

# Transition magnetic moments of Baryons in hot and dense matter

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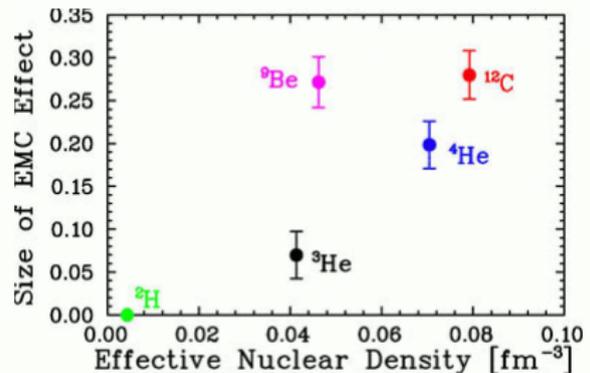
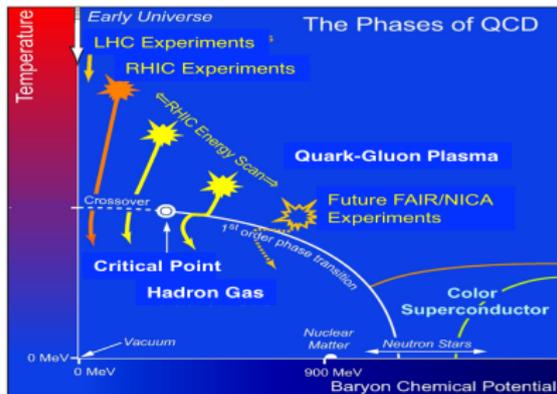
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# Introduction

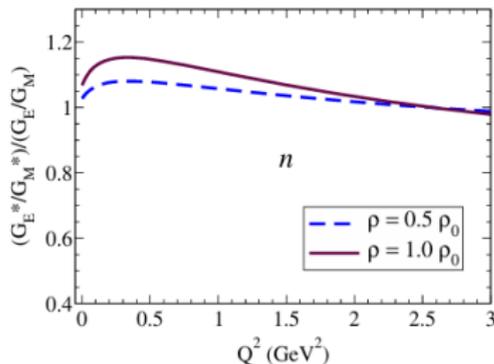
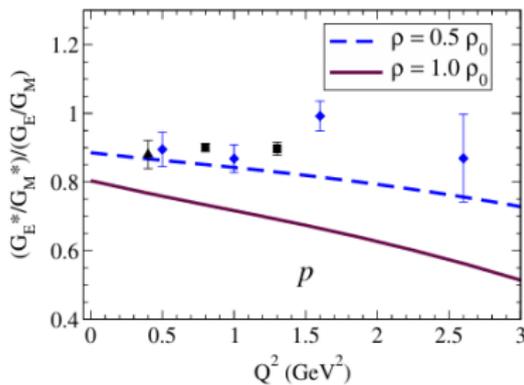
- Understanding different regimes of QCD Phase diagram and exploring the nature of phase transitions is one of the open questions in QCD.
- Experimental facilities like LHC, RHIC, NICA, BES-I and BES-II explore QCD phase diagram at different densities and temperatures.
- EMC effect as observed by European Muon Collaboration provided an evidence that the properties of nucleons are modified in the nuclear medium (Phys. Rev. Lett. 103 (2009) 202301 ).



# Electromagnetic Properties of Baryons

- Electromagnetic properties such as **magnetic moments** and **form factors** provide crucial insights into hadron structure and hence the dynamics of strong interactions.
- They serve as sensitive probes of the **non-perturbative regime of QCD**, where quark confinement and strong correlations dominate.
- Medium modifications of baryon properties reveal how quark-gluon dynamics change inside nuclear matter.

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# Transition Magnetic Moments (TMMs) and Hadron Structure

- TMMs describe processes like  $B^* \rightarrow B + \gamma$ .
- Sensitive to deformation of baryons (spherical vs. oblate). Example:  $\Delta \rightarrow N$  transition reveals nucleon quadrupole deformation.
- Transitions like  $\Sigma^0 \rightarrow \Lambda \gamma$  may highlight  $SU(3)$  symmetry breaking.
- Magnetic moments of baryons may be influenced by the changes in the internal quark and gluon dynamics.
- Many effective approaches like quark models(constituent, bag, light-front), chiral approaches( $\chi$ QSM, ChPT), QCD sum rules etc. have been used to explore TMMs.
- In present work combined approach of chiral  $SU(3)$  mean field model (CQMF) and  $\chi$ CQM is used.

# Chiral Quark Mean Field Model: Introduction

- Based on consideration of quarks and mesons as degrees of freedom.
- Baryons are formed via the confinement of quarks by an effective potential ( $\mathcal{L}_c = \frac{1}{4}k_c r^2(1 + \gamma_0)$ ).
- Interactions among quarks are mediated through  $\sigma$ ,  $\zeta$  and  $\delta$  fields.
- The vector fields ( $\omega$ ,  $\rho$ ,  $\phi$ ) are also incorporated which represent repulsive interactions.
- The total effective Lagrangian density in chiral SU(3) model is :

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{q0} + \mathcal{L}_{qm} + \mathcal{L}_{VV} + \mathcal{L}_{\Sigma\Sigma} + \mathcal{L}_{\chi SB} + \mathcal{L}_{\Delta m} + \mathcal{L}_c$$

- The term  $\mathcal{L}_{q0}$  represents the free part of mass-less quarks.
- Constituent quarks and mesons obtain their masses through spontaneous symmetry breaking ( $\mathcal{L}_{\chi SB}$ ).

# Masses of quarks in Chiral Mean Field Model

- The Dirac equation for quark field,  $\Psi_q$  under the influence of meson mean field is given as

$$[-i\alpha \cdot \nabla + \chi_c + \beta m_i^*] \Psi_q = e_q^* \Psi_q$$

- The in-medium constituent quark mass ( $m_i^*$ ) and effective quark energy ( $e_i^*$ ) are defined in terms of above fields

$$m_q^* = -g_\sigma^q \sigma - g_\zeta^q \zeta - g_\delta^q I^{3q} \delta + m_{q0}$$

$$e_q^* = e_q - g_\omega^q \omega - g_\rho^q I^{3q} \rho - g_\phi^q \phi$$

- The in-medium mass of a baryon is expressed in terms of its effective energy  $E_i^*$  and spurious center of mass momentum  $p_{icm}$  as,

$$M_i^* = \sqrt{E_i^{*2} - \langle p_{icm}^{*2} \rangle} \text{ and } \langle p_{icm}^{*2} \rangle = \frac{(11e_q^* + m_q^*)}{6(3e_q^* + m_q^*)} (e_q^{*2} - m_q^{*2});$$

- $E_i^* = \sum_q n_{qi} e_q^* + E_{ispin}$  : where  $E_{ispin}$  is a correction term obtained by fit to baryon masses.

# Masses of Baryons in Chiral Mean Field Model

- Thermodynamic potential  $\Omega$  is also considered, in order to study the density and temperature effects on baryon masses

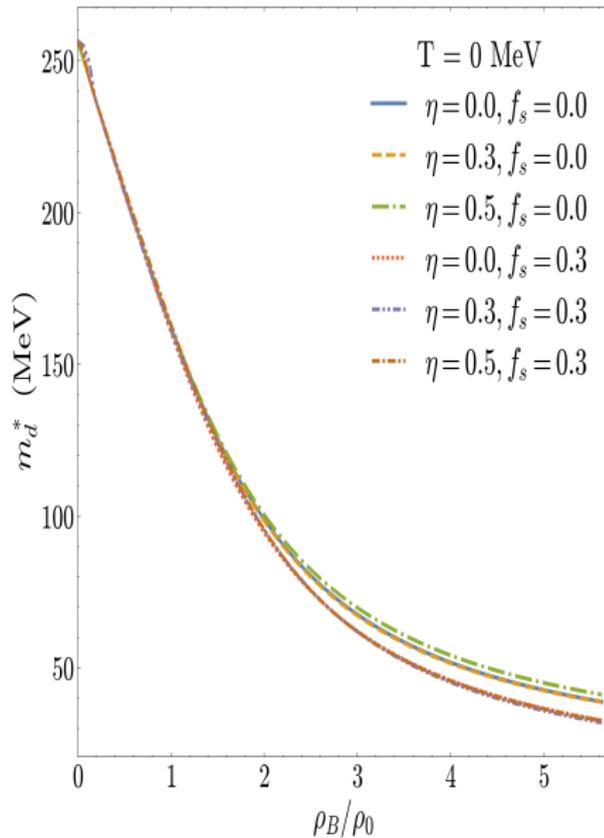
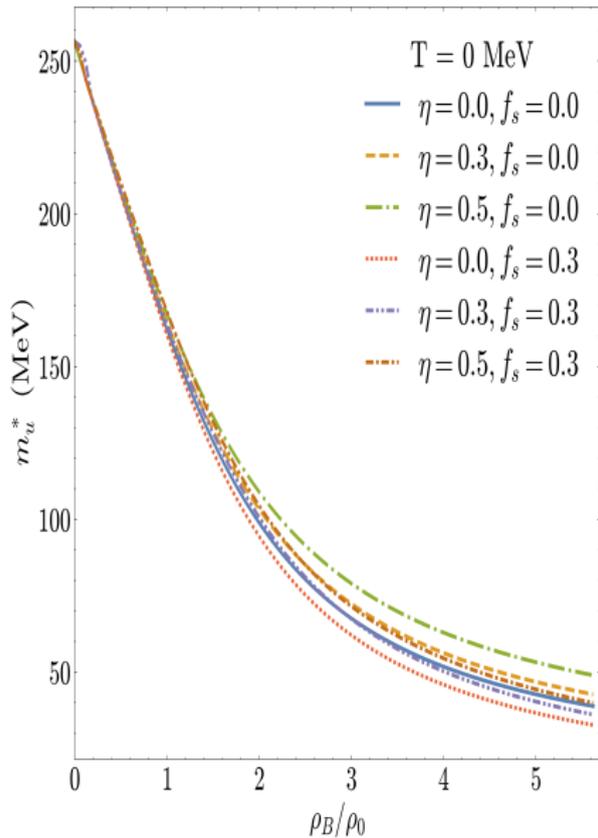
$$-\frac{k_B T}{(2\pi)^3} \sum_i \gamma_i \int_0^\infty d^3 k \left\{ \ln \left( 1 + e^{-\frac{[E_i^*(k) - \nu_i^*]}{k_B T}} \right) + \ln \left( 1 + e^{-\frac{[E_i^*(k) + \nu_i^*]}{k_B T}} \right) \right\} - \mathcal{L}_M - \mathcal{V}_{\text{vac}}$$

- To obtain the density and temperature dependent values of scalar and vector fields, the strange isospin asymmetric thermodynamic potential is minimized with respect to these fields.

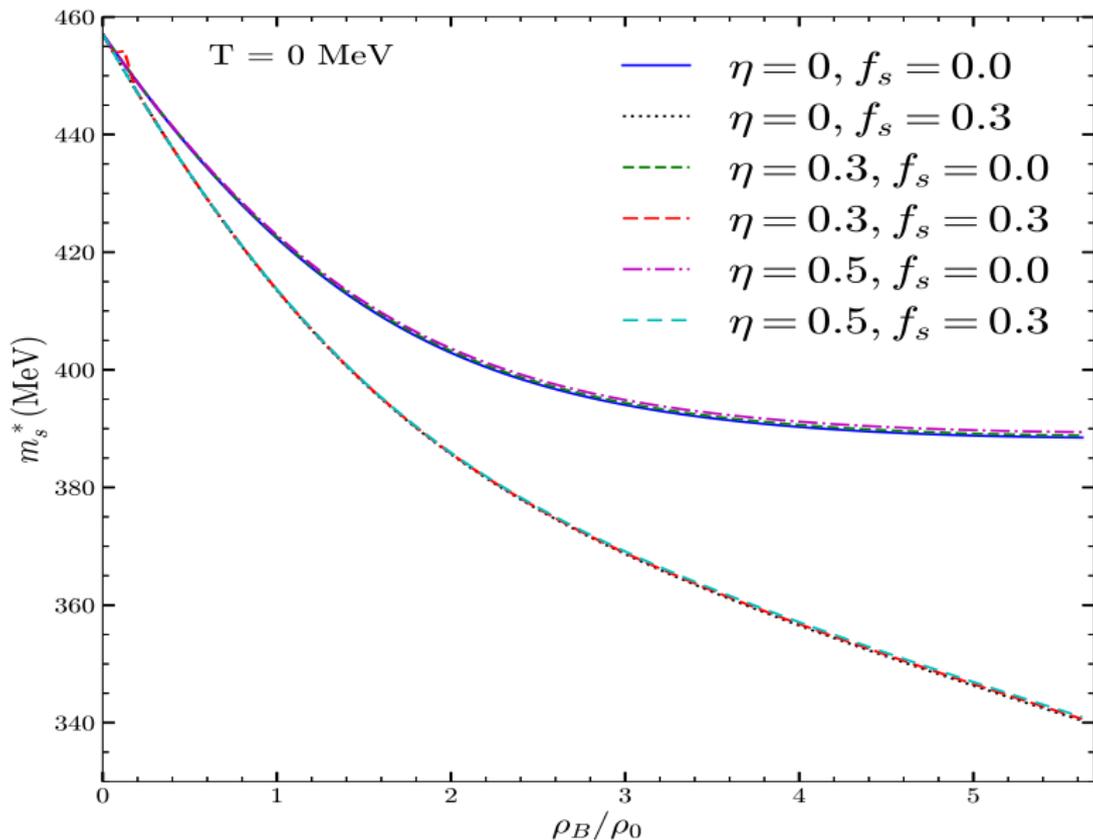
$$\frac{\partial \Omega}{\partial \sigma} = \frac{\partial \Omega}{\partial \zeta} = \frac{\partial \Omega}{\partial \delta} = \frac{\partial \Omega}{\partial \omega} = \frac{\partial \Omega}{\partial \rho} = \frac{\partial \Omega}{\partial \phi} = \frac{\partial \Omega}{\partial \chi} = 0$$

- The above equations are solved for different values of baryonic density ( $\rho_B$ ), temperature, isospin asymmetry ( $\eta$ ) and strangeness fraction ( $f_s$ ) to obtain the values of meson fields and hence the in-medium masses of quarks and baryons.

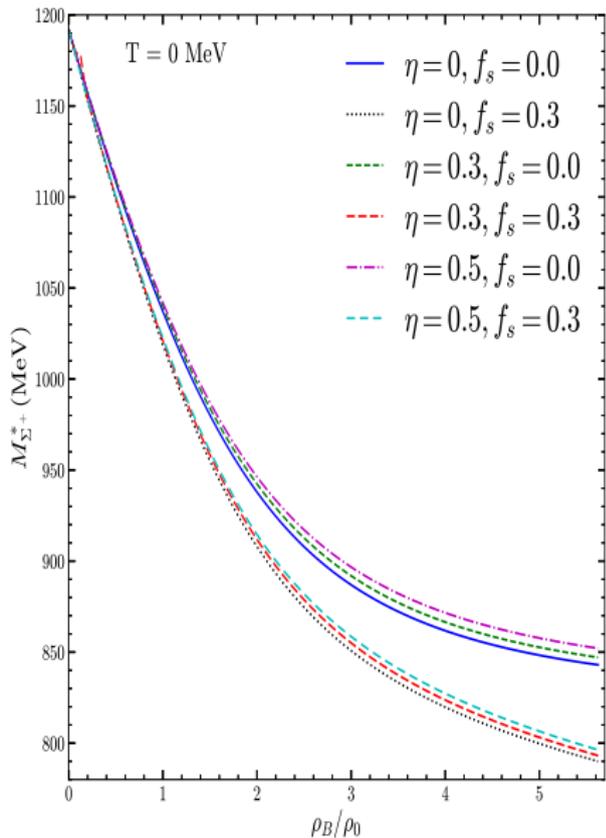
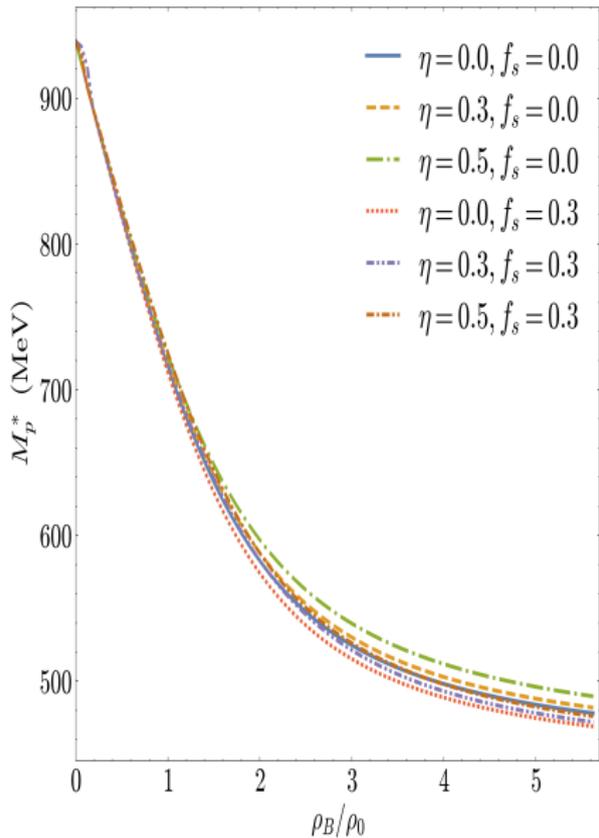
# In-Medium Masses of Light Quarks



# Modification in $s$ -quark mass



# Modification in baryon masses



# Chiral Constituent Quark Model

- $\chi$ CQM initiated by Weinberg and developed by Manohar and Georgi to build upon the successes of Naive Quark Model.
- The fluctuation process describing the effective Lagrangian is

$$q \uparrow\downarrow \rightarrow GB + q' \downarrow\uparrow \rightarrow (q\bar{q}') + q' \downarrow\uparrow$$

$q\bar{q}' + q'$  constitute the sea quarks.

- Incorporates confinement and chiral symmetry breaking.
- “Justifies” the idea of constituent quarks.

- The GB field can be expressed in terms of the GBs and their transition probabilities as  $\Phi' =$

$$\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{P_{\eta\eta}}{\sqrt{6}} + \frac{P_{\eta'\eta'}}{4\sqrt{3}} - \frac{P_D\eta_c}{4} & P_{\pi\pi^+} & P_{KK^+} & P_D\bar{D}^0 \\ P_{\pi\pi^-} & -\frac{P_{\pi\pi^0}}{\sqrt{2}} + \frac{P_{\eta\eta}}{\sqrt{6}} + \frac{P_{\eta'\eta'}}{\sqrt{3}} - \frac{P_D\eta_c}{4} & P_{KK^0} & P_DD^- \\ P_{KK^-} & P_{K\bar{K}^0} & -\frac{2P_{\eta\eta}}{\sqrt{6}} + \frac{P_{\eta'\eta'}}{4\sqrt{3}} - \frac{P_D\eta_c}{4} & P_DD_s^- \\ P_DD^0 & P_DD^+ & P_DD_s^+ & -\frac{3P_{\eta'\eta'}}{4\sqrt{3}} + \frac{3P_D\eta_c}{4} \end{pmatrix}$$

- The chiral fluctuations

$u(d) \rightarrow d(u) + \pi^{+(-)}$ ,  $u(d) \rightarrow s + K^{+(0)}$ ,  $u(d, s) \rightarrow u(d, s) + \eta$ ,  
 $u(d, s) \rightarrow u(d, s) + \eta'$  and  $u(d) \rightarrow c + \bar{D}^0(D^-)$  are given in terms of the transition probabilities  $P_{\pi}$ ,  $P_K$ ,  $P_{\eta}$ ,  $P_{\eta'}$  and  $P_D$  respectively.

# Transition Magnetic Moments

- The transition magnetic moments for the the spin  $\frac{3}{2}^+ \rightarrow \frac{1}{2}^+$  transitions from the radiative decays  $B_i \rightarrow B_f + \gamma$ , where  $B_i$  and  $B_f$  are the initial and final baryons.
- The magnetic moment of a given baryon in the  $\chi$ CQM receives contribution from the valence quark spin, sea quark spin and sea quark orbital angular momentum

$$\mu \left( B_{\frac{3}{2}^+} \rightarrow B_{\frac{1}{2}^+} \right)_{Total} = \mu \left( B_{\frac{3}{2}^+} \rightarrow B_{\frac{1}{2}^+} \right)_V + \mu \left( B_{\frac{3}{2}^+} \rightarrow B_{\frac{1}{2}^+} \right)_S + \mu \left( B_{\frac{3}{2}^+} \rightarrow B_{\frac{1}{2}^+} \right)_O$$

$$\mu \left( B_{\frac{3}{2}^+} \rightarrow B_{\frac{1}{2}^+} \right)_V = \sum_{q=u,d,s} \Delta q \left( \frac{3^+}{2} \rightarrow \frac{1^+}{2} \right)_V \mu q$$

$$\mu \left( B_{\frac{3}{2}^+} \rightarrow B_{\frac{1}{2}^+} \right)_S = \sum_{q=u,d,s} \Delta q \left( \frac{3^+}{2} \rightarrow \frac{1^+}{2} \right)_S \mu q$$

$$\mu \left( B_{\frac{3}{2}^+} \rightarrow B_{\frac{1}{2}^+} \right)_O = \sum_{q=u,d,s} \Delta q \left( \frac{3^+}{2} \rightarrow \frac{1^+}{2} \right)_V \mu(q_+ \rightarrow)$$

- The quark magnetic moments are given as  $\mu_d^* = - \left( 1 - \frac{\Delta M}{M_B^*} \right)$  and  $\mu_u^* = -2\mu_d^*$  in the units of  $\mu_N$  (nuclear magneton).
- $\Delta q \left( \frac{3^+}{2} \rightarrow \frac{1^+}{2} \right)_V$  &  $\Delta q \left( \frac{3^+}{2} \rightarrow \frac{1^+}{2} \right)_S$  represent valance and sea quark spin polarizations respectively.
- $\mu(q_+ \rightarrow)$  is the orbital moment for any chiral fluctuation,  $M_B^*$  is the effective mass of baryon and  $\Delta M = M_B^* - M_{vac}$

- The spin structure of a decuplet to octet transition matrix element is defined as

$$\left\langle B_{\frac{1}{2}^+}, S_z = \frac{1}{2} \left| N(B_{\frac{3}{2}^+} \rightarrow B_{\frac{1}{2}^+}) \right| B_{\frac{3}{2}^+}, S_z = \frac{1}{2} \right\rangle$$

- The number operator measures the number of quarks with spin up ( $\uparrow$ ) or down ( $\downarrow$ ) in the transition ( $\frac{3}{2}^+ \rightarrow \frac{1}{2}^+$ )

$$N(B_{\frac{3}{2}^+} \rightarrow B_{\frac{1}{2}^+}) = \sum_{q=u,d,s} \left( N_{q\uparrow}(B_{\frac{3}{2}^+} \rightarrow B_{\frac{1}{2}^+}) + N_{q\downarrow}(B_{\frac{3}{2}^+} \rightarrow B_{\frac{1}{2}^+}) \right)$$

- The magnetic moment contribution of the angular momentum of a given sea quark

$$\langle L_q \rangle = \frac{M_{GB}}{M_q + M_{GB}} \quad \text{and} \quad \langle L_{GB} \rangle = \frac{M_q}{M_q + M_{GB}}.$$

- The general orbital moment for any quark (q) is given as

$$\mu(q^\uparrow \rightarrow q'^\downarrow) = \frac{e_{q'}}{2M_q} \langle L_q \rangle + \frac{e_q - e_{q'}}{2M_{GB}} \langle L_{GB} \rangle.$$

- The magnetic moment arising from all the possible transitions of a given valence quark to the GBs is obtained by multiplying the orbital moment of each process to the probability for such a process to take place.

- The orbital moments of  $u$ ,  $d$ ,  $s$  and  $c$  quarks after including the transition probabilities  $P_\pi$ ,  $P_K$ ,  $P_\eta$ ,  $P_{\eta'}$  and  $P_D$  as well as the masses of GBs  $M_\pi$ ,  $M_K$ ,  $M_\eta$ ,  $M_{\eta'}$ ,  $M_D$ ,  $M_{D_s}$ , and  $M_{\eta_c}$  can be expressed as (in the units of  $\mu_N$ )

$$[\mu^* (u_\uparrow \rightarrow)] = a \left[ \frac{3m_u^{*2}}{2M_\pi (m_u^* + M_\pi)} - \frac{P_\pi^2 (M_K^2 - 3m_u^{*2})}{2M_K (m_u^* + M_K)} + \frac{P_\eta^2 M_\eta}{6 (m_u^* + M_\eta)} \right. \\ \left. + \frac{P_{\eta'}^2 M_{\eta'}}{48 (m_u^* + M_{\eta'})} + \frac{P_D^2 M_{\eta_c}}{16 (m_u^* + M_{\eta_c})} + \frac{P_D^2 M_D}{m_u^* + M_D} \right] \mu_N,$$

$$[\mu^* (d_\uparrow \rightarrow)] = a \frac{m_u^*}{m_d^*} \left[ \frac{3 (M_\pi^2 - 2m_d^{*2})}{4M_\pi (m_d^{*2} + M_\pi)} - \frac{P_\pi^2 M_K}{2 (m_d^* + M_K)} + \frac{P_D^2 (2M_D^2 - 3m_d^*)}{2M_D (m_d^* + M_D)} \right. \\ \left. - \frac{P_\eta^2 M_\eta}{12 (m_d^* + M_\eta)} - \frac{P_{\eta'}^2 M_{\eta'}}{96 (m_d^* + M_{\eta'})} + \frac{P_D^2 M_{\eta_c}}{32 (m_d^* + M_D)} \right] \mu_N,$$

$$[\mu^* (s_{\uparrow} \rightarrow)] = a \frac{m_u^*}{m_s^*} \left[ \frac{P_{\pi}^2 (M_K^2 - 3m_s^{*2})}{2M_K (m_s^* + M_K)} - \frac{P_{\eta}^2 M_{\eta}}{3(m_s^* + M_{\eta})} + \frac{P_D^2 (2M_{D_s}^2 - 3m_s^{*2})}{2M_D (m_s^* + M_{D_s}^2)} \right. \\ \left. - \frac{P_{\eta'}^2 M_{\eta'}}{96 (m_s^* + M_{\eta'})} - \frac{P_D^2 M_{\eta_c}}{32 (m_s^* + M_D)} \right] \mu_N,$$

$$[\mu^* (c_{\uparrow} \rightarrow)] = a \frac{m_u^*}{m_c} \left[ \frac{P_D^2 (M_D^2 + 3m_c^2)}{2M_D (m_c + M_D^2)} - \frac{P_D^2 (M_{D_s}^2 + 3m_c^2)}{2M_d (m_c + M_{D_s}^2)} \right. \\ \left. + \frac{P_{\eta'}^2 M_{\eta'}}{16 (m_c + M_{\eta'})} + \frac{9P_D^2 M_{\eta_c}}{16(m_c + M_{\eta_c})} \right] \mu_N$$

# Valence and sea transition magnetic moments for $\Delta \rightarrow p$ transitions

- Valence contribution is given as

$$\mu(\Delta \rightarrow p)_V = \frac{2\sqrt{2}}{3}\mu_u^* - \frac{2\sqrt{2}}{3}\mu_d^*$$

- Sea contribution is given as

$$-\frac{2\sqrt{2}}{3}a \left[ 1 + P_\pi^2 + \frac{P_\eta^2}{3} + \frac{P_{\eta'}^2}{24} + \frac{17P_D^2}{16} \right] \mu_u^*$$
$$+\frac{2\sqrt{2}}{3}a \left[ 1 + P_\pi^2 + \frac{P_\eta^2}{3} + \frac{P_{\eta'}^2}{24} + \frac{17P_D^2}{16} \right] \mu_d^*$$

- The orbital contribution to the magnetic moment of the decuplet to octet transition  $\mu \left( B_{\frac{3}{2}}^+ \rightarrow B_{\frac{1}{2}}^+ \right)$  for the baryon of the type  $B(Q_1 Q_2 Q_3)$  is

$$\begin{aligned}
 B(Q_1 Q_2 Q_3) = & \Delta Q_1 \left( \frac{3^+}{2} \rightarrow \frac{1^+}{2} \right)_V \mu(Q_1^\uparrow \rightarrow) \\
 & + \Delta Q_2 \left( \frac{3^+}{2} \rightarrow \frac{1^+}{2} \right)_V \mu(Q_2^\uparrow \rightarrow) \\
 & + \Delta Q_3 \left( \frac{3^+}{2} \rightarrow \frac{1^+}{2} \right)_V \mu(Q_3^\uparrow \rightarrow).
 \end{aligned}$$

Orbital Contribution is given as

$$\frac{2\sqrt{2}}{3} [\mu_u^* (u^\uparrow \rightarrow) - \mu_d^* (d^\uparrow \rightarrow)]$$

- Input parameters: transition probabilities  $P_\pi$ ,  $P_K$ ,  $P_\eta$ ,  $P_{\eta'}$ ,  $P_D$  and masses of GBs  $M_\pi, M_K, M_\eta, M_{\eta'}, M_{\eta_c}$ .
- Hierarchy followed by the probabilities of fluctuations of a constituent quark into pions,  $K$ ,  $\eta$ ,  $\eta'$  and  $D$  mesons.

$$P_D < P_{\eta'} < P_\eta < P_K < P_\pi$$

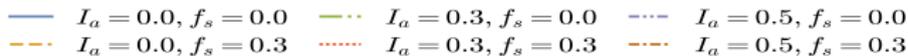
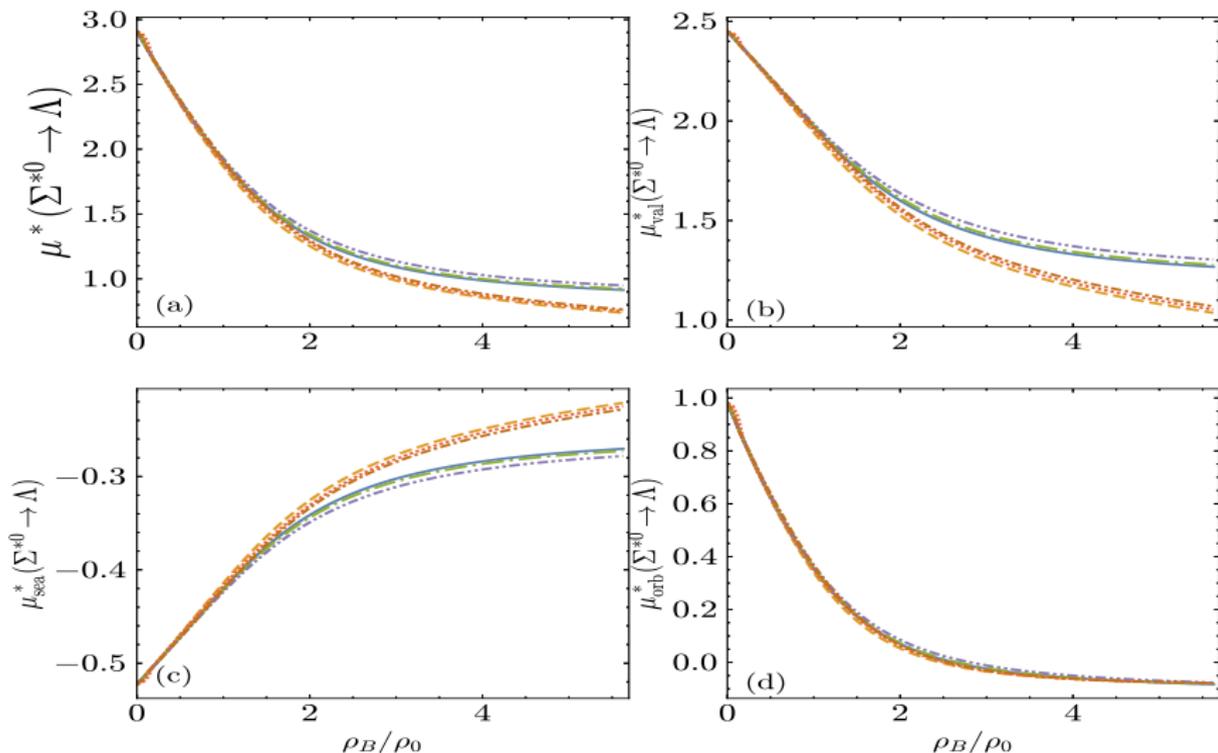
- The transition probabilities are fixed by the experimentally known spin and flavor distribution functions measured from the DIS experiments. A detailed analysis leads to the following probabilities:

$$P_D = 0.01, P_{\eta'} = 0.03, P_\eta = 0.04, P_K = 0.06, P_\pi = 0.12$$

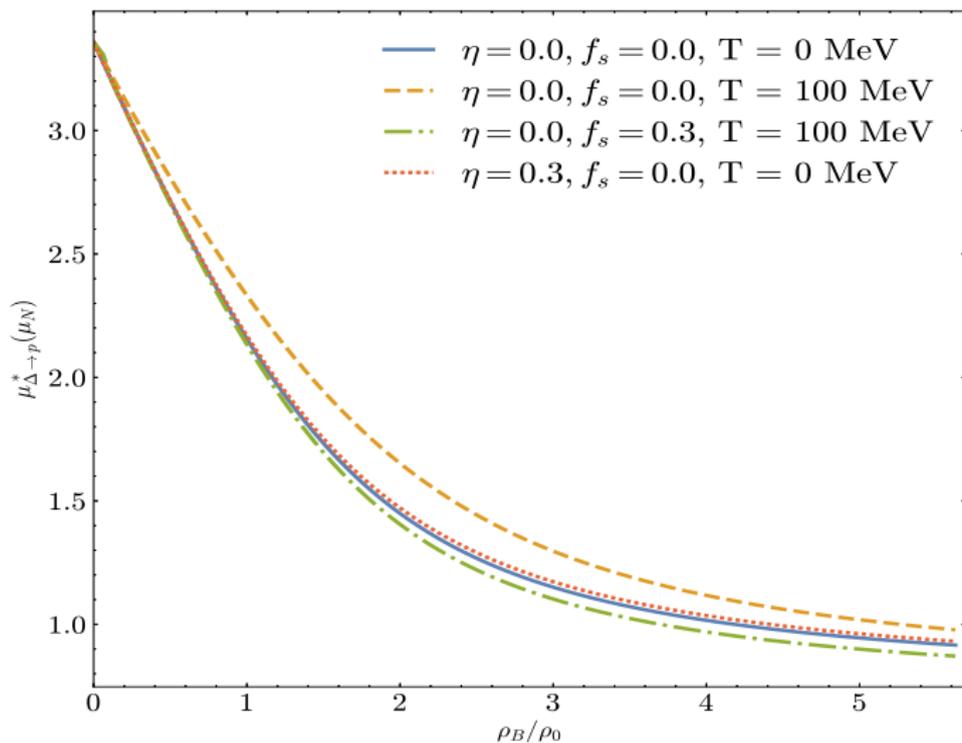
- The on mass shell mass values are used for the orbital angular momentum contributions characterized by the masses of GBs ( $M_{GB}$ ).



# Magnetic moments in for $\Sigma^{*0} \rightarrow \Lambda$ transitions



# Magnetic moments in for $\Delta \rightarrow p$ transitions at High Temperature



# Summary

- Variation of Effective masses of quarks and hadrons with density studied for symmetric and asymmetric nuclear matter along-with in the asymmetric strange matter.
- Masses are found to decrease with density with sharp rate of decrease at lower density values.
- The increase in strangeness fraction decreases the mass while enhancement in the isospin asymmetry increases the mass at fixed density.
- The transition magnetic moments also studied under the same conditions.
- The effective transition magnetic moments decrease with the increase in density. At the fixed density, with  $\eta$  magnetic moments decrease while with  $f_5$  an increase is observed.
- The temperature also have a significant impact on the transition magnetic moments.

# Thank You!

