Probing the Dynamical Conductivity of QGP via Dilepton emission: A Kinetic Theory Approach

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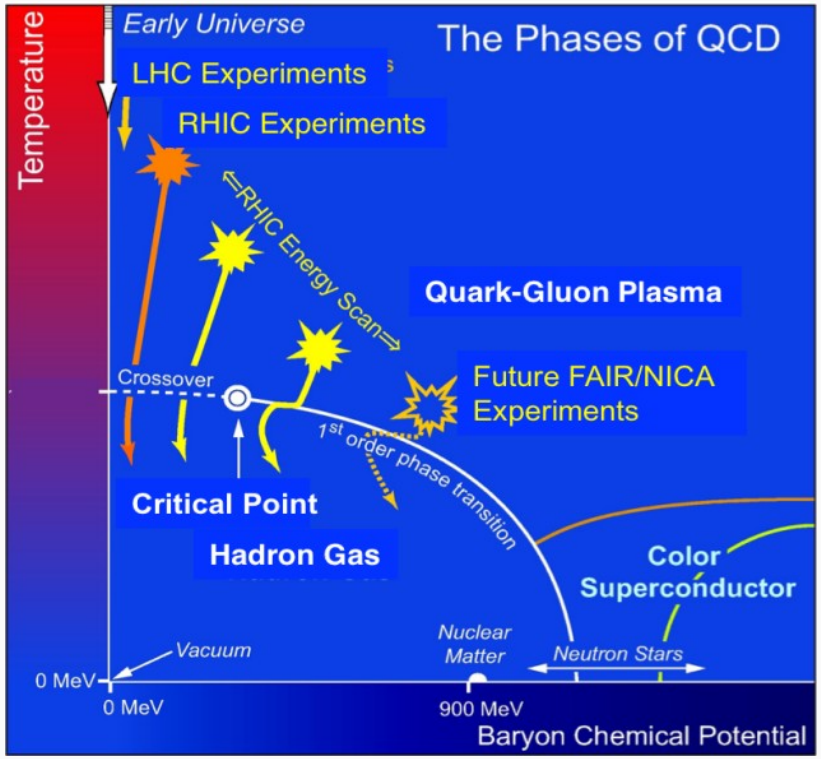
Hot QCD Matter 2025

Outline

QCD phase diagram

- Free quarks and gluons (Quark-Gluon Plasma) $\Longrightarrow$ nuclear matter with high energy density.
- Two ways: Heat nuclei up to very high temperature T or, compress it to very high baryon density ρ_B
- How to implement it practically? Only hope, collide heavy ions with relativistic velocities!

Nuclear matter in $\mu_B - T$ plane [a]



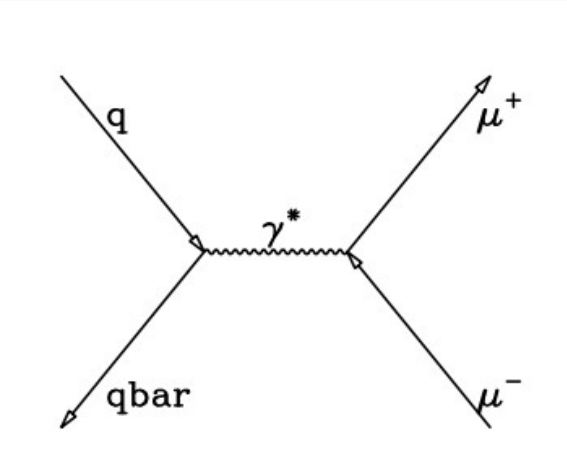
[a] https://doi.org/10.1088/1742-6596/1602/1/012003

Outline

Dileptons as a probe

- Good probe $\Longrightarrow$
  Characterize the medium
  How about electromagnetic probes!
- Dilepton serve as a good probe (minimal final state interaction)
  $q + \bar{q} \rightarrow \gamma^* \rightarrow l^+ + l^-$
  Mean free path
  $\lambda^{l^+ l^-} \gg L$ =medium size

Dilepton emission from QGP



Aim

Studying the dilepton production considering quark-quark interaction

Outline

General Framework from QFT

Dilepton production rate (DPR)

$$R = \frac{d^4N}{d^4x\ d^4q} = \frac{\alpha_e}{12\pi^4}\ \frac{L(q^2)}{q^2}\ f_{\mathrm{BE}}(\omega)\ g^{\mu\nu}\ \mathrm{Im}\tilde{G}^{\mathrm{R}}_{\mu\nu}(\omega,\vec{q})$$

- Phase-space factors for dileptons: $L(q^2) = \left(1+\frac{2m_l^2}{M^2}\right)\sqrt{1-\frac{4m_l^2}{M^2}}$
- $\tilde{G}^{\mathrm{R}}_{\mu\nu}$: retarded propagator (captures the medium dynamics)
- In QFT one express $\tilde{G}^{\mathrm{R}}_{\mu\nu}$ as the thermal expectation of electric current-current correlator[1]

[1] Rapp et al. Adv.Nucl.Phys. 25 (2000) 1

Linking the Green's function (propagator) with conductivity

- Noting that induced electric current $J_\mu(x)$ as a response to an external EM four potential $A^\mu(x)$ as,

$$J_\mu(x) = \int d^4x' \; G^{\mathrm{R}}_{\mu\nu}(x - x') \; A^\nu(x') \; . \tag{1}$$

  In Fourier space, $\tilde{J}_\mu(\omega, \vec{q}) = \tilde{G}^{\mathrm{R}}_{\mu\nu} \; \tilde{A}^\nu(\omega, \vec{q})$.

- Expressing the current $\tilde{J}_\mu$ in terms of electric field,

$$\tilde{A}^\mu(q) = \int d^3\vec{x} \; e^{-i\vec{q}\cdot\vec{x}} \int_{-\infty}^{\infty} e^{i\omega t} A^\mu(t, \vec{x}) \; dt,$$

$$\implies \tilde{A}^j(q) = \frac{1}{i\omega} \tilde{E}^j(q) \; . \tag{2}$$

- Choosing the gauge $A^0 = 0$, and using $\vec{E} = -\partial_t \vec{A}$, we have, $\tilde{\sigma}^{jk} = \frac{\tilde{G}^{(\mathrm{R})\mathrm{jk}}}{i\omega}$.

DPR in terms of dynamical conductivity

$$R = \frac{\alpha}{12\pi^4} \frac{L(q^2)}{q^2} f_{\mathrm{BE}}(\omega) \left[\omega \, \mathrm{Re}\, \tilde{\sigma}^k_k(\omega, \vec{q}) + \mathrm{Im}\, \tilde{G}^{(\mathrm{R})0}_0\right] \tag{3}$$

Evaluating Conductivity from Kinetic Theory

- Electric current from the relativistic kinetic theory,

$$\tilde{J}^k = \sum_j g_j Q_j \int \frac{d^3\vec{p}_j}{(2\pi)^3} \frac{p_j^k}{p_{0j}} \tilde{\delta f}_j \equiv \tilde{\sigma}_l^k \tilde{E}^l \ , \tag{4}$$

$$\tilde{\delta n} = \sum_j g_j Q_j \int \frac{d^3\vec{p}_j}{(2\pi)^3} \tilde{\delta f}_j \equiv \tilde{G}_k^{(R)0} A^k \ , \tag{5}$$

  j stands for different quark species.

- The perturbation δf can be determined from Boltzmann transport equation (BTE) in relaxation time approximation (RTA)

$$\frac{\partial f_j}{\partial t} + \frac{\vec{p}_j}{p_j^0} \cdot \frac{\partial f_j}{\partial \vec{r}} + \vec{F}_j \cdot \frac{\partial f_j}{\partial \vec{p}_j} = -\frac{u_\mu p_j^\mu}{p_j^0} \frac{\delta f_j}{\tau_c} \ . \tag{6}$$

- Obtaining δf from BTE and evaluating the integral in Eqs. (4) and (5) in the limit of massless quarks we get,

$$\tilde{\sigma}_k^k(\omega, \vec{q}) = \frac{e^2 T^2}{3|\vec{q}|} \left[\tan^{-1}\left(\frac{2|\vec{q}|\Gamma}{\Gamma^2 + M^2} \right) + \frac{i}{2} \ln\left(\frac{\Gamma^2 + (\omega+q)^2}{\Gamma^2 + (\omega-q)^2} \right) \right], \tag{7}$$

Relaxation time dependent DPR

$$\tilde{G}_0^{(\mathrm{R})0} = \left[\frac{1}{|\vec{q}|}\left(\frac{\omega}{2}\ln\frac{\Gamma^2+(\omega+|\vec{q}|)^2}{\Gamma^2+(\omega-|\vec{q}|)^2}+\Gamma\tan^{-1}\frac{2|\vec{q}|\Gamma}{\Gamma^2+M^2}-2|\vec{q}|\right)\right.$$
$$\left. - \frac{i}{|\vec{q}|}\left(\omega\tan^{-1}\frac{2|\vec{q}|\Gamma}{\Gamma^2+M^2}-\frac{\Gamma}{2}\ln\frac{\Gamma^2+(\omega+|\vec{q}|)^2}{\Gamma^2+(\omega-|\vec{q}|)^2}\right)\right]\frac{e^2T^2}{3} \quad (8)$$

where we used $\tilde{G}^{\mathrm{R}}_{00} = -\tilde{G}^{\mathrm{R}}_{0k}\frac{q^k}{\omega}$.

DPR from QGP in RTA

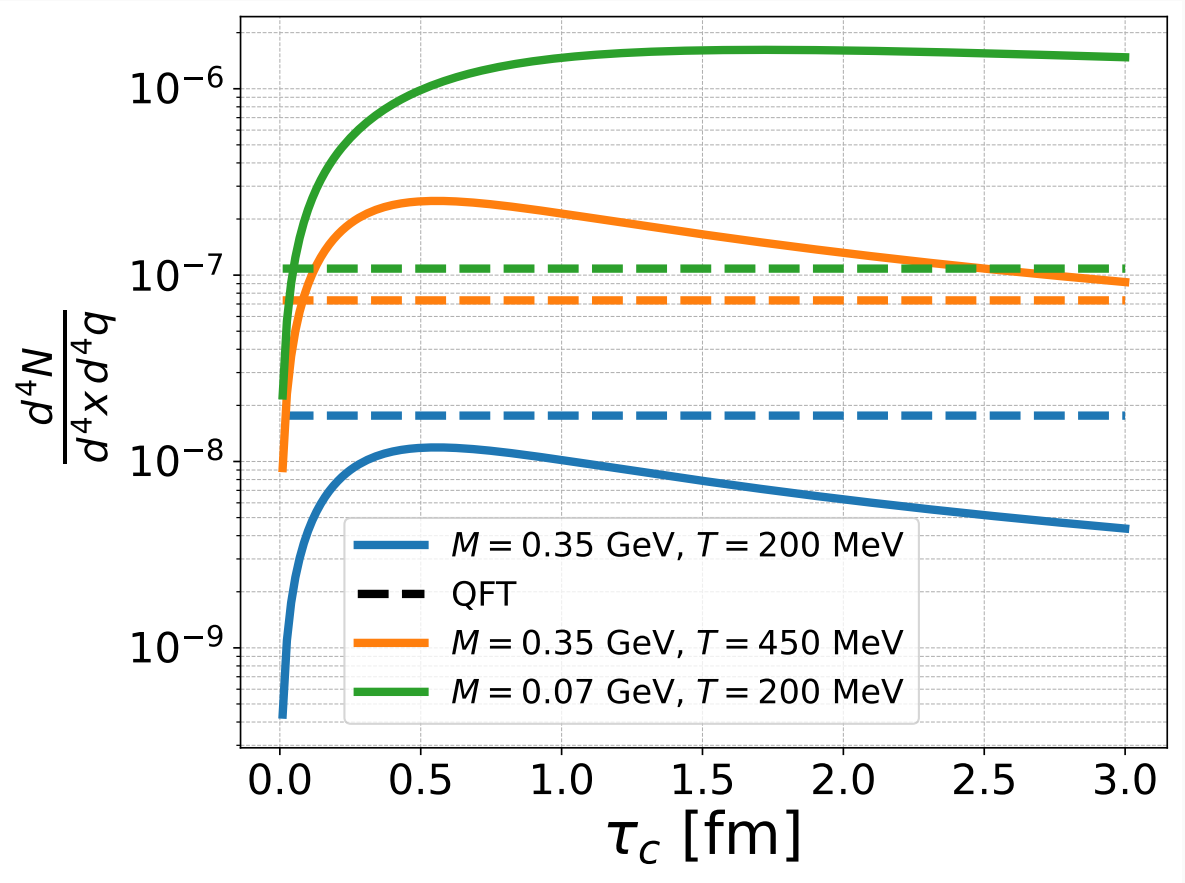
$$R = \frac{d^4N}{d^4x\ d^4q} = \frac{\alpha}{12\pi^4}\ \frac{L(q^2)}{q^2}\ f_{\mathrm{BE}}(\omega)\ 3\sigma_{\mathrm{DC}}\ \mathcal{A}(\vec{q},\omega,\tau_c)$$

where $\sigma_{\mathrm{DC}} = \sum_f \sigma_{\mathrm{DC}(f)} = (1/3)\sum_f Q_f^2\tau_c T^2 = \frac{2e^2}{9}\tau_c T^2$ and $\mathcal{A} = \frac{1}{4|\vec{q}|\tau_c^2}\ln\frac{\tau_c^2(\omega+|\vec{q}|)^2+1}{\tau_c^2(\omega-|\vec{q}|)^2+1}$ with e being the magnitude of the electron's charge.

- Interesting features: 1. Dependent on quark-quark relaxation time τ_c (new expression upto the best of our knowledge)
  2. Proportional to imaginary part of the medium conductivity

Outline

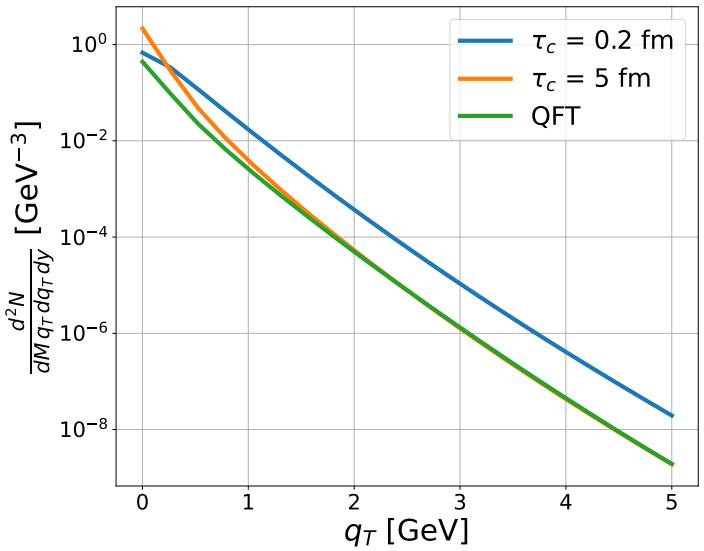
Variation of static rate with relaxation time

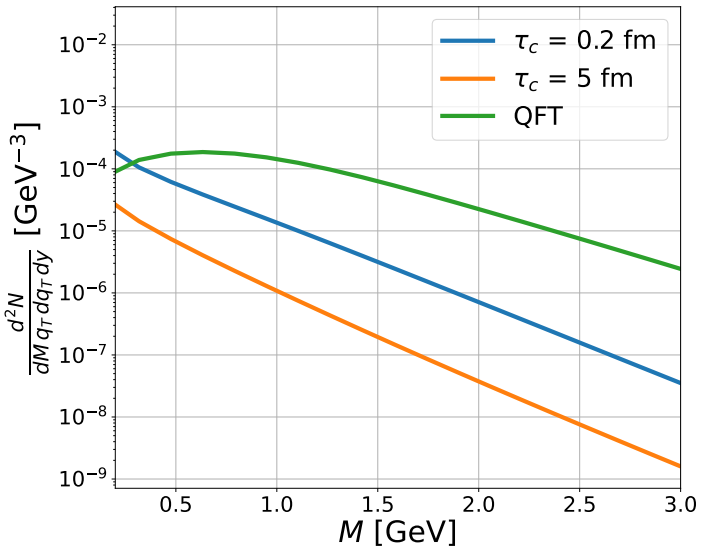


- Thermal production increases with temperature
- QFT rates: $\frac{\alpha^2}{6\pi^4}\ L(q^2)\ f_{\mathrm{BE}}(\omega)$ are relaxation time independent[2].
- Rates evaluated in RTA shows a peak around $\tau_c \sim 1/M$.

[2] Sarkar *et al.*, J.Phys.Conf.Ser. 374 (2012) 012010

Space-time integrated spectra





- For q_T-spectra, $M = 0.1$ GeV and for $M-$spectra, $q_T = 2$ GeV is used.
- In q_T-spectra KTh results lies above (Low mass enhancement in KTh)
- In $M-$spectra QFT lies above for high M whereas reverse is true for low M and higher τ_c. KTh/QFT $\sim \frac{\tau_c}{M(M^2\tau_c^2+1)}$ for $M \gg |\vec{q}|$, whereas KTh/QFT $\sim \frac{1}{M^2|\vec{q}|\tau_c}\ln[(2|\vec{q}|\tau_c)^2+1]$ for $M/|\vec{q}| \ll 1$.
- See arXiv:2508.16988 for more results like v_2 vs M, v_2 vs p_T and modified rate in presence of magnetic fields.

Outline

Summary

- We evaluate the retarded Green's function from the BTE in RTA.
- We establish that the DPR is proportional to the imaginary part of the dynamical conductivity.
- As a result we find an expression of DPR which is a function of quark-antiquark scattering rate.
- We compare our rate and spectra with that of QFT results.

Thank You!

Questions or Comments?

Outline

Enegy density and Temperature profile

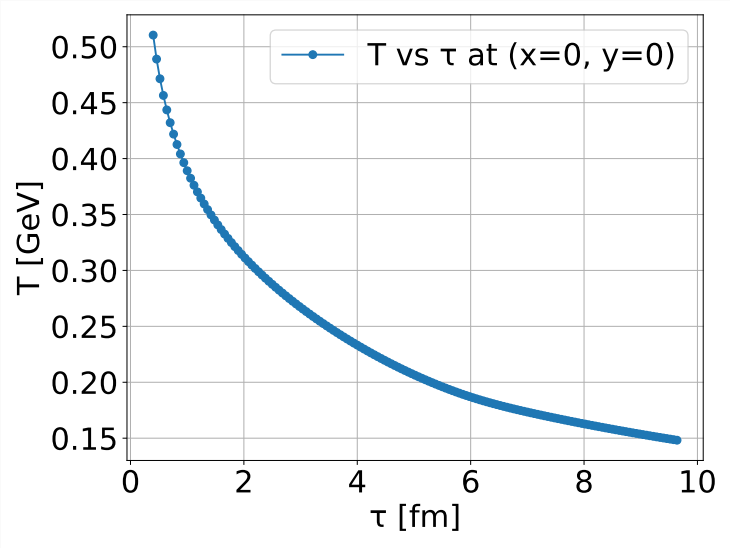


Figure 1: The temperature evolution at the center of the collision zone for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV and impact parameter $b = 7$ fm.

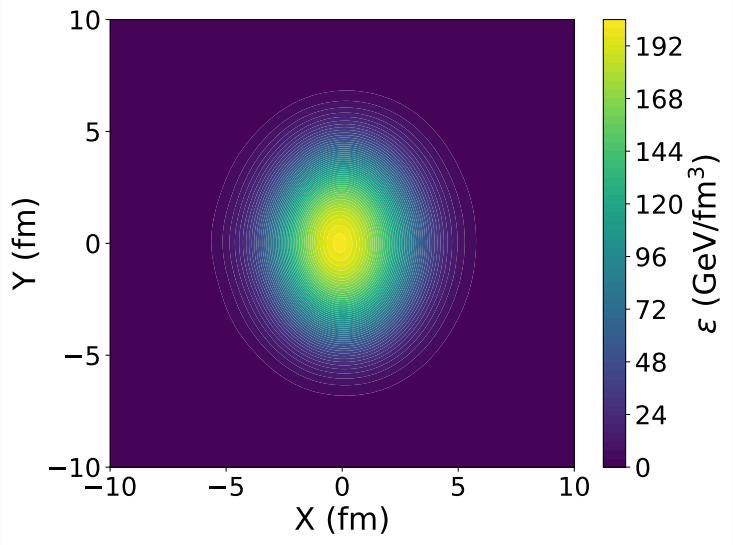


Figure 2: Initial energy density profile averaged over 2000 Monte Carlo Glauber events for Au+Au collisions at $\sqrt{s_{\mathrm{NN}}} = 200$ GeV with impact parameter $b = 7$ fm.

MUSIC is a 3+1 dimensional viscous hydrodynamic framework capable of solving the full relativistic conservation equations. In the current study we employ a 2+1D MUSIC hydrodynamic code with shear viscosity $\eta/s = 0.1$ and zero bulk viscosity.