

# Hot QCD Matter 2025

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## PHENOMENOLOGICAL INVESTIGATION OF CONVENTIONAL AND NON-CONVENTIONAL HADRONS

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# Outline

## Introduction

## Baryons

### Models

Singly, doubly and triply heavy baryons

## Meson

Spectroscopic properties of  $B$ ,  $B_s$  and  $B_c$ , bottomonium mesons

Spectroscopy of all bottom and heavy-light tetraquarks

Conclusion and Future Work

## 1 Introduction

## 2 Baryons

- Models
- Singly, doubly and triply heavy baryons

## 3 Meson

- Spectroscopic properties of  $B$ ,  $B_s$  and  $B_c$ , bottomonium mesons

## 4 Spectroscopy of all bottom and heavy-light tetraquarks

## 5 Conclusion and Future Work

# Review of The Standard Model

## Introduction

### Baryons

#### Models

Singly, doubly and triply heavy baryons

### Meson

Spectroscopic properties of  $B$ ,  $B_s$  and  $B_c$ , bottomonium mesons

Spectroscopy of all bottom and heavy-light tetraquarks

Conclusion and Future Work

Particle Physics is the study of:

- **MATTER:** the fundamental constituents of the universe- the elementary particles
- **FORCE:** the fundamental forces of nature, i.e. the interactions between the elementary particles

# Review of The Standard Model

## Introduction

### Baryons

#### Models

Singly, doubly and triply heavy baryons

### Meson

Spectroscopic properties of  $B$ ,  $B_c$  and  $B_c'$ , bottomonium mesons

Spectroscopy of all bottom and heavy-light tetraquarks

Conclusion and Future Work

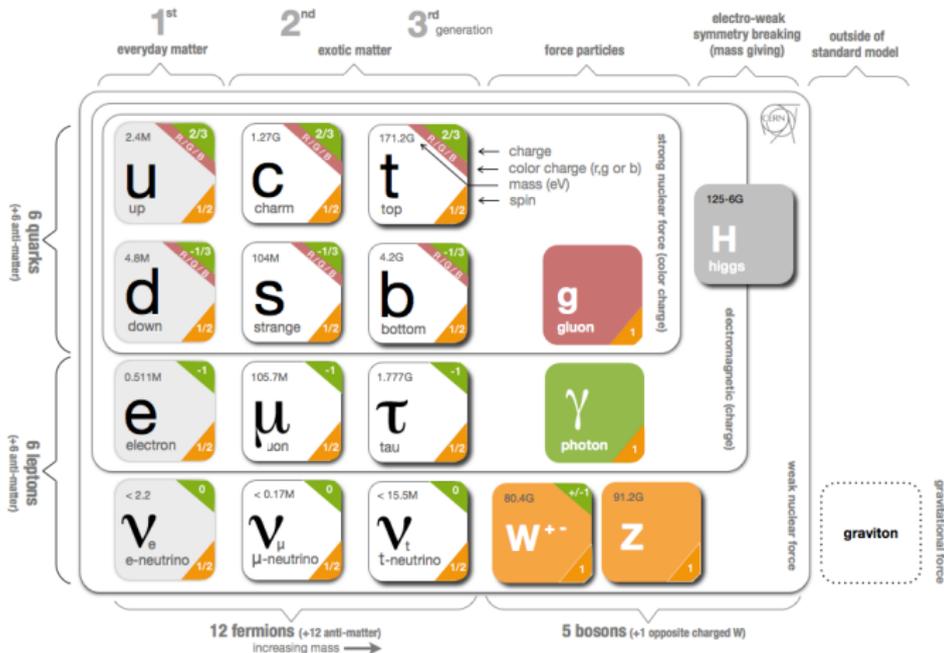


Figure: The Standard model [<http://united-states.cern>]

# Review of The Standard Model

## Introduction

### Baryons

#### Models

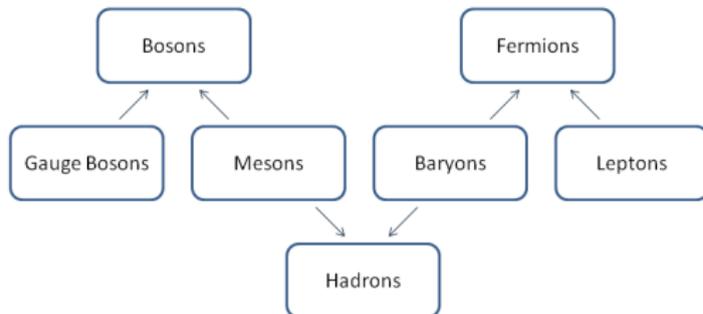
Singly, doubly and triply heavy baryons

### Meson

Spectroscopic properties of  $B_c$ ,  $B_s$  and  $B_c$ , bottomonium mesons

Spectroscopy of all bottom and heavy-light tetraquarks

Conclusion and Future Work



## Baryons

- Light sector
- Light-Heavy Sector
- Heavy Sector

## Mesons

- Light sector
- Heavy-Light Sector
- Heavy Sector

## Exotic States

- Glueballs
- Tetraquarks
- Pentaquarks
- Hexaquarks

# Heavy baryons

## Introduction

## Baryons

### Models

Singly, doubly and triply heavy baryons

## Meson

Spectroscopic properties of  $B$ ,  $B_s$  and  $B_c$ , bottomonium mesons

Spectroscopy of all bottom and heavy-light tetraquarks

Conclusion and Future Work

Resonance	Mass (MeV)	$J^P$	Status	Resonance	Mass (MeV)	$J^P$	Status
$\Lambda_c^+$	$2286.46 \pm 0.14$	$\frac{1}{2}^+$	****	$\Xi_c(2790)^0$	$2793.9 \pm 0.5$	$\frac{1}{2}^-$	***
$\Lambda_c(2595)^+$	$2592.25 \pm 0.28$	$\frac{1}{2}^-$	***	$\Xi_c(2815)^+$	$2816.51 \pm 0.25$	$\frac{1}{2}^-$	***
$\Lambda_c(2625)^+$	$2628.11 \pm 0.19$	$\frac{1}{2}^-$	***	$\Xi_c(2815)^0$	$2819.79 \pm 0.30$	$\frac{1}{2}^-$	***
$\Lambda_c(2765)^+$	$2766.6 \pm 2.4$	$??$	*	$\Xi_c(2923)^0$	$2923.04 \pm 0.35$	$??$	**
$\Lambda_c(2860)^+$	$2856.1^{+2.3}_{-6.0}$	$\frac{1}{2}^+$	***	$\Xi_c(2930)^+$	$2942 \pm 5$	$??$	**
$\Lambda_c(2880)^+$	$2881.63 \pm 0.24$	$\frac{1}{2}^+$	***	$\Xi_c(2930)^0$	$2938.55 \pm 0.30$	$??$	**
$\Lambda_c(2940)^+$	$2939.6^{+1.3}_{-1.5}$	$\frac{1}{2}^-$	***	$\Xi_c(2970)^+$	$2964.3 \pm 1.5$	$\frac{1}{2}^+$	***
$\Sigma_c(2455)^{++}$	$2453.97 \pm 0.14$	$\frac{1}{2}^+$	****	$\Xi_c(2970)^0$	$2967.1 \pm 1.7$	$\frac{1}{2}^+$	***
$\Sigma_c(2455)^+$	$2452.65^{+0.22}_{-0.16}$	$\frac{1}{2}^+$	****	$\Xi_c(3055)^+$	$3055.9 \pm 0.4$	$??$	***
$\Sigma_c(2455)^0$	$2453.75 \pm 0.14$	$\frac{1}{2}^+$	****	$\Xi_c(3080)^+$	$3077.2 \pm 0.4$	$??$	***
$\Sigma_c(2520)^{++}$	$2518.41^{+0.22}_{-0.18}$	$\frac{1}{2}^+$	***	$\Xi_c(3080)^0$	$3079.9 \pm 1.4$	$??$	***
$\Sigma_c(2520)^+$	$2517.4^{+0.7}_{-0.5}$	$\frac{1}{2}^+$	***	$\Xi_c(3123)^+$	$3122.9 \pm 1.3$	$??$	*
$\Sigma_c(2520)^0$	$2518.48 \pm 0.20$	$\frac{1}{2}^+$	***	$\Omega_c^0$	$2695.2 \pm 1.7$	$\frac{1}{2}^+$	***
$\Sigma_c(2800)^{++}$	$2801^{+4}_{-6}$	$??$	***	$\Omega_c(2770)^0$	$2765.9 \pm 2.0$	$\frac{1}{2}^+$	***
$\Sigma_c(2800)^+$	$2792^{+14}_{-5}$	$??$	***	$\Omega_c(3000)^0$	$3000.41 \pm 0.22$	$??$	***
$\Sigma_c(2800)^0$	$2806^{+5}_{-7}$	$??$	***	$\Omega_c(3050)^0$	$3050.19 \pm 0.13$	$??$	***
$\Xi_c^+$	$2467.71 \pm 0.23$	$\frac{1}{2}^+$	***	$\Omega_c(3065)^0$	$3065.54 \pm 0.26$	$??$	***
$\Xi_c^0$	$2470.44 \pm 0.28$	$\frac{1}{2}^+$	****	$\Omega_c(3090)^0$	$3090.1 \pm 0.5$	$??$	***
$\Xi_c^+$	$2578.2 \pm 0.5$	$\frac{1}{2}^+$	***	$\Omega_c(3120)^0$	$3119.1 \pm 1.0$	$??$	***
$\Xi_c^0$	$2578.7 \pm 0.5$	$\frac{1}{2}^+$	***				
$\Xi_c(2645)^+$	$2645.10 \pm 0.30$	$\frac{1}{2}^+$	***				
$\Xi_c(2645)^0$	$2646.16 \pm 0.25$	$\frac{1}{2}^+$	***				

# Heavy baryons

Introduction

**Baryons**

Models

Singly, doubly and triply heavy baryons

Meson

Spectroscopic properties of  $B$ ,  $B_c$  and  $B_s$ , bottomonium mesons

Spectroscopy of all bottom and heavy-light tetraquarks

Conclusion and Future Work

Resonance	Mass (MeV)	$J^P$	Status
$\Lambda_b^0$	$5619.60 \pm 0.17$	$\frac{1}{2}^+$	***
$\Lambda_b(5920)^0$	$5920.09 \pm 0.17$	$\frac{3}{2}^-$	***
$\Lambda_b(6070)^0$	$6072.3 \pm 2.9$	$?$	?
$\Lambda_b(6146)^0$	$6146.2 \pm 0.4$	$\frac{3}{2}^+$	***
$\Lambda_b(6152)^0$	$6152.5 \pm 0.4$	$\frac{3}{2}^+$	***
$\Sigma_b^+$	$5810.56 \pm 0.25$	$\frac{1}{2}^+$	***
$\Sigma_b^-$	$5815.64 \pm 0.27$	$\frac{1}{2}^+$	***
$\Sigma_b^{*+}$	$5830.32 \pm 0.25$	$\frac{3}{2}^+$	***
$\Sigma_b^{*-}$	$5834.74 \pm 0.30$	$\frac{3}{2}^+$	***
$\Sigma_b(6097)^+$	$6095.8 \pm 1.7$	$?$	***
$\Sigma_b(6097)^-$	$6098.0 \pm 1.8$	$?$	***
$\Xi_b^0$	$5791.9 \pm 0.5$	$\frac{1}{2}^+$	***
$\Xi_b^-$	$5797.0 \pm 0.6$	$\frac{1}{2}^+$	***
$\Xi_b(5935)^-$	$5935.02 \pm 0.05$	$\frac{1}{2}^+$	***
$\Xi_b(5945)^0$	$5952.3 \pm 0.6$	$\frac{3}{2}^+$	***
$\Xi_b(5955)^-$	$5955.33 \pm 0.13$	$\frac{3}{2}^+$	***
$\Xi_b(6227)^-$	$6227.9 \pm 0.9$	$?$	***
$\Omega_b^-$	$6046.1 \pm 1.7$	$\frac{1}{2}^+$	***
$\Omega_b(6316)^-$	$6315.6 \pm 0.6$	$?$	*
$\Omega_b(6330)^-$	$6330.3 \pm 0.6$	$?$	*
$\Omega_b(6340)^-$	$6339.7 \pm 0.6$	$?$	*
$\Omega_b(6350)^-$	$6349.8 \pm 0.6$	$?$	*

# Different Approaches

Introduction

Baryons

Models

Singly, doubly and triply heavy baryons

Meson

Spectroscopic properties of  $B_c$ ,  $B_s$  and  $B_c$ , bottomonium mesons

Spectroscopy of all bottom and heavy-light tetraquarks

Conclusion and Future Work

The excited states of the nucleon have been studied experimentally since the 1950's. They contributed to the discovery of the quark model in 1964 by Gell-Mann, and Zweig, and were critical for the discovery of "color" degrees of freedom as introduced by Greenberg.

## Excited heavy baryon masses: Theoretical study

Approach	Authors
relativistic quark model	Ebert et al.
variational approach	Roberts et al.
Fadeev approach	Valcarce et al.
Hamiltonian model	Yoshida et al.
Lattice QCD	Padmanath et al., Brown et al., Paula et al.
HQET	Chen et al.
Sum rules	Aliev et al.
Regge Phenomology	Wei et al.

# The Hypercentral Constituent Quark Model

The relevant degrees of freedom for the relative motion of the three constituent quarks are provided by the relative Jacobi coordinates ( $\vec{\rho}$  and  $\vec{\lambda}$ ) which are given by

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2) \quad \vec{\lambda} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 - (m_1 + m_2) \vec{r}_3}{\sqrt{m_1^2 + m_2^2 + (m_1 + m_2)^2}} \quad (1)$$

The confining three-body potential is chosen within a string-like picture, where the quarks are connected by gluonic strings and the potential strings increases linearly with a collective radius  $r_{3q}$ . We define hyper radius  $x$  and hyper angle  $\xi$  in terms of the absolute values  $\rho$  and  $\lambda$  of the Jacobi coordinates,

$$x = \sqrt{\rho^2 + \lambda^2} \quad \text{and} \quad \xi = \arctan\left(\frac{\rho}{\lambda}\right) \quad (2)$$

The hyper radius  $x$  is a collective coordinate and therefore the hypercentral potential contains also the three-body effects. The Hamiltonian of three body baryonic system in the hCQM is then expressed as

$$H = \frac{P_x^2}{2m} + V(x) \quad (3)$$

where,  $m = \frac{2m_\rho m_\lambda}{m_\rho + m_\lambda}$ , is the reduced mass.

Introduction

Baryons

Models

Singly, doubly and triply heavy baryons

Meson

Spectroscopic properties of  $B$ ,  $B_c$  and  $B_s$ , bottomonium mesons

Spectroscopy of all bottom and heavy-light tetraquarks

Conclusion and Future Work

# The Hypercentral Constituent Quark Model

Introduction

Baryons

Models

Singly, doubly and triply heavy baryons

Meson

Spectroscopic properties of  $B_c$ ,  $B_s$  and  $B_c$ , bottomonium mesons

Spectroscopy of all bottom and heavy-light tetraquarks

Conclusion and Future Work

## The hypercentral potential $V(x)$

$$V(x) = V_{SI}(x) + V_{SD}(x) \quad (4)$$

$$V_{SI}(x) = -\frac{2}{3} \frac{\alpha_s}{x} + a \left( \frac{1 - e^{-\mu x}}{\mu} \right) \quad (5)$$

$$V_{SD}(x) = V_{SS}(x) \left[ S(S+1) - s_\rho(s_\rho+1) - \frac{3}{4} \right] \quad (6)$$
$$+ V_{LS}(x)(\vec{L} \cdot \vec{S}) + V_T(x) \left[ S(S+1) - \frac{3(\vec{S} \cdot \vec{x})(\vec{S} \cdot \vec{x})}{x^2} \right]$$

R. Bijkar et al., Ann. Phys. (N. Y.) 236, 69 (1994).

M. B. Voloshin, Prog. Part. Nucl. Phys. 61, 455 (2008).

Z. Shah et al., Few-Body Systems 64 (2), 40 (2023).

A. Kakadiya et al., IJMP A, 2341003 (2023).

# Regge phenomenology

## Introduction

## Baryons

### Models

Singly, doubly and triply heavy baryons

## Meson

Spectroscopic properties of  $B$ ,  $B_s$  and  $B_c$ , bottomonium mesons

Spectroscopy of all bottom and heavy-light tetraquarks

Conclusion and Future Work

In 1978, Nambu presented the most general form of linear Regge trajectories<sup>a</sup> and postulated the uniform interaction of quark and antiquark pairs creates a strong flux tube, with light quarks revolving at the speed of light at a radius  $R$  at the tube's end. The mass originating in this flux tube is given by,

$$M = 2 \int_0^R \frac{\sigma}{\sqrt{1 - \nu^2(r)}} dr = \pi \sigma R, \quad (7)$$

where  $\sigma$  is the string tension, represents the mass density per unit length. The angular momentum of this flux tube is given by,

$$J = 2 \int_0^R \frac{\sigma r \nu(r)}{\sqrt{1 - \nu^2(r)}} dr = \frac{\pi \sigma R^2}{2} + c'. \quad (8)$$

One can also write

$$J = \frac{M^2}{2\pi\sigma} + c'', \quad (9)$$

where  $c'$  and  $c''$  are constants of integration. Hence, we can say that  $J$  and  $M^2$  are linearly related to each other.

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<sup>a</sup>Y. Nambu, PRD **10**, 4262 (1974); PLB **80B**, 372 (1979).

# Regge phenomenology

Introduction

Baryons

Models

Singly, doubly and triply heavy baryons

Meson

Spectroscopic properties of  $B_c$ ,  $B_s$  and  $B_c$ , bottomonium mesons

Spectroscopy of all bottom and heavy-light tetraquarks

Conclusion and Future Work

The most general form of the linear Regge trajectories is given as,

$$J = \alpha(M) = a(0) + \alpha' M^2, \quad (10)$$

where  $a(0)$  and  $\alpha'$  represents the intercept and slope of the trajectory, respectively. For a baryon multiplet, these Regge parameters for different flavors can be related by the following relations<sup>a</sup> :

$$a_{iiq}(0) + a_{jjq}(0) = 2a_{ijq}(0), \quad (11)$$

$$\frac{1}{\alpha'_{iiq}} + \frac{1}{\alpha'_{jjq}} = \frac{2}{\alpha'_{ijq}}, \quad (12)$$

where  $i, j, q$  represent quark flavors.

Using equations (4) and (6) and after solving them we obtains,

$$\alpha'_{iiq} M_{iiq}^2 + \alpha'_{jjq} M_{jjq}^2 = 2\alpha'_{ijq} M_{ijq}^2. \quad (13)$$

Combining Eqs. (13) and (14) and solving the quadratic equation, we obtain two pairs of solutions as,

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<sup>a</sup>A. B. Kaidalov, Z. Phys. C **12**, 63 (1982);  
L. Burakovsky et al. PRD **56**, 7124 (1997);  
L. Burakovsky and T. Goldman, PLB **434**, 251 (1998);  
V.V. Dixit and L. A. P. Balazs, PRD**20**, 816 (1979).

# Regge phenomenology

## Introduction

## Baryons

### Models

Singly, doubly and triply heavy baryons

## Meson

Spectroscopic properties of  $B$ ,  $B_s$  and  $B_c$ , bottomonium mesons

Spectroscopy of all bottom and heavy-light tetraquarks

Conclusion and Future Work

$$\frac{\alpha'_{jjq}}{\alpha'_{iiq}} = \frac{1}{2M_{jjq}^2} \times [(4M_{ijq}^2 - M_{iiq}^2 - M_{jjq}^2) \pm \sqrt{(4M_{ijq}^2 - M_{iiq}^2 - M_{jjq}^2)^2 - 4M_{iiq}^2 M_{jjq}^2}]. \quad (14)$$

and

$$\frac{\alpha'_{ijq}}{\alpha'_{iiq}} = \frac{1}{4M_{ijq}^2} \times [(4M_{ijq}^2 + M_{iiq}^2 - M_{jjq}^2) \pm \sqrt{(4M_{ijq}^2 - M_{iiq}^2 - M_{jjq}^2)^2 - 4M_{iiq}^2 M_{jjq}^2}]. \quad (15)$$

Also,

$$\frac{\alpha'_{jjq}}{\alpha'_{iiq}} = \frac{\alpha'_{kkq}}{\alpha'_{iiq}} \times \frac{\alpha'_{jjq}}{\alpha'_{kkq}}. \quad (16)$$

where  $k$  can be any quark flavor. We have,

$$\frac{[(4M_{ijq}^2 - M_{iiq}^2 - M_{jjq}^2) + \sqrt{(4M_{ijq}^2 - M_{iiq}^2 - M_{jjq}^2)^2 - 4M_{iiq}^2 M_{jjq}^2}]}{2M_{jjq}^2} \quad (17)$$

This is a general relation in terms of baryon masses that can be used to predict the mass of any baryon state if we know all other masses.

With the aid of relations (15) and (16) we can obtain the values of Regge slopes ( $\alpha'$ ) of baryon systems.

**Table:** Regge slopes of ground state  $\frac{1}{2}^+$  ( $\alpha'$ ) and  $\frac{3}{2}^+$  ( $\alpha'^*$ ) trajectories of singly, doubly, and triply heavy baryons (in  $\text{GeV}^{-2}$ ).

Baryon	$\alpha'$	$\alpha'^*$
$\Lambda_b$ ( <i>nnb</i> )	0.2852	-
$\Sigma_b$ ( <i>nnb</i> )	0.2852	0.2906
$\Xi_b$ ( <i>nsb</i> )	0.2795	0.2846
$\Xi'_b$ ( <i>nsb</i> )	0.2795	-
$\Omega_b$ ( <i>ssb</i> )	0.2740	0.2789
$\Omega_c$ ( <i>ssc</i> )	0.5013	0.6069
$\Xi_{cc}$ ( <i>ncc</i> )	0.3866	0.3692
$\Xi_{bb}$ ( <i>nbb</i> )	0.1792	0.1695
$\Xi_{bc}$ ( <i>ncb</i> )	0.2007	0.1994
$\Omega_{cc}$ ( <i>scc</i> )	0.5721	0.4085
$\Omega_{bb}$ ( <i>sbb</i> )	0.1656	0.1688
$\Omega_{bc}$ ( <i>scb</i> )	0.3095	0.2260
$\Omega_{ccc}$ ( <i>ccc</i> )	-	0.4199
$\Omega_{bbb}$ ( <i>bbb</i> )	-	0.1216

- Now, from eq. (11), we can write

$$M_{J+1} = \sqrt{M_J^2 + \frac{1}{\alpha'}} \quad (18)$$

From the above Eq. and the values of extracted Regge slopes, we calculate the orbitally excited state ( $L = 1, 2, 3, \dots$ ) masses of all the baryons.

J. Oudichya et al., PRD **104**, 114027 (2021).

J. Oudichya et al., NPA **1035**, 122658 (2023)

J. Oudichya et al., EPJA **59** (6), 123 (2023)

# Heavy Baryon Spectrum

Introduction

Baryons

Models

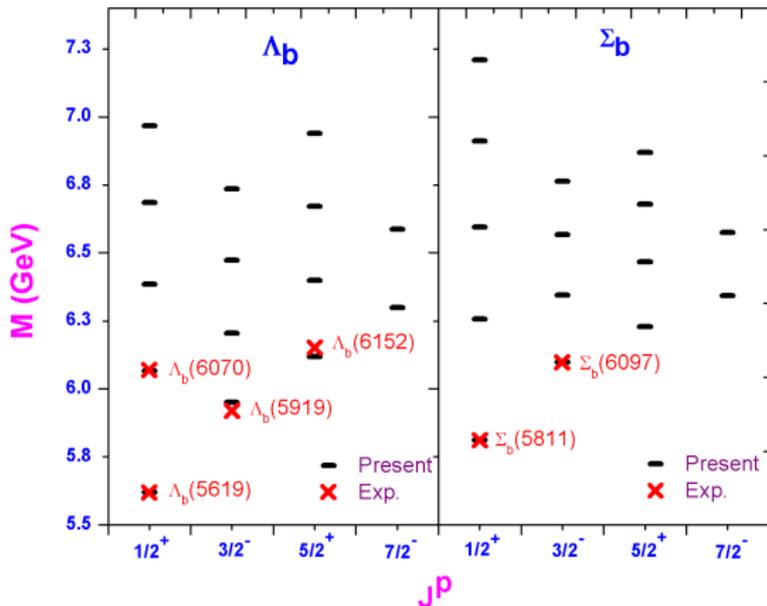
Singly, doubly and triply heavy baryons

Meson

Spectroscopic properties of  $B$ ,  $B_s$  and  $B_c$ , bottomonium mesons

Spectroscopy of all bottom and heavy-light tetraquarks

Conclusion and Future Work



A Kakadiya, Z Shah, K Gandhi, AK Rai, Few body syst. **63**, 29, (2022).

# Heavy Baryon Spectrum

Introduction

Baryons

Models

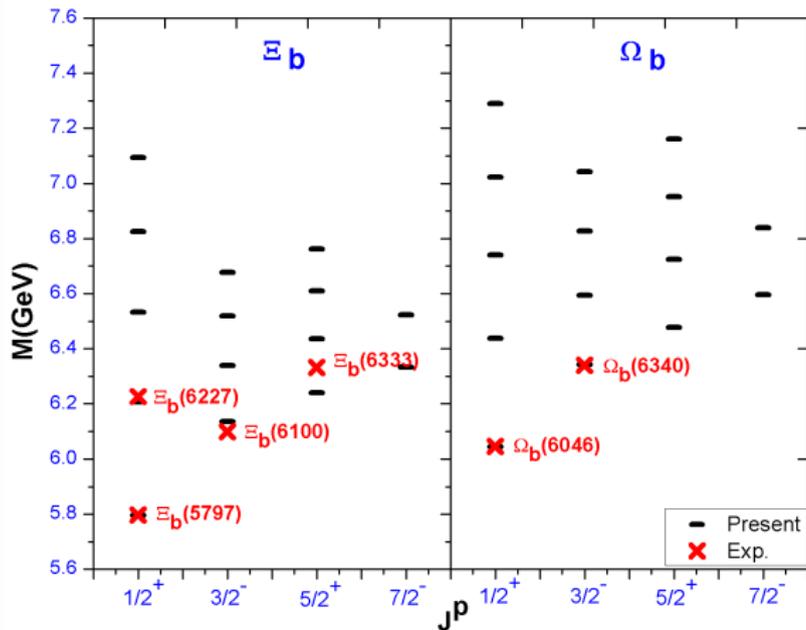
Singly, doubly and triply heavy baryons

Meson

Spectroscopic properties of  $B$ ,  $B_s$  and  $B_c$ , bottomonium mesons

Spectroscopy of all bottom and heavy-light tetraquarks

Conclusion and Future Work



A Kakadiya, Z Shah, AK Rai, IJMPA 37, 2250053 (2022)

# Heavy Baryon Spectrum

Introduction

Baryons

Models

Singly, doubly and triply heavy baryons

Meson

Spectroscopic properties of  $B$ ,  $B_c$  and  $B_s$ , bottomonium mesons

Spectroscopy of all bottom and heavy-light tetraquarks

Conclusion and Future Work

**Table:** Masses of excited states of the  $\Xi_{cc}$  baryon in the  $(J, M^2)$  plane (in GeV).

$N^{2S+1}L_J$	Present		Others		$N^{2S+1}L_J$	Present		Others	
	$\Xi_{cc}^{++}$	$\Xi_{cc}^+$	PDG	Yoshida		$\Xi_{bb}^0$	$\Xi_{bb}^-$	Wei	Roberts
$1^2S_{\frac{1}{2}}$	3.579	3.581	3.621	3.685	$1^2S_{\frac{1}{2}}$	10.225	10.230	10.199	10.340
$1^2P_{\frac{3}{2}}$	3.924	3.926		3.949	$1^2P_{\frac{3}{2}}$	10.494	10.499	10.474	10.495
$1^2D_{\frac{5}{2}}$	4.241	4.243		4.115	$1^2D_{\frac{5}{2}}$	10.757	10.761	10.742	10.676
$1^2F_{\frac{7}{2}}$	4.536	4.538			$1^2F_{\frac{7}{2}}$	11.013	11.017	11.004	
$1^2G_{\frac{9}{2}}$	4.813	4.815			$1^2G_{\frac{9}{2}}$	11.263	11.267	11.259	
$1^2H_{\frac{11}{2}}$	5.075	5.077			$1^2H_{\frac{11}{2}}$	11.508	11.512		
$1^4S_{\frac{3}{2}}$	3.729	3.726		3.754	$1^4S_{\frac{3}{2}}$	10.330	10.333	10.316	10.367
$1^4P_{\frac{5}{2}}$	4.076	4.074		4.163	$1^4P_{\frac{5}{2}}$	10.612	10.615	10.588	10.731
$1^4D_{\frac{7}{2}}$	4.396	4.394			$1^4D_{\frac{7}{2}}$	10.886	10.889	10.853	10.608
$1^4F_{\frac{9}{2}}$	4.694	4.692			$1^4F_{\frac{9}{2}}$	11.154	11.157	11.112	
$1^4G_{\frac{11}{2}}$	4.974	4.973			$1^4G_{\frac{11}{2}}$	11.415	11.418	11.365	
$1^4H_{\frac{13}{2}}$	5.240	5.239			$1^4H_{\frac{13}{2}}$	11.670	11.673		

J. Oudichya, K. Gandhi, and A.K. Rai, Phys. Scr. **97**, 054001 (2022).

J. Oudichya, K. Gandhi, and A. K. Rai, Phys. Rev. D **104**, 114027

(2021)

9/27

◀ Back

Forward ▶

# Heavy Baryon Spectrum

Introduction

Baryons

Models

Singly, doubly and triply heavy baryons

Meson

Spectroscopic properties of  $B_c$ ,  $B_s$  and  $B_c$ , bottomonium mesons

Spectroscopy of all bottom and heavy-light tetraquarks

Conclusion and Future Work

**Table:** Masses of excited states of the  $\Omega_{cc}$  baryon in the  $(J, M^2)$  plane (in GeV). **Table:** Masses of excited states of  $\Omega_{bb}$  baryon in the  $(J, M^2)$  plane (in GeV).

$N^{2S+1}L_J$	Present	Shah	Ebert	Wei	$N^{2S+1}L_J$	Present	Shah	Wei	Roberts
$1^2S_{\frac{1}{2}}$	3.719	3.650	3.778	3.650	$1^2S_{\frac{1}{2}}$	10.350	10.446	10.320	10.454
$1^2P_{\frac{3}{2}}$	3.947	3.972	4.102	3.910	$1^2P_{\frac{3}{2}}$	10.638	10.641	10.593	10.619
$1^2D_{\frac{5}{2}}$	4.162	4.141		4.153	$1^2D_{\frac{5}{2}}$	10.918	10.792	10.858	10.720
$1^2F_{\frac{7}{2}}$	4.367	4.387		4.383	$1^2F_{\frac{7}{2}}$	11.191	10.930	11.118	
$1^2G_{\frac{9}{2}}$	4.562				$1^2G_{\frac{9}{2}}$	11.458		11.372	
$1^2H_{\frac{11}{2}}$	4.749				$1^2H_{\frac{11}{2}}$	11.718			
$1^4S_{\frac{3}{2}}$	3.847	3.810	3.872	3.809	$1^4S_{\frac{3}{2}}$	10.449	10.467	10.431	10.486
$1^4P_{\frac{5}{2}}$	4.153	3.958	4.303	4.058	$1^4P_{\frac{5}{2}}$	10.729	10.637	10.700	10.766
$1^4D_{\frac{7}{2}}$	4.438	4.122		4.294	$1^4D_{\frac{7}{2}}$	11.002	10.786	10.964	10.732
$1^4F_{\frac{9}{2}}$	4.706	4.274	4.516		$1^4F_{\frac{9}{2}}$	11.268	10.924	11.221	
$1^4G_{\frac{11}{2}}$	4.960				$1^4G_{\frac{11}{2}}$	11.528		11.472	
$1^4H_{\frac{13}{2}}$	5.201				$1^4H_{\frac{13}{2}}$	11.782			

Oudichya, K. Gandhi, and A.K. Rai, Phys. Scr. 97, 054001 (2022).

J. Oudichya, K. Gandhi, and A. K. Rai, Phys. Rev. D 104, 114027

(2021).

# Heavy Baryon Spectrum

Introduction

Baryons

Models

Singly, doubly and triply heavy baryons

Meson

Spectroscopic properties of  $B$ ,  $B_c$  and  $B_s$ , bottomonium mesons

Spectroscopy of all bottom and heavy-light tetraquarks

Conclusion and Future Work

**Table:** Masses of excited states of the  $\Xi_{bc}$  baryon in the  $(J, M^2)$  plane (in GeV).

$N^{2S+1}L_J$	Present		Others	
	$\Xi_{bc}^+$	$\Xi_{bc}^0$	Eakins	Roberts
$1^2S_{\frac{1}{2}}$	6.902	6.906	7.014	7.011
$1^2P_{\frac{3}{2}}$	7.254	7.258	7.394	
$1^2D_{\frac{5}{2}}$	7.590	7.594		
$1^2F_{\frac{7}{2}}$	7.912	7.916		
$1^2G_{\frac{9}{2}}$	8.221	8.225		
$1^2H_{\frac{11}{2}}$	8.519	8.523		
$1^4S_{\frac{3}{2}}$	7.030	7.029	7.064	7.074
$1^4P_{\frac{5}{2}}$	7.378	7.377		
$1^4D_{\frac{7}{2}}$	7.710	7.709	7.292	
$1^4F_{\frac{9}{2}}$	8.029	8.028		
$1^4G_{\frac{11}{2}}$	8.335	8.334		
$1^4H_{\frac{13}{2}}$	8.631	8.630		

**Table:** Masses of excited states of  $\Omega_{bc}$  baryon in the  $(J, M^2)$  plane (in GeV).

$N^{2S+1}L_J$	Present	Ebert	Giannuzzi	Roberts
$1^2S_{\frac{1}{2}}$	7.035	7.136	7.088	6.994
$1^2P_{\frac{3}{2}}$	7.261	7.373		
$1^2D_{\frac{5}{2}}$	7.480	7.547		
$1^2F_{\frac{7}{2}}$	7.693	7.705		
$1^2G_{\frac{9}{2}}$	7.900			
$1^2H_{\frac{11}{2}}$	8.102			
$1^4S_{\frac{3}{2}}$	7.149	7.187	7.130	7.017
$1^4P_{\frac{5}{2}}$	7.452	7.363		
$1^4D_{\frac{7}{2}}$	7.744	7.534		
$1^4F_{\frac{9}{2}}$	8.025	7.690		
$1^4G_{\frac{11}{2}}$	8.296			
$1^4H_{\frac{13}{2}}$	8.559			

J. Oudichya, K. Gandhi, and A.K. Rai, Phys. Scr. 97, 054001 (2022).

## Introduction

## Baryons

## Models

## Singly, doubly and triply heavy baryons

## Meson

Spectroscopic properties of  $B$ ,  $B_c$  and  $B_s$ , bottomonium mesons

## Spectroscopy of all bottom and heavy-light tetraquarks

## Conclusion and Future Work

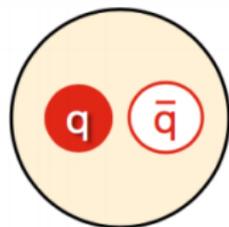
**Table:** Masses of excited states of the  $\Omega_{ccc}$  baryon in the  $(J, M^2)$  plane (in GeV). **Table:** Masses of excited states of  $\Omega_{bbb}$  baryon in the  $(J, M^2)$  plane (in GeV).

$N^{2S+1}L_J$	Present	Roberts	Wei
$1^4S_{\frac{3}{2}}$	4.841	4.965	4.818
$1^4P_{\frac{5}{2}}$	5.081		
$1^4D_{\frac{7}{2}}$	5.310	5.311	5.302
$1^4F_{\frac{9}{2}}$	5.530		
$1^4G_{\frac{11}{2}}$	5.741		
$1^4H_{\frac{13}{2}}$	5.945		

$N^{2S+1}L_J$	Present	Shah	Wei	Roberts
$1^4S_{\frac{3}{2}}$	14.822	14.496	14.788	14.834
$1^4P_{\frac{5}{2}}$	15.097	14.931		
$1^4D_{\frac{7}{2}}$	15.367	15.286	15.318	15.101
$1^4F_{\frac{9}{2}}$	15.632	15.631		
$1^4G_{\frac{11}{2}}$	15.893		15.831	
$1^4H_{\frac{13}{2}}$	16.150			

J. Oudichya, K. Gandhi, and A. K. Rai, Phys. Rev. D, **103**, 114030 (2021). J. Oudichya, K. Gandhi, and A. K. Rai, PRD **104**, 114027 (2021).

# Mesons



Combination of quark-antiquark (Bosons)  
Classified by  $J^{PC}$

$$J = L + S$$

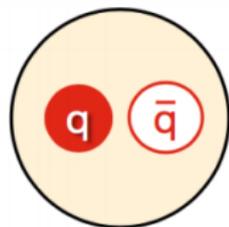
$$P = (-1)^{L+1}$$

$$C = (-1)^{L+S}$$

## Light Mesons

Meson	$\pi^\pm$	$\pi^0$	$K^\pm$	$K^0$	$\eta$
Mass	139.57	134.97	493.6	497.6	547.8
$I^G(J^{PC})$	$1^-(0^-)$	$1^-(0^{-+})$	$\frac{1}{2}(0^-)$	$\frac{1}{2}(0^-)$	$0^+(0^{-+})$
Meson	$\eta'$	$\rho$	$\omega$	$K^*$	$\phi$
Mass	957.7	775.4	782.6	895.9	1019.4
$I^G(J^{PC})$	$0^+(0^{-+})$	$1^+(1^{--})$	$0^-(1^{--})$	$\frac{1}{2}(1^-)$	$0^-(1^{--})$

# Mesons



Combination of quark-antiquark (Bosons)  
Classified by  $J^{PC}$

$$J = L + S$$

$$P = (-1)^{L+1}$$

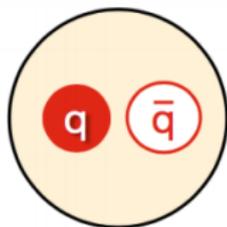
$$C = (-1)^{L+S}$$

## Heavy-light Mesons

Meson	$D^\pm$	$D^0$	$D_s^\pm$	$B^\pm$	$B^0$	$B_s$
Mass	1864.86	1968.49	5279.25	5279.58	5366.77	2006.98
$I^G(J^{PC})$	$\frac{1}{2}(0^-)$	$0(0^-)$	$\frac{1}{2}(0^-)$	$\frac{1}{2}(0^-)$	$0(0^-)$	$\frac{1}{2}(1^-)$

Meson	$D^{*0}$	$D^{*\pm}$	$D_s^{*\pm}$	$B^*$	$B_s^*$
Mass	1869.62	2010.28	2112.30	5325.20	5415.40
$I^G(J^{PC})$	$\frac{1}{2}(0^-)$	$\frac{1}{2}(1^-)$	$0(?)^?$	$\frac{1}{2}(1^-)$	$0(1^-)$

# Mesons



Combination of quark-antiquark (Bosons)  
Classified by  $J^{PC}$

$$J = L + S$$

$$P = (-1)^{L+1}$$

$$C = (-1)^{L+S}$$

## Heavy Mesons

Meson	$B_c^\pm$	$\eta_c$	$J/\psi$	$\eta_b$	$\Upsilon$
Mass	6277	2981	3096.91	9399.8	9460.30
$I^G(J^{PC})$	$0(0^-)$	$0^+(0^{-+})$	$0^-(1^{--})$	$0^+(0^{-+})$	$0^-(1^{--})$

# Theoretical Overview-Screened potential

The Hamiltonian used to calculate the  $B$  and  $B_s$  meson mass spectrum <sup>1</sup>,

$$H = \sqrt{p^2 + m_1^2} + \sqrt{p^2 + m_2^2} + V(\mathbf{r}); \quad (19)$$

$m_1$  and  $m_2$  are the heavy and light quark masses,  $p$  is the relative momentum of the quark and anti-quark system, and  $V(\mathbf{r})$  is the quark anti-quark interaction potential.

$$V(r) = V^{(0)}(r) + \left( \frac{1}{m_1} + \frac{1}{m_2} \right) V^{(1)}(r) + \mathcal{O}\left(\frac{1}{m^2}\right); \quad (20)$$

$$V^{(0)}(r) = V_v(r) + V_s(r) + V_0; \quad V^{(1)}(r) = -\frac{C_F C_A \alpha_s^2}{4r^2} \quad (21)$$

$$V_v(r) = -\frac{\alpha_c}{r}, \quad V_s(r) \equiv \begin{cases} Ar & \text{linear} \\ \frac{A}{\mu}(1 - e^{-\mu r}) & \text{screened;} \end{cases} \quad (22)$$

$$\begin{aligned} V_{spin}(r) &= \left( \frac{L \cdot S_1}{2m_1^2} + \frac{L \cdot S_2}{2m_2^2} \right) \left( -\frac{dV^{(0)}(r)}{dr} + \frac{8}{3} \alpha_s \frac{1}{r^3} \right) + \\ &\frac{4}{3} \alpha_s \frac{1}{m_1 m_2} \frac{L \cdot S}{r^3} + \frac{4}{3} \alpha_s \frac{2}{3m_1 m_2} S_1 \cdot S_2 4\pi \delta(r) \\ &+ \frac{4}{3} \alpha_s \frac{1}{m_1 m_2} \left\{ 3(S_1 \cdot n)(S_2 \cdot n) - (S_1 \cdot S_2) \right\} \frac{1}{r^3}, \end{aligned}$$

**Table:**  $B$  and  $B_s$  mesons with experimental states, masses and experimental facilities where they were first observed.

State	$J^P$	Mass (MeV)	Observed Mode	Experiments
$B^\pm$	$0^-$	$5279.31 \pm 0.15$	$D\pi & D^*\pi\pi$	CLEO
$B_s^0$	$0^-$	$5366.82 \pm 0.22$		CUSB-II
$B^0$	$0^-$	$5279.58 \pm 0.15 \pm 0.28$	$D\pi\pi & D^*\pi$	CLEO
$B^*$	$1^-$	$5324.65 \pm 0.25$	$B\gamma$	CUSB
$B_s^*$	$1^-$	$5415.4^{+1.8}_{-1.5}$	$B_s\gamma$	CUSB-II
$B_1(5721)^*\pm$	$1^+$	$5725^{+2.5}_{-2.7}$	$B^{*0}\pi^-$	LHCb
$B_1(5721)^0$	$1^+$	$5726 \pm 1.3$	$B^{*+}\pi^-$	CDF
$B_{s1}(5830)$	$1^+$	$5828.40 \pm 0.04 \pm 0.04 \pm 0.41$	$B^*K$	LHCb
$B_{s2}^*(5840)^0$	$1^+$	$5839.99 \pm 0.05 \pm 0.11 \pm 0.17$	$B^*K$	LHCb
$B_J(5732)$	$?^?$	$5695^{+17}_{-19}$	$B^*\pi$	ALEPH
$B_2^*(5747)^+$	$2^+$	$5737.20 \pm 0.72 \pm 0.40 \pm 0.17$	$B^{*0}\pi^+$	LHCb
$B_{sJ}^*(5850)$	$?$	$5853 \pm 15$		OPAL
$B_2^*(5747)^0$	$2^+$	$5739.44 \pm 0.37 \pm 0.33 \pm 0.17$	$B^{*+}\pi^+$	LHCb
$B_J(5840)^+$	$?^?$	$5850.3 \pm 12.7 \pm 13.7 \pm 0.2$	$B\pi$	LHCb
$B_J(5840)^0$	$?^?$	$5862.9 \pm 5.0 \pm 6.7 \pm 0.2$	$B\pi$	LHCb
$B_J(5970)^+$	$?^?$	$5961 \pm 5 \pm 12$	$B^0\pi^+$	CDF
$B_J(5970)^0$	$?^?$	$5978 \pm 5 \pm 12$	$B^+\pi^-$	CDF
$B_{sJ}(6064)^0$	$?^?$	$6063.5 \pm 1.2 \pm 0.8$	$B^+K^-$	LHCb
$or B_{sJ}(6109)^0$	$?^?$	$6108.8 \pm 1.1 \pm 0.7$	$B^{*+}K^-$	LHCb
$B_{sJ}(6114)^0$	$?^?$	$6114 \pm 3 \pm 5$	$B^+K^-$	LHCb
$or B_{sJ}(6158)^0$	$?^?$	$6158.5 \pm 4 \pm 5$	$B^{*+}K^-$	LHCb

# Mass spectrum of $B_s$ meson (in MeV).

Introduction

Baryons

Models

Singly, doubly and triply heavy baryons

Meson

Spectroscopic properties of  $B$ ,  $B_c$  and  $B_c$ , bottomonium mesons

Spectroscopy of all bottom and heavy-light tetraquarks

Conclusion and Future Work

State	$J^P$	Meson	Present <sup>2</sup>	3	4	5	6	7	8
$1^1S_0$	$0^-$	$B_S^0$	5359	$5366 \pm 0.19$	5367	5366	5372	5394	5355
$2^1S_0$	$0^-$		5980		6003	5939	5976	5984	5962
$1^3S_1$	$1^-$	$B_S^*$	5415	$5415^{+1.8}_{-1.5}$	5413	5415	5414	5440	5416
$2^3S_1$	$1^-$		5993		6029	5956	5992	6012	5999
$1^3P_0$	$0^+$		5798		5812	5799	5833	5831	5782
$1^1P_1$	$1^+$	$B_{S1} (5830)^0$	5818	$5828 \pm 0.27$	5828	5819	5865	5857	5833
$1^3P_1$	$1^+$	$B_{sJ}^*$	5846	5850	5842	5854	5831	5861	5843
$1^3P_2$	$2^+$	$B_{S2}^* (5840)^0$	5838	$5839 \pm 0.17$	5840	5849	5842	5876	5848
$1^3D_1$	$1^-$		6144		6119	6226	6209	6179	6155
$1^1D_2$	$2^-$	$B_{sJ} (6064)$	6139		6128	6177	6218	6169	6079
$1^3D_2$	$2^-$		6135		6157	6209	6189	6196	6172
$1^3D_3$	$3^-$		6139		6172	6145	6191	6179	6088
$1^3F_2$	$2^+$		6392					6454	
$1^1F_3$	$3^+$		6384					6425	
$1^3F_3$	$3^+$		6385					6462	
$1^3F_4$	$4^+$		6381					6432	

<sup>2</sup>V. Patel et al., arXiv:2201.01120 (2022)

<sup>3</sup>M. Tanabashi, et al., PRD **98**(3), 030001 (2018)

<sup>4</sup>V. Kher et al., Chin. Phys. C **41**(9), 093101 (2017).

<sup>5</sup>M. Shah et al., PRD **93**(9), 094028 (2016).

<sup>6</sup>D. Ebert et al., EPJC **66**, 197 (2010).

<sup>7</sup>S. Godfrey et al., PRD **94**(5), 054025 (2016).

<sup>8</sup>J.B. Liu, C.D. Lu, EPJC **77**(5), 312 (2017).

# Regge-trajectories of $B_s$ meson

Introduction

Baryons

Models

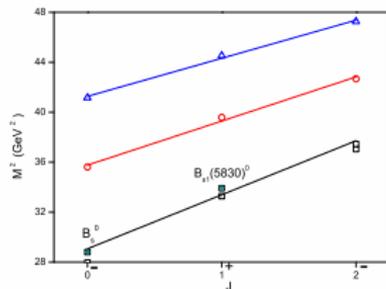
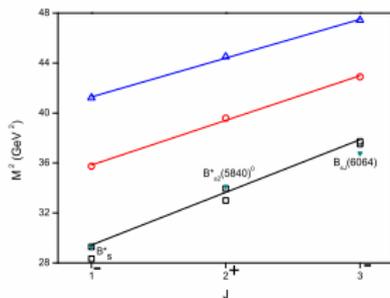
Singly, doubly and triply heavy baryons

Meson

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Spectroscopy of all bottom and heavy-light tetraquarks

Conclusion and Future Work



**Figure:**  $J \rightarrow M^2$  Regge-trajectory of  $B_s$  mesons for natural and unnatural parity states

# $B_c$ meson spectroscopy motivated by general features of pNRQCD

Introduction

Baryons

Models

Singly, doubly and triply heavy baryons

Meson

Spectroscopic properties of  $B$ ,  $B_s$  and  $B_c$ , bottomonium mesons

Spectroscopy of all bottom and heavy-light tetraquarks

Conclusion and Future Work

- Analysis and interpretation of huge data of Quarkonia, coming from experiments is the most crucial and challenging task for theoreticians.
- Many theoretical studies have tried to explain new observed states, but the investigations are still open due to impecunious interpretations.
- Of all available theoretical models we use potential model approach because it takes into account asymptotic freedom enjoyed by quark anti-quark interaction, also the B.E  $\ll$  compared to the rest energy of the constituents.
- To calculate the masses we include corrections from EFT's (pNRQCD), because the short distance and long distance contributions can be taken care individually.

$$V(r) = -\frac{4\alpha_s}{3} \frac{1}{r} + Ar + \frac{1}{m} V_{pNRQCD} + \frac{1}{m^2} V_{SD} \quad (24)$$

- The potential has been solved numerically using the Mathematica notebook utilizing Runge Kutta method.

# Results

We use the following potential parameters to calculate the mass spectra of quarkonia and  $B_c$  meson.

**Table:** Potential parameters for quarkonium

$\alpha_c$	$m_q$	$\epsilon$	A	$\alpha_s$	C	a
0.4	1.321GeV	0.12	0.191 $\frac{\text{GeV}}{\text{fm}}$	0.318	0.12	-0.165 $\text{GeV}^2$
0.19	4.81GeV	0.2	0.28 $\frac{\text{GeV}}{\text{fm}}$	0.216	0.1	-0.523 $\text{GeV}^2$

State	S wave of $B_c$ (GeV)	
	9	10
$1^1S_0$	6.274	$6.274 \pm 0.008$
$1^3S_1$	6.332	-
$2^1S_0$	6.851	$6.871 \pm 0.016$
$2^3S_1$	6.888	-
$3^1S_0$	7.275	-
$3^3S_1$	7.306	-
$4^1S_0$	7.639	-
$4^3S_1$	7.666	-
$5^1S_0$	7.967	-
$5^3S_1$	7.992	-

<sup>9</sup>Raghav Chaturvedi et al., Eur.Phys.J.A 58, (2022) 11.

<sup>10</sup>P. A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020).

# Bottomonium spectroscopy using Coulomb plus linear (Cornell) potential

Introduction

Baryons

Models

Singly, doubly and triply heavy baryons

Meson

Spectroscopic properties of  $B$ ,  $B_s$  and  $B_c$ , bottomonium mesons

Spectroscopy of all bottom and heavy-light tetraquarks

Conclusion and Future Work

- Bottomonia has maximum number of experimental available states off all the mesons.
- Analysis and interpretation of huge data of Bottomonia, coming from experiments is the most crucial and challenging task for theoreticians.
- Bottom quarks being heavy, the analysis of bottomonium is comparatively easy and can be made using non-relativistic quark models.
- Of all available theoretical models we use potential model approach because it takes into account asymptotic freedom enjoyed by quark anti-quark interaction, also the B.E  $\ll$  compared to the rest energy of the constituents.
- To calculate the masses we use Cornell potential, and add spin dependent terms perturbatively.

$$V^{(0)}(r) = -\frac{4\alpha_S (M^2)}{3r} + Ar + V_0 \quad (25)$$

- we employ variational method with a single Gaussian trial wave function with one parameter; both in position space as well as momentum space.

# Results

Introduction

Baryons

Models

Singly, doubly and triply heavy baryons

Meson

Spectroscopic properties of  $B_c$ ,  $B_s$  and  $B_c$ , bottomonium mesons

Spectroscopy of all bottom and heavy-light tetraquarks

Conclusion and Future Work

**Table:** Potential parameters for quarkonium

State	$J^P$	11	12
$1^1S_0$	$0^{-+}$	9.423	9.399
$1^3S_1$	$1^{--}$	9.463	9.460
$2^1S_0$	$0^{-+}$	9.983	9.999
$2^3S_1$	$1^{--}$	10.001	10.023
$3^1S_0$	$0^{-+}$	10.342	
$3^3S_1$	$1^{--}$	10.354	10.355
$4^1S_0$	$0^{-+}$	10.638	
$4^3S_1$	$1^{--}$	10.650	10.579
$5^1S_0$	$0^{-+}$	10.901	
$5^3S_1$	$1^{--}$	10.912	10.885
$6^1S_0$	$0^{-+}$	11.140	
$6^3S_1$	$1^{--}$	11.151	11.000

<sup>11</sup>R. L. Workman et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2022, 083C01 (2022).

<sup>12</sup>V. H. Kher et.al., Eur.Phys.J.Plus 137, (2022) 3.

# Exotic Hadrons

Introduction

Baryons

Models

Singly, doubly and triply heavy baryons

Meson

Spectroscopic properties of  $B_c$ ,  $B_s$  and  $B_c$ , bottomonium mesons

Spectroscopy of all bottom and heavy-light tetraquarks

Conclusion and Future Work

- Exotic hadrons are those which have quark combinations other than the conventional baryons ( $qqq$ ) and mesons ( $q\bar{q}$ ).
- The simplest multiquark system is a tetraquark, composed of two diquarks and two anti-diquarks. Heavy tetraquarks are of particular interest, since the presence of a heavy quark increases the binding energy of the bound system and, as a result, the possibility that such tetraquarks will have masses below the thresholds for decays to mesons with open heavy flavour.
- Significantly different interpretations for the  $qq\bar{q}\bar{q}$  candidates were proposed: molecules composed from two mesons  $[q\bar{q}][q\bar{q}]$  loosely bound by the meson exchange, compact tetraquarks composed from a diquark  $[qq]$  and antidiquark  $[\bar{q}\bar{q}]$  bound by strong forces, hadroquarkonia composed of a heavy quarkonium embedded in a light meson, hybrids, etc.
- In the 21st century there are numerous exotic hadrons namely XYZ, have been observed theoretically as well as experimentally, e.g.  $X(3872)$  aka  $\chi_{c1}(3872)$ ,  $X(6900)$ ,  $Y(4260)$ ,  $Z_b(10650)$  and  $Z_b(10610)^*$  etc.
- Previously we have also studied about the mass-spectra of tetraquarks namely  $\psi(4260)$ ,  $Z(4430)$  and  $[cc][\bar{c}\bar{c}]^{**}$  now known as  $X(6900)$ .

R. Tiwari, D. P. Rathaud, and A. K. Rai, Indian J Phys 1-22 (2022), Rohit Tiwari, D. P. Rathaud, and A. K. Rai Eur. Phys. J. A, **57**, 289 (2021), R. Tiwari, A. K. Rai, Few-Body Systems 64 (2), 20 (2023).

# Theoretical Framework

## Introduction

## Baryons

### Models

Singly, doubly and triply heavy baryons

## Meson

Spectroscopic properties of  $B$ ,  $B_s$  and  $B_c$ , bottomonium mesons

## Spectroscopy of all bottom and heavy-light tetraquarks

## Conclusion and Future Work

- One of the most common functional forms of the potential,  $V(r)$ , employed in heavy quarkonium spectroscopy is the Coulomb plus linear potential. The potential is given by;

$$V_{C+L}(r) = V_v + V_s \implies \frac{k_s \alpha_s}{(r)} + b(r), \quad (26)$$

- where  $k_s$ , is a color factor,  $\alpha_s$  is the QCD fine structure constant and  $b$  is string tension. The Coulomb term arises from the One gluon exchange (OGE) and the linear term is responsible for confinement.
- The final form of central potential is given by;

$$V(r) = V_{C+L}(r) + V^1(r) \left( \frac{1}{m_1} + \frac{1}{m_2} \right) + \mathcal{O} \left( \frac{1}{m^2} \right) \quad (27)$$

$$V^1(r) = -\frac{C_F C_A}{4} \frac{\alpha_s^2}{(r)^2} \quad (28)$$

- where,  $C_F = \frac{4}{3}$  and  $C_A = 3$  are the Casimir charges of the fundamental and the adjoint representation respectively.

# Spin-dependent Terms

Introduction

Baryons

Models

Singly, doubly and triply heavy baryons

Meson

Spectroscopic properties of  $B$ ,  $B_c$  and  $B_s$ , bottomonium mesons

Spectroscopy of all bottom and heavy-light tetraquarks

Conclusion and Future Work

- Based on the Breit-Fermi Hamiltonian for one-gluon exchange, the spin-spin dependent terms may be defined as;

$$V_{SS}(r) = C_{SS}(r)S_1 \cdot S_2, \quad (29)$$

$$V_{LS}(r) = C_{LS}(r)L \cdot S, \quad (30)$$

$$V_T(r) = C_T(r)S_{12}, \quad (31)$$

- where,

$$C_{SS}(r) = \frac{2}{3m^2} \nabla^2 V_V(r) = \frac{8k_s \alpha_s \pi}{3m^2} \left( \frac{\sigma}{\sqrt{\pi}} \right)^3 \exp^{-\sigma^2 r^2} \quad (32)$$

$$C_{LS}(r) = -\frac{3k_s \alpha_s \pi}{2m^2} \frac{1}{(r)^2} - \frac{b}{2m^2} \frac{1}{(r)} \quad (33)$$

$$C_T(r) = -\frac{12k_s \alpha_s \pi}{4m^2} \frac{1}{(r)^3} \quad (34)$$

# Bottomonium

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**Table:** The mass-spectra of bottomonium states  $[b\bar{b}]$ , obtained from Data Set I. ( $m_i^{exp}$ ) corresponds to mass predicted in recent PDG.

$N^{2S+1}L_J$	$\langle E \rangle$	$\langle V_V \rangle$	$\langle V_S \rangle$	$m_i^f$	$m_i^{exp}$	Meson	$J^{PC}$
$1^1S_0$	-133.0	-757.8	165.5	9409	$9398.7 \pm 2.0$	$\eta_b(1S)$	$0^{-+}$
$1^3S_1$	-132.9	-757.8	165.5	9440	$9460.30 \pm 0.26$	$\Upsilon(1S)$	$1^{--}$
$2^1S_0$	428.3	-352.1	404.5	9987	...	...	$0^{-+}$
$2^3S_1$	429.6	-352.1	404.5	9997	$10023.26 \pm 0.31$	$\Upsilon(2S)$	$1^{--}$
$3^1S_0$	764.1	-258.2	596.3	10325	...	...	$0^{-+}$
$3^3S_1$	764.1	-258.2	596.3	10331	$10355.2 \pm 0.5$	$\Upsilon(3S)$	$1^{--}$
$4^1S_0$	1034.1	-213.1	761.0	10596	...	...	$0^{-+}$
$4^3S_1$	1034.1	-213.1	761.0	10601	$10579.4 \pm 1.2$	$\Upsilon(4S)$	$1^{--}$
$1^3P_0$	335.3	-313.6	329.1	9863	$9859.44 \pm 0.42$	$\chi_{b0}(1P)$	$0^{++}$
$1^3P_1$	335.3	-313.6	329.1	9893	$9892.78 \pm 0.26$	$\chi_{b1}(1P)$	$1^{++}$
$1^1P_1$	335.3	-313.6	329.1	9898	$9899.3 \pm 0.8$	$h_b(1P)$	$1^{+-}$
$1^3P_2$	335.3	-313.6	329.1	9915	$9912.21 \pm 0.26$	$\chi_{b2}(1P)$	$2^{++}$
$2^3P_0$	679.2	-227.2	529.5	10213	$10232.5 \pm 0.4$	$\chi_{b0}(2P)$	$0^{++}$
$2^3P_1$	679.2	-227.2	529.5	10239	$10255.46 \pm 0.22$	$\chi_{b1}(2P)$	$1^{++}$
$2^1P_1$	679.2	-227.2	529.5	10243	-	-	$1^{+-}$
$2^3P_2$	679.2	-227.2	529.5	10257	$10268.65 \pm 0.22$	$\chi_{b2}(2P)$	$2^{++}$
$1^3D_1$	576.4	-213.0	455.2	10135	-	-	$1^{--}$
$1^3D_2$	576.4	-213.0	455.2	10141	$10163.7 \pm 1.4$	$\Upsilon_2(1D)$	$2^{--}$
$1^1D_2$	576.4	-213.0	455.2	10142	-	-	$2^{+-}$
$1^3D_3$	576.4	-213.0	455.2	10146	-	-	$3^{--}$

# All bottom tetraquark

**Table:** Mass-Spectra of S and P-wave tetraquark states  $T_{4b}$ , obtained from Data Set I.

$N^{2S+1}L_J$	$\langle E \rangle$	$\langle V_V \rangle$	$\langle V_S \rangle$	$m_i^f$	$m_{th}$	Threshold	$J^{PC}$
$1^1S_0$	-503.4	-1252.3	83.4	18749	18798	$\eta_b(1S)\eta_b(1S)$	$0^{++}$
$1^3S_1$	-503.4	-1252.3	83.4	18764	18859	$\eta_b(1S)\Upsilon(1S)$	$1^{+-}$
$1^5S_2$	-503.4	-1252.3	83.4	18792	18920	$\Upsilon(1S)\Upsilon(1S)$	$2^{++}$
$2^1S_0$	136.1	-531.0	245.1	19414	19998	$\eta_b(2S)\eta_b(2S)$	$0^{++}$
$2^3S_1$	136.1	-531.0	245.1	19416	...	...	$1^{+-}$
$2^5S_2$	136.1	-531.0	245.1	19421	...	...	$2^{++}$
$1^1P_1$	84.2	-428.2	200.0	19361	...	...	$1^{--}$
$1^3P_0$	84.2	-428.2	200.0	19327	19258	$\eta_b(1S)\chi_{b0}(1P)$	$0^{-+}$
$1^3P_1$	84.2	-428.2	200.0	19361	19292	$\eta_b(1S)\chi_{b1}(1P)$	$1^{-+}$
$1^3P_2$	84.2	-428.2	200.0	19373	19311	$\eta_b(1S)\chi_{b2}(1P)$	$2^{-+}$
$1^5P_1$	84.2	-428.2	200.0	19325	19298	$\eta_b(1S)h_b(1P)$	$1^{--}$
$1^5P_2$	84.2	-428.2	200.0	19369	19353	$\Upsilon(1S)\chi_{b1}(1P)$	$2^{--}$
$1^5P_3$	84.2	-428.2	200.0	19388	19372	$\Upsilon(1S)\chi_{b2}(1P)$	$3^{--}$

**Table:** Comparison of tetraquarks masses obtained from present work with others.

State	$J^{PC}$	Ours	PRD	Berezhnoy	Ming	Karliner	Guang
$bb\bar{b}\bar{b}$	$0^{++}$	18719	18723	18754	19322	$18826 \pm 25$	19247
	$1^{+-}$	18734	18738	18808	19329	.....	19247
	$2^{++}$	18764	18768	18916	19341	$18956 \pm 25$	19249

# Singly Charm Tetraquark mass spectra

**Table:** Mass spectra for singly charm tetraquark for S and P waves in MeV

2*State	2*J <sup>PC</sup>	$\bar{3} - 3$		$6 - \bar{6}$	
		Mass <sub>NR</sub>	Mass <sub>SR</sub>	Mass <sub>NR</sub>	Mass <sub>SR</sub>
$1^1S_0$	$0^+$	$2580.65 \pm 0.64$	$2674.99 \pm 0.63$	$1680.38 \pm 0.46$	$1805.63 \pm 0.48$
$2^1S_0$	$0^+$	$3034.34 \pm 0.54$	$3114.17 \pm 0.52$	$3101.64 \pm 0.75$	$3212.74 \pm 0.56$
$1^3S_1$	$1^+$	$2652.11 \pm 0.59$	$2743.73 \pm 0.58$		
$2^3S_1$	$1^+$	$3053.43 \pm 0.54$	$3133.04 \pm 0.52$		
$1^5S_2$	$2^+$	$2795.05 \pm 0.51$	$2876.96 \pm 0.50$		
$2^5S_2$	$2^+$	$3091.61 \pm 0.53$	$3170.43 \pm 0.52$		
$1^1P_1$	$1^{--}$	$3021.69 \pm 0.53$	$3101.21 \pm 0.51$	$3102.19 \pm 0.57$	$3206.61 \pm 0.53$
$2^1P_1$	$1^{--}$	$3209.80 \pm 0.55$	$3286.74 \pm 0.53$	$3375.97 \pm 0.61$	$3486.24 \pm 0.57$
$1^3P_0$	$0^-$	$2956.35 \pm 0.54$	$3045.26 \pm 0.52$	$3004.22 \pm 0.55$	$3103.71 \pm 0.55$
$2^3P_0$	$0^-$	$3159.67 \pm 0.55$	$3242.77 \pm 0.53$	$3318.13 \pm 0.61$	$3426.55 \pm 0.56$
$1^3P_1$	$1^-$	$3020.20 \pm 0.53$	$3100.47 \pm 0.51$	$3182.64 \pm 0.57$	$3291.63 \pm 0.56$
$2^3P_1$	$1^-$	$3209.12 \pm 0.56$	$3286.68 \pm 0.53$	$3423.32 \pm 0.60$	$3534.96 \pm 0.61$
$1^3P_2$	$2^-$	$3042.43 \pm 0.53$	$3119.24 \pm 0.51$	$3111.27 \pm 0.55$	$3216.11 \pm 0.59$
$2^3P_2$	$2^-$	$3226.58 \pm 0.56$	$3301.82 \pm 0.54$	$3381.25 \pm 0.60$	$3491.44 \pm 0.61$
$1^5P_1$	$1^-$	$2953.36 \pm 0.54$	$3043.77 \pm 0.52$		
$2^5P_1$	$1^-$	$3158.32 \pm 0.55$	$3242.56 \pm 0.53$		
$1^5P_2$	$2^-$	$2952.73 \pm 0.54$	$3042.93 \pm 0.52$		
$2^5P_2$	$2^-$	$3157.97 \pm 0.55$	$3242.06 \pm 0.53$		
$1^5P_3$	$3^-$	$3033.59 \pm 0.53$	$3112.65 \pm 0.51$		
$2^5P_3$	$3^-$	$3220.7 \pm 0.56$	$3297.58 \pm 0.54$		

C. Lodha, M. Parmar, A. K. Rai, Int. J.Mod. Phys. A.  
[doi.org/10.1142/S0217751X25501258](https://doi.org/10.1142/S0217751X25501258) (2025).

**Table:** Mass spectra for singly charm tetraquark for D waves in MeV

2*State	2* $J^{PC}$	$\bar{3} - 3$		$6 - \bar{6}$	
		Mass <sub>NR</sub>	Mass <sub>SR</sub>	Mass <sub>NR</sub>	Mass <sub>SR</sub>
$1^1D_2$	$2^+$	$3165.66 \pm 0.54$	$3242.44 \pm 0.53$	$3360.45 \pm 0.58$	$3472.86 \pm 0.59$
$2^1D_2$	$2^+$	$3315.92 \pm 0.58$	$3391.44 \pm 0.55$	$3532.30 \pm 0.62$	$3644.47 \pm 0.62$
$1^3D_1$	$1^+$	$3154.7 \pm 0.54$	$3232.98 \pm 0.52$	$3356.65 \pm 0.55$	$3468.23 \pm 0.59$
$2^3D_1$	$1^+$	$3305.71 \pm 0.58$	$3382.49 \pm 0.55$	$3529.21 \pm 0.61$	$3641.33 \pm 0.61$
$1^3D_2$	$2^+$	$3164.09 \pm 0.54$	$3241.19 \pm 0.53$	$3365.61 \pm 0.56$	$3477.57 \pm 0.58$
$2^3D_2$	$2^+$	$3314.5 \pm 0.58$	$3390.29 \pm 0.55$	$3536.32 \pm 0.62$	$3648.59 \pm 0.62$
$1^3D_3$	$3^+$	$3171.00 \pm 0.54$	$3247.00 \pm 0.53$	$3359.98 \pm 0.57$	$3472.01 \pm 0.65$
$2^3D_3$	$3^+$	$3321.18 \pm 0.58$	$3396.04 \pm 0.55$	$3532.35 \pm 0.61$	$3644.24 \pm 0.69$
$1^5D_0$	$0^+$	$3148.24 \pm 0.54$	$3227.62 \pm 0.53$		
$2^5D_0$	$0^+$	$3299.72 \pm 0.58$	$3377.44 \pm 0.55$		
$1^5D_1$	$1^+$	$3144.17 \pm 0.54$	$3223.94 \pm 0.52$		
$2^5D_1$	$1^+$	$3296.04 \pm 0.57$	$3374.07 \pm 0.55$		
$1^5D_2$	$2^+$	$3149.82 \pm 0.54$	$3228.91 \pm 0.53$		
$2^5D_2$	$2^+$	$3301.3 \pm 0.58$	$3378.76 \pm 0.55$		
$1^5D_3$	$3^+$	$3159.18 \pm 0.54$	$3237.08 \pm 0.51$		
$2^5D_3$	$3^+$	$3310.05 \pm 0.58$	$3386.53 \pm 0.55$		
$1^5D_4$	$4^+$	$3169.53 \pm 0.54$	$3247.51 \pm 0.52$		
$2^5D_4$	$4^+$	$3319.84 \pm 0.58$	$3395.44 \pm 0.57$		

# Closure

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## Conclusion and Future Work

- The hadron spectroscopy has been studied in light, heavy-light and heavy-heavy sector using constituent quark models.
- The mass spectra for the excited states of all heavy flavored baryons are determined using Regge phenomenology.
- The Regge trajectories are useful to determine the unknown states. Thus, we plot graphs in baryon spectra as well as in meson spectra.
- The mass spectra of heavy-light and heavy heavy mesons are determined.
- Some exotic states are also predicted.
- Looking for the new results from future experiments.

# Acknowledgement

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Conclusion and Future Work

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