

Electric, thermal, and thermoelectric response of a hot pion gas in a time-dependent background magnetic field

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Authors: Ankit Kumar¹, Gowthama K K², Vinod Chandra¹, Sadhana Dash²

¹*Indian Institute of Technology Gandhinagar*

²*Indian Institute of Technology Bombay*



Presented by: Ankit Kumar
IIT Gandhinagar

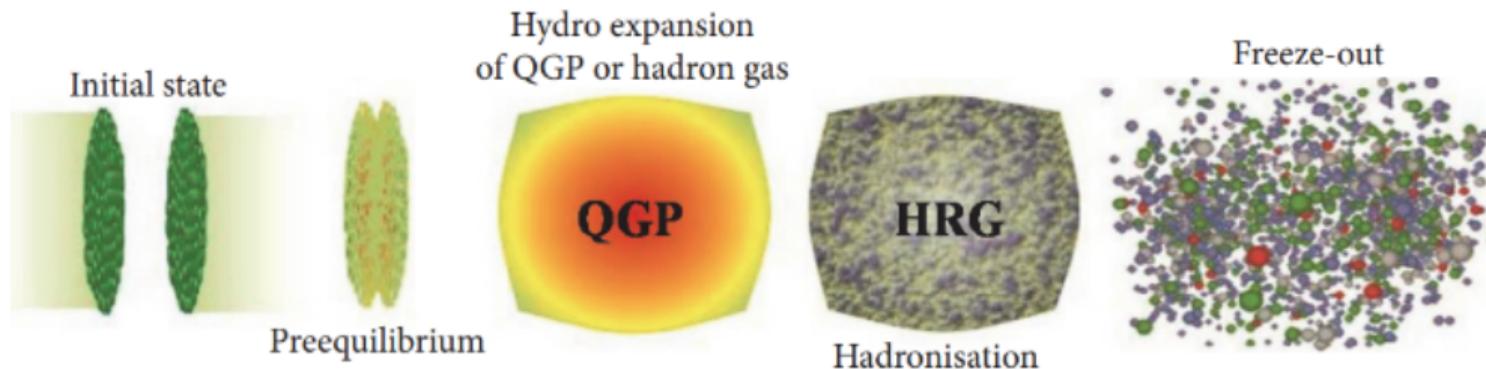
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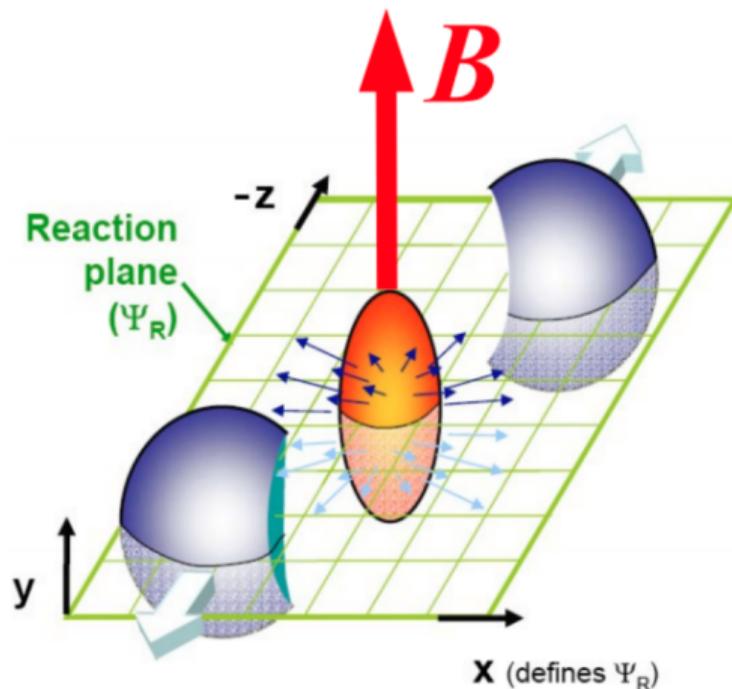
1. Introduction
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Introduction

- When atomic nuclei (heavy ions) collide at nearly the speed of light in particle accelerators like LHC and RHIC, they can create an exotic state of matter called **Quark-Gluon Plasma (QGP)**.



- In non-central collisions, strong magnetic fields of the order of $\sim 10^{18-19}$ G are generated because of the fast moving charged **spectators**¹.



¹Kharzeev et al., Nucl. Phys. A 803, 227 (2008)

- Theoretical modeling of the decay of the magnetic field in a conducting medium like QGP indicates that the fields may persist beyond the hadronization of the medium².

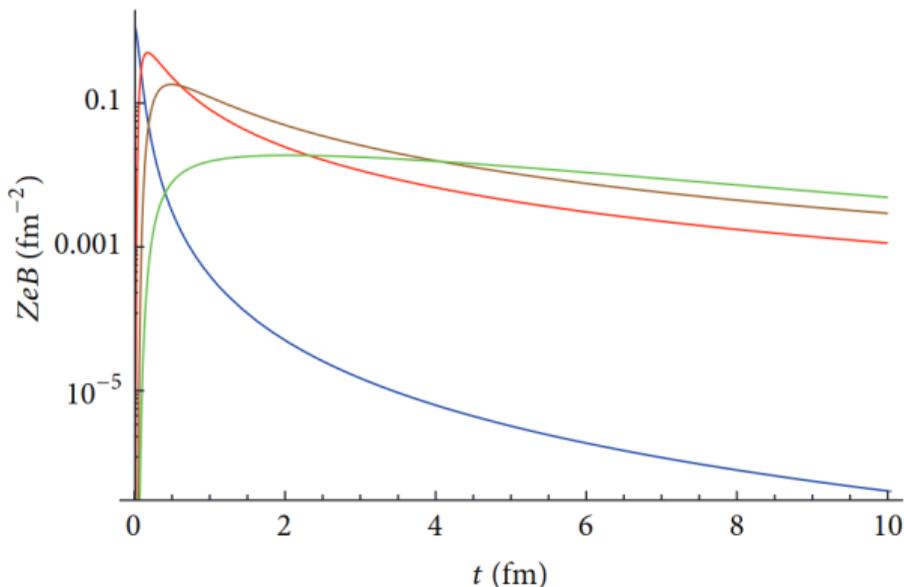


Figure: Relaxation of magnetic field for $Z = 79$ (Gold nucleus), $b = 7$ fm in Vacuum, Medium ($\sigma = 5.8$ MeV), Medium ($\sigma = 16$ MeV), and Expanding Medium.

²Tuchin, Adv. High Energy Phys. 2013, 490495 (2013)

- Among the hadrons produced during hadronization, pions ($\pi^{\pm,0}$) are the lightest and most abundant constituents of the resulting hadron gas. Hence, they govern the late time dynamics in HICs.
- The persistence of the magnetic field beyond the hadronization may significantly influence the properties of the pion gas, and hence, the final state observables.
- Our work titled "Electric, thermal, and thermoelectric response of a hot pion gas in a time-dependent background magnetic field" sheds light on how a time-dependent background magnetic field influences the properties of the pion gas.

- Only π^\pm are influenced by the EM fields.
- EM fields induce an electric current in the medium given by

$$\mathbf{j} = \sum_k \int dP_k \mathbf{p}_k q_k \delta f_k, \quad (k = \pi^\pm), \quad (1)$$

where $dP_k = \frac{d^3 \mathbf{p}_k}{(2\pi)^3 \epsilon_k}$ with $\epsilon_k = \sqrt{p_k^2 + m_k^2}$, and δf_k is the shift in the distribution function of the pion gas due to external time-dependent electromagnetic fields.

- δf_k can be obtained by solving the Boltzmann equation under the relaxation time approximation

$$\frac{\partial f_k}{\partial t} + \mathbf{v} \cdot \frac{\partial f_k}{\partial \mathbf{x}} + q_k [\mathbf{E} + \mathbf{v} \times \mathbf{B}] \cdot \frac{\partial f_k}{\partial \mathbf{p}} = -\frac{\delta f_k}{\tau_R} \quad (2)$$

- τ_R is the relaxation time, which has been calculated by considering a $2 \rightarrow 2$ pion-pion scattering process with interactions occurring through the exchange of vector and scalar mesons in a thermal background, and fitting it with a polynomial function given by³

$$\tau_R = \sum_{i=0}^3 a_i \left(\frac{m}{T}\right)^i \frac{1}{T}$$

where $a_0 = 0.0145 \text{ fm GeV}^3$, $a_1 = -0.0109 \text{ fm GeV}^3$, $a_2 = 0.0058 \text{ fm GeV}^3$, and $a_3 = 0.0026 \text{ fm GeV}^3$.

³Kalikotay et al., Phys. Rev. D 102, 076007 (2020)

- The fields cause the distribution function of the pion gas to shift slightly away from the equilibrium, and hence, this shift can be written in terms of the electromagnetic fields and their derivatives.
- We adapt the following ansatz for δf_k to solve the Boltzmann equation:

$$\delta f_k = (\mathbf{p}_k \cdot \Xi) \frac{\partial f_k^0}{\partial \epsilon_k} \quad (3)$$

with

$$\begin{aligned} \Xi = & \alpha_1 \mathbf{E} + \alpha_2 \dot{\mathbf{E}} + \alpha_3 (\mathbf{E} \times \mathbf{B}) + \alpha_4 (\dot{\mathbf{E}} \times \mathbf{B}) + \alpha_5 (\mathbf{E} \times \dot{\mathbf{B}}) \\ & + \alpha_6 (\nabla \times \mathbf{E}) + \alpha_7 \mathbf{B} + \alpha_8 \dot{\mathbf{B}} + \alpha_9 (\nabla \times \mathbf{B}) \end{aligned} \quad (4)$$

- We consider the case with vanishing chiral chemical potential and CP symmetry $\Rightarrow \alpha_i = 0$ for $i = (6, 7, 8)$ ⁴.

⁴Satow, Phys. Rev. D 90, 034018 (2014)

- Using (3) and (4) in the Boltzmann equation, simplifying, and comparing the coefficients of the tensorial structures on both sides, we obtain the following expressions for α_j :

$$\alpha_1 = -\frac{\tilde{\Omega}_k}{2}(l_1 e^{\eta_1} + l_2 e^{\eta_2}),$$

$$\alpha_3 = \frac{q_k^i}{2\epsilon_k}(l_1 e^{\eta_1} - l_2 e^{\eta_2}),$$

$$\alpha_5 = -\frac{\tau_R q_k^i}{2\epsilon_k}(l_1 e^{\eta_1} - l_2 e^{\eta_2}),$$

$$\alpha_2 = \frac{(\frac{\tilde{\Omega}_k \tau_R}{2} + \frac{i\Omega_k^2 \tau_R^2}{2})l_1 e^{\eta_1} + (\frac{\tilde{\Omega}_k \tau_R}{2} - \frac{i\Omega_k^2 \tau_R^2}{2})l_2 e^{\eta_2}}{1 + \Omega_k^2 \tau_R^2}$$

$$\alpha_4 = \frac{(\frac{\tilde{\Omega}_k^2 \tau_R^2}{2F} - \frac{i q_k \tau_R}{2\epsilon_k})l_1 e^{\eta_1} + (\frac{\tilde{\Omega}_k^2 \tau_R^2}{2F} + \frac{i q_k \tau_R}{2\epsilon_k})l_2 e^{\eta_2}}{1 + \Omega_k^2 \tau_R^2}.$$

- Where,

$$\eta_j = -\frac{t}{\tau_R} + a_j \frac{q_k i}{\epsilon_k} \int F dt, \quad I_j = \int \frac{e^{-\eta_j}}{F}, \quad (j = 1, 2) \quad (5)$$

with $a_1 = 1$ and $a_2 = -1$. $\Omega_k = \frac{q_k B}{\epsilon_k}$ is the cyclotron frequency and $\tilde{\Omega}_k = \frac{q_k F}{\epsilon_k}$ with $F = \sqrt{B(B - \tau_R \dot{B})}$.

- We consider the electric and magnetic field magnitudes of the form $E = E_0 e^{-t/\tau_E}$ and $B = B_0 e^{-t/\tau_B}$, where τ_E and τ_B are the decay parameters of electric and magnetic fields respectively.
- The induced current can then be decomposed into Ohmic and Hall currents as $\mathbf{j} = j_e \hat{\mathbf{e}} + j_H(\hat{\mathbf{e}} \times \hat{\mathbf{b}})$, with

$$j_e = j_e^{(0)} + j_e^{(1)}, \quad j_H = j_H^{(0)} + j_H^{(1)} + j_H^{(2)}. \quad (6)$$

- These currents are given as

$$j_e^{(0)} = \frac{Ee}{3} \int dP p^2 \left\{ \left(\alpha_1 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^+} - \left(\alpha_1 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^-} \right\},$$

$$j_e^{(1)} = \frac{\dot{E}e}{3} \int dP p^2 \left\{ \left(\alpha_2 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^+} - \left(\alpha_2 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^-} \right\},$$

$$j_H^{(0)} = \frac{EBe}{3} \int dP p^2 \left\{ \left(\alpha_3 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^+} - \left(\alpha_3 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^-} \right\},$$

$$j_H^{(1)} = \frac{\dot{E}Be}{3} \int dP p^2 \left\{ \left(\alpha_4 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^+} - \left(\alpha_4 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^-} \right\},$$

$$j_H^{(2)} = \frac{E\dot{B}e}{3} \int dP p^2 \left\{ \left(\alpha_5 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^+} - \left(\alpha_5 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^-} \right\}.$$

- The Ohmic and Hall conductivities are given as $\sigma_e = \frac{j_e^{(0)} + j_e^{(1)}}{E}$ and $\sigma_H = \frac{j_H^{(0)} + j_H^{(1)} + j_H^{(2)}}{E}$ respectively.

Results of Electric Response

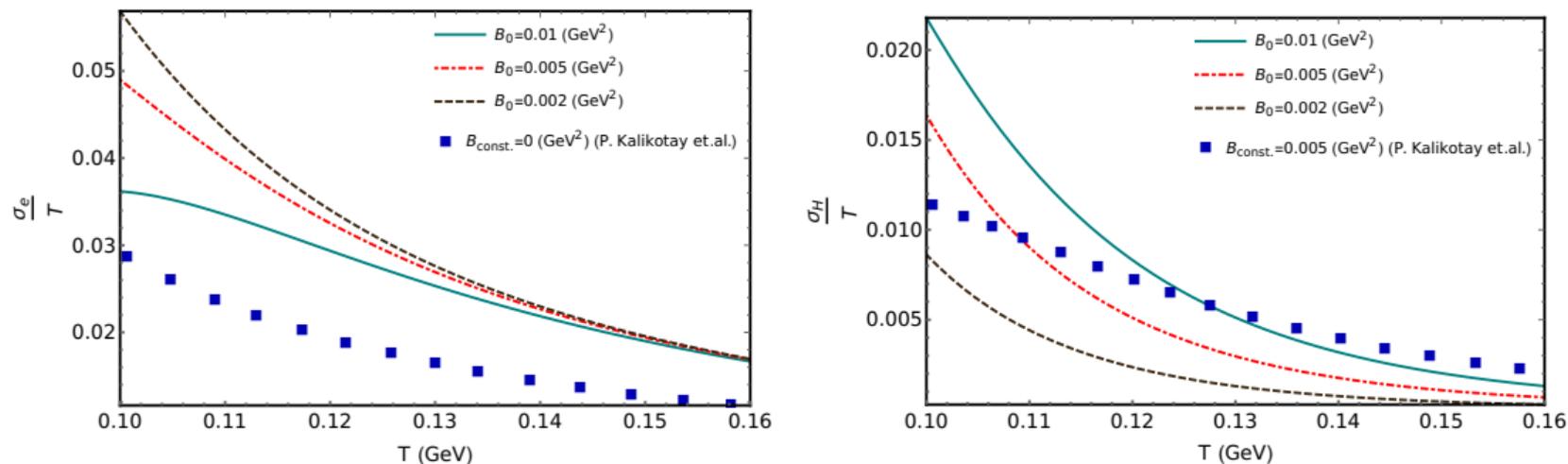


Figure: Ohmic conductivity (left panel) and Hall conductivity (Right panel) as a function of temperature for different amplitudes of magnetic field at $t = 2.5$ fm and $(\mu)_{\pi\pm} = \pm 0.1$ GeV. The results have been compared with ⁵.

⁵P. Kalikotay et al., Phys. Rev. D 102, 076007 (2020)

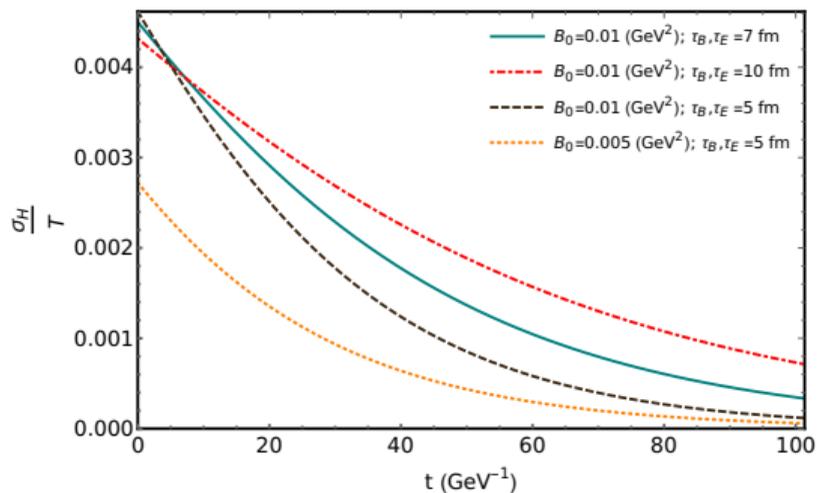
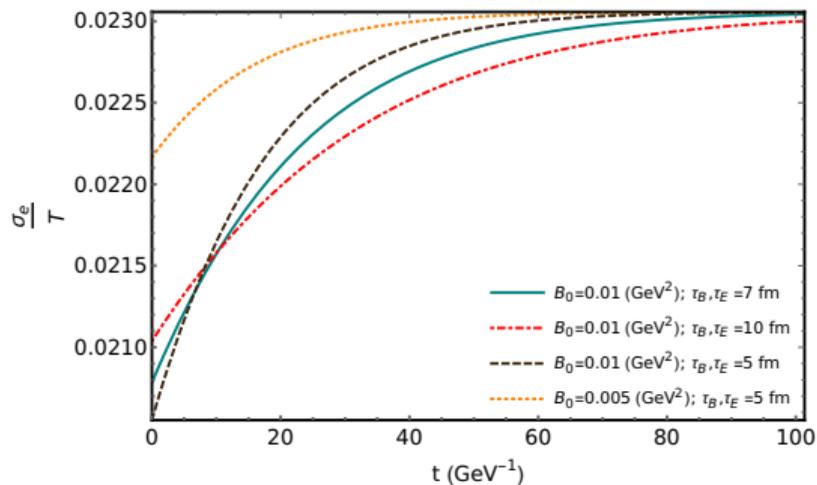


Figure: Ohmic conductivity (left panel) and Hall conductivity (Right panel) as a function of time for different amplitudes and decay parameters of magnetic and electric fields at $T = 0.14$ GeV and $(\mu)_{\pi^\pm} = \pm 0.1$ GeV.

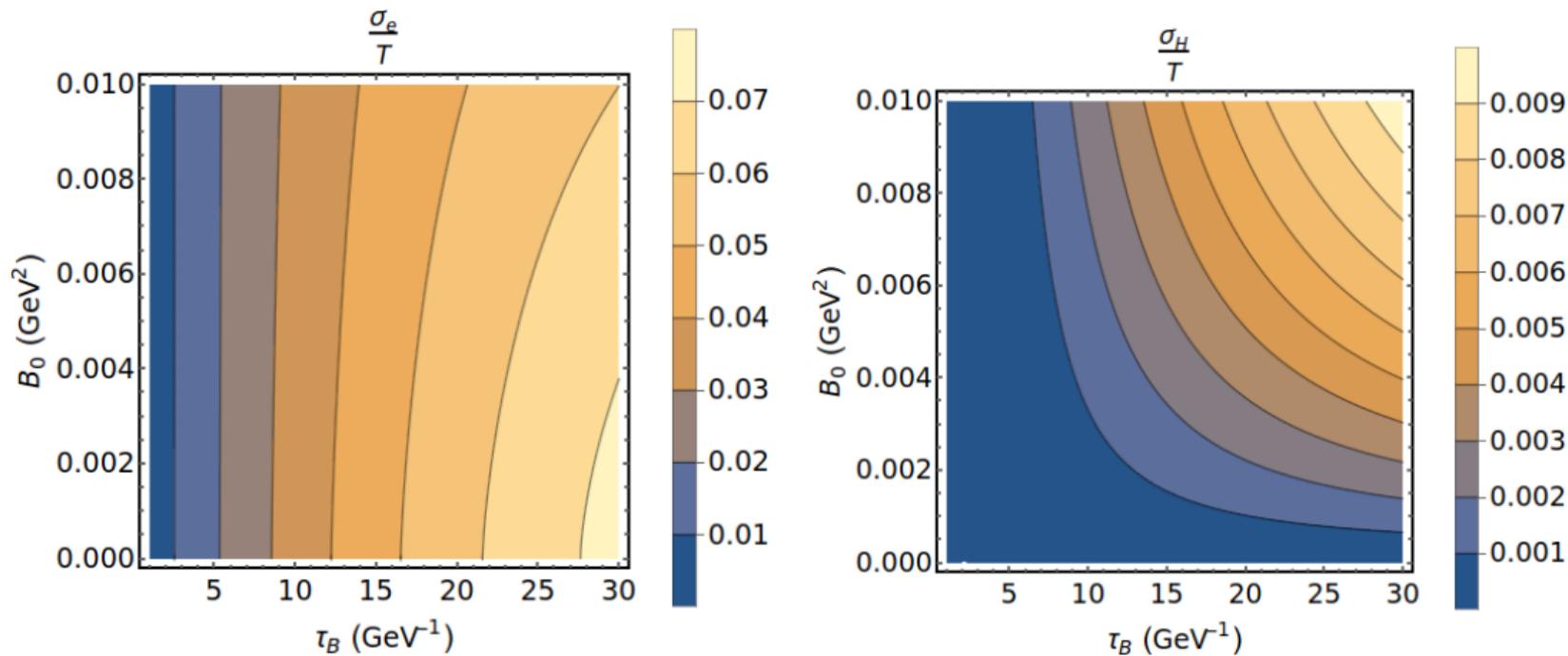


Figure: Ohmic conductivity (left panel) and Hall conductivity (Right panel) as a function of amplitude and decay parameter of the magnetic field at $t = 2.5$ fm, $T = 0.12$ GeV and $(\mu)_{\pi\pm} = \pm 0.1$ GeV.

Thermal Response

- The thermal response of the system is studied through dissipative net heat flow given by the expression

$$\mathbf{l}_k = \sum_k \int dP_k \mathbf{p}_k (\epsilon_k - h) \delta f_k. \quad (7)$$

where h is the enthalpy per particle given by $h = \frac{\epsilon - P}{n}$.

- The shift in the distribution function, δf_k , can again be written as

$$\delta f_k = (\mathbf{p} \cdot \Xi) \frac{\partial f_k^0}{\partial \epsilon_k}. \quad (8)$$

- The system is driven away from equilibrium due to the external time dependent magnetic field and the local thermal driving force, \mathbf{X} , defined as $X_i = \frac{\partial_i T}{T} - \frac{\partial_i P}{nh}$. The vector Ξ can be written as

$$\Xi = \beta_1 \mathbf{B} + \beta_2 \mathbf{X} + \beta_3 (\mathbf{X} \times \mathbf{B}) + \beta_4 \dot{\mathbf{B}} + \beta_5 (\mathbf{X} \times \dot{\mathbf{B}}). \quad (9)$$

- Following the same steps as for the case of electric response, we obtain the following expressions for β_i

$$\beta_1 = -\frac{i(\mathbf{B}\cdot\mathbf{X})}{F\epsilon_k} l_0 e^{\eta_0} + \frac{i(\mathbf{B}\cdot\mathbf{X})}{2F\epsilon_k} l_1 e^{\eta_1} + \frac{i(\mathbf{B}\cdot\mathbf{X})}{2F\epsilon_k} l_2 e^{\eta_2},$$

$$\beta_2 = (\epsilon_k - h) \frac{F}{2\epsilon_k} l_1 e^{\eta_1} + (\epsilon_k - h) \frac{F}{2\epsilon_k} l_2 e^{\eta_1},$$

$$\beta_3 = i(\epsilon_k - h) \frac{l_1}{2\epsilon_k} e^{\eta_1} - i(\epsilon_k - h) \frac{l_2}{2\epsilon_k} e^{\eta_1},$$

$$\beta_4 = -\tau_R \beta_1 - \frac{q_k \tau_R^2}{\epsilon_k} \beta_3(\mathbf{B}\cdot\mathbf{X}),$$

$$\beta_5 = -\tau_R \beta_3,$$

- We assume $\mathbf{B}\cdot\mathbf{X} = 0 \Rightarrow \beta_1, \beta_4 = 0$.

- The heat current of the pion gas in the presence of time time-decaying magnetic field takes the form:

$$\mathbf{I} = -\kappa T \mathbf{X} + (\bar{\kappa}_1 + \bar{\kappa}_2) T (\mathbf{X} \times \mathbf{b}), \quad (10)$$

where

$$\begin{aligned} \kappa = -\frac{1}{3T} \int dP p^2 (\epsilon - h) & \left\{ \left(\beta_2 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^+} \right. \\ & \left. + \left(\beta_2 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^-} \right\} - (\kappa)_{\pi^0}, \end{aligned}$$

with⁶

$$(\kappa)_{\pi^0} = \frac{1}{3T} \int dP \frac{p^2}{\epsilon} (\epsilon - h)^2 \tau_R \left(\frac{\partial f^0}{\partial \epsilon} \right)_{\pi^0}.$$

⁶Mitra & Sarkar, Phys. Rev. D 89, 054013 (2014)

- The coefficients $\bar{\kappa}_1$ and $\bar{\kappa}_2$ represent Hall-like thermal conductivities that arise purely due to the magnetic field and are of the form,

$$\bar{\kappa}_1 = \frac{B}{3T} \int dP p^2 (\epsilon - h) \left\{ \left(\beta_3 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^+} + \left(\beta_3 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^-} \right\},$$

$$\bar{\kappa}_2 = \frac{\dot{B}}{3T} \int dP p^2 (\epsilon - h) \left\{ \left(\beta_5 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^+} + \left(\beta_5 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^-} \right\}.$$

$$\kappa_H = \bar{\kappa}_1 + \bar{\kappa}_2$$

Results for Thermal Response

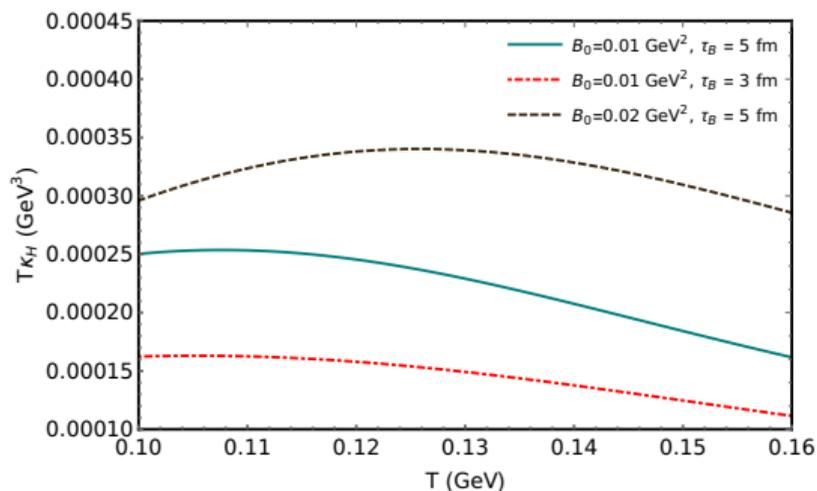
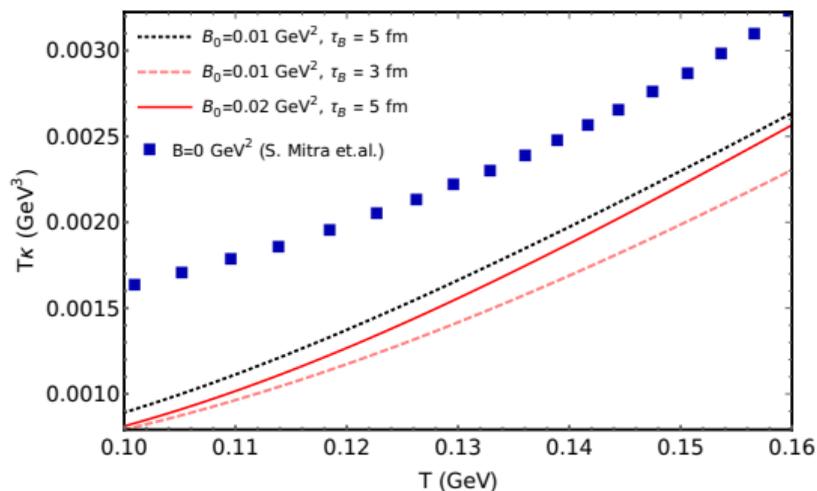


Figure: Thermal conductivity (left panel) and Hall-like Thermal conductivity (right panel) as a function of temperature for different values of amplitude and decay parameter of the magnetic field at $t = 2.5$ fm, $(\mu)_{\pi^\pm} = \pm 0.1$ GeV and $(\mu)_{\pi^0} = 0$ GeV. The results of the thermal conductivity have been compared with ⁷.

⁷Mitra & Sarkar, Phys. Rev. D 89, 054013 (2014)

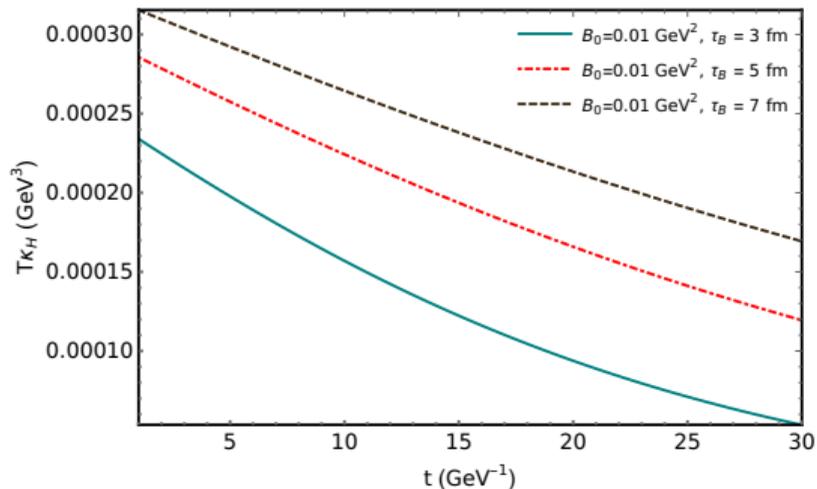
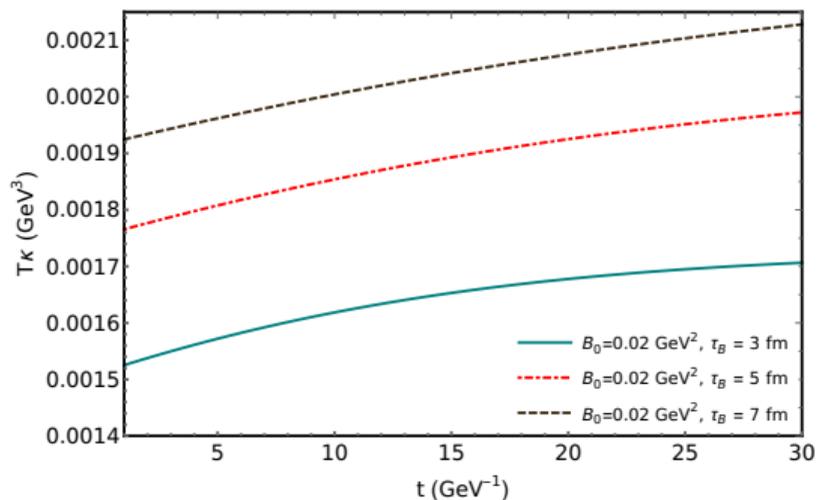


Figure: Thermal conductivity (left panel) and Hall-like Thermal conductivity (right panel) as a function of time for different values of decay parameter of the magnetic field at $T = 0.14$ GeV, $(\mu)_{\pi^\pm} = \pm 0.1$ GeV and $(\mu)_{\pi^0} = 0$ GeV.

Thermoelectric Response

Seebeck Effect

- The Seebeck effect is the generation of an electric voltage across a material when there is a temperature gradient along it.
- Characterized by magneto-Seebeck coefficient, S_B , in presence of a magnetic field.

Nernst Effect

- The Nernst effect is the generation of a transverse electric voltage in a material when a temperature gradient is applied in the presence of a magnetic field.
- Characterized by normalized Nernst coefficient, NB .

- The current density induced in the medium because of the thermal gradients is given by

$$\mathbf{j} = \sum_k \int dP_k \mathbf{p}_k q_k \delta f_k, \quad (11)$$

where,

$$\begin{aligned} \delta f_k = \mathbf{p} \cdot [& \gamma_1 \mathbf{E} + \gamma_2 \mathbf{B} + \gamma_3 \mathbf{X} + \gamma_4 (\mathbf{X} \times \mathbf{B}) + \gamma_5 \dot{\mathbf{B}} + \gamma_6 (\mathbf{X} \times \dot{\mathbf{B}}) \\ & + \gamma_7 (\mathbf{E} \times \mathbf{B}) + \gamma_8 (\mathbf{E} \times \dot{\mathbf{B}})] \frac{\partial f_k^0}{\partial \epsilon_k}. \end{aligned} \quad (12)$$

- Following the same steps as before, while considering $\mathbf{B}\cdot\mathbf{X} = 0$, we obtain the following expressions for γ_i :

$$\gamma_1 = -\frac{\tilde{\Omega}_k}{2}(l_1 e^{\eta_1} + l_2 e^{\eta_2}),$$

$$\gamma_2 = 0,$$

$$\gamma_3 = (\epsilon_k - h) \frac{F}{2\epsilon_k} l_1 e^{\eta_1} + (\epsilon_k - h) \frac{F}{2\epsilon_k} l_2 e^{\eta_1},$$

$$\gamma_4 = i(\epsilon_k - h) \frac{l_1}{2\epsilon_k} e^{\eta_1} + i(\epsilon_k - h) \frac{l_2}{2\epsilon_k} e^{\eta_1},$$

$$\gamma_5 = 0,$$

$$\gamma_6 = -\tau_R \gamma_4,$$

$$\gamma_7 = \frac{q_k^i}{2\epsilon_k} (l_1 e^{\eta_1} - l_2 e^{\eta_2}),$$

$$\gamma_8 = -\frac{\tau_R q_k^i}{2\epsilon_k} (l_1 e^{\eta_1} - l_2 e^{\eta_2}),$$

- Once we obtain the expression for \mathbf{j} , we project it in two directions, along the direction of ∇T and $\nabla T \times \mathbf{B}$, which we call directions x and y , respectively.
- The induced electric field can be obtained by considering the steady-state solution ($j_x, j_y = 0$).
- This gives two coupled equations relating the induced electric fields (E_x, E_y) and the temperature gradients ($\frac{dT}{dx}, \frac{dT}{dy}$) along x and y directions, which we can write in matrix form as

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} S_B & NB \\ -NB & S_B \end{pmatrix} \begin{pmatrix} \frac{dT}{dx} \\ \frac{dT}{dy} \end{pmatrix}$$

- S_B and NB take the following forms

$$S_B = -\frac{L_1L_4 + L_2L_5 + L_3L_5 + L_2L_6 + L_3L_6}{T(L_1^2 + L_2^2 + L_3^2 + 2L_2L_3)},$$

$$NB = \frac{L_2L_4 + L_3L_4 - L_1L_5 - L_1L_6}{T(L_1^2 + L_2^2 + L_3^2 + 2L_2L_3)}$$

where

$$\begin{aligned}L_1 &= \frac{e}{3} \int dP p^2 \left\{ \left(\gamma_1 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^+} - \left(\gamma_1 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^-} \right\}, \\L_2 &= \frac{Be}{3} \int dP p^2 \left\{ \left(\gamma_7 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^+} - \left(\gamma_7 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^-} \right\}, \\L_3 &= \frac{\dot{B}e}{3} \int dP p^2 \left\{ \left(\gamma_8 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^+} - \left(\gamma_8 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^-} \right\}, \\L_4 &= \frac{e}{3} \int dP p^2 \left\{ \left(\gamma_3 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^+} - \left(\gamma_3 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^-} \right\}, \\L_5 &= \frac{Be}{3} \int dP p^2 \left\{ \left(\gamma_4 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^+} - \left(\gamma_4 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^-} \right\}, \\L_6 &= \frac{\dot{B}e}{3} \int dP p^2 \left\{ \left(\gamma_6 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^+} - \left(\gamma_6 \frac{\partial f^0}{\partial \epsilon} \right)_{\pi^-} \right\}.\end{aligned}$$

Results for Thermoelectric Response

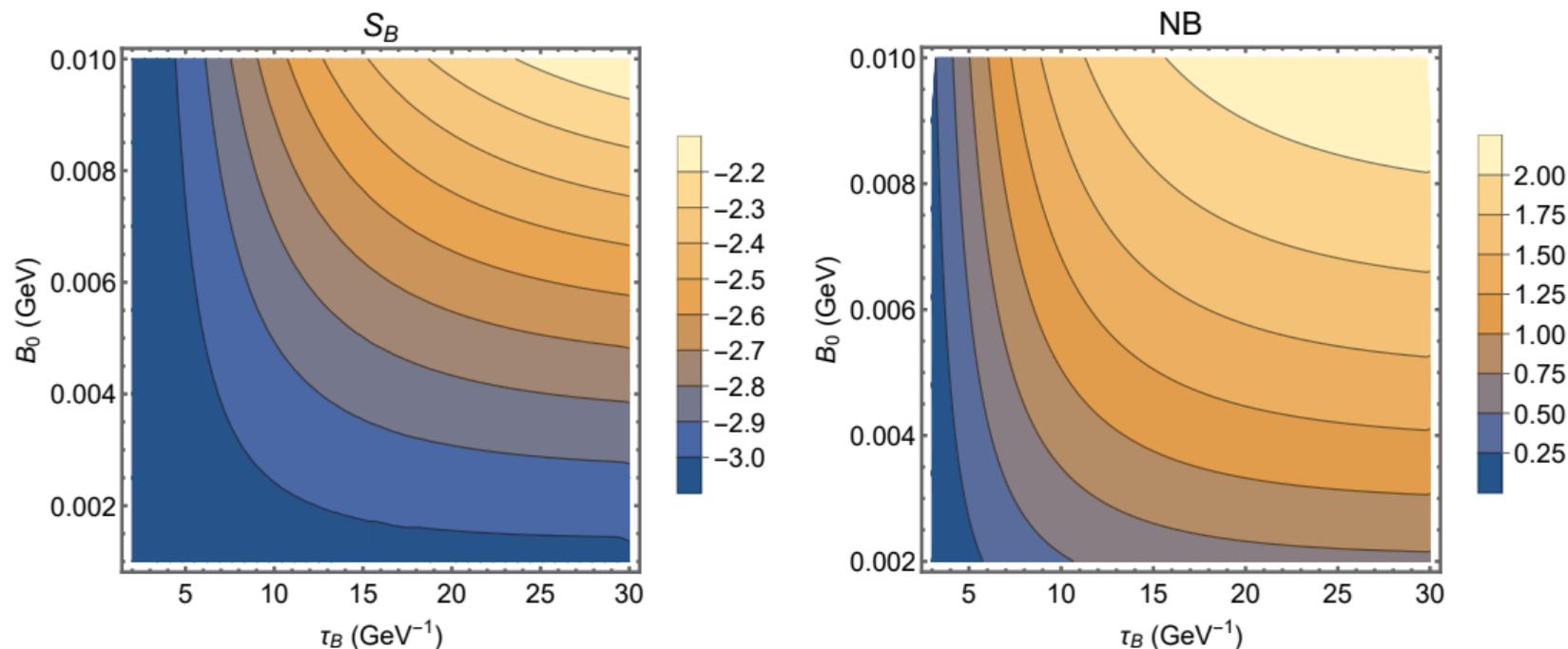


Figure: Magneto-Seebeck coefficient (left panel) and normalized Nernst coefficient (right panel) as a function of amplitude and decay parameter of the magnetic field at $t = 2.5$ fm, $T = 0.12$ GeV, $(\mu)_{\pi^\pm} = \pm 0.1$ GeV and $(\mu)_{\pi^0} = 0$ GeV.

Knudsen number (Kn)

Knudsen number, Kn , is defined as the ratio of the mean path (λ) of the constituent particle to the size of the system, l .

$$Kn = \frac{\lambda}{l}$$

- It determines the applicability of hydrodynamics to the system.
- The mean free path is related to the thermal conductivity as $\lambda = \frac{3\kappa}{vC_v}$.
- Hence, the Knudsen number can be expressed in terms of thermal conductivity as

$$Kn = \frac{3\kappa}{lvC_v}$$

Elliptic flow coefficient (v_2)

v_2 is the second-order Fourier coefficient in the azimuthal distribution of produced particles in HICs given by

$$v_2 = \langle \cos 2\phi \rangle$$

- The overlap region of the two nuclei in the transverse plane is shaped like an almond.
- Pressure gradient is larger along the short axis. This pressure difference pushes more particles in the short-axis direction.
- This momentum-space anisotropy is what v_2 measures.

- The elliptic flow v_2 can be expressed in terms of the Knudsen number as⁸

$$v_2 = \frac{v_2^h}{1 + \frac{Kn}{Kn_0}},$$

where v_2^h is the elliptic flow at the hydrodynamical limit ($Kn \rightarrow 0$) and the quantity Kn_0 is a number obtained to fit the Monte-Carlo simulations of the relativistic Boltzmann equation.

- In our present analysis, we have taken⁹ $v_2^h = 0.154 \pm 0.014$ and¹⁰ $Kn_0 = 0.7$.

⁸Bhalerao et al., Phys. Lett. B 627, 49 (2005)

⁹PHENIX Collaboration, Phys. Rev. Lett. 91, 182301 (2003)

¹⁰C. Gombeaud and J.-Y. Ollitrault, Phys. Rev. C 77, 054904 (2008)

Results for Kn and v_2

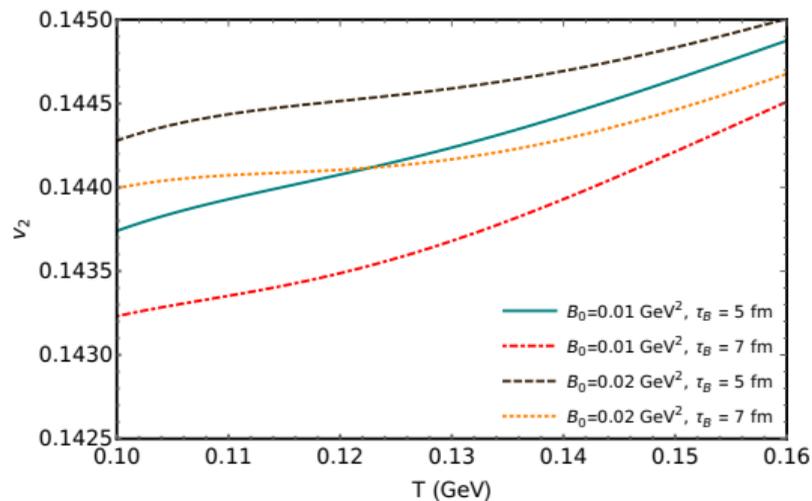
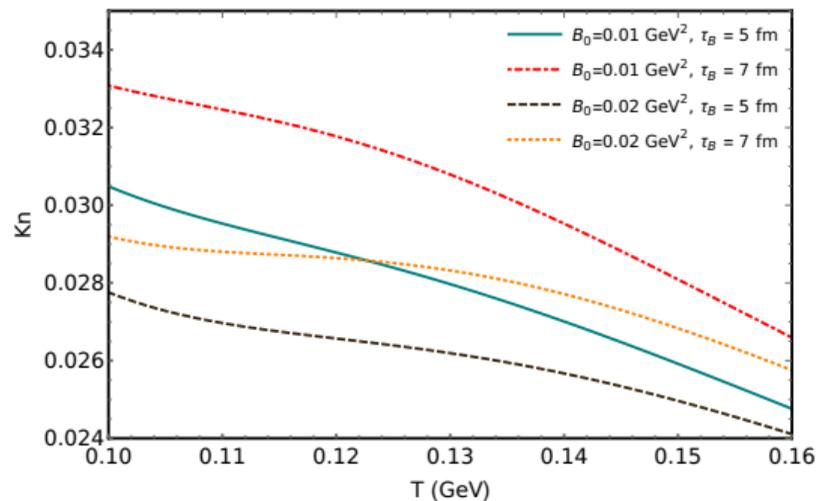


Figure: Knudsen number (left panel) and v_2 (right panel) as a function of temperature for different values of amplitude and decay parameter of the magnetic field at $t = 2.5 \text{ fm}$, $(\mu)_{\pi^\pm} = \pm 0.1 \text{ GeV}$ and $(\mu)_{\pi^0} = 0 \text{ GeV}$.

Conclusion and Outlook

- This work investigated the transport properties of a hot pion gas in a more realistic, time-dependent magnetic field.
- It was found that both the strength and decay rate of the magnetic field have a significant and non-trivial influence on the electrical, thermal, and thermoelectric transport coefficients.
- The model shows that these time-dependent effects introduce meaningful corrections to phenomenologically important observables, including the Knudsen number and the elliptic flow coefficient, v_2 .
- The current model can be improved by using ERTA for the collision kernel and by considering the magnetic field's effect on the relaxation time itself.

Thank You