Thermodynamics of strongly magnetized dense quark matter from hard dense loop perturbation theory

Hot QCD Matter 2025,

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Based on PRD 111, 116025 (2025)

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Outline of Talk

1. Off-central heavy-ion collisions, Magnetic field

2. Dense quark matter in magnetic field

3. Thermodynamics of dense quark matter in strong magnetic field

4. Results

5. Conclusions & Future Prospects

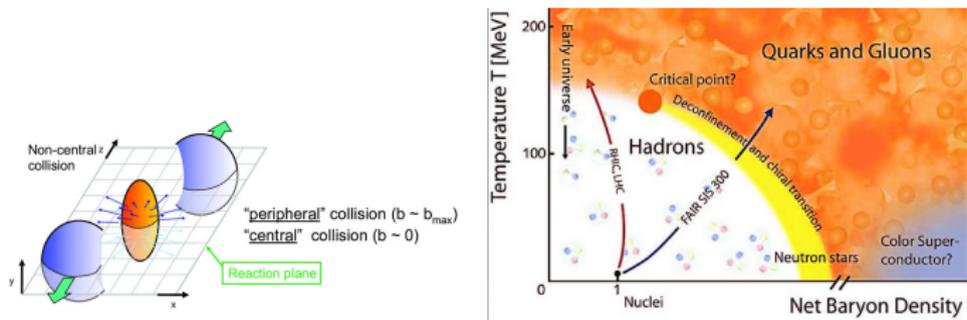Figure: (a) Left panel: Off-central collision of nucleus. "b" is the impact parameter. (b) Right Panel: QCD Phase diagram

➤ Estimated strength of magnetic field in off-central HIC: $10^{18} - 10^{20}$ **Gauss** : K. Tuchin , *Particle Production in Strong Electromagnetic Fields in Heavy Ion collisions*, Adv. High Energy Phys 2013, 490495 (2013).

➤ Oscillatory transport : S. Satapathy, S. Ghosh and S. Ghosh, "Quantum field theoretical structure of electrical conductivity of cold and dense fermionic matter in the presence of a magnetic field," Phys. Rev. D **106**, no.3, 036006 (2022) [arXiv:2112.08236 [hep-ph]].

➤ **Thermodynamics** : Considering magnetic field in z direction
$k^\mu = k^\mu_\perp + k^\mu_\parallel, \quad k^\mu_\perp = (0, k_x, k_y, 0), \quad k^\mu_\parallel = (k_0, 0, 0, k_z)$
Anisotropy is induced in Transport coefficients & Thermodynamics :
Eg. Pressure $(P) \in \{P_\parallel, P_\perp\}$, Speed of sound $c_s \in \{c_{s,\parallel}, c_{s,\perp}\}$.

Energy gets **Landau quantized** i.e without magnetic field $E = \sqrt{\sum_i k_i^2 + m^2}$, $i = x, y, z$, with magnetic field
$E_l = \sqrt{k_z^2 + 2l|q_f B| + m^2}$, where $|k_\perp^2| = 2l|q_f B|$.

Dense quark matter in magnetic field

Energy scales : T (GeV), μ (GeV), $q_f B$ (GeV^2)

Study by : E. Annala, T. Gorda, A. Kurkela, J. Nättilä and A. Vuorinen, "Evidence for quark-matter cores in massive neutron stars," Nature Phys. **16**, no.9, 907-910 (2020).

- At $\mu/T \to$ high $(10^2 - 10^3)$, the Fermi-Dirac distribution function $\left[\exp(\beta(E-\mu))+1\right]^{-1} \to \Theta(\mu - E)$.
- Sets a **Fermi-Energy** of the system : μ or Fermi momentum k_F
- In Magnetic Field - dynamics is more complicated (Landau quantization)

For Fermions (quarks) : $E \to E_l = \sqrt{m^2 + k_z^2 + 2l|q_f B|}$, $\quad l \in [0, \infty)$ is the Landau level

In very strong magnetic field, $l = 0$ contributes only

- For dense systems number of "l" is finite which leads to :
$$l_{max} = \left\lfloor \frac{\mu^2}{2|q_f B|} \right\rfloor \implies l_{max} = 0 \text{ for very strong magnetic field.}$$

Technical Issues in studying dense matter :

- LQCD does not work at high densities
- Alternative Techniques : Renormalization Group Optimized Perturbation Theory.
E. S. Fraga, L. F. Palhares and T. E. Restrepo, "Cold and dense perturbative QCD in a very strong magnetic background," Phys. Rev. D **109**, no.5, 5 (2024).
- **Our approach :** Hard Dense Loop Perturbation Theory (HDLpt) - A perturbative approach with clear hierarchy of energy scales.

Studies of dense matter

- SdH oscillations were experimentally verified long time back in 1930. In condensed matter physics :
  L. Schubnikow and W.J. De Haas, *Magnetic resistance increase in single crystals of bismuth at low temperatures,* Proceedings of the Royal Netherlands Academy of Arts and Science. **33**: 130–133, (1930).
  $\to \lim_{T \to 0} f_{\text{FD}} \to \Theta(\mu - E_l)$. Everytime a **Landau level** "l" is encountered, an oscillatory amplitude is formed.

- Oscillatory electrical conductivity : S. Satapathy, S. Ghosh and S. Ghosh, "Quantum field theoretical structure of electrical conductivity of cold and dense fermionic matter in the presence of a magnetic field," Phys. Rev. D **106**, no.3, 036006 (2022) [arXiv:2112.08236 [hep-ph]].

- Recent work of Podo & Santoni : A. Podo and L. Santoni, "Fermions at finite density in the path integral approach," JHEP **02**, 182 (2024) [arXiv:2312.14753 [hep-th]].
  $\to$ This work rigorously treats the path integral formulation of dense fermions and studies the origin of SdH oscillations which is a part of **de Haas-van Alphen effect**.

- Issues with evaluation of Dense Sum-integrals :
  - T. Gorda, J. Österman and S. Säppi, "Augmenting the residue theorem with boundary terms in finite-density calculations," Phys. Rev. D **106**, no.10, 105026 (2022).
  - J. Österman, P. Schicho and A. Vuorinen, "Integrating by parts at finite density," JHEP **08**, 212 (2023) [arXiv:2304.05427 [hep-ph]].

Thermodynamics of dense quark matter in strong magnetic field

S. Satapathy, Sumit and S. A. Khan, "Thermodynamics of strongly magnetized dense quark matter from hard dense loop perturbation theory," Phys. Rev. D **111**, no.11, 116025 (2025)
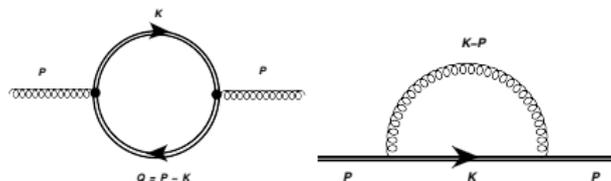
• Diagrams considered :

Figure: One-loop gluon (left) and quark (right) self-energies. Doubled fermion lines indicate modification in magnetic field

HDLpt in strong magnetic field

• Applied for $\mu/T \to 10^n$, $n \geq 2$
• External fermion momentum $\sim g\mu$ and loop momentum $\sim \mu$. HDLpt perturbation theory can be arranged then.
• Lowest landau level : $l_{\max} = \lfloor \mu^2/2|q_f B| \rfloor = 0$. (Very strong magnetic field)
• Hierarchy of scales : $g\mu < \mu < \sqrt{2|q_f B|}$
• Dynamics of the system is in $1+1d$ instead of $3+1d$ (in weak magnetic field and without magnetic field)

Self-energy and propagators in magnetic field

- Only "u" and "d" quarks have been considered.
- General structure of fermion self-energy in the massless limit :

$$\Sigma(p_0, p_3) = a(p_0, p_3)\slashed{p} + b(p_0, p_3)\slashed{p} + c(p_0, p_3)\gamma_5\slashed{p} + d(p_0, p_3)\gamma_5\slashed{p}, \quad \slashed{V} = \gamma^\mu V_\mu$$

- Anisotropy is induced at the level of the fermion propagator. Breaks translational symmetry $S(x, y) = e^{i\Phi(x,y)}S(x-y)$
- From Schwinger's proper time formalism calculation (after gauging away the Schwinger phase factor in a particular gauge, as physical quantities should be independent of any gauge !) :

$$iS(k) = ie^{-\frac{k_\perp^2}{q_f B}} \sum_{l=0}^{\infty} \frac{(-1)^l D_l(q_f B, k)}{k_\parallel^2 - m_f^2 + i\epsilon} \tag{1}$$

where $l \in [0, \infty)$ - Landau levels. $D_l(q_f B, k)$ is given by

$$D_l(q_f B, k) = (\slashed{k}_\parallel + m)\left[(1 - i\gamma^1\gamma^2)L_l\left(\frac{2k_\perp^2}{q_f B}\right) - (1 + i\gamma^1\gamma^2)L_{l-1}\left(\frac{2k_\perp^2}{q_f B}\right)\right] - 4\slashed{k}_\perp L_{l-1}^1\left(\frac{2k_\perp^2}{q_f B}\right) \tag{2}$$

- Strong Magnetic field limit : $iS_F(k) = ie^{-\frac{k_\perp^2}{q_f B}} \frac{\slashed{k}_\parallel + m}{k_\parallel^2 - m^2}(1 - i\gamma^1\gamma^2)$
- **Form Factors** : $a = \frac{1}{4}\text{Tr}[\Sigma\slashed{p}], \quad b = \frac{1}{4}\text{Tr}[\Sigma\slashed{p}], \quad c = \frac{1}{4}\text{Tr}[\gamma_5\Sigma\slashed{p}], \quad d = \frac{1}{4}\text{Tr}[\gamma_5\Sigma\slashed{p}]$
- **Free Energy of quarks** :

$$F_f = -N_c \sum_{\{P\}}\!\!\!\!\!\!\!\!\!\!\int \ln\left(\det\left[S_{\text{eff}}^{-1}(p_0, p_3)\right]\right), \quad S_{\text{eff}}^{-1}(p_0, p_3) = (p_0 + a)\gamma^0 + (b - p_3)\gamma^3 + c\gamma_5\gamma^0 + d\gamma_5\gamma^3. \tag{3}$$

Free-energy of quarks and dense sum-integrals

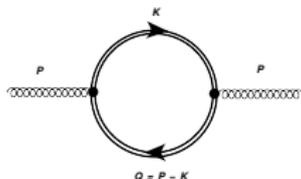$$F_f = F_{0,f} + F'_f, \qquad F_{0,f} = -N_c \frac{|q_f B|}{4\pi^2} \mu^2$$

$$F'_f(\mu, q_f B) = -N_c \frac{|q_f B|}{(2\pi)^2} \Big[-\frac{1}{8}\Big(\frac{|q_f B|}{\mu}\Big)^2 \Big(g^2 C_F \frac{|q_f B|}{4\pi^2}\Big)^2 \{\mathcal{I}_{630} + \mathcal{I}_{603} - \mathcal{I}_{612} - \mathcal{I}_{621}\}$$

$$-\frac{1}{2}\Big(g^2 C_F \frac{|q_f B|}{4\pi^2}\Big) \log\Big(\frac{e^{\gamma_E}\Lambda^2}{4\pi\mu^2}\Big)\{\mathcal{I}_{210} - \mathcal{I}_{201}\} - \frac{1}{8}\Big(g^2 C_F \frac{|q_f B|}{4\pi^2}\Big)^2 \Big\{\log\Big(\frac{e^{\gamma_E}\Lambda^2}{4\pi\mu^2}\Big)\Big\}^2$$

$$\times \{\mathcal{I}_{420} - 2\mathcal{I}_{411} + \mathcal{I}_{402}\}\Big], \tag{4}$$

$\mathcal{I}_{\alpha\beta\omega}, \ \alpha, \beta, \omega \in \mathbb{Z}^+ \cup \{0\}$ are the generalized dense sum-integrals. $T \to 0$ and $T = 0$ limits are not the same.

T. Gorda, J. Österman and S. Säppi, "Augmenting the residue theorem with boundary terms in finite-density calculations," Phys. Rev. D **106**, no.10, 105026 (2022)

General structure of the dense sum-integral $\mathcal{I}_{\alpha\beta\omega}(\mu)$

$$
\begin{aligned}
\mathcal{I}_{\alpha\beta\omega}(\mu) &= \lim_{T \to 0} \sum\!\!\!\!\!\!\int_{\{P\}} \frac{p_0^{2\beta} p^{2\omega}}{P^{2\alpha}} \\
&= \Big(\frac{e^{\gamma_E}\Lambda^2}{4\pi}\Big)^\epsilon \frac{i\mu}{2\pi} \frac{\Gamma(\alpha - \omega - d/2)\Gamma(d/2 + \omega)(i\mu)^{d+2\omega-2\alpha+2\beta}}{(4\pi)^{d/2}\Gamma(\alpha)\Gamma(d/2)(1 + d + 2\omega - 2\alpha + 2\beta)} \Big\{(-1)^{d+2\omega-2\alpha+2\beta} - 1\Big\}. \tag{5}
\end{aligned}
$$

HTLpt corrections to free energy of gluons

$$F_g' = -(N_c^2 - 1) \sum_P \left[\frac{b+c+d}{2P^2} + \frac{b^2+c^2+d^2}{4P^4} \right]. \qquad (6)$$

The $T \to 0$ limit : $F_g' = -\frac{(N_c^2-1)}{2} \sum_{f_1,f_2} q_{f_1} q_{f_2} \left(\frac{g^2 B}{4\pi^2} \right)^2 \frac{1}{(4\pi)^2} \left[\frac{1}{2\epsilon} + \frac{\ln 4}{2} + \gamma_E \right], \quad (7)$

Counterterm Free-energy : $F_g^{\text{ct}} = \frac{(N_c^2-1)}{4\epsilon} \sum_{f_1,f_2} q_{f_1} q_{f_2} \left(\frac{g^2 B}{4\pi^2} \right)^2 \frac{1}{(4\pi)^2}$

Renormalized Free Energy : $F_g = F_g' + F_g^{\text{ct}}$

• **NOTE :** F_g is independent of μ. One-loop gluon self-energy contains fermion & anti-fermion lines. The cancellation is exact at the level of μ.

Longitudinal Pressure

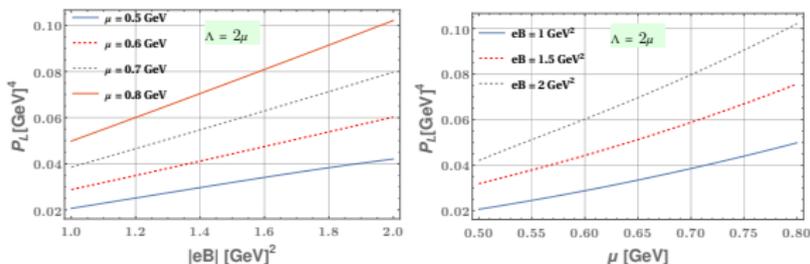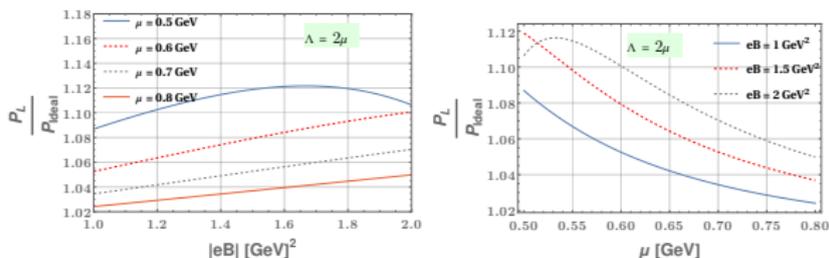$$P_L = -F_f, \quad P_\perp = -F_f - eB \cdot \mathcal{M}$$

Figure: (a) The left panel shows the variation with magnetic field for different chemical potentials, and (b) the right panel shows the variation with chemical potential for different magnetic fields.

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Longitudinal Speed of Sound

• The Transverse component ($c_{s,\perp}$) : suppressed in LLL. The Longitudinal component ($c_{s,\parallel}$) dominates.

Focussing only on the longitudinal component of the speed of sound here : $c_{s,\parallel}^2 = \frac{\partial P_\parallel}{\partial \varepsilon}$.
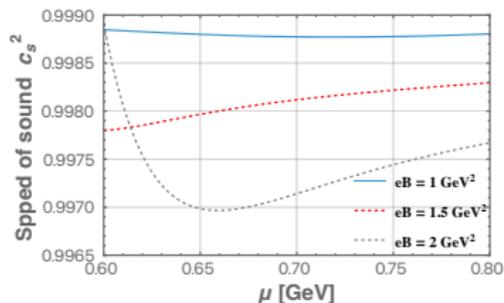

Figure: $c_{s,\parallel}^2$ vs μ

• *Y. B. Zel'dovich, "The equation of state at ultrahigh densities and its relativistic limitations," Zh. Eksp. Teor. Fiz.* **41**, *1609-1615 (1961).*
• Speed of sound approaches the speed of light in the extremely high dense limit. (*Kapusta & Gale, Finite-Temperature Field Theory Principles and Applications*)
• This claim has been verified for the first time in the QCD sector (in dense QCD subjected to strong magnetic field).
• Attribution : *E. J. Ferrer and A. Hackebill, "Speed of sound for hadronic and quark phases in a magnetic field," Nucl. Phys. A* **1031**, *122608 (2023).*

If $F \sim \mathcal{O}(\mu^{d+1})$, d is the number of spatial dimensions, then $c_s \sim 1/d$. Here $F_{0,f} = -N_c \frac{|q_f B|}{4\pi^2}\mu^2$

➤ Thermodynamic quantities increase with μ and B.

➤ Speed of sound approaches speed of light at high μ in strong magnetic field.

➤ Extension to higher loop diagrams will improve the results.

➤ Study of Isospin QCD is reserved for future work.

➤ Study in nonuniform magnetic field.

THANK YOU EVERYONE !