

Neutron star matter equation of state from the Bayesian analysis



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arXiv: [2505.02618](https://arxiv.org/abs/2505.02618) (under review) and arXiv: [2505.18888](https://arxiv.org/abs/2505.18888) (accepted in PRD)

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Introduction: Neutron star



• Compact stars, (NS/HS)

- ❖ Macroscopic properties of such NS like mass, radius, moment of inertia, tidal deformability depend on the equation of state of matter of NS.

$$\frac{dp}{dr} = -(\epsilon + p) \frac{m + 4\pi r^3 p}{r(r - 2m)}, \quad \frac{dm}{dr} = 4\pi r^2 \epsilon$$

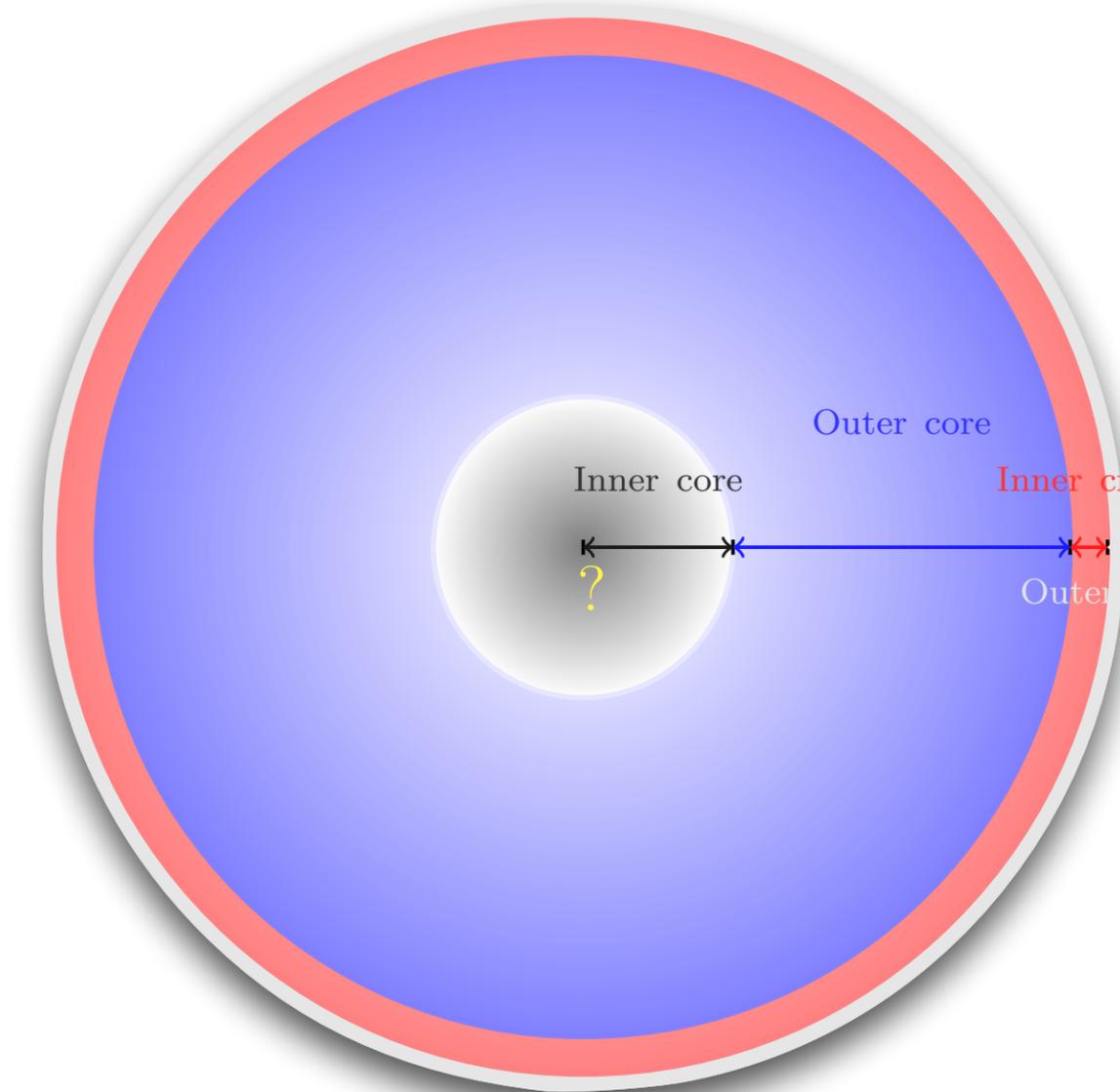
The boundary conditions:

JCAP 02 (2023) 015

$$m(0) = p(R) = 0, \quad p(r = 0) = p_0,$$

- ❖ Astrophysical observations:

1. PSR J 1614-2230 $M = 1.97 \pm 0.04 M_{\odot}$ ApJ 8,32,167 (2016)
2. PSR J0348+0432 $M = 2.01 \pm 0.04 M_{\odot}$ Science 340, 1233232
3. PSR J1810+1744 $M = 2.13 \pm 0.07 M_{\odot}$ Apj L 908 L46 (2021)
4. PSR J0740+6620 $R = 13.7^{+2.6}_{-1.5}$ km 2105.06980



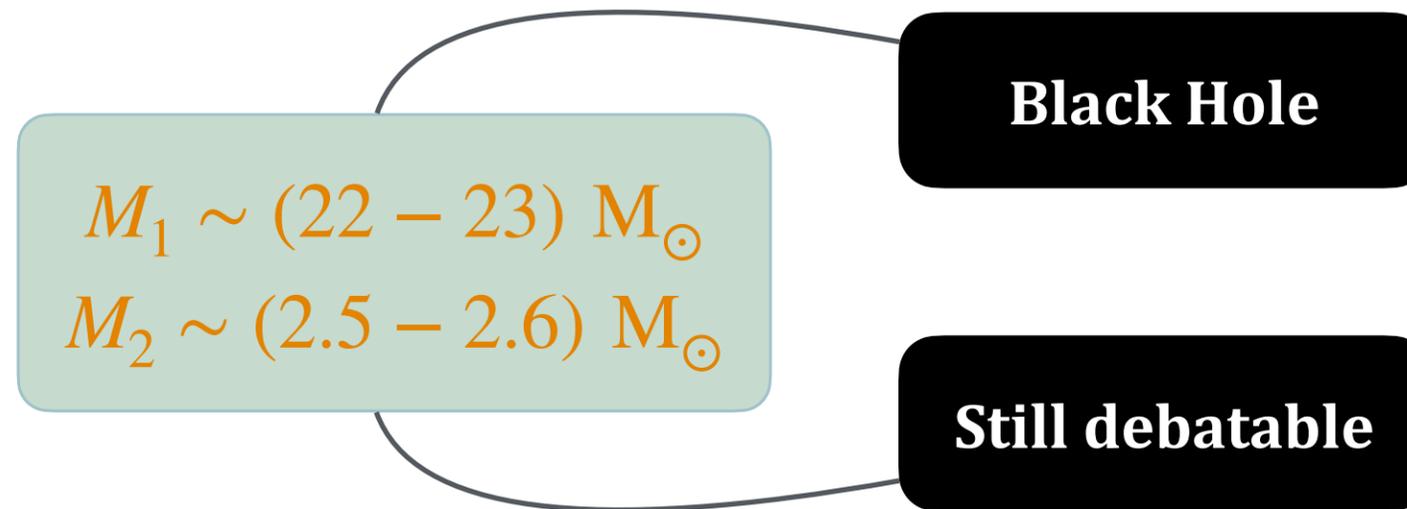
$$M \sim 2 M_{\odot} \quad \text{and} \quad R \sim 10 \text{ km}$$

$$\rho_c \sim (5 - 10) \rho_0$$

$$T_{\text{PNS}} \sim (50 - 100) \text{ MeV}$$



Recent observation (Binary merger): GW190814



Abbott et al., *Astrophys. J. Lett.*, 892(1):L3, 2020

Abbott et al., *Astrophys. J. Lett.*, 896(2):L44, 2020

- ❖ It falls in the so-called mass-gap between heaviest neutron star and lightest known black hole.
- ❖ Astrophysical Implications: (i) If it's a neutron star, it places stringent constraints on the equation of state (EoS) of dense matter, (ii) If it's a black hole, it challenges current formation models

Gives challenges to revisit the theoretical models for high-mass NS's equations of states i.e. $p = p(\epsilon)$.

Introduction: Equation of state



✓ Walecka's mean field model

- ❖ The relativistic mean field (RMF) model is one of the most widely used frameworks for describing dense nuclear matter, where nucleons interact via meson exchange.

scalar meson exchange \Rightarrow *Attraction* while vector meson exchange \Rightarrow *Repulsion*

$$\mathcal{L} = \sum_i \bar{\Psi}_i \left(i\gamma_\mu \partial^\mu - m_i + g_\sigma \sigma - g_\omega \gamma_\mu \omega^\mu - g_\rho \gamma_\mu \vec{I}_i \vec{\rho}^\mu \right) \Psi_i + \mathcal{L}_{\text{mes}}$$

$$\mathcal{L}_{\text{mes}} = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{4} \Omega^{\mu\nu} \Omega_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \vec{R}^{\mu\nu} \vec{R}_{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \vec{\rho}^\mu$$

$$+ \frac{\kappa}{3!} (g_{\sigma N} \sigma)^3 + \frac{\lambda}{4!} (g_{\sigma N} \sigma)^4 - \frac{\xi}{4!} (g_{\omega N}^2 \omega_\mu \omega^\mu)^2 - \Lambda_{\omega\rho} (g_{\omega N}^2 \omega_\mu \omega^\mu) (g_{\rho N}^2 \rho_\mu \rho^\mu)$$

$$\epsilon = -\gamma \sum_{i=n,p,e} \int \frac{d^3\mathbf{k}}{(2\pi)^3} E_i^* (1 - f_-^i - f_+^i)$$

$$+ \frac{1}{2} m_\sigma^2 \sigma_0^2 + \frac{1}{3} \kappa \sigma_0^3 + \frac{1}{4} \lambda \sigma_0^4 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_0^2 + \frac{\xi}{8} (g_{\omega N} \omega)^4 + 3\Lambda_{\omega\rho} (g_{\omega N} g_{\rho N} \omega_0 \rho_0^3)^2$$

and,

$$p = \sum_{i=n,p,e} \mu_i n_i - \epsilon$$

The predictive power of the RMF approach crucially depends on the choice of these coupling constants, which are usually calibrated to reproduce nuclear saturation properties and finite nuclei observables.

Introduction: Bayesian inference



To systematically quantify uncertainties in the coupling constants and propagate them to macroscopic observables (mass–radius relations, tidal deformabilities, etc.), **Bayesian inference** offers a powerful framework.

☑ Bayesian analysis:

$$P(\theta | \mathbf{D}) = \frac{\mathcal{L}(\mathbf{D} | \theta) \mathbf{P}(\theta)}{\mathcal{Z}}$$

- ❖ where θ and \mathbf{D} denote the set of model parameters and the fit data.
- ❖ $\mathbf{P}(\theta)$ is the prior for the model parameters and \mathcal{Z} is the evidence.
- ❖ The $\mathbf{P}(\theta | \mathbf{D})$ is the joint posterior distribution of the parameters,
- ❖ $\mathcal{L}(\mathbf{D} | \theta)$ is the likelihood

**Prior
distribution**

based on physical considerations (e.g., nuclear saturation constraints, symmetry energy bounds)

Likelihood

constructed using experimental and observational data (finite nuclei properties, astrophysical measurements such as neutron star masses and radii)



☑ Nuclear saturation properties (Numerical values):

Saturation density, $\rho_0 = 0.16 \pm 0.02 \text{ fm}^{-3}$

Binding energy per nucleon, $BE = -16.5 \pm 0.05 \text{ MeV}$

Incompressibility, $K_0 = 240 \pm 20 \text{ MeV}$

Symmetry energy, $J_0 = 32.5 \pm 1.5 \text{ MeV}$

Slope of Symmetry energy, $L_0 = 40 \pm 20 \text{ MeV}$

Curvature of Symmetry energy, $K_{\text{sym},0} = -100 \pm 50 \text{ MeV}$

Mass of neutron star, $M = 2.01 \pm 0.04 M_\odot$

Radius of Neutron star of Mass $1.4 M_\odot$, $R_{1.4} = 11 \text{ km}$

Maximum mass, $M_{\text{max}} = 2.6 \pm 0.04 M_\odot$

Miller et al., *Astrophys. J. Lett.*, 918(2):L28, 2021

Li et al., *Phys. Rev. C*, 102(4):045807, 2020

Dittmann et al., *Astrophys. J.*, 974(2):295, 2024

Vinas et al., *Symmetry*, 16(2), 2024.

Garg et al., *Prog. Part. Nucl. Phys.*, 101:55–95, 2018

Shlomo et al., *Eur. Phys. J. A*, 30(1):23–30, 2006

Oertel et al., *Rev. Mod. Phys.*, 89(1):015007, 2017

De et al., *Phys. Rev. Lett.*, 121(9):091102, 2018.

☑ The equation of state (EOS) could be written as follows:

$$\epsilon(\rho, \delta) = \epsilon(\rho, 0) + S_{\text{sym}}(\rho) \delta^2$$

$$\delta = \frac{\rho_n - \rho_p}{\rho}$$

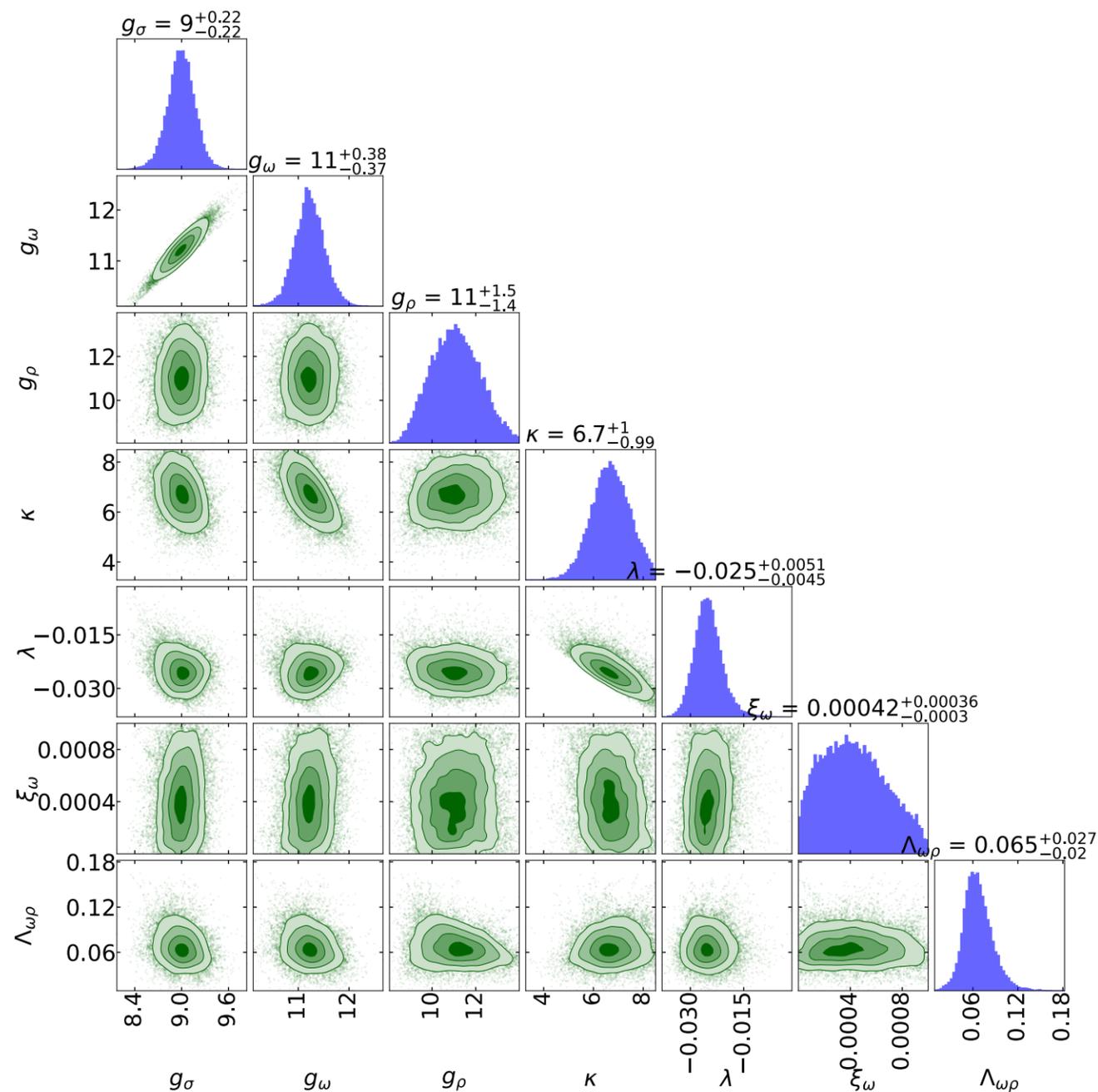
Symmetric matter

Symmetry energy

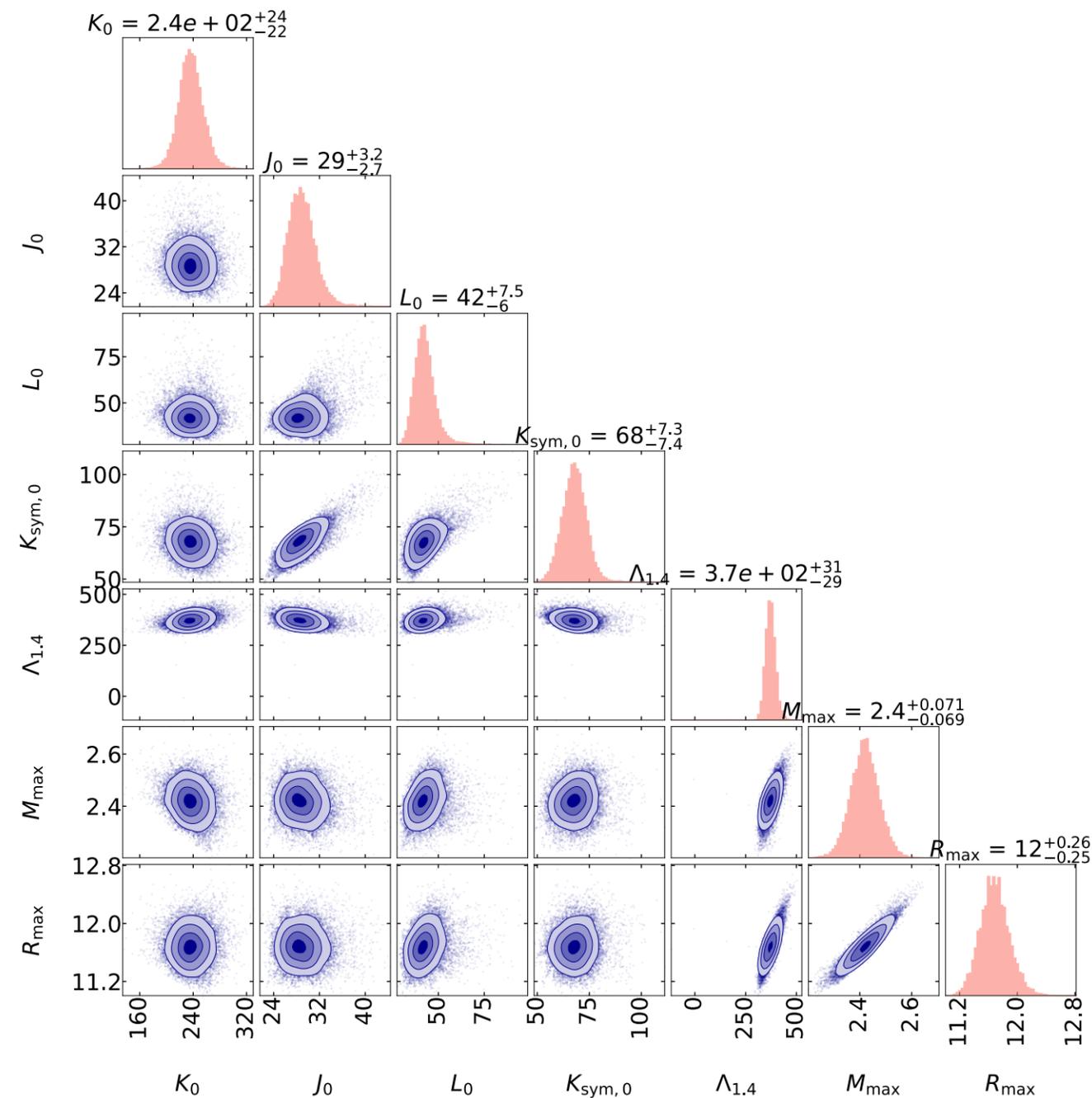
$$X_0^{(n)} = 3^n \rho_0^n \left(\frac{\partial^n \epsilon(\rho, 0)}{\partial \rho^n} \right)_{\rho_0}, \quad n = 2, 3, 4;$$

$$X_{\text{sym},0}^{(n)} = 3^n \rho_0^n \left(\frac{\partial^n S(\rho)}{\partial \rho^n} \right)_{\rho_0}, \quad n = 1, 2, 3, 4.$$

Results: Bayesian analysis

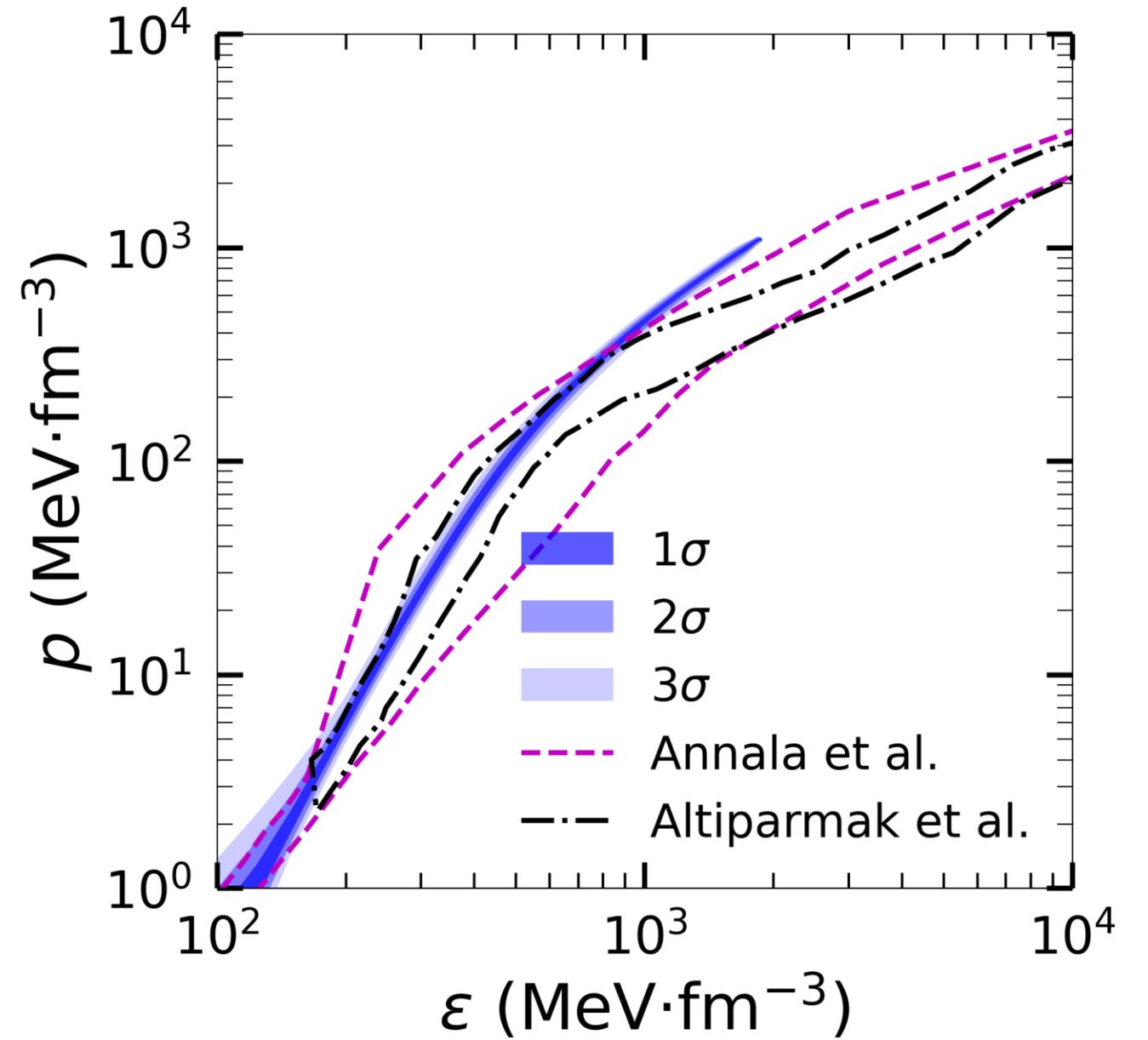
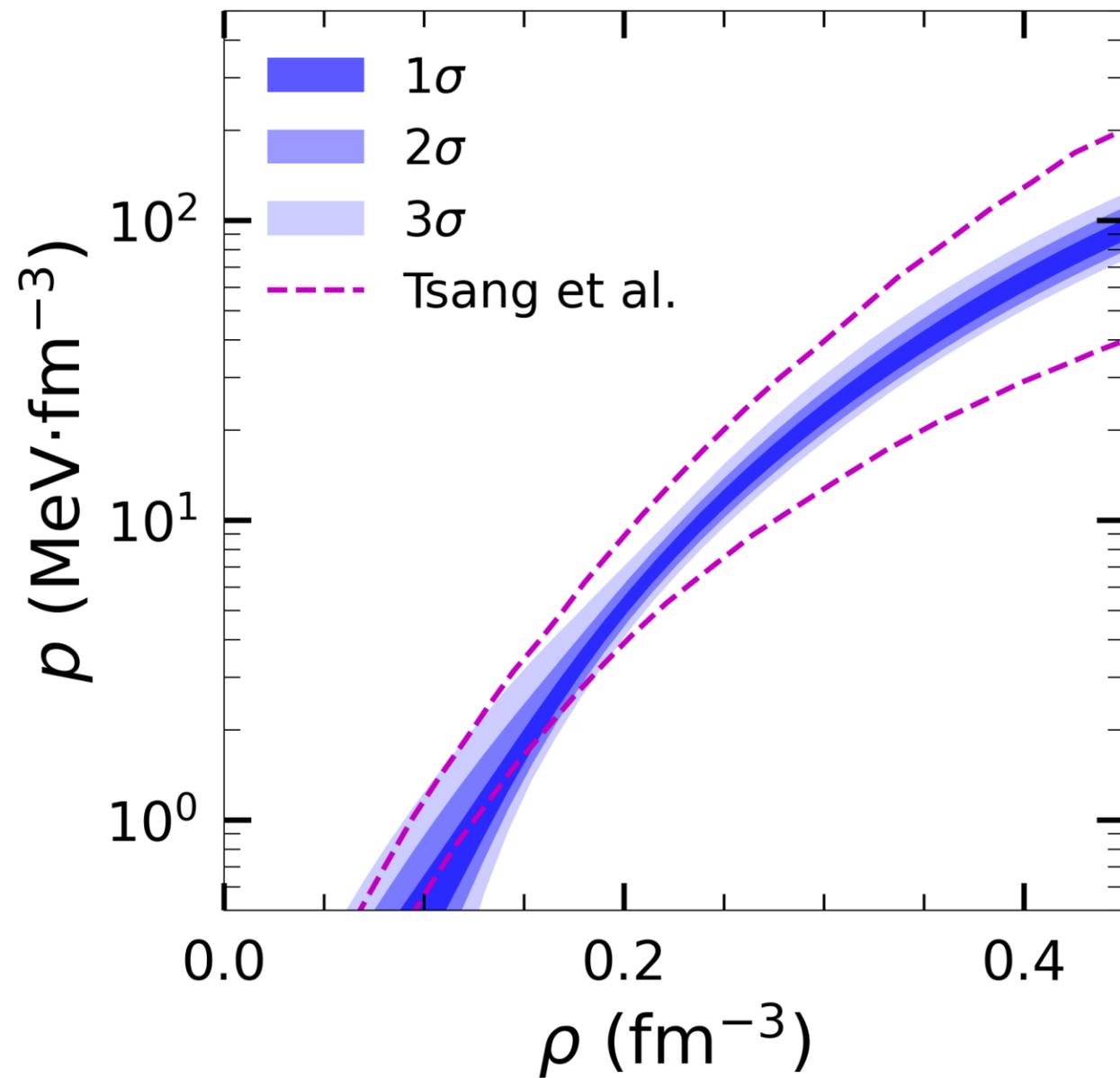


coupling constants

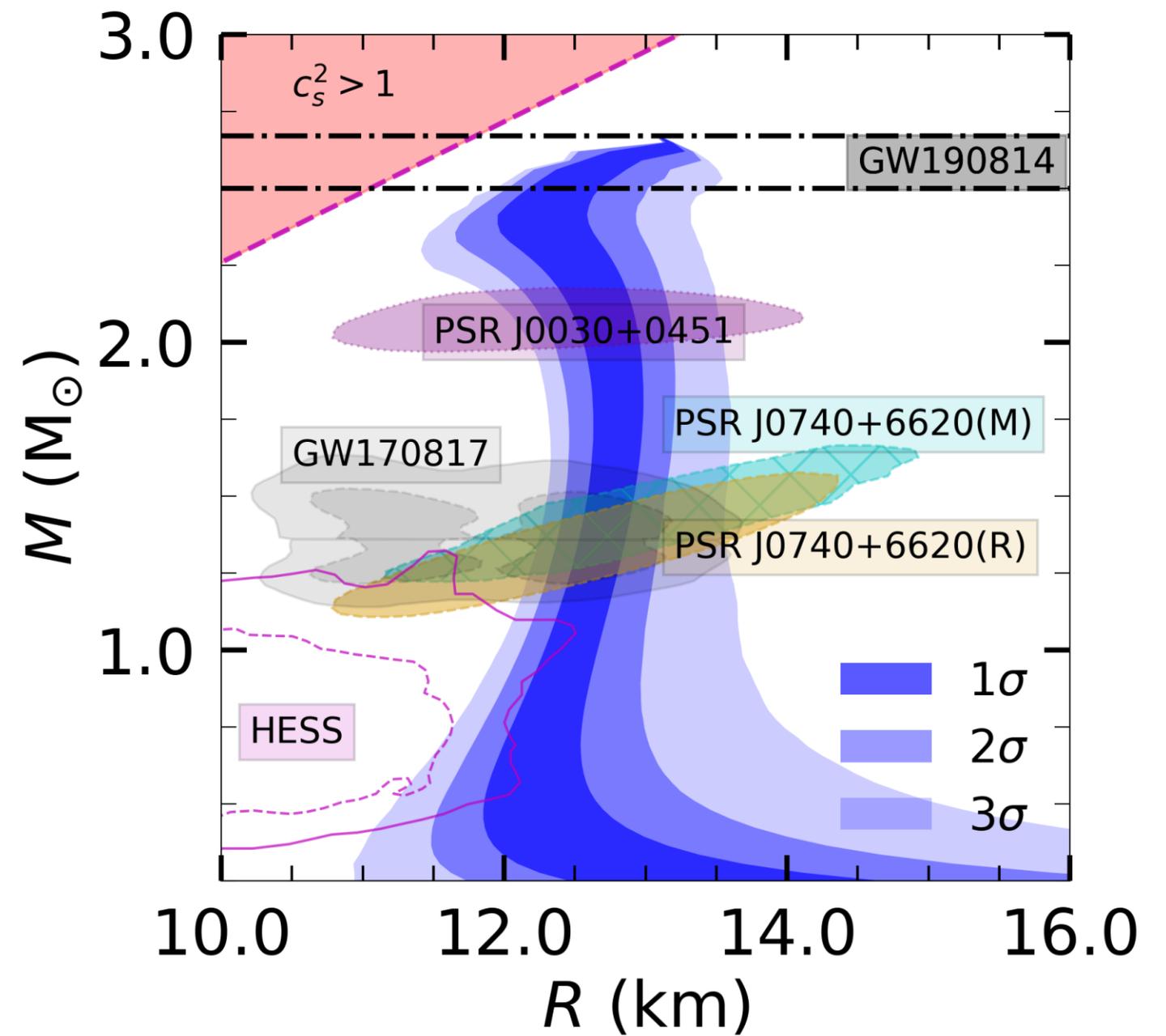
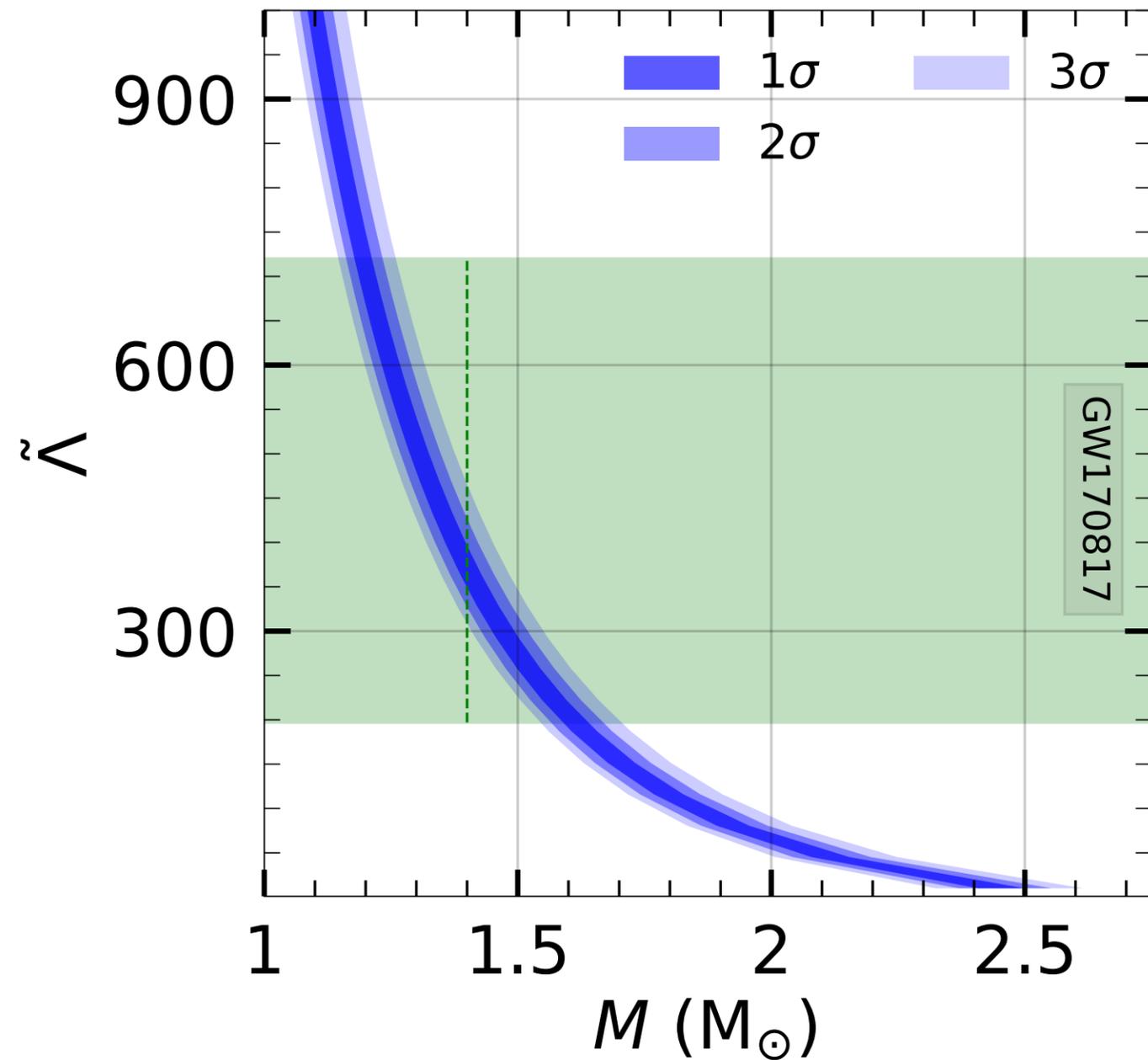


Nuclear saturation and NS properties

Results: Equation of state



Results: Neutron star properties



EOS: with hyperons and PNS



- ❖ Hyperonic meson coupling refers to the way in which hyperons interact with mesons such as σ , ω , and ρ .
- ❖ These couplings are typically determined by fitting theoretical models to experimental data, such as the properties of hypernuclei, or by using symmetry arguments.

Y	$R_{\sigma Y}$	$R_{\omega Y}$	$R_{\rho Y}$
Λ	0.6613	2/3	0
Σ	0.4673	2/3	1
Ξ	0.3305	1/3	1

- ☑ **Proto-neutron stars (PNS)**: gets formed after a successful supernova when the stellar remnant decouples from the ejecta.

Neutrinos are produced in large quantities

$$S = 1, Y_e + Y_{\nu_e} = 0.4$$

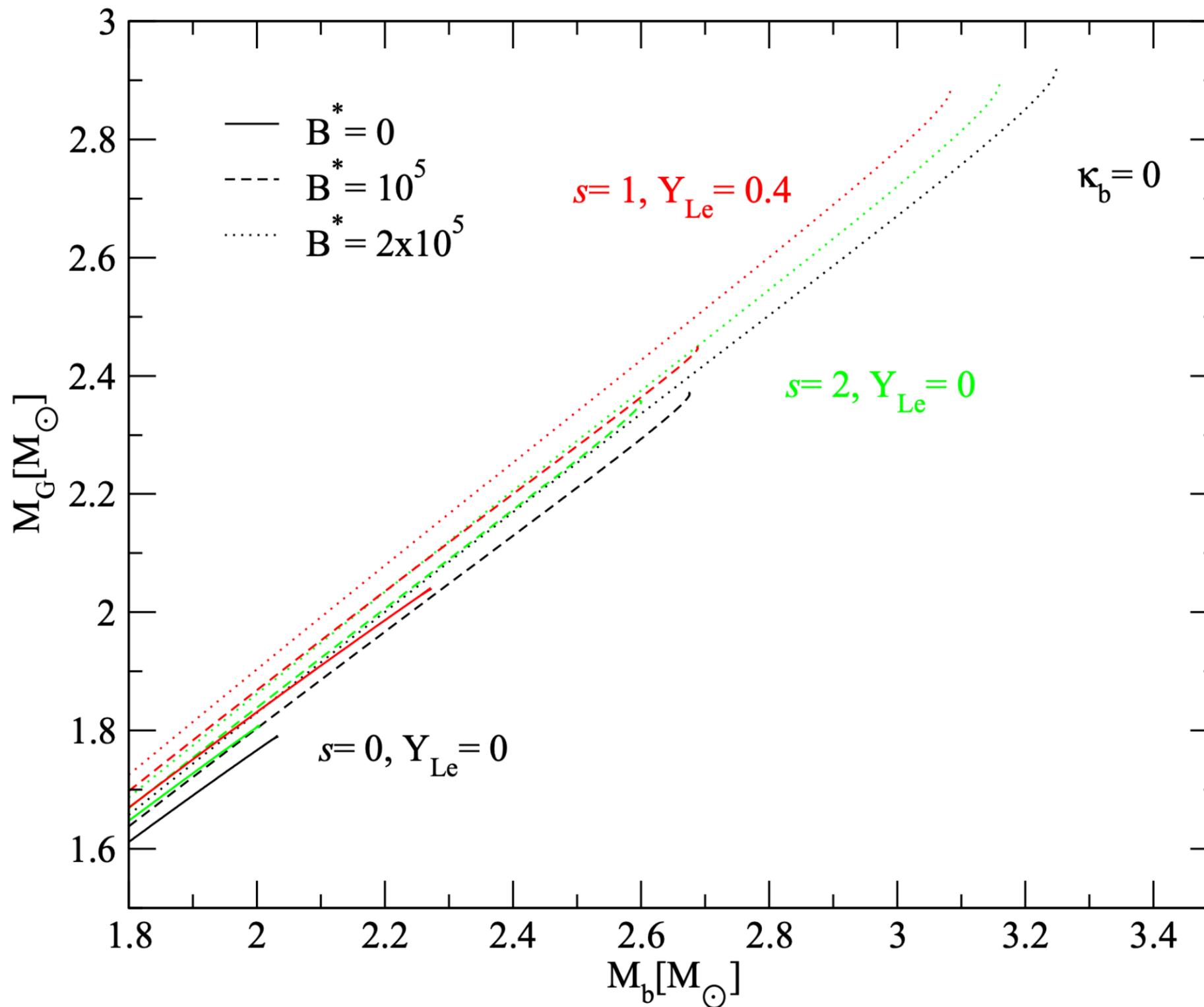
Deleptonization starts

$$S = 2$$

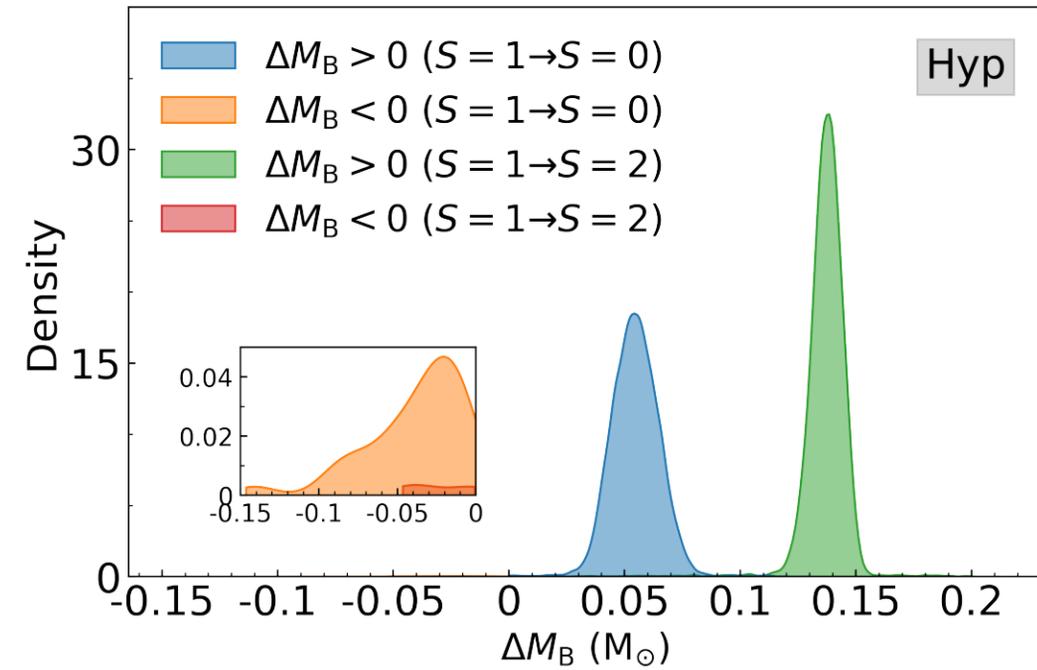
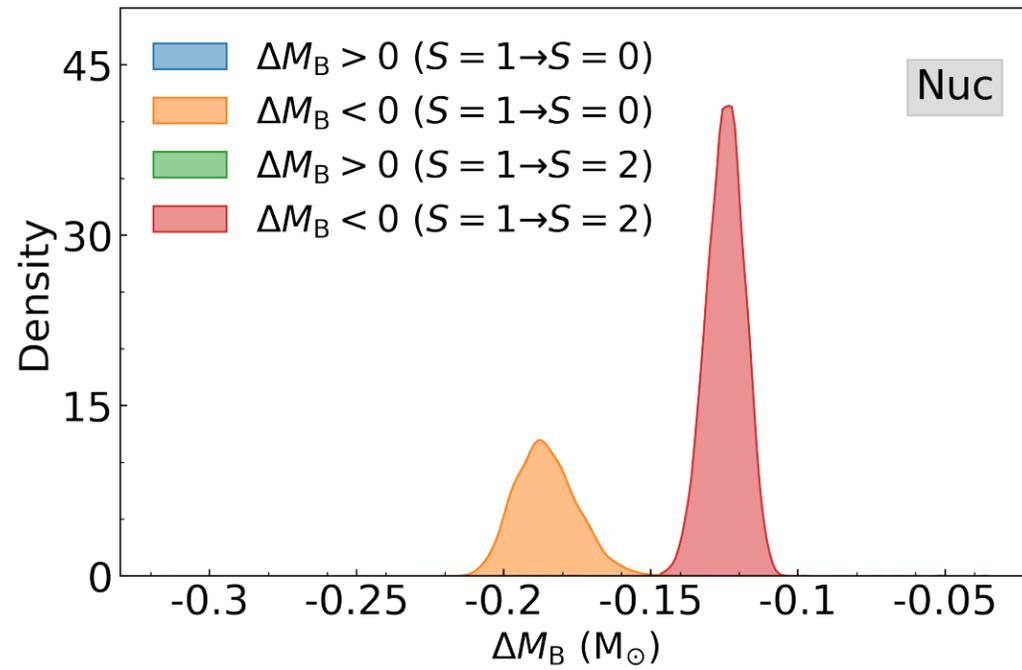
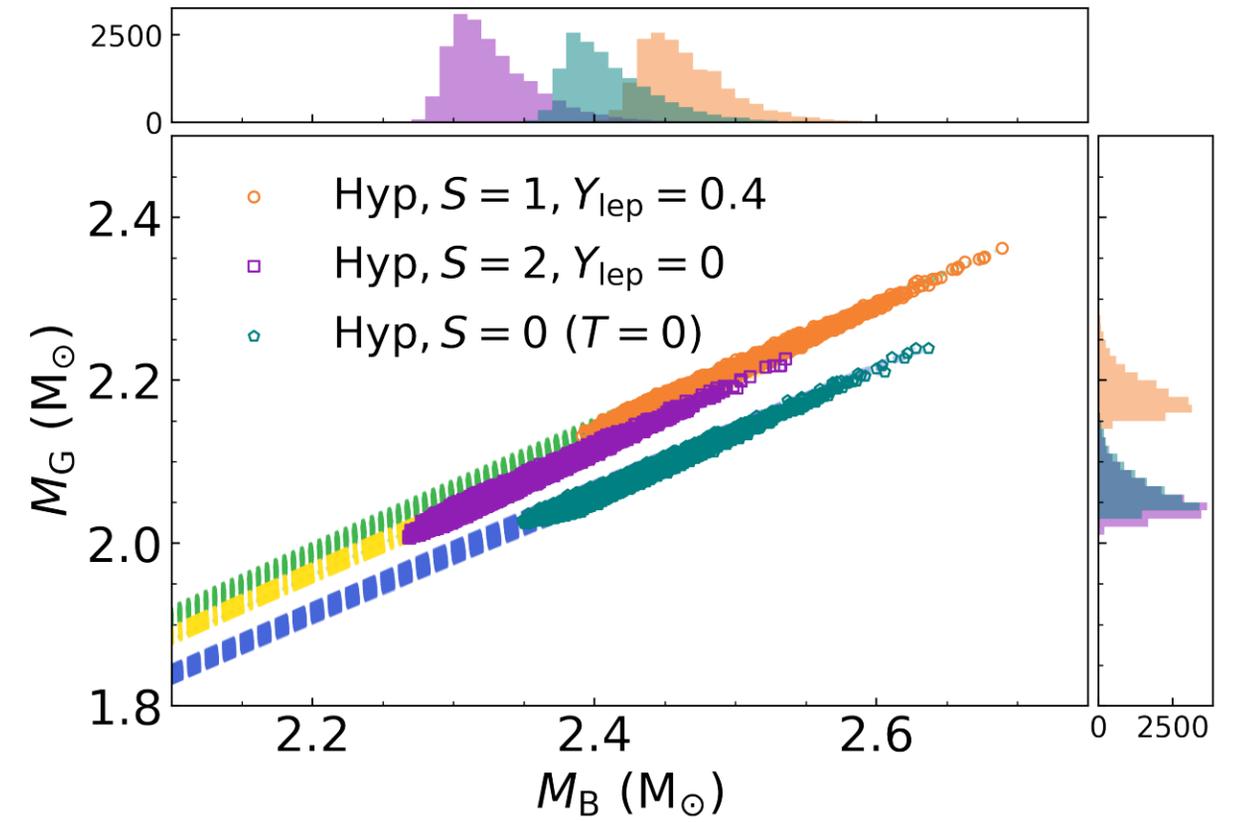
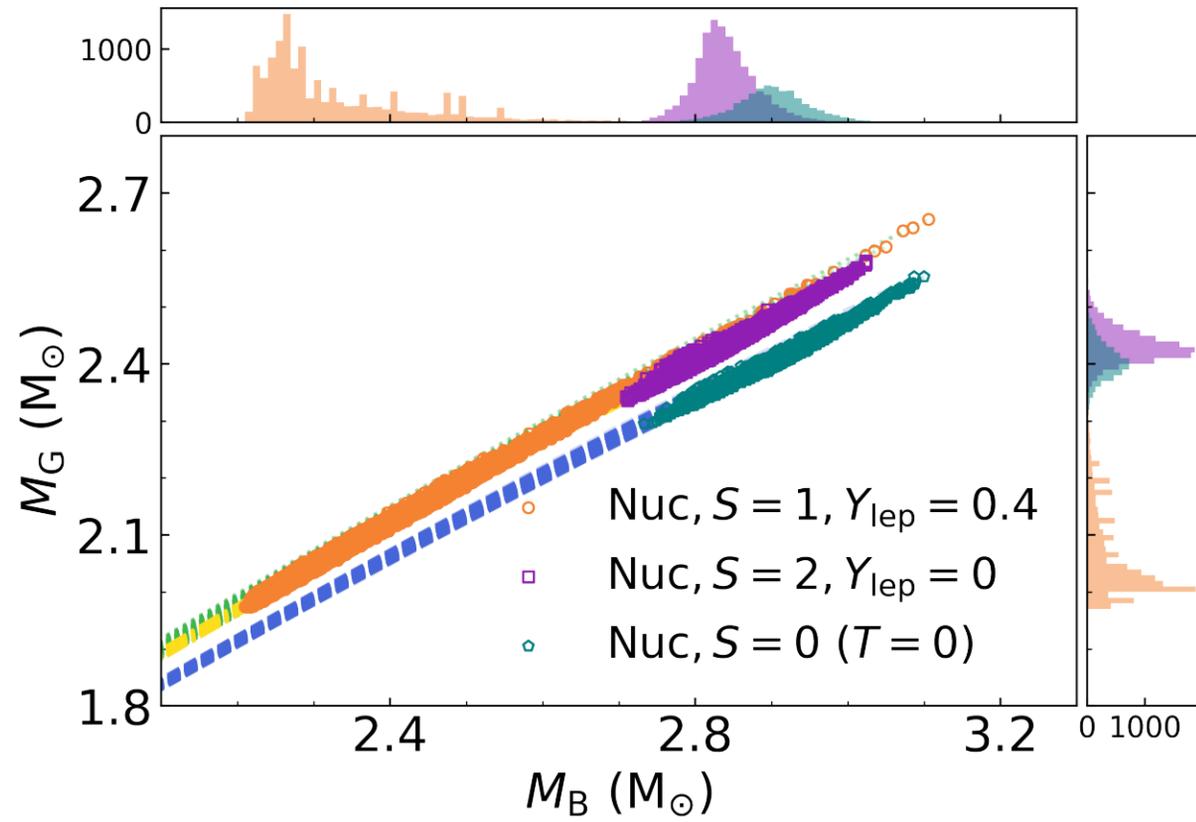
Metastable PNS

Collapse to a BH or stable NS

EOS: evolution of a PNS



Results: Evolution of NS





- ❖ We discussed the basics of neutron stars where neutron stars are natural laboratories to study high-density interaction.
- ❖ Nuclear matter is described within the relativistic mean field model.
- ❖ Bayesian approach is used to extract coupling parameters in such a way that nuclear saturation properties along with neutron stars observations must be satisfy.
- ❖ From the neutron star evolution, we found that hyperonic stars are most likely to collapse to black as compared to nucleonic star.



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thank you your kind attention





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