

Extracting Baryon initial condition and Diffusion using Bayesian Approach

(Hot QCD Matter - 2025)

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Outline

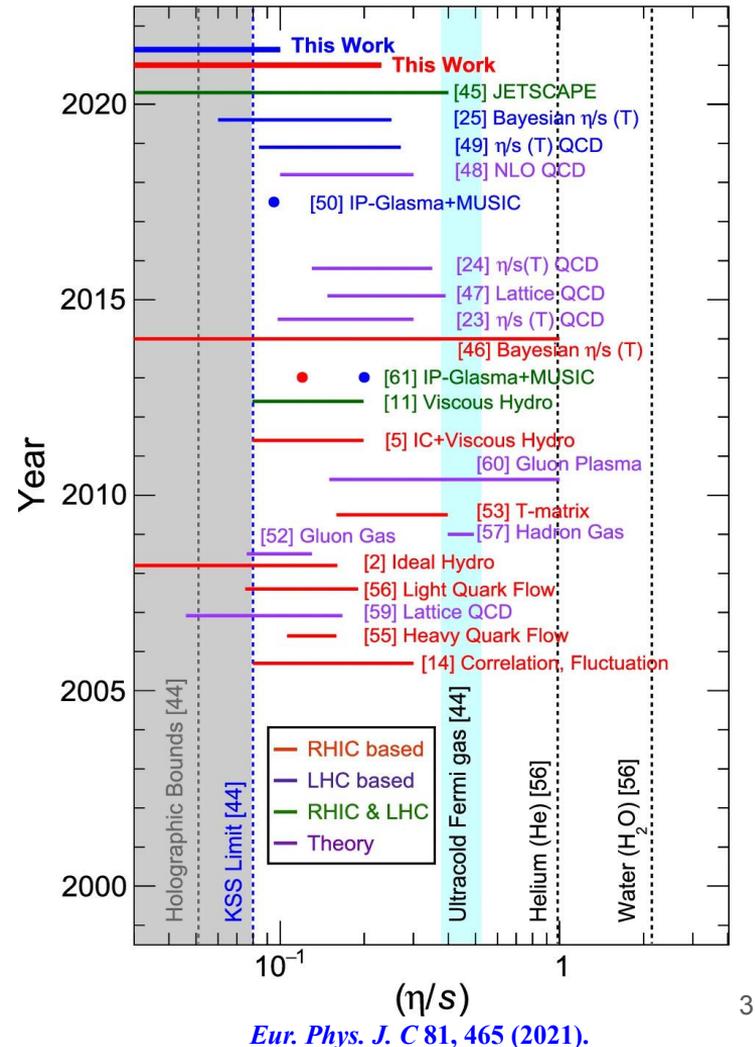
- Motivation
- Bayesian Framework
- Analysis steps
- Results
- Summary

Motivation

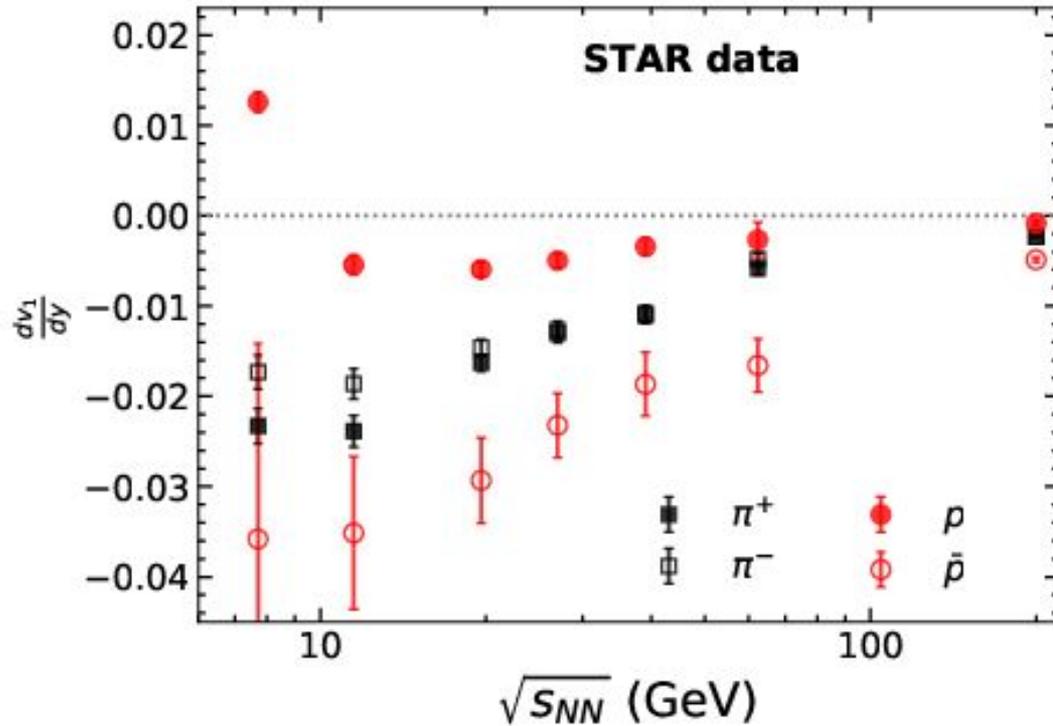
- Year by year progressing work in the direction of other transportation coefficients like η/s , ζ/s , etc.
- Not much work is done towards transportation coefficients related to the conserved charges B, Q, and S.

$$\begin{pmatrix} q_B^\mu \\ q_S^\mu \\ q_Q^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BS} & \kappa_{BQ} \\ \kappa_{SB} & \kappa_{SS} & \kappa_{SQ} \\ \kappa_{QB} & \kappa_{QS} & \kappa_{QQ} \end{pmatrix} \begin{pmatrix} \nabla^\mu (\mu_B/T) \\ \nabla^\mu (\mu_S/T) \\ \nabla^\mu (\mu_Q/T) \end{pmatrix}$$

Phys. Rev. D 104, 034014 (2021)
 Phys. Rev. D 106, 014013 (2022)



Motivation



- The rapidity odd directed flow splitting of proton and anti-proton observed in relativistic heavy ion collisions STAR energies.

Hydro Framework

To describe the splitting in directed flow of baryon and its anti-particle, initial conditions proposed for

- Energy deposition:

$$\epsilon(x, y, \eta_s) = \epsilon_0 \left[(N_+(x, y) f_+(\eta_s) + N_-(x, y) f_-(\eta_s)) \times (1 - \alpha) + N_{coll}(x, y) \epsilon_{\eta_s}(\eta_s) \alpha \right]$$

- Baryon deposition:

$$n_B(x, y, \eta_s) = N_B \left[W_+^B(x, y) f_+^B(\eta_s) + W_-^B(x, y) f_-^B(\eta_s) \right]$$

- Weight profile:

$$W_{\pm}^B(x, y) = (1 - \omega) N_{\pm}(x, y) + \omega N_{coll}(x, y)$$

Hydro Framework

- Rapidity profile:

$$f_+^{n_B}(\eta_s) = \left[\theta(\eta_s - \eta_0^{n_B}) \exp - \frac{(\eta_s - \eta_0^{n_B})^2}{2\sigma_{B,+}^2} + \theta(\eta_0^{n_B} - \eta_s) \exp - \frac{(\eta_s - \eta_0^{n_B})^2}{2\sigma_{B,-}^2} \right]$$

and

$$f_-^{n_B}(\eta_s) = \left[\theta(\eta_s + \eta_0^{n_B}) \exp - \frac{(\eta_s + \eta_0^{n_B})^2}{2\sigma_{B,-}^2} + \theta(-\eta_s - \eta_0^{n_B}) \exp - \frac{(\eta_s + \eta_0^{n_B})^2}{2\sigma_{B,+}^2} \right]$$

- Baryon diffusion coefficient:

$$\kappa_B = C_B \frac{n_B}{T} \left(\frac{1}{3} \coth \left(\frac{\mu_B}{T} \right) - \frac{n_B T}{\epsilon + P} \right)$$

arXiv:2305.10371v1

- **Parameters:**

1. Matter eta plateau η_0
2. Matter eta fall σ_η
3. Matter tilt η_m
4. Two component baryon deposition ω
5. Baryon rapidity profile eta peak $\eta_0^{n_B}$
6. Baryon rapidity profile sigma eta plus $\sigma_{B,+}$
7. Baryon rapidity profile sigma eta minus $\sigma_{B,-}$
8. Normalization factor ϵ_0
9. Baryon diffusion $\kappa_B \sim C_B$

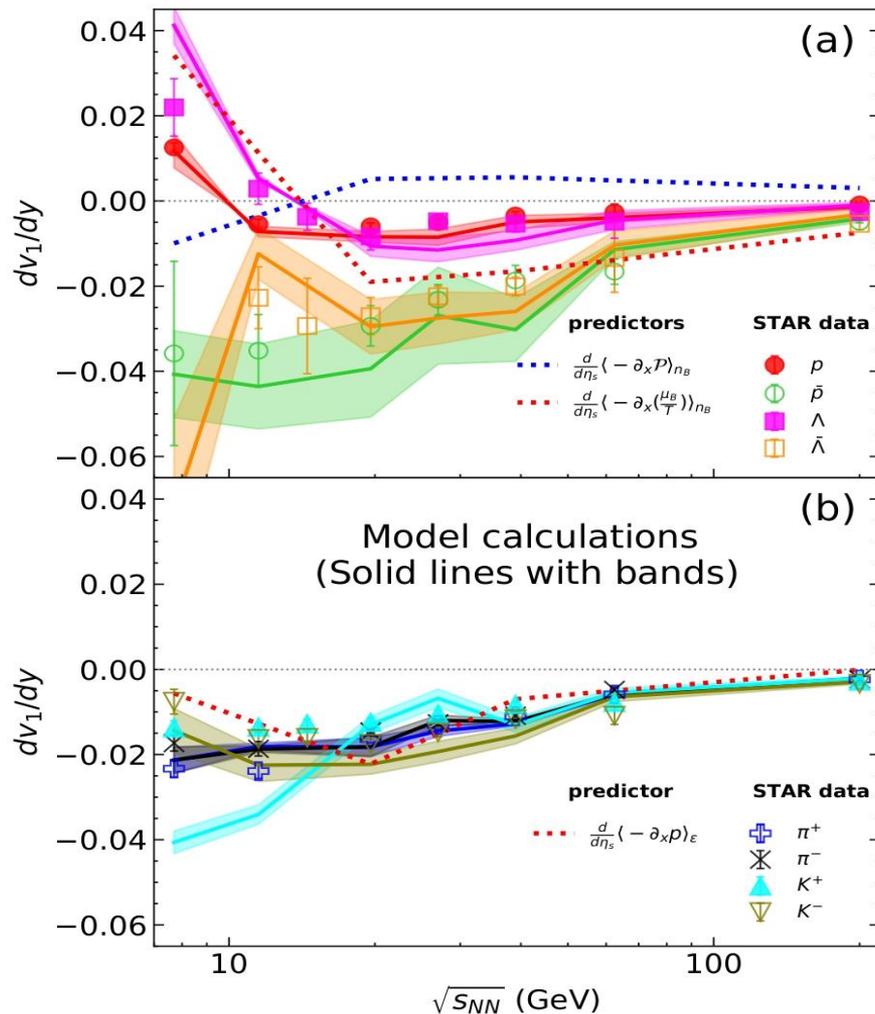
We aim to constrain C_B

Experimental (and Hydro simulated) data used:

- $dN_{ch}/d\eta$ vs η @19.6 GeV (0-6%) (PHOBOS)
[Rev. Lett. 91, 052303](#)
- dN/dy of proton, and anti-proton net-proton @17.3 GeV (0-5%) (NA49) [Phys. Rev. C 83, 014901](#)
- v_1 vs y @19.6 GeV (STAR) (10-40%)
[Phys. Rev. Lett. 112, 162301](#)

Hydro Framework

- These initial conditions implementation helps describing the splitting in rapidity odd directed flow.
- Hydro data generated @19.6 GeV in 0-5% and 10-40% centralities in Au+Au system for this analysis.



Motivation for Bayesian approach

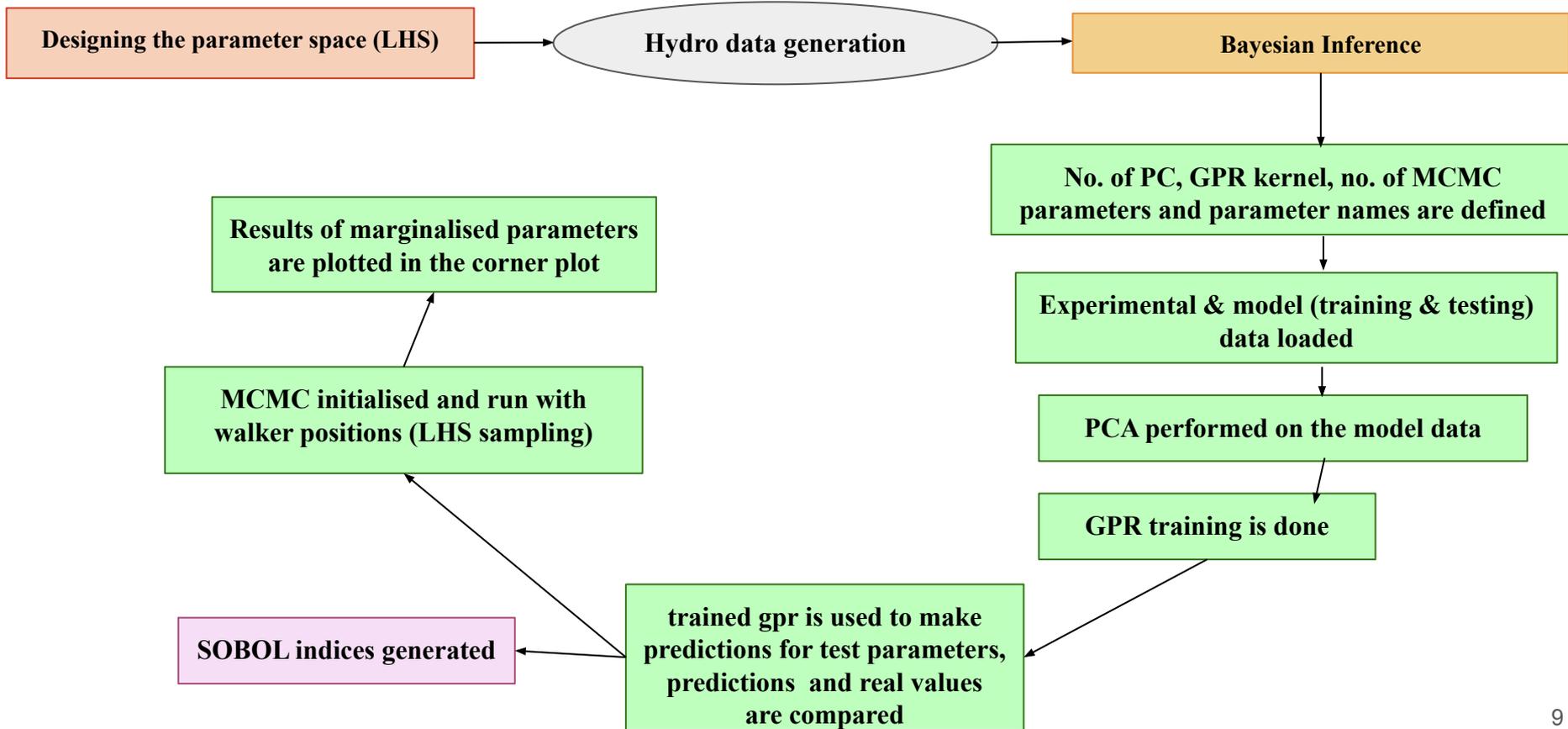
Extracting this coefficient through simulations is:

- **Computationally intensive**
- **Numerically challenging**
- **and demands rigorous uncertainty quantification** to be credible.

Bayesian Framework:

- **Statistically robust**
- **Sophisticated**
- Lets us confront the hydro output with data **efficiently**

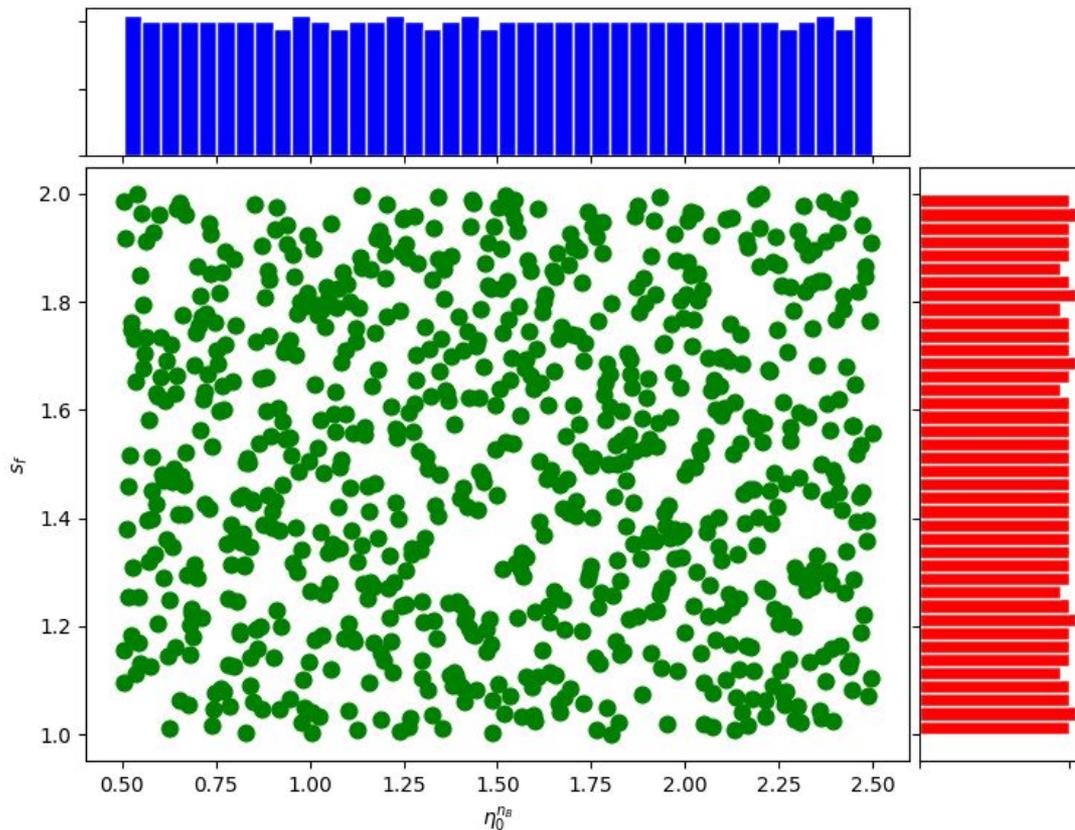
Bayesian Framework



Analysis

Latin Hypercube Sampling for parameters

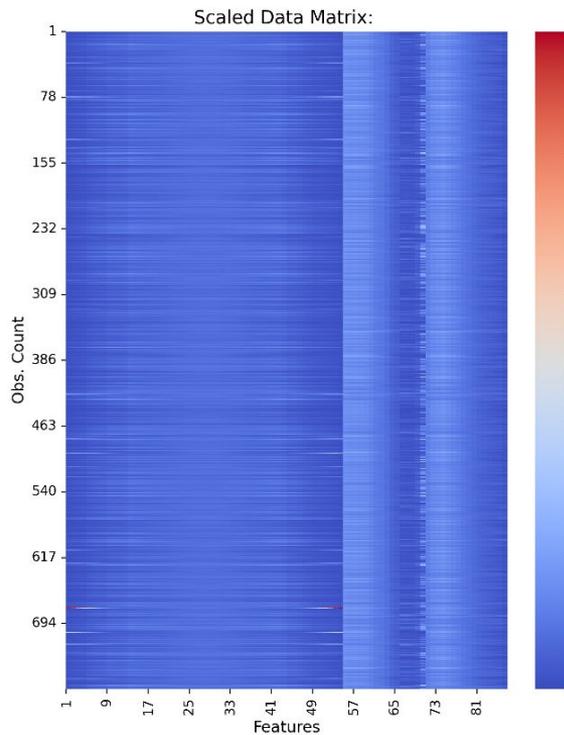
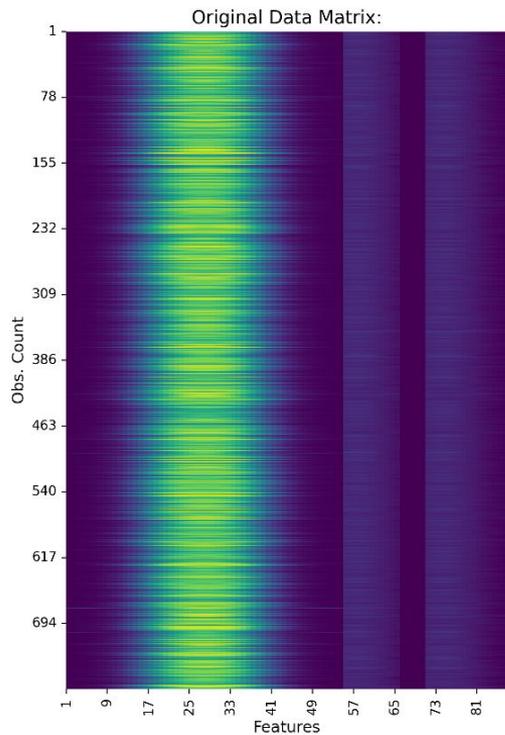
- Parameters sampled with well coverage within the desired ranges.



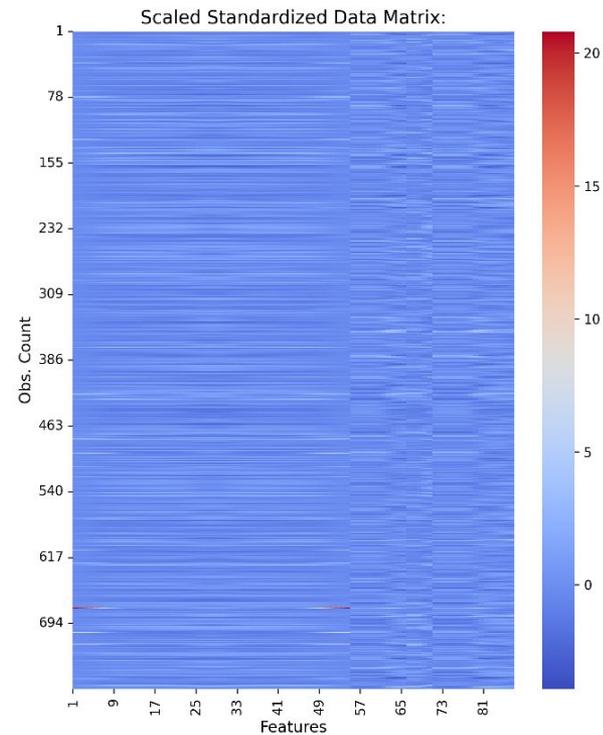
Analysis

Data Matrix

$$x_{i,j}^{std} = \frac{x_{i,j} - \mu_j}{\sigma_j}$$



**Scaled by experimental data to
bring each feature to same footing**



**To keep mean = 0 and
std dev = 1 of each feature**

Analysis:

Principal Component Analysis

Standardised data matrix is decomposed as

$$X_{n \times m} = U_{n \times n} \Sigma_{n \times m} V^T_{m \times m}$$

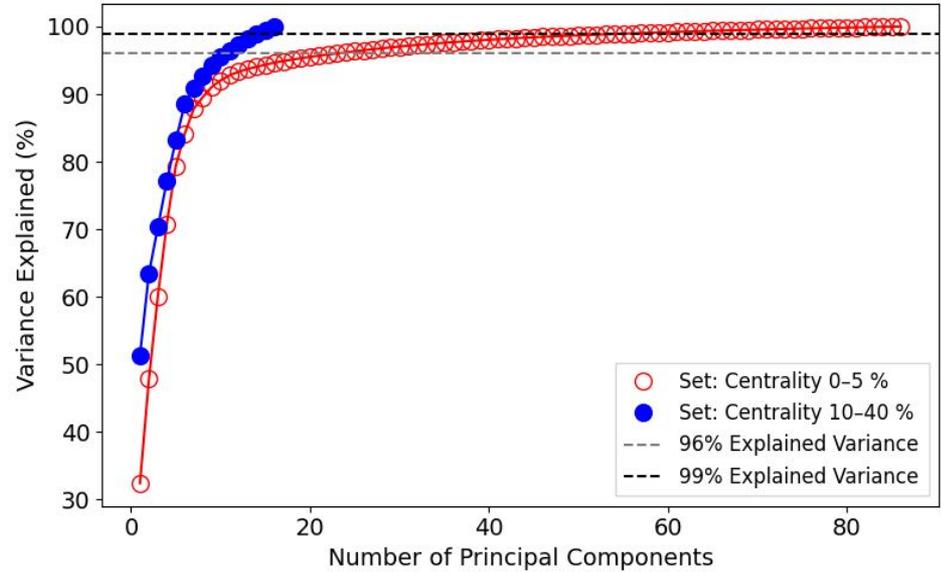
Where:

1. U : left singular matrix. It's columns are eigenvectors of square matrix $X_{n \times m} X_{n \times m}^T$
2. Σ : a diagonal matrix, diagonal elements of $\Sigma^T \Sigma$ are the eigenvalues of $X_{n \times m}^T X_{n \times m}$
3. V : right singular matrix. It's columns are eigenvectors of square matrix $X_{n \times m}^T X_{n \times m}$

$$\text{s.t. } X^T X = (U \Sigma V^T)^T (U \Sigma V^T) = V \Sigma^2 V^T$$

V and Σ^2 give the eigenvectors and eigenvalues of covariance matrix $X_{n \times m}^T X_{n \times m}$

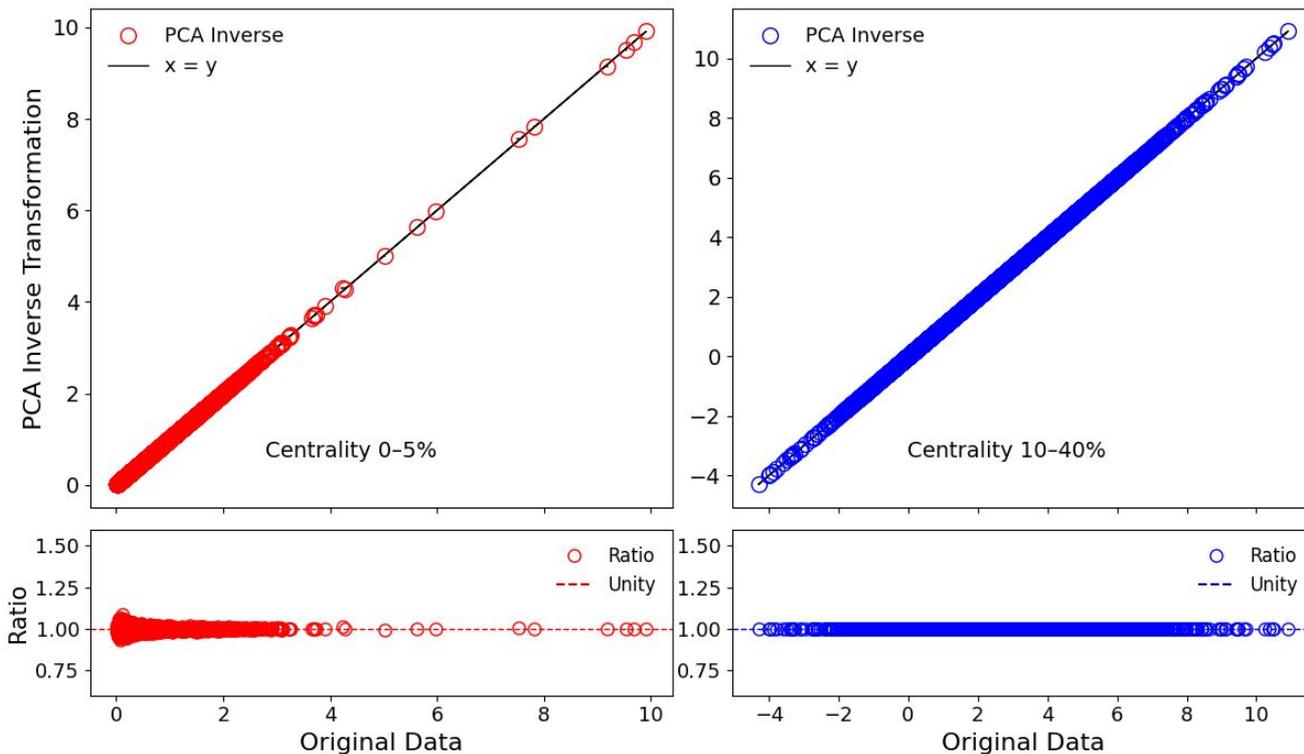
PCs taken are: 14 for 10-40% and 42 for 0-5%



$$\text{Variance Ratio} = \frac{\sum_{i=1}^k \lambda_i}{\sum_{i=1}^n \lambda_i}$$

Analysis:

Principal Component Analysis



PCA validation for both data sets

Analysis

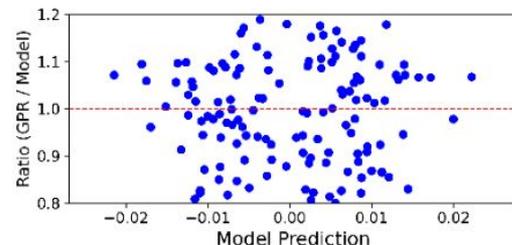
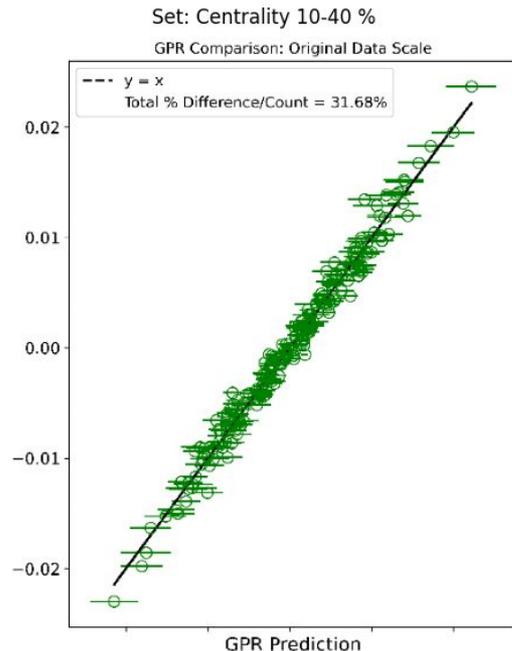
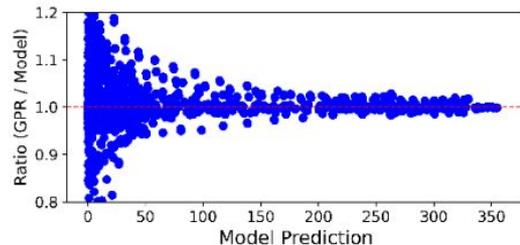
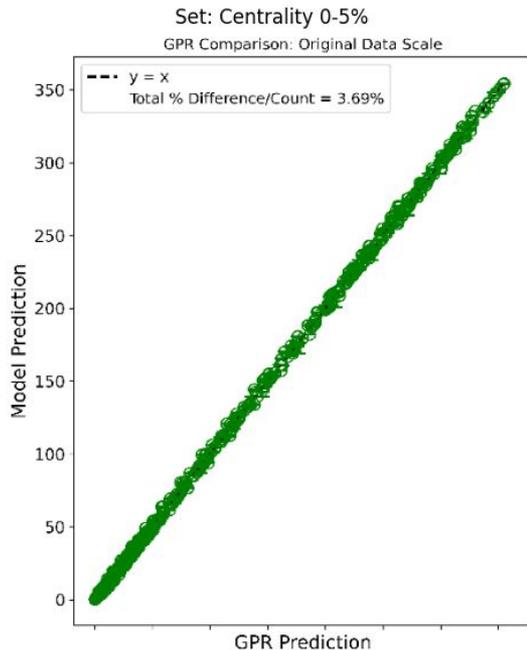
Gaussian Process Regression

- Models a distribution over functions and uses a kernel function (here matern kernel) to compute the covariance between the data points.

- Matern kernel:

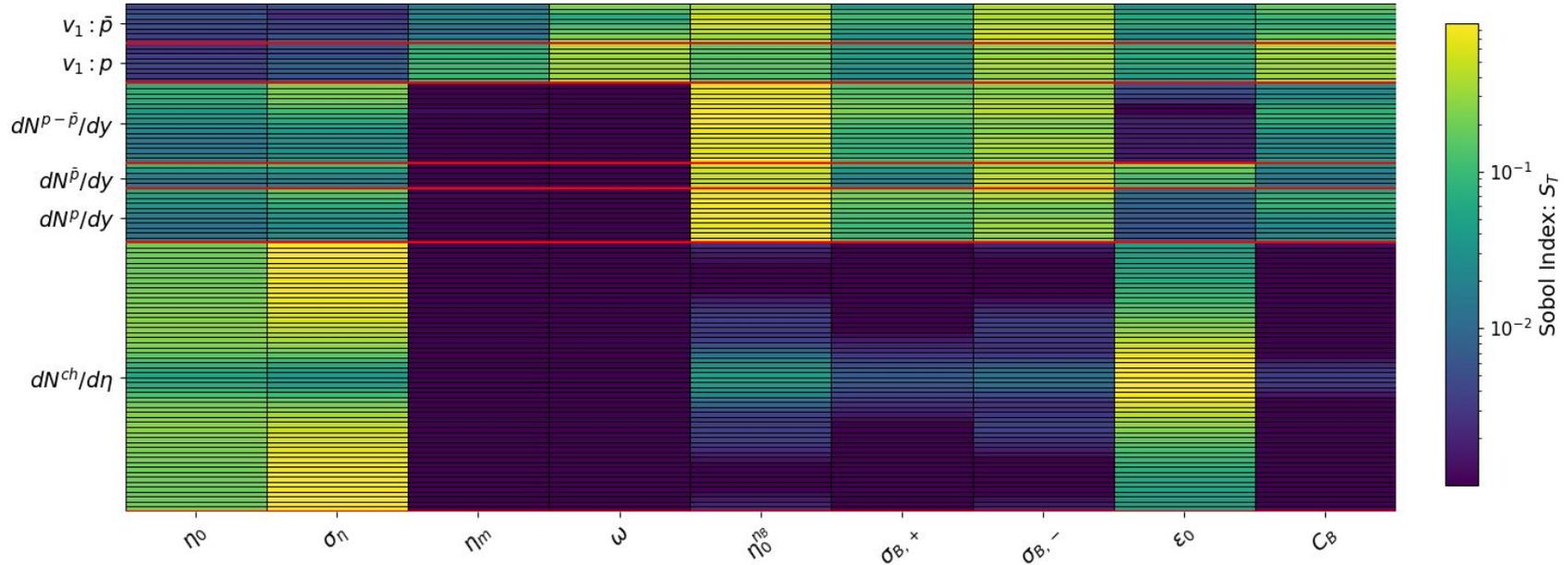
$$k(x, x') = \sigma^2 \frac{2^{1-\nu}}{\Gamma(\nu)} \left(\frac{\sqrt{2\nu} \|x - x'\|}{\ell} \right)^\nu \times K_\nu \left(\frac{\sqrt{2\nu} \|x - x'\|}{\ell} \right)$$

- ν : smoothness parameter
- σ^2 : signal variance
- ℓ : length scale parameter
- K_ν : modified Bessel function



Results

Sensitivity Analysis



Visualisation of the sensitivity of variables on the parameters

Results

Markov Chain Monte Carlo

The MCMC algorithm **samples from the posterior distribution**, which follows Bayes' theorem:

$$P(\theta | \text{data}) \propto P(\text{data} | \theta)P(\theta)$$

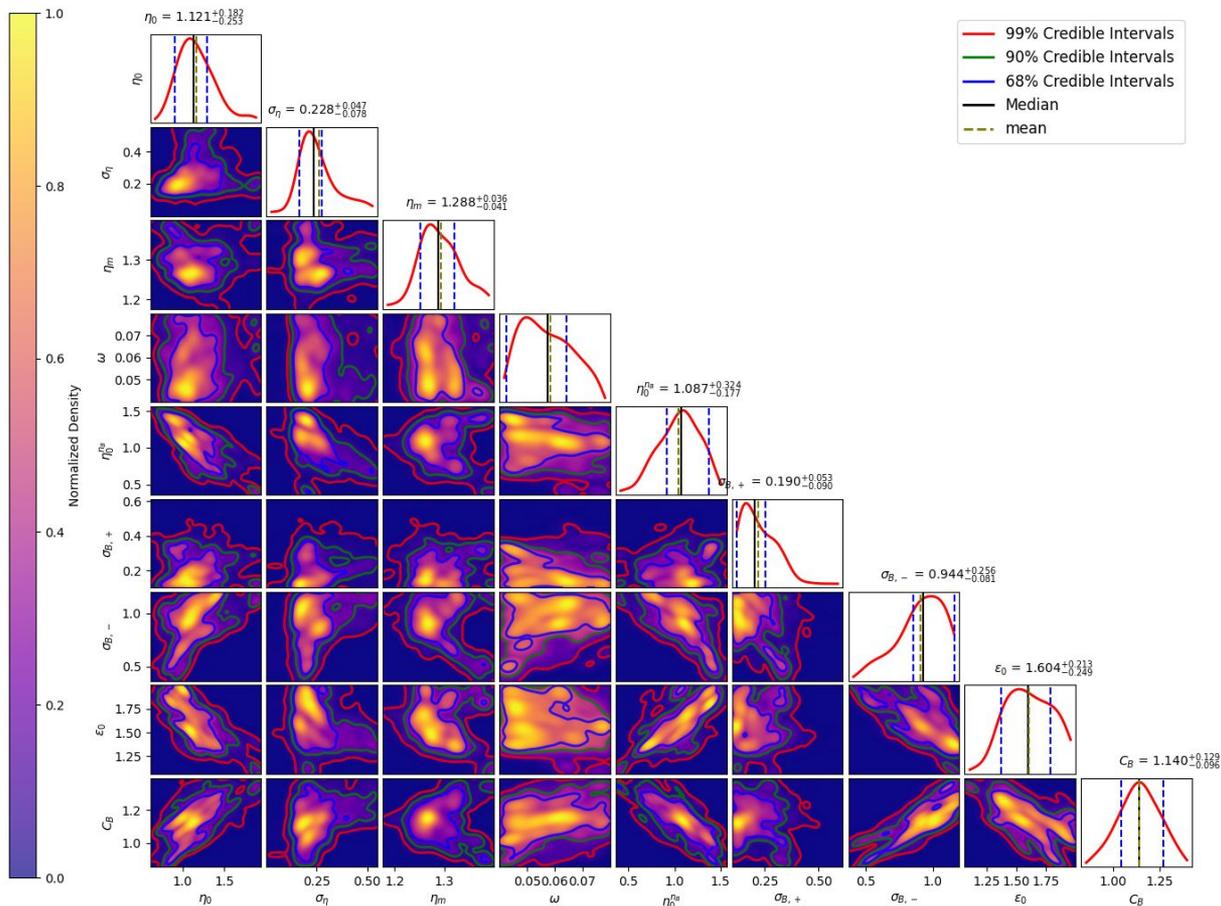
where:

- $P(\theta | \text{data}) =$ **posterior probability**.
- $P(\text{data} | \theta) =$ **likelihood**.
- $P(\theta) =$ **prior probability**.
- Full posterior distribution of model parameters computed using MCMC.
- Log-likelihood defined as

$$P(D|\theta) \propto \exp\left[-\frac{1}{2}(y_m(\theta) - y_e)^T \Sigma^{-1}(y_m(\theta) - y_e)\right]$$

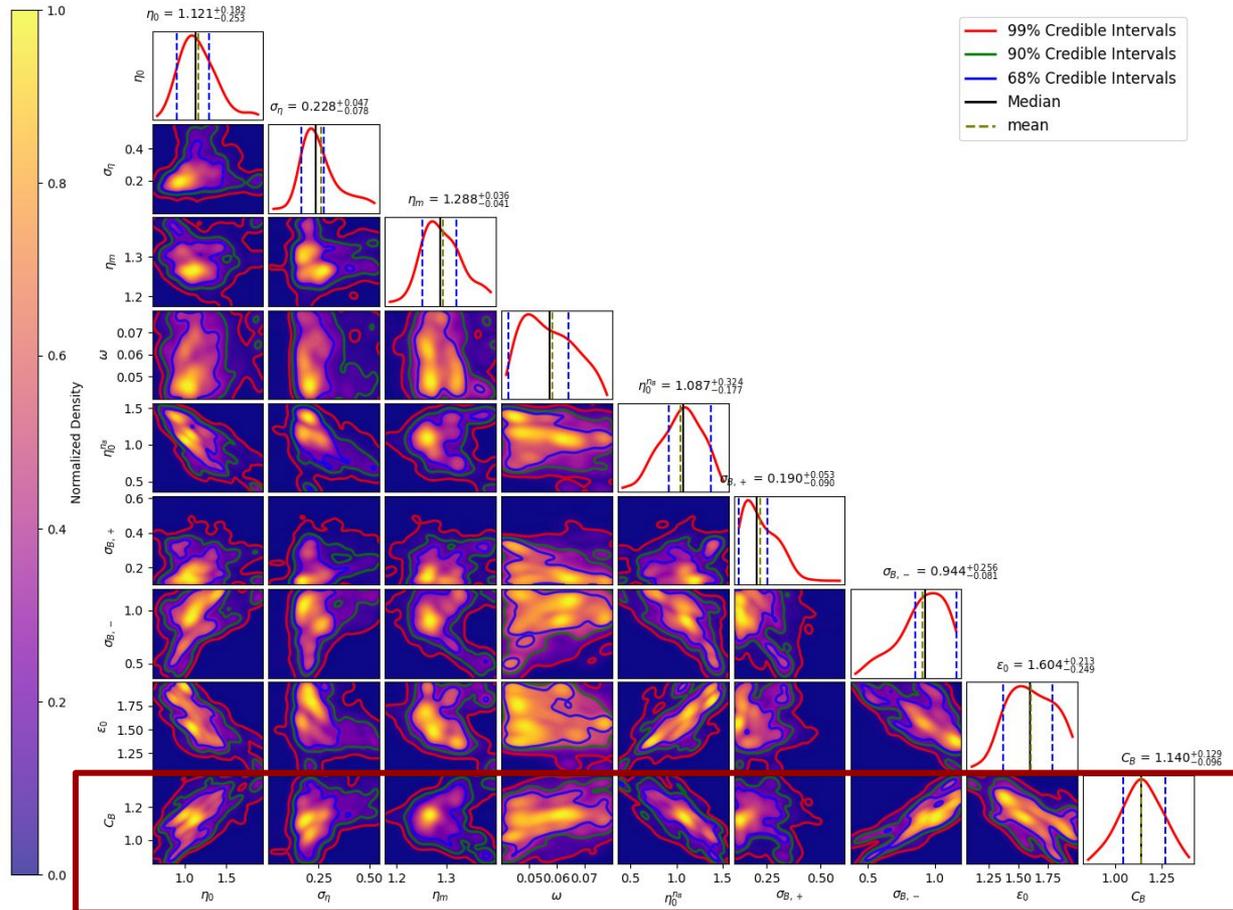
Results

Markov Chain Monte Carlo



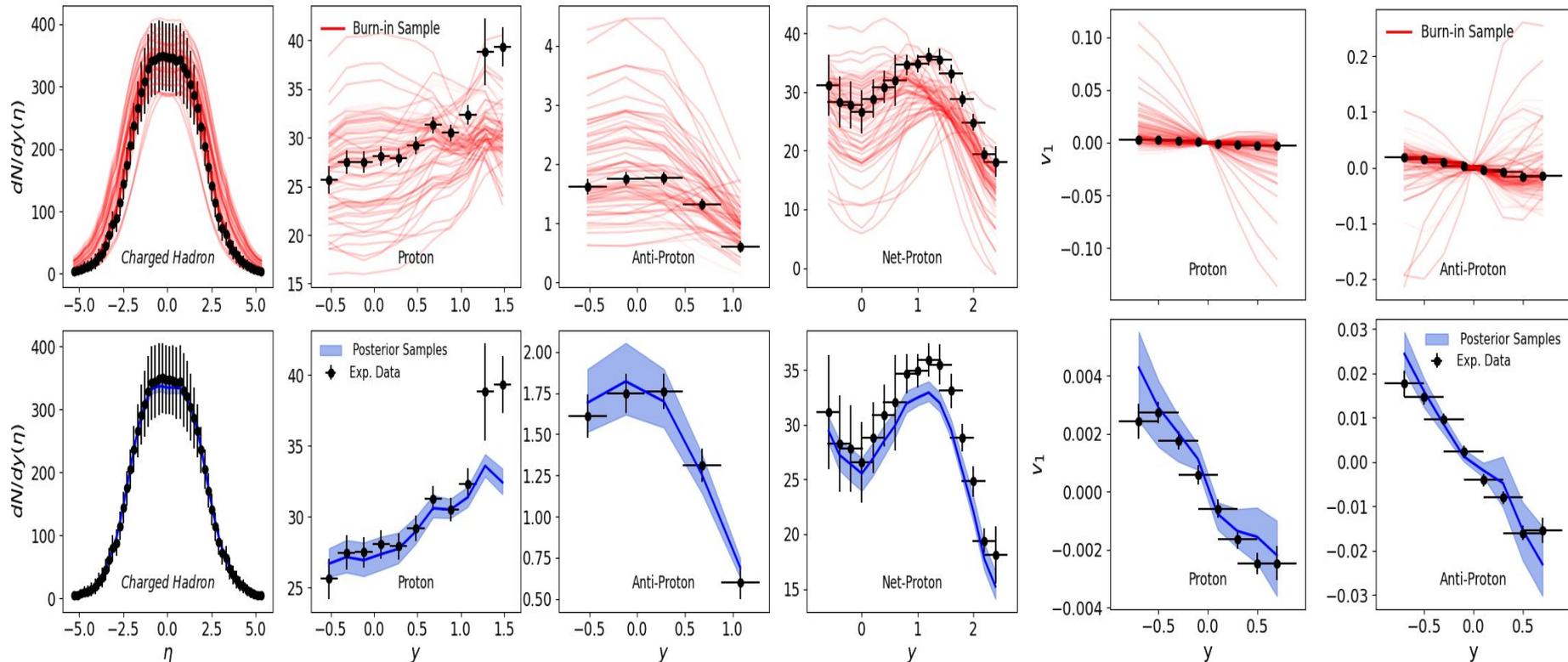
Results

Markov Chain Monte Carlo

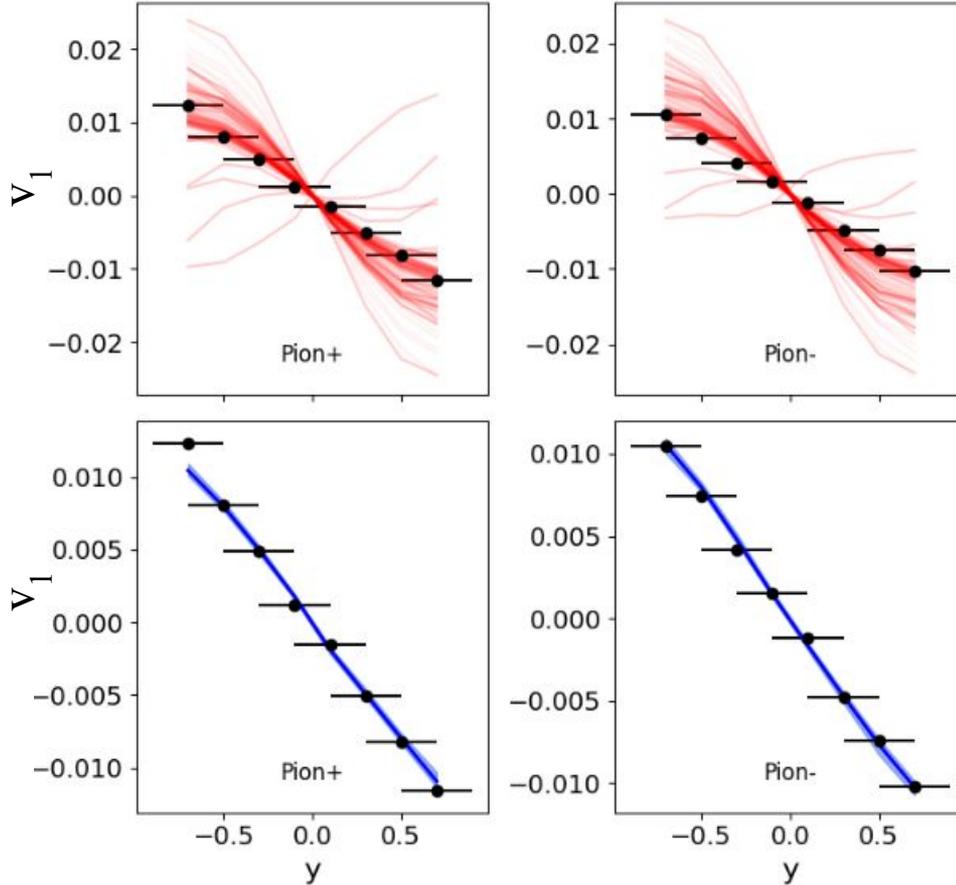


Results

Markov Chain Monte Carlo



Closure test



Summary

- Presented work of constraining Baryon diffusion coefficient via Bayesian inference.
- Analysis provides a good constrain on the Baryon diffusion coefficient as well as other parameters related to hydrodynamic evolution of the heavy ion collisions.

Thank you for your attention ... 😊

Back-up

Parameter	Mean	Median	68% CI	90% CI	95% CI	99% CI
η_0	1.16	1.121	[0.868, 1.303]	[0.766, 1.460]	[0.762, 1.660]	[0.766, 1.914]
σ_{η}	0.258	0.228	[0.150, 0.276]	[0.150, 0.429]	[0.150, 0.497]	[0.150, 0.550]
η_m	1.294	1.288	[1.247, 1.324]	[1.233, 1.370]	[1.226, 1.382]	[1.217, 1.399]
ω	0.058	0.057	[0.041, 0.065]	[0.041, 0.076]	[0.041, 0.078]	[0.041, 0.080]
$\eta^{n_B}_0$	1.055	1.087	[0.909, 1.410]	[0.684, 1.433]	[0.627, 1.449]	[0.398, 1.483]
$\sigma_{B,+}$	0.207	0.19	[0.100, 0.242]	[0.100, 0.325]	[0.100, 0.346]	[0.100, 0.397]
$\sigma_{B,-}$	0.922	0.944	[0.863, 1.200]	[0.629, 1.200]	[0.524, 1.200]	[0.429, 1.200]
ϵ_0	1.617	1.604	[1.356, 1.817]	[1.351, 1.994]	[1.310, 1.994]	[1.124, 2.000]
C_B	1.139	1.14	[1.044, 1.269]	[0.974, 1.346]	[0.924, 1.357]	[0.881, 1.369]

