

Wiedemann-Franz law violation in Graphene and Quark Matter

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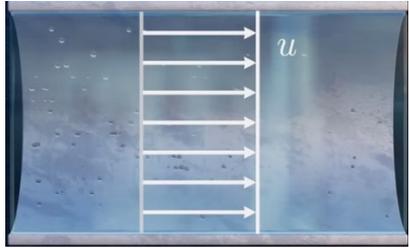


Outline

- **Introduction & Motivation**
- **Mathematical Framework**
- **Results and Discussion**
- **Summary**

Introduction & Motivation

Electron flow in Metals and Graphene

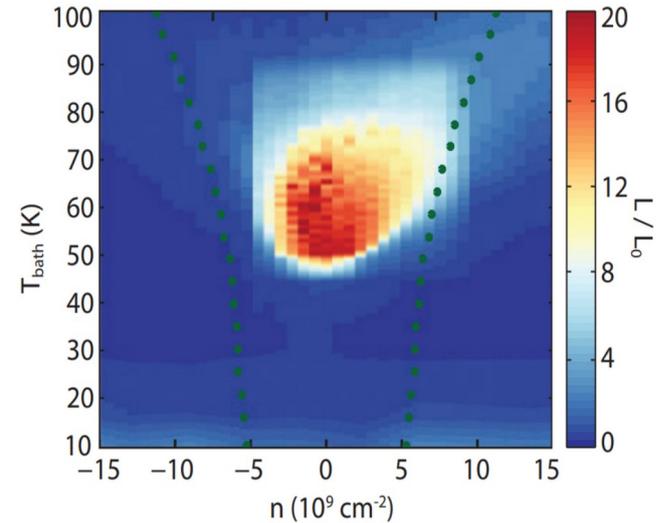


Electron *ohmic* Flow in metals.



Poiseuille hydrodynamics Flow in graphene[1].

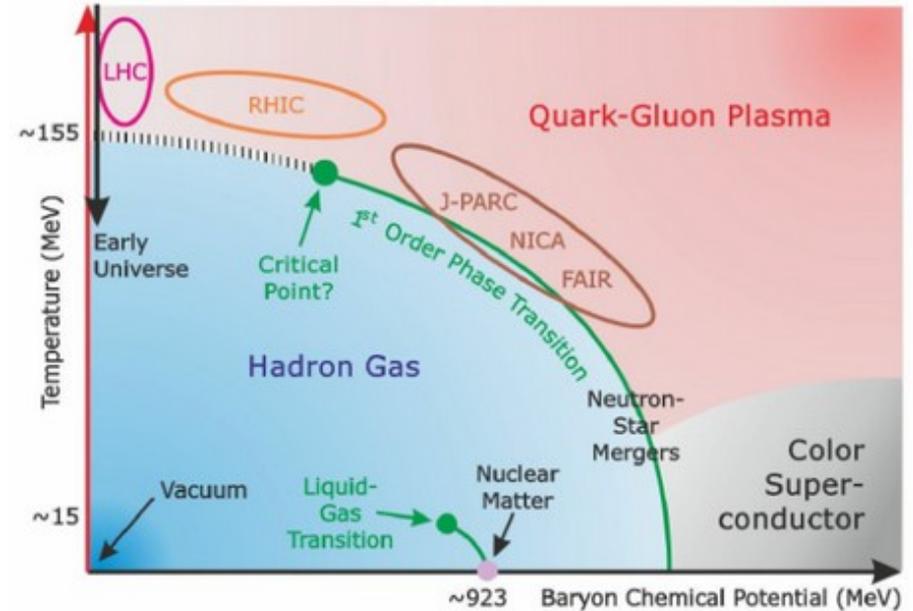
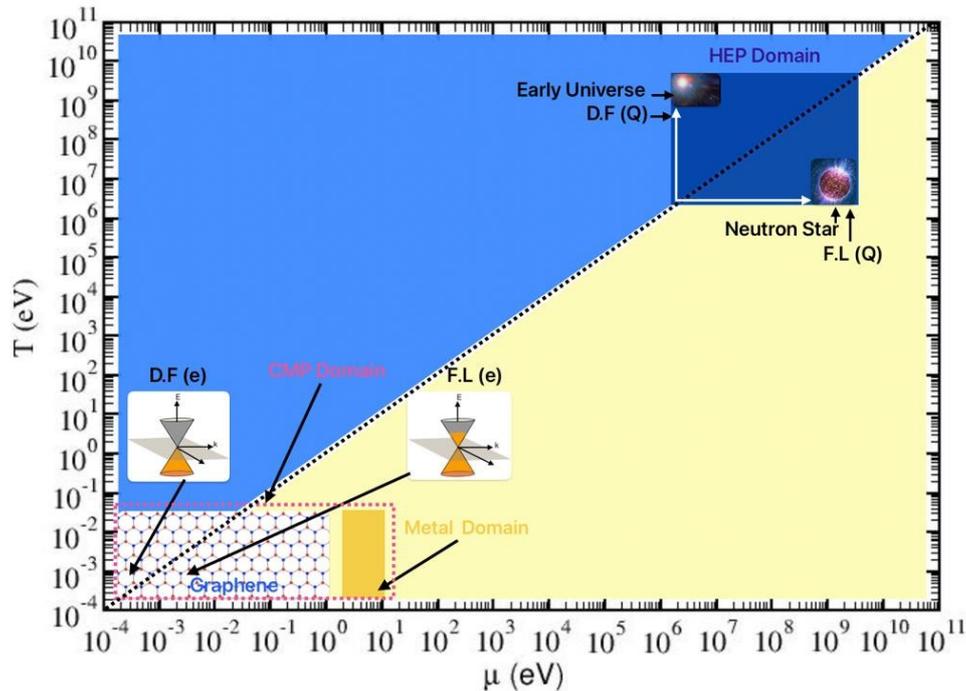
- Electrons inside graphene move like a water inside a pipe.
- The poiseuille flow pattern of electrons inside graphene was observed by the experimental group of researchers[1].



$$\frac{\kappa}{\sigma T} = \frac{\pi^2}{3} \left(\frac{k_B}{e} \right)^2$$

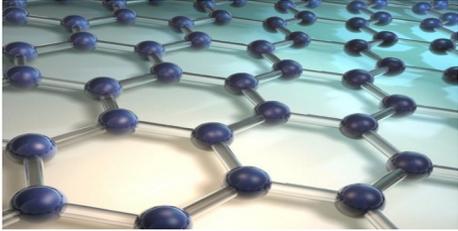
$$L_0 = 2.445 \times 10^{-8} \text{ watt} \frac{\Omega}{K^2}$$

Introduction & Motivation



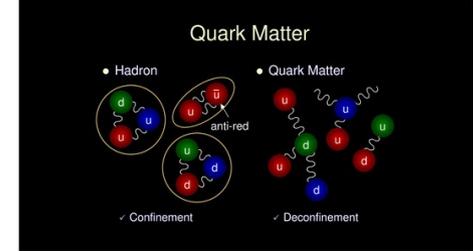
Connecting between Condensed Matter domain and High-energy Physics domain

Mathematical Framework (Drude's free electron gas model)



$$\mathbf{E} = \rho \mathbf{v}_g.$$

$$\mathbf{E} = \rho \mathbf{c}.$$



- The ratio of thermal and electrical conductivity

$$\frac{\kappa_G^{2D}}{\sigma_G^{2D}} = \frac{1}{2} \frac{\epsilon_F}{e^2} {}^{2D}_G [C_V]$$

$$\frac{\kappa_Q^{3D}}{\sigma_Q^{3D}} = \frac{1}{3} \frac{\mu_q}{e^2} {}^{3D}_Q [C_V]$$

- The specific heat

$${}^{2D}_G [C_V] = 2k_B \left[3 \frac{f_3(A)}{f_2(A)} - 2 \frac{f_2(A)}{f_1(A)} \right]$$

$${}^{3D}_Q [C_V] = 3k_B \left[4 \frac{f_4(A)}{f_3(A)} - 3 \frac{f_3(A)}{f_2(A)} \right]$$

- Lorenz Ratios

$$\frac{L_G^{2D}}{L_0} = \frac{\kappa_G^{2D}}{\sigma_G^{2D} T L_0} = \frac{3}{\pi^2} \frac{\epsilon_F}{k_B T} \left[3 \frac{f_3(A)}{f_2(A)} - 2 \frac{f_2(A)}{f_1(A)} \right]$$

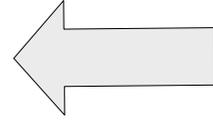
$$\frac{L_Q^{3D}}{L_0} = \frac{\kappa_Q^{3D}}{\sigma_Q^{3D} T L_0} = \frac{3}{\pi^2} \frac{\mu_q}{k_B T} \left[4 \frac{f_4(A)}{f_3(A)} - 3 \frac{f_3(A)}{f_2(A)} \right] \quad 5$$

Mathematical Framework (BTE)

The Boltzmann transport equation,

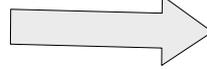
$$\frac{\partial f}{\partial t} + \frac{\partial x^i}{\partial t} \frac{\partial f}{\partial x^i} + \frac{\partial p^i}{\partial t} \frac{\partial f}{\partial p^i} = -\frac{\delta f}{\tau_c}$$

$$f_0(T) = \frac{1}{e^{(\epsilon - \epsilon_F)/k_B T} + 1}$$



$$\delta f_\kappa = v^i \left(-\frac{\partial f_0}{\partial \epsilon} \right) [C_V] \left(\frac{\partial T}{\partial x^i} \right) \tau_c$$

Fluid (F)



$$[C_V]_F^{G/Q} = \frac{u + P}{nT}$$

2D G system

3D Q system

Non_Fluid (NF)

Fermi Integral Function $f_n(A)$

$$[C_V]_{NF}^G = 2k_B \left[3 \frac{f_3(A)}{f_2(A)} - 2 \frac{f_2(A)}{f_1(A)} \right]$$

$$[C_V]_{NF}^Q = 3k_B \left[4 \frac{f_4(A)}{f_3(A)} - 3 \frac{f_3(A)}{f_2(A)} \right]$$

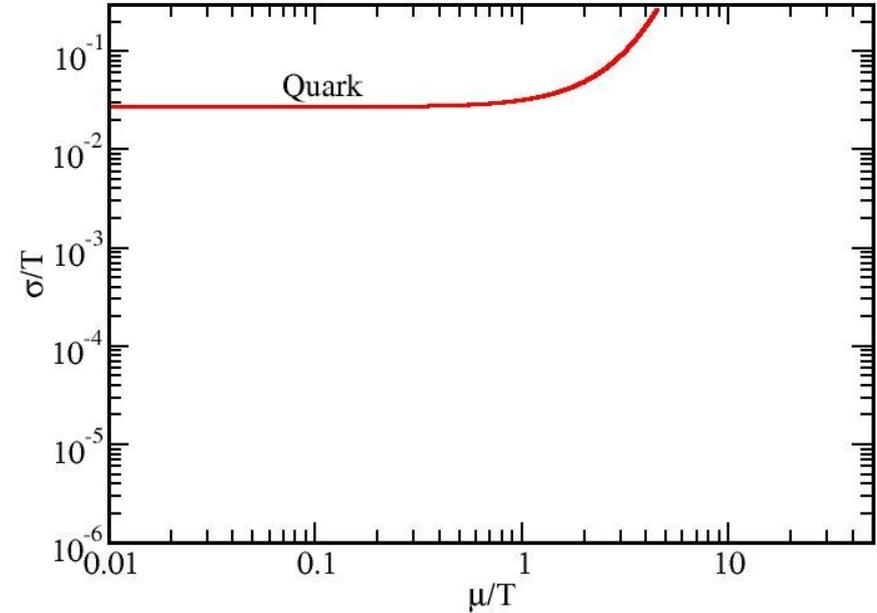
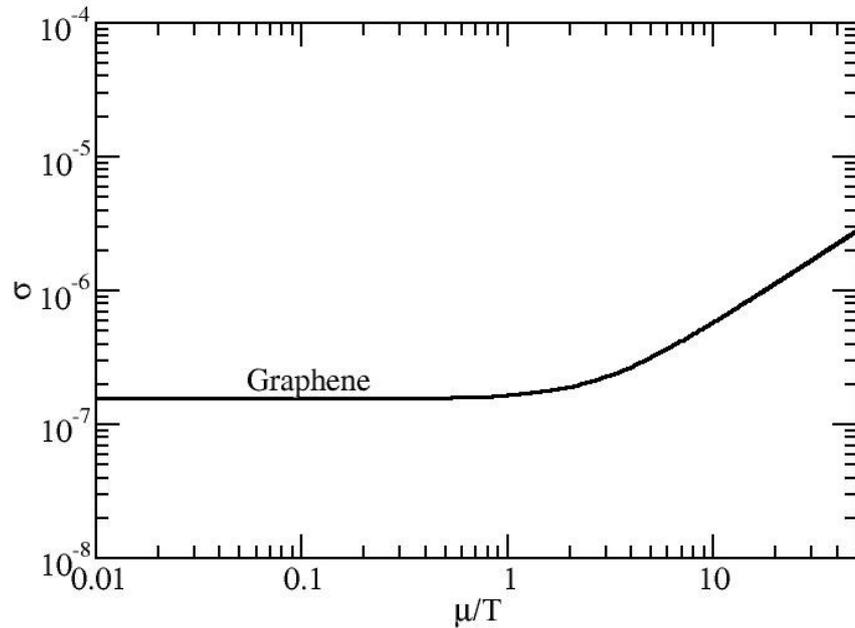
$$[C_V] = \left. \frac{\partial}{\partial T} \left(\frac{U}{N} \right) \right|_{V, \epsilon_F}$$



Results and Discussion (Electrical Conductivity)

$$\sigma_G^{2D} = \frac{ne^2\tau_c v_g^2}{\mu}$$

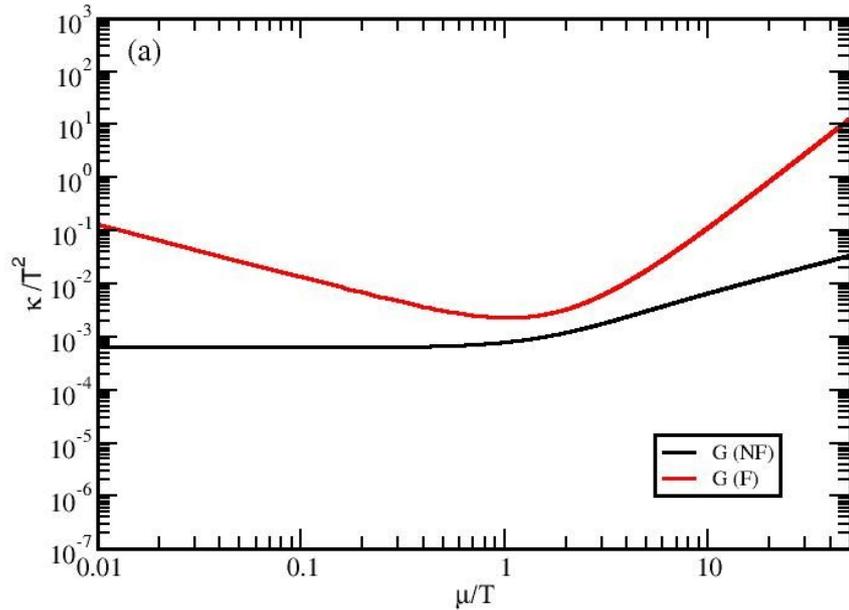
$$\sigma_Q^{3D} = \frac{ne^2\tau_c c^2}{\mu_q}$$



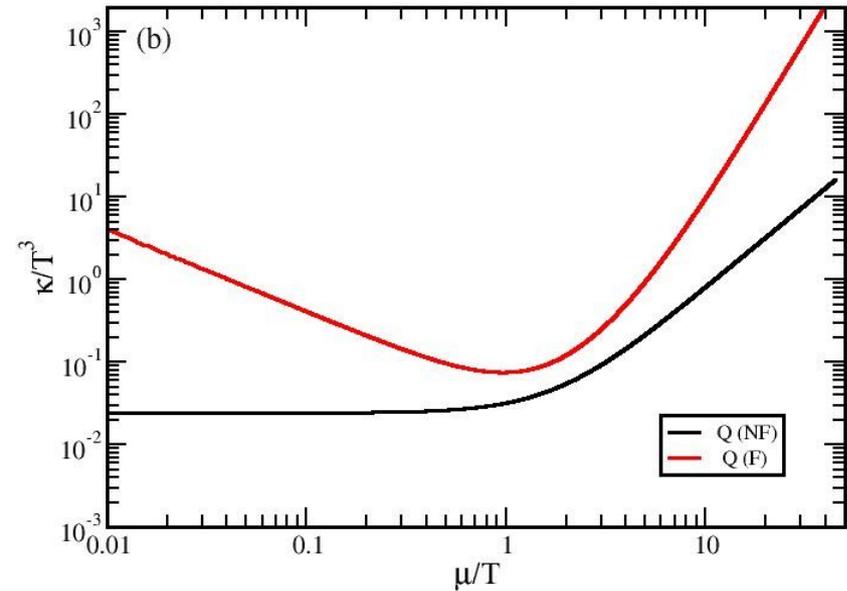
The electrical conductivity in terms of chemical potential for graphene and quark matter.

Results and Discussion (Thermal Conductivity)

$$\kappa_{F/NF}^G = \frac{\tau_c}{\pi} [C_V]_{F/NF} f_2(A) T^2$$



$$\kappa_{F/NF}^Q = \frac{24 \tau_c}{\pi^2} [C_V]_{F/NF} f_3(A) T^3$$

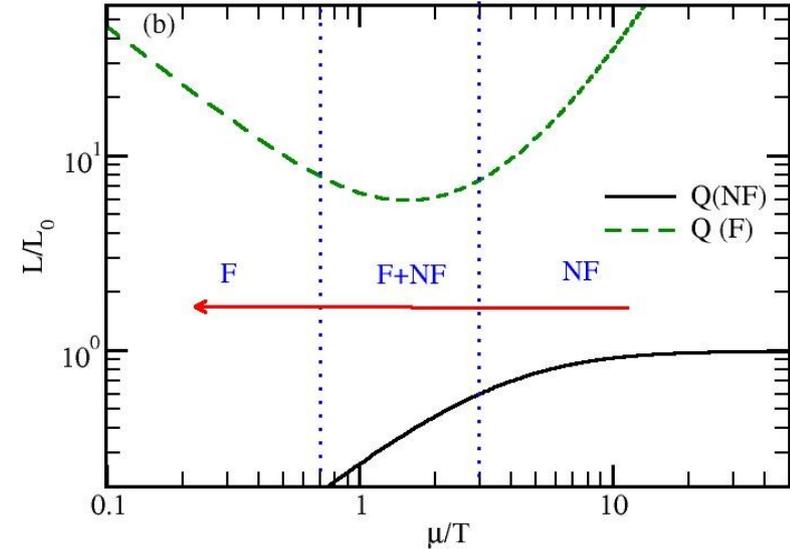
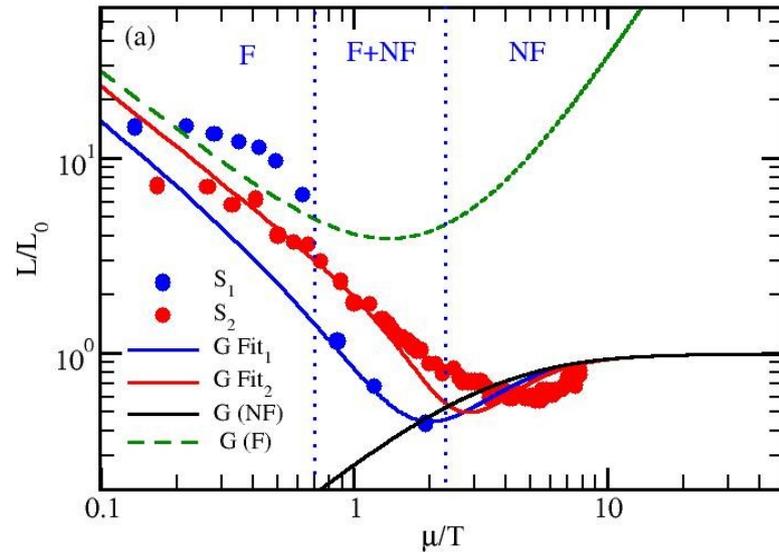


The thermal conductivity in terms of chemical potential for (a) graphene and (b) quark matter for fluid and non-fluid description.

Results and Discussion (Lorenz ratio)

$$\left(\frac{L}{L_0}\right)_{F/NF}^G = \frac{6}{\pi^2} [C_V]_{F/NF} \frac{f_2(A)}{f_1(A)} \quad (A = e^x)$$

$$\left(\frac{L}{L_0}\right)_{F/NF}^Q = \frac{9}{\pi^2} [C_V]_{F/NF} \frac{f_3(A)}{f_2(A)}$$



Fig(3). The Lorenz ratio in terms of chemical potential for (a) graphene and (b) quark matter.

Summary

- ✓ We have gone through comparative research on Lorenz ratios for graphene and quark systems to check the WF law violation domain by playing the chemical potential or baryon chemical potential.
- ✓ The specific heat is replaced by enthalpy density per particle in fluid case because enthalpy density per particle the fundamental role in determining the fluid dynamics.
- ✓ Our conclusion is that fluid nature is the possible reason for the violation.



Thank You

Switching function

$$\frac{L}{L_0} = \left(\frac{L}{L_0} \right)_F \times a_F + \left(\frac{L}{L_0} \right)_{NF} \times a_{NF}$$

$$a_F(x) = \frac{1}{e^{b_F(x-x_F)} + 1}$$

$$a_{NF}(x) = \frac{1}{e^{b_{NF}(x-x_{NF})} + 1}$$

$b_F=2, b_{NF}=0.7, x_F=0.2, x_{NF}=0.8$ (for red)

$b_F=2, b_{NF}=0.7, x_F=0.9, x_{NF}=2$ (for blue)