* - \bar{f}_i^*) p^2 dp, \quad (5)$$

where we have the proton (neutron) distribution $f_i^* = 1/[e^{(E_i^* - \mu_i)/T} + 1]$ and anti-proton (anti-neutron) distribution $\bar{f}_i^* = 1/[e^{(E_i^* + \mu_i)/T} + 1]$ with $E_i^* = \sqrt{p^2 + m_i^{*2}}$.

The expression for the quark chemical potential μ_q can be obtained self-consistently from,

$$\rho_q = 3\rho_B = g \frac{4\pi}{(2\pi)^3} \int p^2 (f^0 - \bar{f}^0) dp, \quad (6)$$

where $f^0 = 1/[e^{(E - \mu_q)/T} + 1]$ and $\bar{f}^0 = 1/[e^{(E + \mu_q)/T} + 1]$ stands for quark and anti-quark distributions respectively with $E = \sqrt{p^2 + M_q^2(\rho_B, T)}$.

Electrical conductivity

The electrical current density for a system of quarks, in the microscopic kinetic theory, is defined as

$$J^i = gQ \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{p^i}{E} \delta f, \quad (7)$$

The kinetic evaluation of δf and the subsequent determination σ^{ij} can be carried out by solving the BTE in presence of magnetic field \vec{B} for the quarks,

$$-\frac{f^0(1-f^0)}{ET} Q \tilde{E}^j p^j + Q \epsilon^{ijk} v^j B^k \frac{\partial \delta f}{\partial p^i} = -\frac{\delta f}{\tau_c}, \text{ for particles} \quad (8)$$

where $v^j = p^j/E$ and $f^0 = 1/(e^{(E-\mu_q)/T} + 1)$. the conductivity tensor has the form,

$$\sigma^{ij} = \frac{gQ^2}{T} \int \frac{d^3\vec{p}}{(2\pi)^3} [f^0(1-f^0) + \bar{f}^0(1-\bar{f}^0)] \frac{\tau_c}{1 + (\tau_c/\tau_B)^2} \frac{p^i p^k}{E^2} \left[\delta^{jk} + \left(\frac{\tau_c}{\tau_B} \right)^2 b^j b^k + \frac{\tau_c}{\tau_B} \epsilon^{kjm} b^m \right], \quad (9)$$

i.e., magnetic fields create anisotropy. For a magnetic field along the z-axis,

- parallel conductivity ($\sigma^{\parallel} = \sigma^{zz}$)
- perpendicular conductivity ($\sigma^{\perp} = \sigma^{xx} = \sigma^{yy}$)
- Hall conductivity ($\sigma^x = \sigma^{xy} = -\sigma^{yx}$)

Classical expressions of electrical conductivity

For a relatively weak magnetic field B , one can obtain the classical components of the conductivity tensor σ^{ij} as,

$$\sigma^{ij} = \frac{gQ^2}{T} \int \frac{d^3p}{(2\pi)^3} [f^0(1-f^0) + \bar{f}^0(1-\bar{f}^0)] \frac{\tau_c}{1 + (\tau_c/\tau_B)^2} \frac{p^2}{3E^2} \left[\delta^{ij} + \left(\frac{\tau_c}{\tau_B} \right)^2 b^i b^j + \frac{\tau_c}{\tau_B} \epsilon^{ijk} b^k \right]. \quad (10)$$

$$\sigma_{CM}^{\parallel} = \frac{gQ^2}{T} \int \frac{d^3p}{(2\pi)^3} \tau_c \frac{p^2}{3E^2} [f^0(1-f^0) + \bar{f}^0(1-\bar{f}^0)], \quad (11)$$

$$\sigma_{CM}^{\perp} = \frac{gQ^2}{T} \int \frac{d^3p}{(2\pi)^3} \frac{\tau_c}{1 + (\tau_c/\tau_B)^2} \frac{p^2}{3E^2} [f^0(1-f^0) + \bar{f}^0(1-\bar{f}^0)], \quad (12)$$

$$\sigma_{CM}^{\times} = \frac{gQ^2}{T} \int \frac{d^3p}{(2\pi)^3} \frac{\tau_c(\tau_c/\tau_B)}{1 + (\tau_c/\tau_B)^2} \frac{p^2}{3E^2} [f^0(1-f^0) - \bar{f}^0(1-\bar{f}^0)], \quad (13)$$

Quantized expressions of electrical conductivity

The conductivity tensor σ^{ij} in Eq. (10) can be re-expressed by modifying the phase space integrals to incorporate the quantization of the quark momenta in the xy -plane, *i.e.*,

$$g_s \int \frac{d^3 \vec{p}}{(2\pi)^3} \rightarrow \frac{QB}{(2\pi)^2} \sum_{l=0}^{\infty} \alpha_l \int d\phi \int \frac{dp^z}{2\pi} \quad \text{for } E = \sqrt{p_z^2 + 2lQB + M^2}. \quad (14)$$

$$\sigma_{QM}^{\parallel} = \frac{3Q^2}{T} \sum_{l=0}^{\infty} \alpha_l \frac{|Q|B}{(2\pi)^2} \int_{-\infty}^{\infty} [f^0(E_l)(1 - f^0(E_l)) + \bar{f}^0(E_l)(1 - \bar{f}^0(E_l))] \tau_c \frac{(p^z)^2}{E_l^2} dp^z, \quad (15)$$

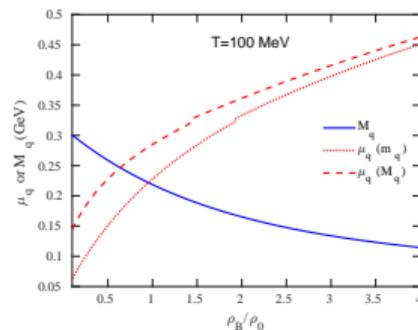
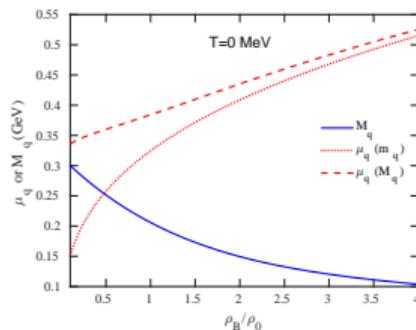
$$\sigma_{QM}^{\perp} = \frac{3Q^2}{T} \sum_{l=0}^{\infty} l \alpha_l \frac{(|Q|B)^2}{(2\pi)^2} \int_{-\infty}^{\infty} [f^0(E_l)(1 - f^0(E_l)) + \bar{f}^0(E_l)(1 - \bar{f}^0(E_l))] \frac{\tau_c}{1 + (\tau_c/\tau_{B,l})^2} \frac{dp^z}{E_l^2}, \quad (16)$$

$$\sigma_{QM}^{\times} = \frac{3Q^2}{T} \sum_{l=0}^{\infty} l \alpha_l \frac{(|Q|B)^2}{(2\pi)^2} \int_{-\infty}^{\infty} [f^0(E_l)(1 - f^0(E_l)) - \bar{f}^0(E_l)(1 - \bar{f}^0(E_l))] \frac{\tau_c(\tau_c/\tau_{B,l})}{1 + (\tau_c/\tau_{B,l})^2} \frac{dp^z}{E_l^2}, \quad (17)$$

Results

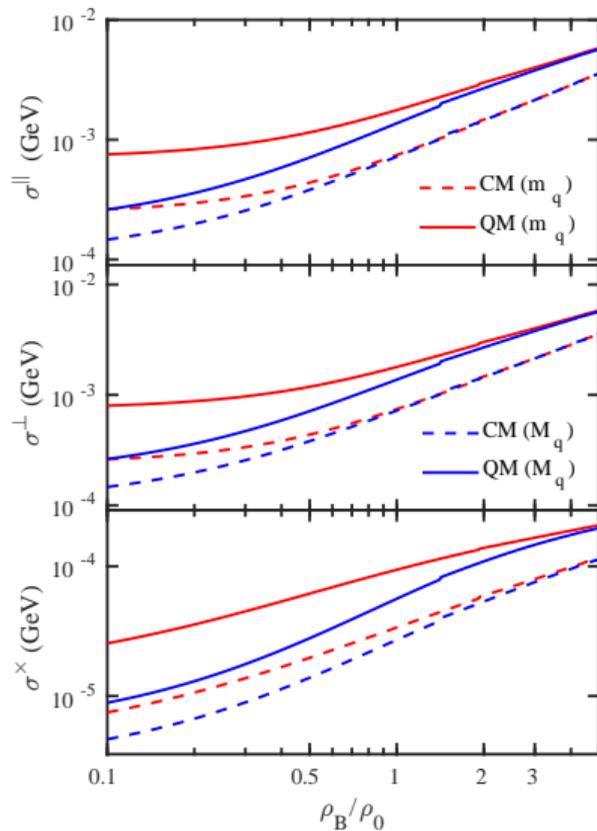
A quantitative comparison of the conductivity components obtained from the relaxation time approximated (RTA) Boltzmann Transport Equation (BTE) at finite baryon density ρ_B .

- A simplified system consisting of a single quark flavour - the u quark.
- We compare the conductivity components in two different mass scenarios:
 - current quark mass
 $m_q = (m_u + m_d)/2 = 4.63 \text{ MeV}$
 - density-dependent in-medium constituent quark mass $M_q(\rho_B)$ obtained using the chiral effective model.

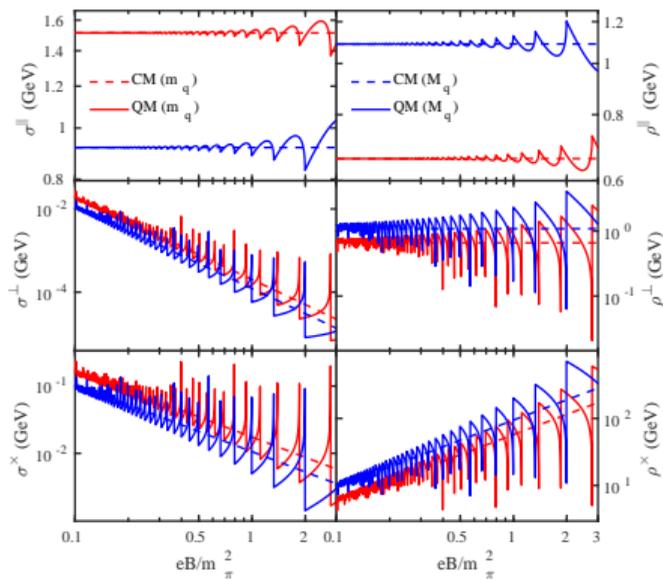
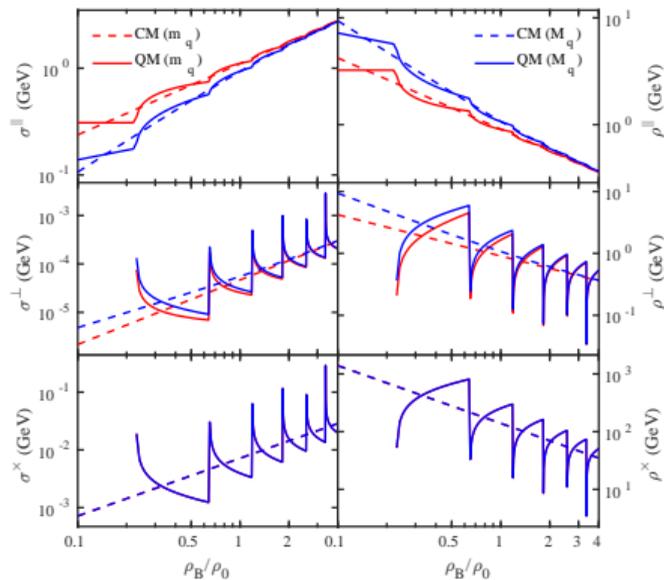


Results

- Variation of electrical conductivity components as a function of baryon density at a constant magnetic field of $eB = 1 m_\pi^2$ and temperature $T = 100$ MeV
- Expected in the reaction zone of upcoming CBM/NICA experiments.
- The current quark mass calculations overestimates the conductivities, particularly at low baryon densities.
- The quantized estimates differ significantly from the classical (unquantized) estimates over the entire range of densities.



Results



We observe Shubnikov-de Haas (SdH) type oscillations in the conductivity and resistivity components as a function of baryon density and magnetic fields. Quantization of Hall conductivity and resistivity are also observed.

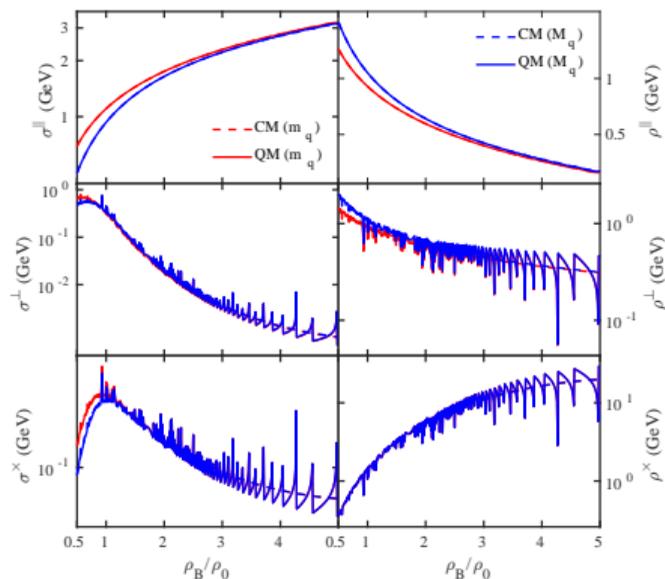
Results

As we move from the surface to the core of a neutron star, the density and the magnetic field increases gradually.

$$B(\rho_B/\rho_0) = B_{surf} + B_0[1 - \exp(-\beta(\rho_B/\rho_0)^\gamma)]^7$$

$$\beta=0.01, \gamma=3, B_{surf}= 10^{14} \text{ G}, B_0 = 5 \times 10^{18} \text{ G}.$$

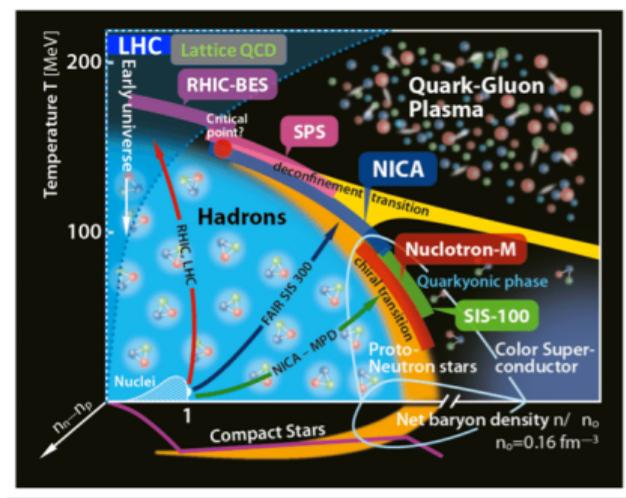
The seemingly opposite effects of baryon density and magnetic field with regard to the quantization of electrical conductivity and resistivity can give rise to a rather unique conductivity/resistivity profile in neutron stars.



⁵D. Bandyopadhyay *et al.*, *Phys. Rev. Lett.* **79**, 2176–2179, arXiv: astro-ph/9703066 (1997).

Summary and Outlook

- We study the transport properties of dense medium — electrical conductivity — including the effects of quantization.
- This can be applied to study observables such as the dilepton spectrum and provide inputs for hydrodynamic evolution, offering insight into matter at high baryon density and moderate temperature.



This region of the QCD phase diagram remains largely unexplored experimentally.

⁶R. Sahoo, T. Nayak, *Possible early universe signals in proton collisions at the Large Hadron Collider*, Jan. 2022.

Thank you!