

# Higher-order thermoelectric effects in hot and dense QCD matter in the presence of a magnetic field

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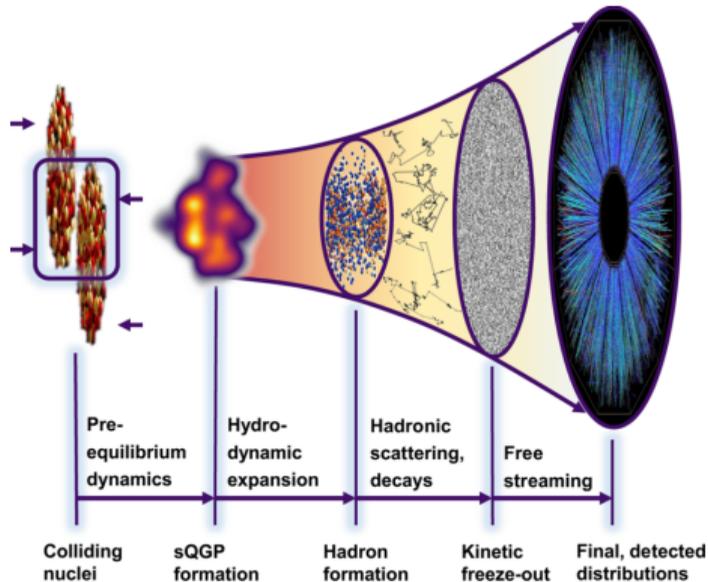


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# Dynamics of heavy-ion collisions

Heavy-ion collisions act as a “**Little Bang**” generating a fireball of quarks and gluons that mimics the early universe microseconds after the Big Bang.



## Evolution of HICs

- Collision of two Lorentz contracted nuclei
- Pre-equilibrium stage
- Quark-gluon plasma phase
- Chemical freeze out
- Hadron gas phase
- Kinetic freeze out
- Free streaming

Dániel Kincses, Communications Physics. 8. 10.1038/s42005-025-01973-x.

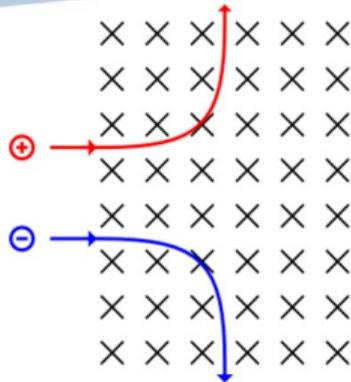




# Motivation and objectives of my current work

Study the off-equilibrium dynamics of QCD matter via its leading and higher-order thermoelectric coefficients with and without a magnetic field.

## Charged Particles in a Magnetic Field

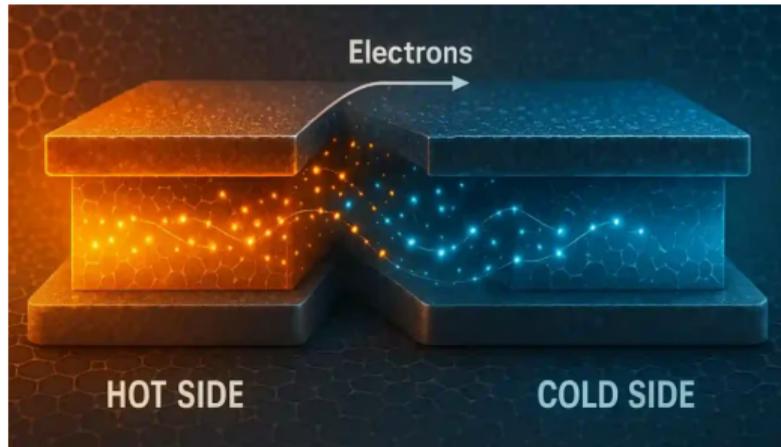


- QCD matter is composed of both **charged and neutral** particles
- In the **QGP** phase, quarks are charged and gluons are neutral
- In the **HRG** phase, both charged and neutral hadrons are there
- Charged constituents experience Lorentz force due to the spectator-induced magnetic field
- Hence, it affects the off-equilibrium dynamics of the QCD matter



# Concept of thermoelectricity

A medium composed of **charged particles** exhibits thermoelectric behavior in the presence of temperature gradients.



[chemengcalc.com/thermoelectric-materials-electricity-from-waste/](http://chemengcalc.com/thermoelectric-materials-electricity-from-waste/)

- The **gradients of temperature** in a system lead to the transportation of heat.
- If this transportation of heat is conducted by charged particles, the net electric current created in the system (**provided-unequal number of positive and negative charges**) due to the diffusion of charged particles from the **hotter to the colder** region.



# Various thermoelectric phenomenon

Input Configuration	Heat current	Charge current	Heat and charge currents
Longitudinal	<b>a</b> Seebeck effect (1821) 	<b>b</b> Peltier effect (1834) 	<b>c</b> Thomson effect (1856) 
	<b>d</b> Magneto-Seebeck effect (1950s) 	<b>e</b> Magneto-Peltier effect (1950s) 	<b>f</b> Magneto-Thomson effect (2020) 
	<b>g</b> Nernst effect (1886) 	<b>h</b> Ettingshausen effect (1887) 	<b>i</b> Transverse Thomson effect (2025) 
Transverse			

<sup>0</sup>A. Takahagi et al. Nat. Phys. (2025).<https://doi.org/10.1038/s41567-025-02936-3>



# Formalism - Boltzmann transport equation

- **Total single particle distribution function**  $f_i = f_i^0 + \delta f_i$ . Where,  $f_i^0$  is equilibrium distribution function and  $\delta f_i$  represents the deviation of equilibrium
- To find the expression of  $\delta f_i$ , we solve the **Boltzmann transport equation** with the help of relaxation time approximation

$$\frac{\partial f_i}{\partial \tau} + \frac{\vec{k}_i}{\omega_i} \cdot \frac{\partial f_i}{\partial \vec{x}_i} + q_i \left( \vec{E} + \frac{\vec{k}_i}{\omega_i} \times \vec{B} \right) \cdot \frac{\partial f_i}{\partial \vec{k}_i} = -\frac{\delta f_i}{\tau_R^i}.$$

$\tau_R^i \rightarrow$  relaxation time,  $\omega_i \rightarrow$  energy and  $k_i \rightarrow$  momentum of the  $i$ th particle.

- **Ansatz**  $\rightarrow \delta f_i = (\vec{k}_i \cdot \vec{\Omega}) \frac{\partial f_i^0}{\partial \omega_i}$
- $\vec{\Omega} = \alpha_1 \vec{E} + \alpha_2 \vec{B} + \alpha_3 (\vec{E} \times \vec{B}) + \alpha_4 \vec{\nabla} T + \alpha_5 (\vec{\nabla} T \times \vec{B}) + \alpha_6 (\vec{\nabla} T \times \vec{E})$ .



# Formalism - Thermoelectricity (In the absence of a magnetic field)

- In kinetic theory, **electric current density** for such a system can be expressed as  $\vec{j} = \sum_i q_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i}{\omega_i} \delta f_i$  Here,  $q_i \rightarrow$  the electric charge, and  $g_i \rightarrow$  the degeneracy of the  $i$ th species particles.

- A general form of unknown vector  $\vec{\Omega}$  can be assumed as  $\vec{\Omega} = \alpha_1 \vec{E} + \alpha_2 \vec{\nabla} T$

- Using the expressions of  $\delta f_i$ , we can express electric current as

$$\vec{j} = \sum_i \frac{q_i g_i}{3} \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} v_i^2 \tau_R^i \left[ -q_i \vec{E} + \frac{(\omega_i - b_i \hbar)}{T} \vec{\nabla} T \right] \frac{\partial f_i^0}{\partial \omega_i}$$

- For a **open circuit** system i.e.  $\vec{j} = 0$ , we get  $\vec{E} \propto \vec{\nabla} T$

- The constant of proportionality is **Seebeck coefficient**  $S$  i.e.  $\vec{E} = S \vec{\nabla} T$



# Seebeck and Thomson coefficients

Finally, the expression for the **Seebeck coefficient** is:

$$S = \frac{\sum_i \frac{g_i}{3T} \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \tau_R^i q_i \left(\frac{\vec{k}_i}{\omega_i}\right)^2 (\omega_i - b_i h) f_i^0 (1 \mp f_i^0)}{T \sum_i \frac{g_i}{3T} \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \tau_R^i q_i^2 \left(\frac{\vec{k}_i}{\omega_i}\right)^2 f_i^0 (1 \mp f_i^0)}$$

The **Thomson coefficient** describes the continuous absorption or release of heat in the charge-carrying medium in the presence of temperature gradients, which remains largely unexplored in QCD matter.

$$Th = T \frac{dS}{dT}.$$

The above relation is usually known as the first Thomson relation.



# Magneto-Seebeck coefficient

$$S_B = \frac{\sum_i q_i^2 H_{1i} \sum_i q_i H_{3i} + \sum_i q_i^2 H_{2i} \sum_i q_i H_{4i}}{T \left[ \left( \sum_i q_i^2 H_{1i} \right)^2 + \left( \sum_i q_i^2 H_{2i} \right)^2 \right]}$$

where,

$$H_{1i} = \frac{g_i}{3T} \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} f_i^0 (1 - f_i^0) \tau_R^i \times \frac{(1 + \chi_i^2) + \chi_i(2 + \chi_i)}{(1 + \chi_i)(1 + \chi_i^2)(1 + \chi_i + \chi_i^2)},$$

$$H_{2i} = \frac{g_i}{3T} \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} f_i^0 (1 - f_i^0) \tau_R^i \times \chi_i \frac{(1 + \chi_i)(1 + \chi_i^2) + \chi_i(2 + \chi_i)}{(1 + \chi_i)(1 + \chi_i^2)(1 + \chi_i + \chi_i^2)},$$

$$H_{3i} = \frac{g_i}{3T} \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i h) f_i^0 (1 - f_i^0) \tau_R^i \times \frac{1}{(1 + \chi_i + \chi_i^2)},$$

$$H_{4i} = \frac{g_i}{3T} \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i h) f_i^0 (1 - f_i^0) \tau_R^i \times \frac{\chi_i}{(1 + \chi_i + \chi_i^2)} ; \chi_i = \frac{\tau_R^i}{\tau_B}$$



## Normalized Nernst coefficient

The Hall-like component in presence of magnetic field is:

$$NB = \frac{\sum_i q_i^2 H_{1i} \sum_i q_i H_{4i} - \sum_i q_i^2 H_{2i} \sum_i q_i H_{3i}}{T \left[ \left( \sum_i q_i^2 H_{1i} \right)^2 + \left( \sum_i q_i^2 H_{2i} \right)^2 \right]}$$

Here, we can define the **transverse Thomson coefficient** as

$$Th_N = T \frac{d(NB)}{dT} + 2NB.$$

In the absence of a magnetic field, the coefficient  $Th_N$  vanishes because of the vanishing  $NB$



## Quasiparticle model

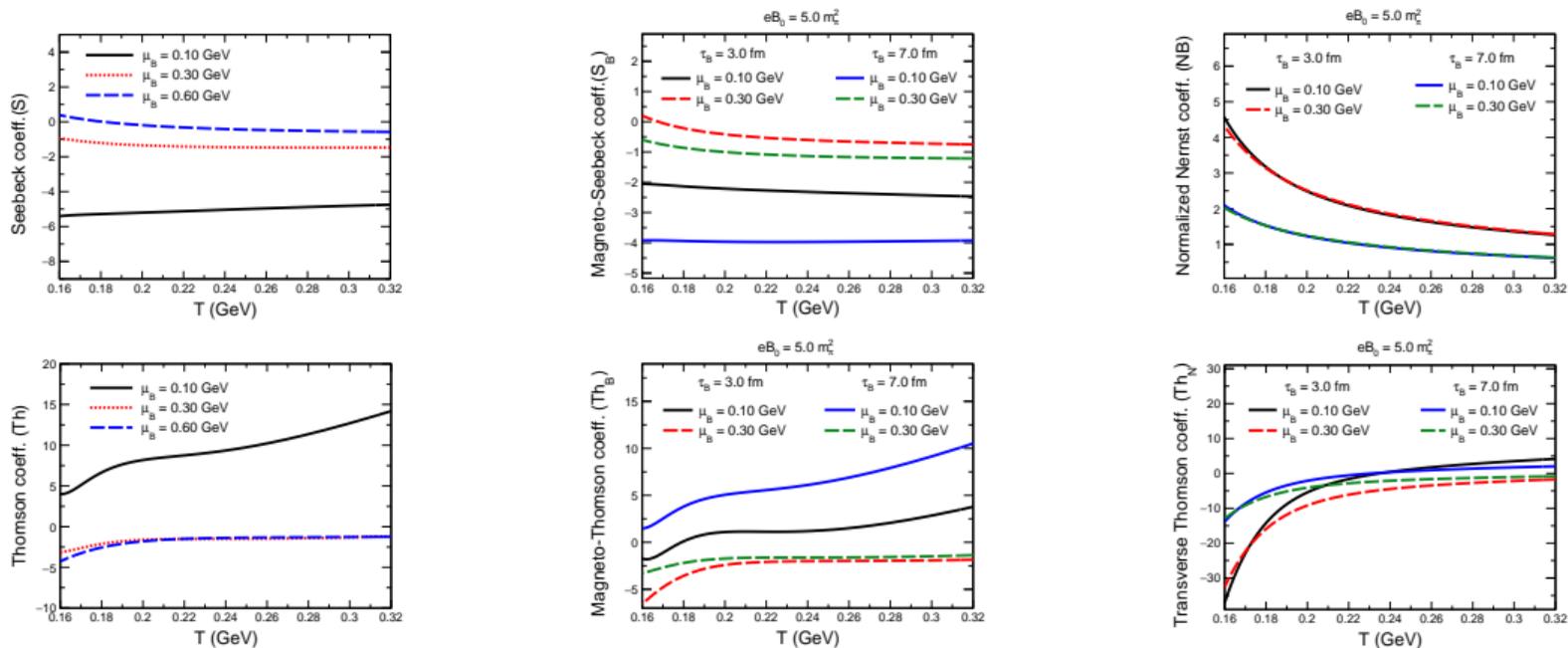
- Describes the QGP phase using effective quasi-particles (quarks and gluons) with thermal masses.
- Captures interaction effects by modifying dispersion relations.
- Successfully reproduces lattice QCD thermodynamics above  $T_c$ .
- Useful for transport coefficient calculations in the deconfined phase.

## Hadron Resonance Gas model

- **IHRG:** Non-interacting gas of hadrons and resonances.
- **EV-HRG:** Includes finite-size (hard-core) corrections.
- **VDW-HRG:** Adds attractive and repulsive interactions (van der Waals approach).
- **RMF-HRG:** Accounts for baryon interactions via meson exchange mean-fields.



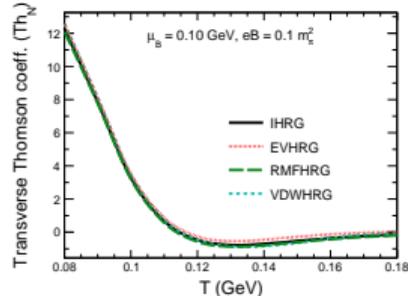
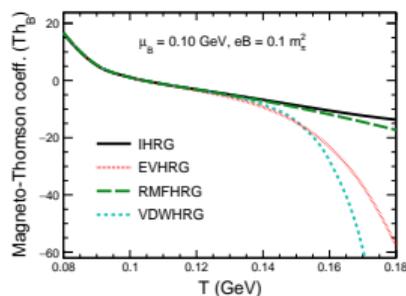
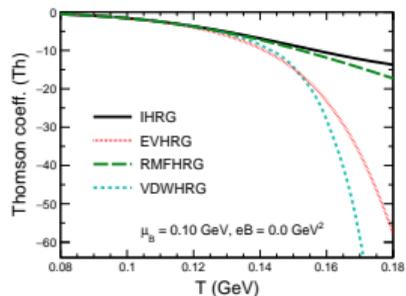
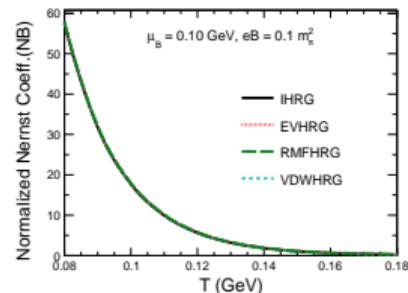
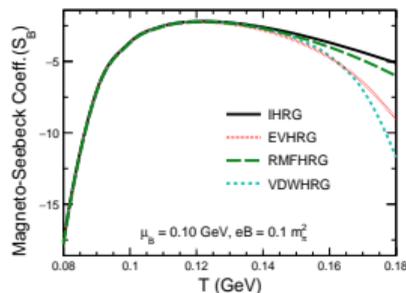
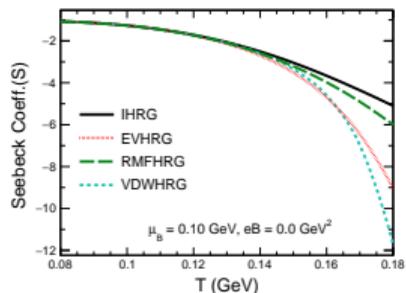
# Results: For QGP phase



**Left column:** Seebeck ( $S$ ), and Thomson coefficients ( $Th$ ); **Middle column:** magneto-Seebeck ( $S_B$ ), and magneto-Thomson coefficients ( $Th_B$ ); **Right column:** Nernst ( $NB$ ), and transverse Thomson coefficients ( $Th_N$ ) as a function of temperature ( $T$ ) for different values of  $\mu_B = 0.10, 0.30,$  and  $0.60$  GeV.

<sup>0</sup>K. Singh, and R. Sahoo, Phys. Rev. D 112, 034032 (2025). <https://doi.org/10.1103/lx5-43qg>

# Results: For HRG phase



**Left column:** Seebeck ( $S$ ), and Thomson coefficients ( $Th$ ); **Middle column:** magneto-Seebeck ( $S_B$ ), and magneto-Thomson coefficients ( $Th_B$ ); **Right column:** Nernst ( $NB$ ), and transverse Thomson coefficients ( $Th_N$ ) as a function of temperature ( $T$ ) at  $\mu_B = 0.10$  GeV.

<sup>0</sup>K. Singh, K. K. Pradhan and R. Sahoo. arXiv:2506.22086

# Summary

- **Spectators** in the non-central heavy-ion collisions produces the **magnetic field**
- This magnetic field creates **Hall-like** the transport properties of produced medium
- **Thermoelectric properties** get significantly affected by the presence of magnetic field
- Temperature dependence of leading order thermoelectric coefficients gives rise to higher order thermoelectric coefficients
- It is the first time that we have studied **higher-order** thermoelectric coefficients in QCD matter



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